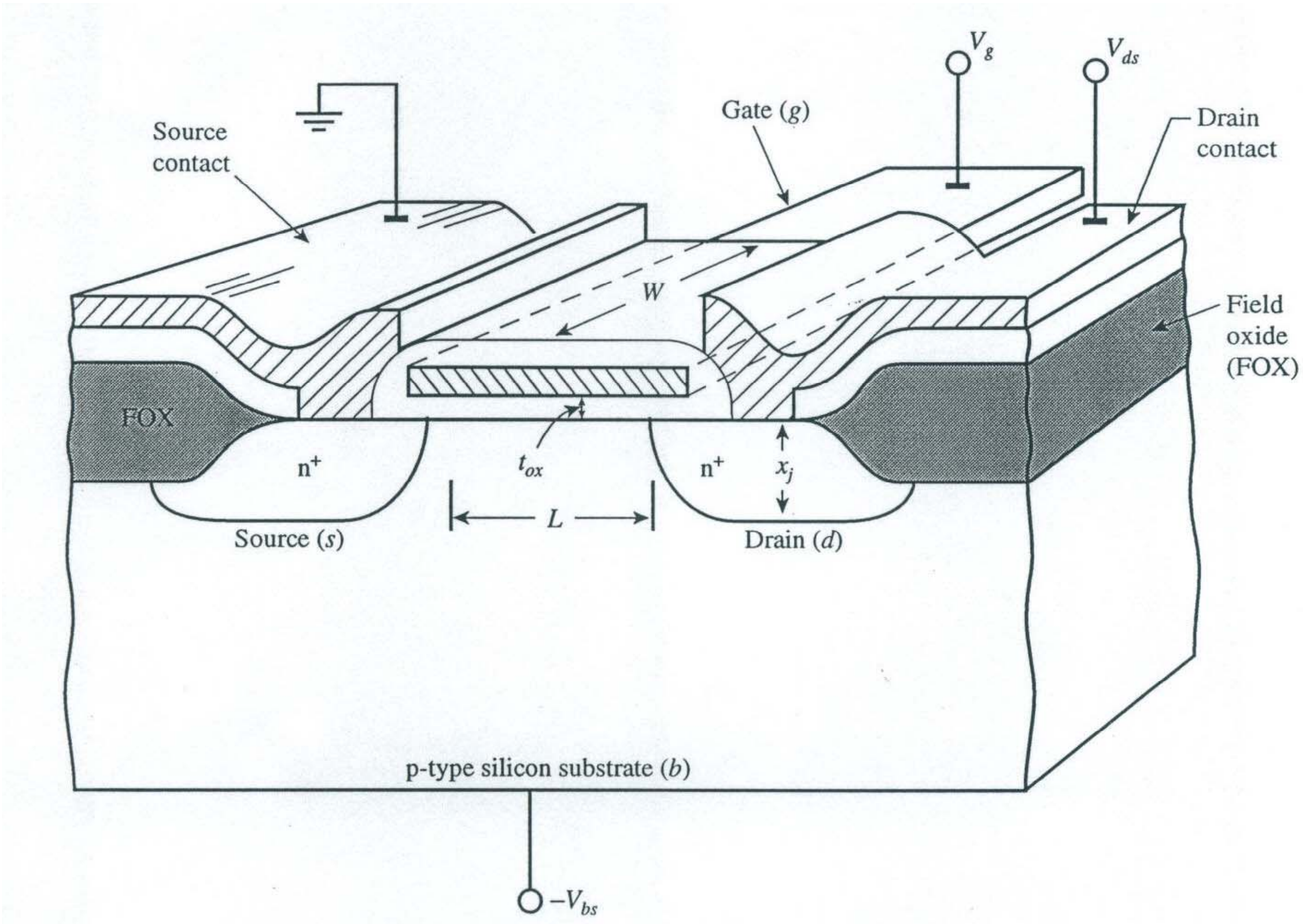
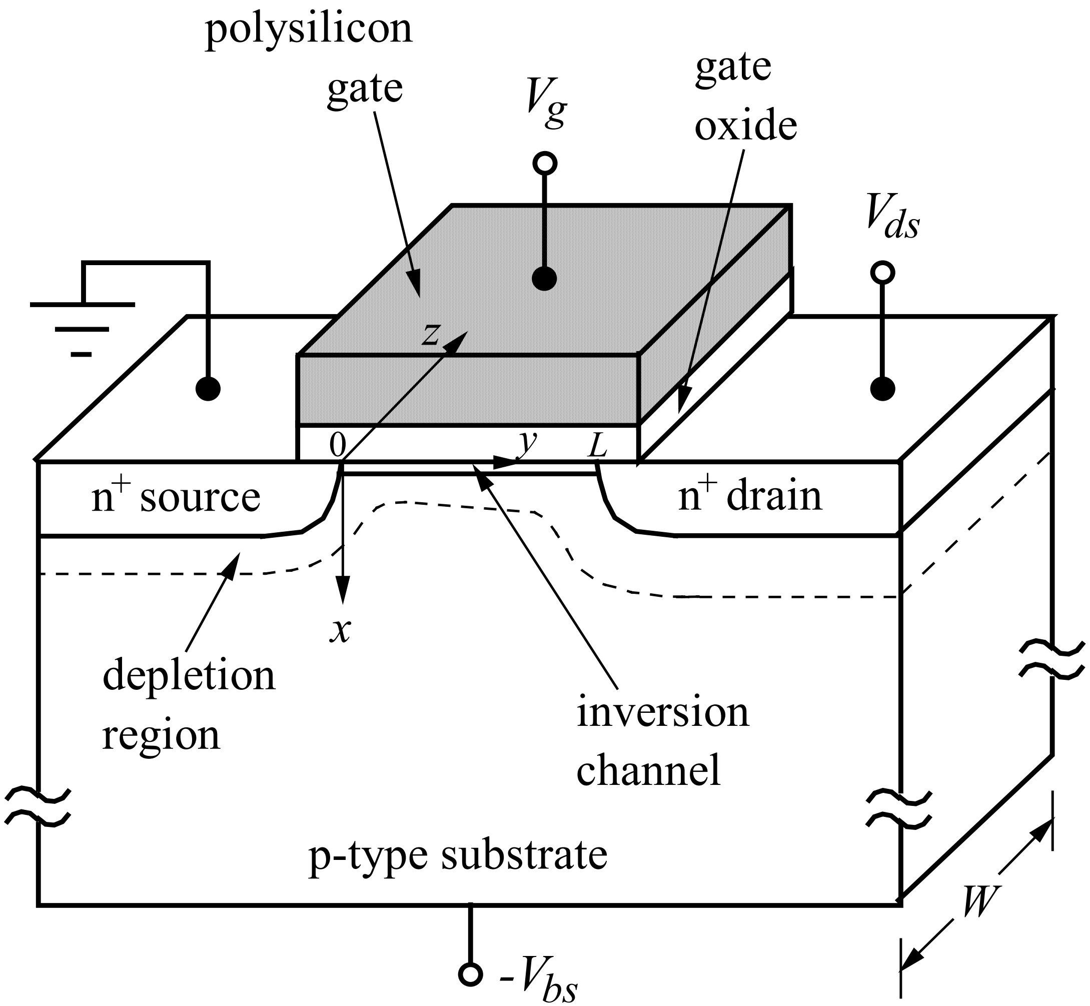
nMOSFET Schematic



- Four structural masks: Field, Gate, Contact, Metal.
- Reverse doping polarities for pMOSFET in N-well.

nMOSFET Schematic



- Source terminal: Ground potential.
- Gate voltage: V_g
- Drain voltage: V_{ds}
- Substrate bias voltage: $-V_{bs}$

- $\psi(x,y)$: Band bending at any point (x,y).
- $V(y)$: Quasi-Fermi potential along the channel.
- $V(y=0) = 0$, $V(y=L) = V_{ds}$.

Drain Current Model

Electron concentration:

$$n(x,y) = \frac{n_i^{\,2}}{N_a} e^{q(\psi - V)kT}$$

Electric field:

$$\mathcal{E}^2(x,y) = \left(\frac{d\psi}{dx}\right)^2 = \frac{2kTN_a}{\varepsilon_{si}}\left[\left(e^{-q\psi/kT} + \frac{q\psi}{kT} - 1\right) + \frac{n_i^{\,2}}{N_a^{\,2}}\left(e^{-qV/kT}(e^{q\psi/kT} - 1) - \frac{q\psi}{kT}\right)\right]$$

Condition for surface inversion:

$$\psi(0,y) = V(y) + 2\psi_B$$

Maximum depletion layer width at inversion:

$$W_{dm}(y) = \sqrt{\frac{2\varepsilon_{si}\left[V(y) + 2\psi_B\right]}{qN_a}}$$

Gradual Channel Approximation

Assumes that vertical field is stronger than lateral field in the channel region, thus 2-D Poisson's eq. can be solved in terms of 1-D vertical slices.

Current density eq. (both drift and diffusion):

$$J_n(x,y) = -q\mu_n n(x,y)\frac{dV(y)}{dy}$$

Integrate in *x*- and *z*-directions,

$$I_{ds}(y) = -\mu_{eff} W \frac{dV}{dy} Q_i(y) = -\mu_{eff} W \frac{dV}{dy} Q_i(V)$$

where $Q_i(y) = -q\int_0^{x_i} n(x,y)dx$ is the inversion charge/area.

Current continuity requires I_{ds} independent of *y*, integration with respect to *y* from 0 to *L* yields

$$I_{ds} = \mu_{eff}\frac{W}{L}\int_0^{V_{ds}}\left(-Q_i(V)\right)dV$$

Pao-Sah's Double Integral

Change variable from (x,y) to (ψ,V),

$$n(x,y) = n(\psi,V) = \frac{n_i^{\,2}}{N_a} e^{q(\psi-V)/kT}$$

$$Q_i(V) = -q\int_{\psi_s}^{\psi_B} n(\psi,V)\frac{dx}{d\psi}d\psi = -q\int_{\psi_B}^{\psi_s}\frac{(n_i^{\,2}/N_a)e^{q(\psi-V)/kT}}{\mathcal{E}(\psi,V)}d\psi$$

Substituting into the current expression,

$$I_{ds} = q\mu_{eff}\frac{W}{L}\int_0^{V_{ds}}\left[\int_{\psi_B}^{\psi_s}\frac{(n_i^{\,2}/N_a)e^{q(\psi-V)/kT}}{\mathcal{E}(\psi,V)}d\psi\right]dV$$

where $\psi_s(V)$ is solved by the gate voltage eq. for a vertical slice of the MOSFET:

$$V_g = V_{fb} + \psi_s - \frac{Q_s}{C_{ox}} = V_{fb} + \psi_s + \frac{\sqrt{2\varepsilon_{si}kTN_a}}{C_{ox}}\left[\frac{q\psi_s}{kT} + \frac{n_i^{\,2}}{N_a^{\,2}}e^{q(\psi_s-V)/kT}\right]^{1/2}$$

Charge Sheet Approximation

Assumes that all the inversion charges are located at the silicon surface like a sheet of charge and that there is no potential drop across the inversion layer.

After the onset of inversion, the surface potential is pinned at $\psi_s = 2\psi_B + V(y)$.

- Depletion charge: $Q_d = -qN_a W_{dm} = -\sqrt{2\varepsilon_{si} qN_a (2\psi_B + V)}$
- Total charge: $Q_s = -C_{ox}(V_g - V_{fb} - \psi_s) = -C_{ox}(V_g - V_{fb} - 2\psi_B - V)$
- Inv. charge: $Q_i = Q_s - Q_d = -C_{ox}(V_g - V_{fb} - 2\psi_B - V) + \sqrt{2\varepsilon_{si} qN_a (2\psi_B + V)}$

Substituting in $I_{ds} = \mu_{eff} \dfrac{W}{L} \int_0^{V_{ds}} \left(-Q_i(V)\right) dV$ and integrate:

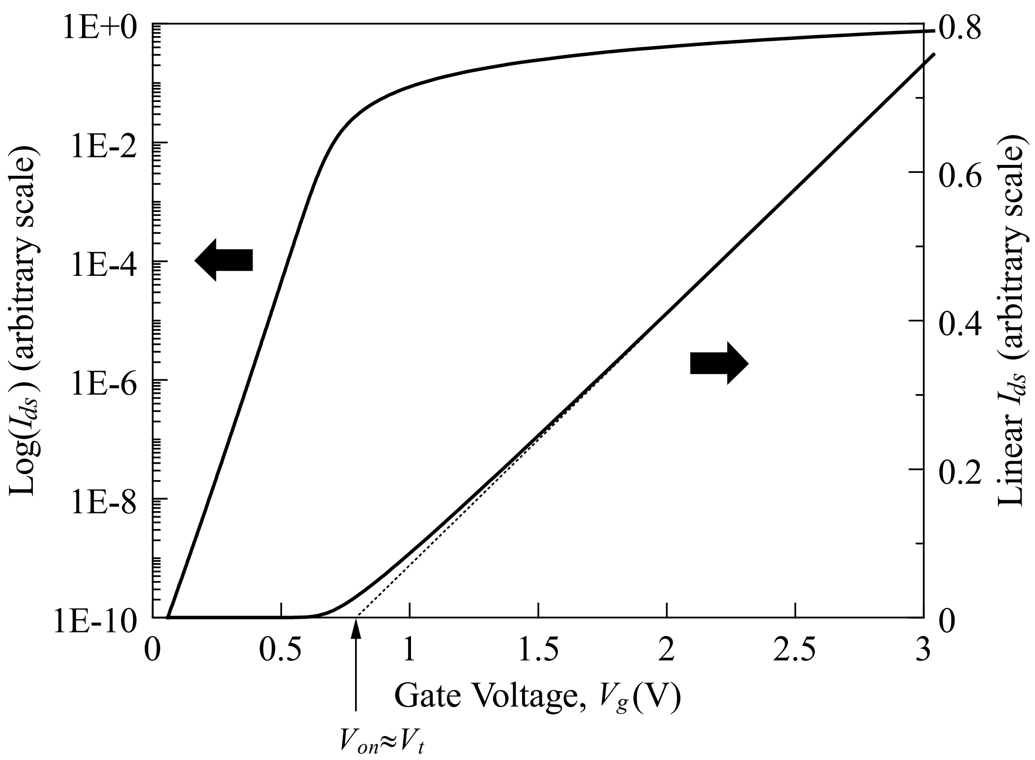
$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L} \left\{ \left(V_g - V_{fb} - 2\psi_B - \frac{V_{ds}}{2} \right) V_{ds} - \frac{2\sqrt{2\varepsilon_{si} qN_a}}{3C_{ox}} \left[(2\psi_B + V_{ds})^{3/2} - (2\psi_B)^{3/2} \right] \right\}$$

Linear Region I-V Characteristics

For $V_{ds} << V_g$,

$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L}\left(V_g - V_{fb} - 2\psi_B - \frac{\sqrt{4\varepsilon_{si} q N_a \psi_B}}{C_{ox}} \right) V_{ds} = \mu_{eff} C_{ox} \frac{W}{L}(V_g - V_t) V_{ds}$$

where $V_t = V_{fb} + 2\psi_B + \frac{\sqrt{4\varepsilon_{si} q N_a \psi_B}}{C_{ox}}$ is the MOSFET ***threshold voltage***.



Saturation Region I-V Characteristics

Keeping the 2nd order terms in V_{ds}: $I_{ds} = \mu_{eff} C_{ox} \frac{W}{L}\left[(V_g - V_t)V_{ds} - \frac{m}{2}V_{ds}^{\ 2}\right]$

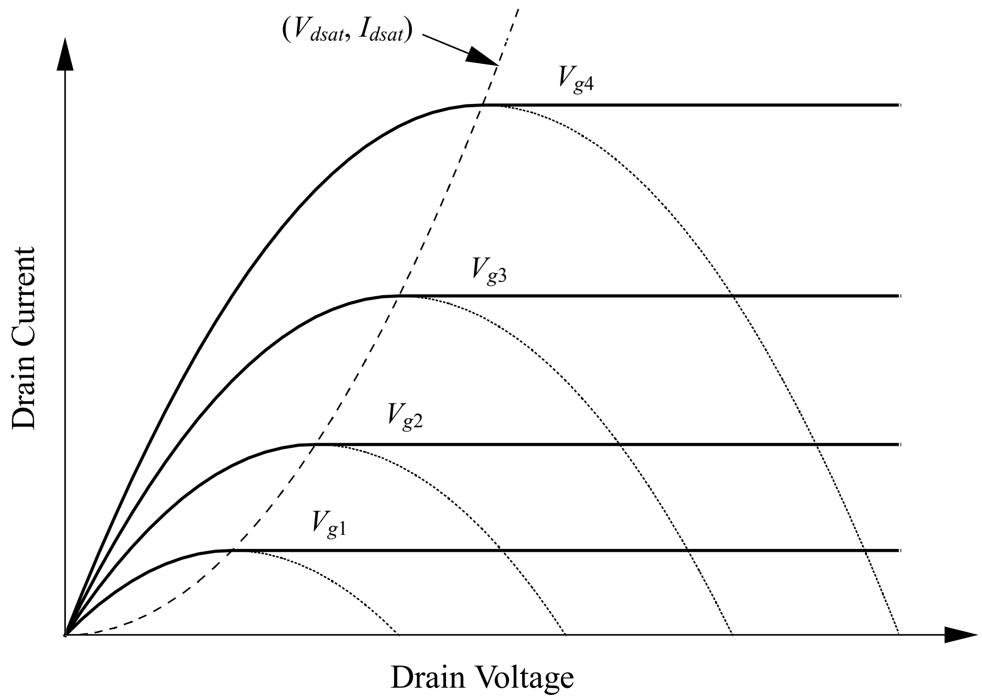
where $m = 1 + \frac{\sqrt{\varepsilon_{si} q N_a / 4\psi_B}}{C_{ox}} = 1 + \frac{C_{dm}}{C_{ox}} = 1 + \frac{3t_{ox}}{W_{dm}}$ is the body-effect coefficient.

$$I_{ds} = I_{dsat} = \mu_{eff} C_{ox} \frac{W}{L} \frac{(V_g - V_t)^2}{2m}$$

when

$V_{ds} = V_{dsat} = (V_g - V_t)/m$.

Typically, $m \approx 1.2$.

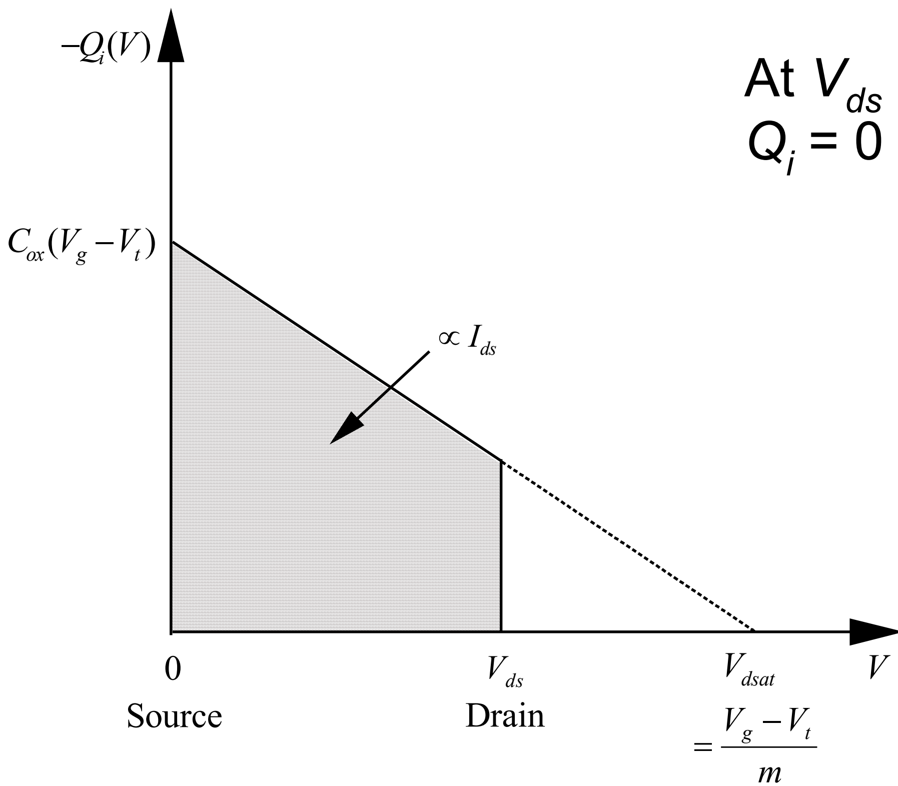


Pinch-off Condition

From inversion charge density point of view,

$$Q_i(V) = -C_{ox}(V_g - V_t - mV)$$

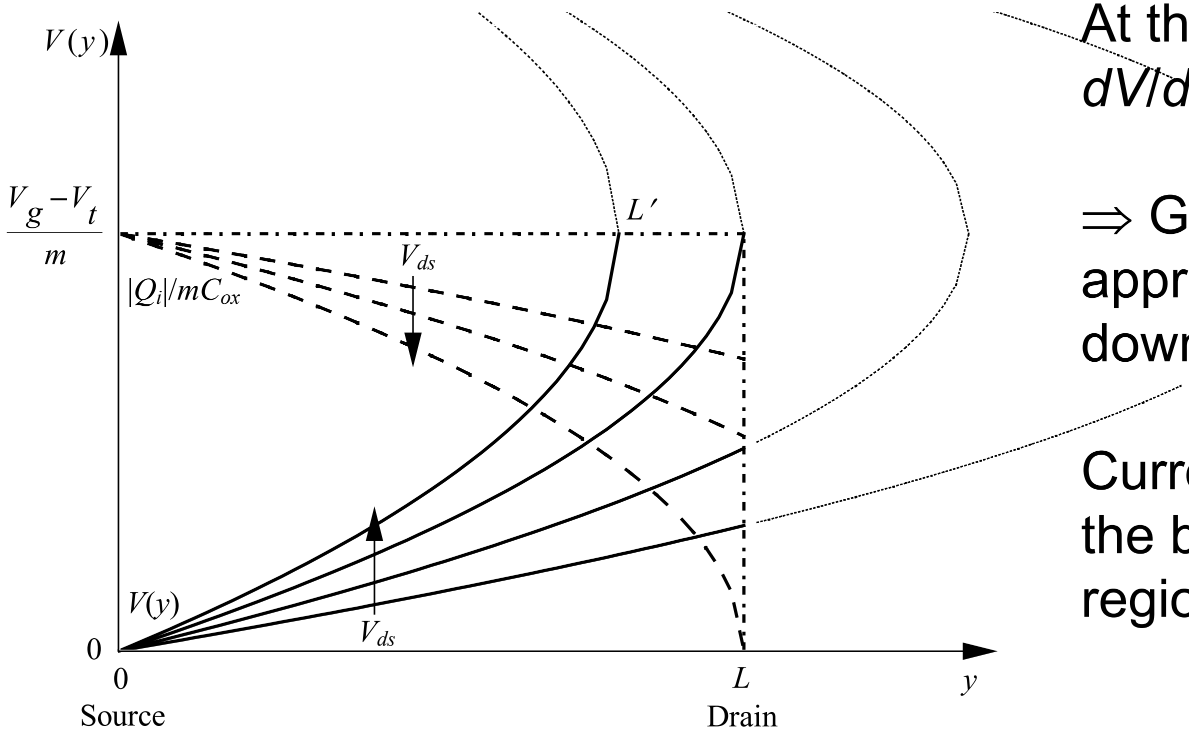
while $I_{ds} = \mu_{eff} \frac{W}{L} \int_0^{V_{ds}} \left(-Q_i(V)\right) dV$



At $V_{ds} = V_{dsat} = (V_g - V_t)/m$,
$Q_i = 0$ and I_{ds} = max.

Pinch-off from Potential Point of View

$$V(y) = \frac{V_g - V_t}{m} - \sqrt{\left(\frac{V_g - V_t}{m}\right)^2 - 2\frac{y}{L}\left(\frac{V_g - V_t}{m}\right)V_{ds} + \frac{y}{L}V_{ds}^{\ 2}}$$

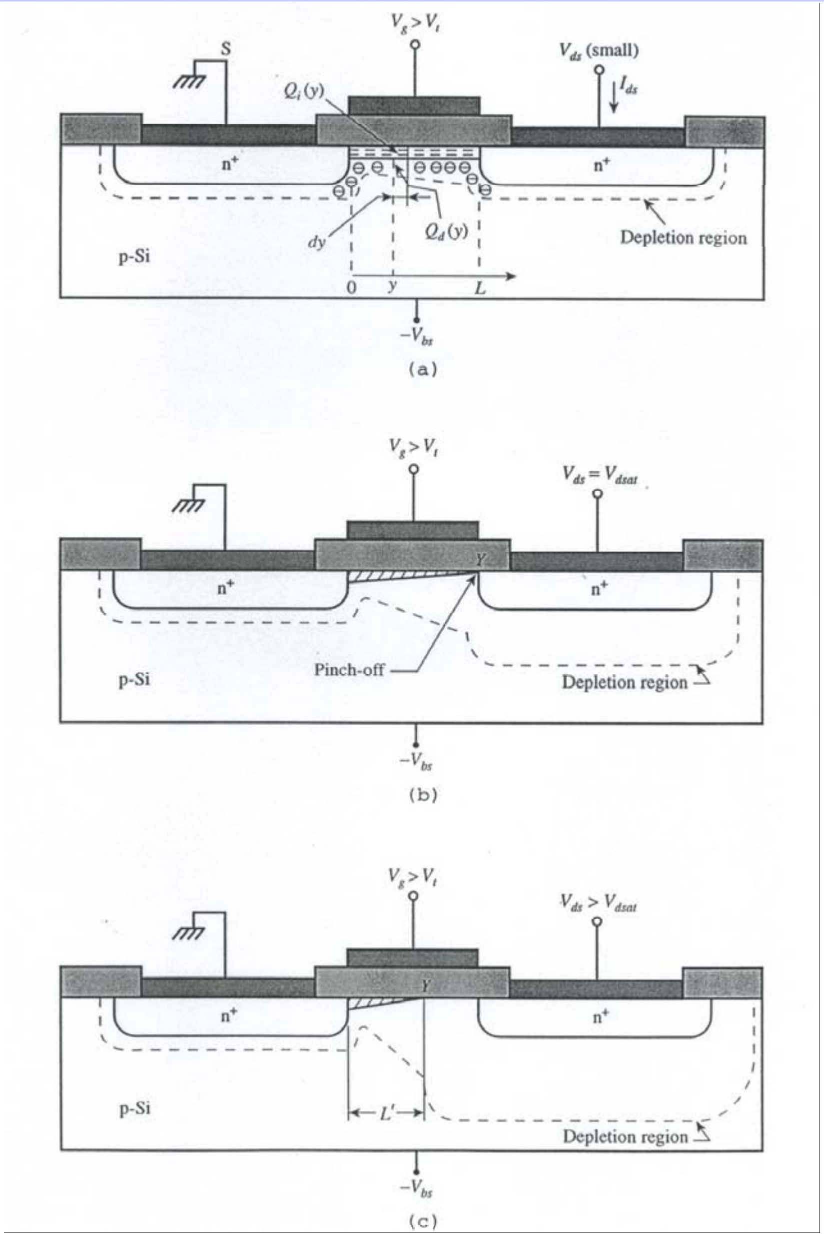


At the pinch-off point, $dV/dy \rightarrow \infty$

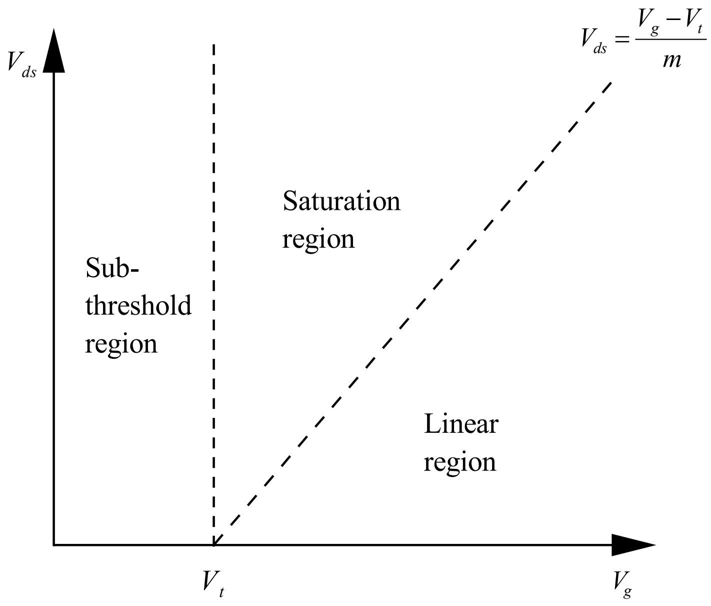
$\Rightarrow$ Gradual channel approximation breaks down.

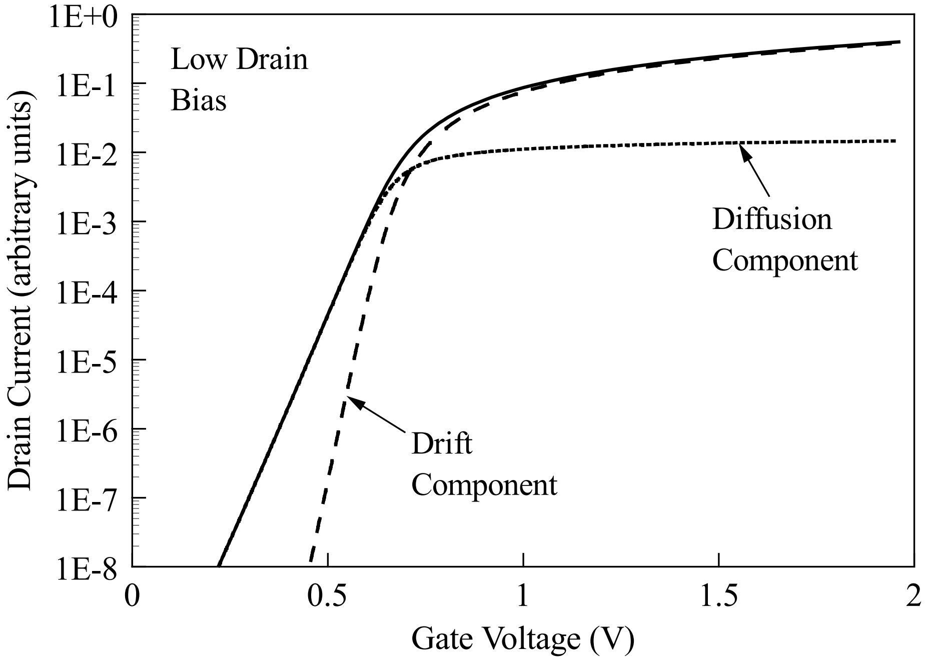
Current is injected into the bulk depletion region.

Beyond Pinch-off



Subthreshold Region





Subthreshold Currents

$$-Q_s = \varepsilon_{si}\mathcal{E}_s = \sqrt{2\varepsilon_{si}kTN_a}\left[\frac{q\psi_s}{kT} + \frac{n_i^{\,2}}{N_a^{\,2}}e^{q(\psi_s - V)/kT}\right]^{1/2}$$

Power series expansion: 1st term Q_d, 2nd term Q_i,

$$-Q_i = \sqrt{\frac{\varepsilon_{si}qN_a}{2\psi_s}}\left(\frac{kT}{q}\right)\left(\frac{n_i}{N_a}\right)^2 e^{q(\psi_s - V)/kT}$$

$$\Rightarrow \; I_{ds} = \mu_{eff}\frac{W}{L}\sqrt{\frac{\varepsilon_{si}qN_a}{2\psi_s}}\left(\frac{kT}{q}\right)^2\left(\frac{n_i}{N_a}\right)^2 e^{q\psi_s/kT}\left(1 - e^{-qV_{ds}/kT}\right)$$

or, $$I_{ds} = \mu_{eff}C_{ox}\frac{W}{L}(m-1)\left(\frac{kT}{q}\right)^2 e^{q(V_g - V_t)/mkT}\left(1 - e^{-qV_{ds}/kT}\right)$$

Inverse subthreshold slope:

$$S = \left(\frac{d(\log I_{ds})}{dV_g}\right)^{-1} = 2.3\frac{mkT}{q} = 2.3\frac{kT}{q}\left(1 + \frac{C_{dm}}{C_{ox}}\right)$$

Body Effect: Dependence of Threshold Voltage on Substrate Bias

If $V_{bs} \neq 0$,

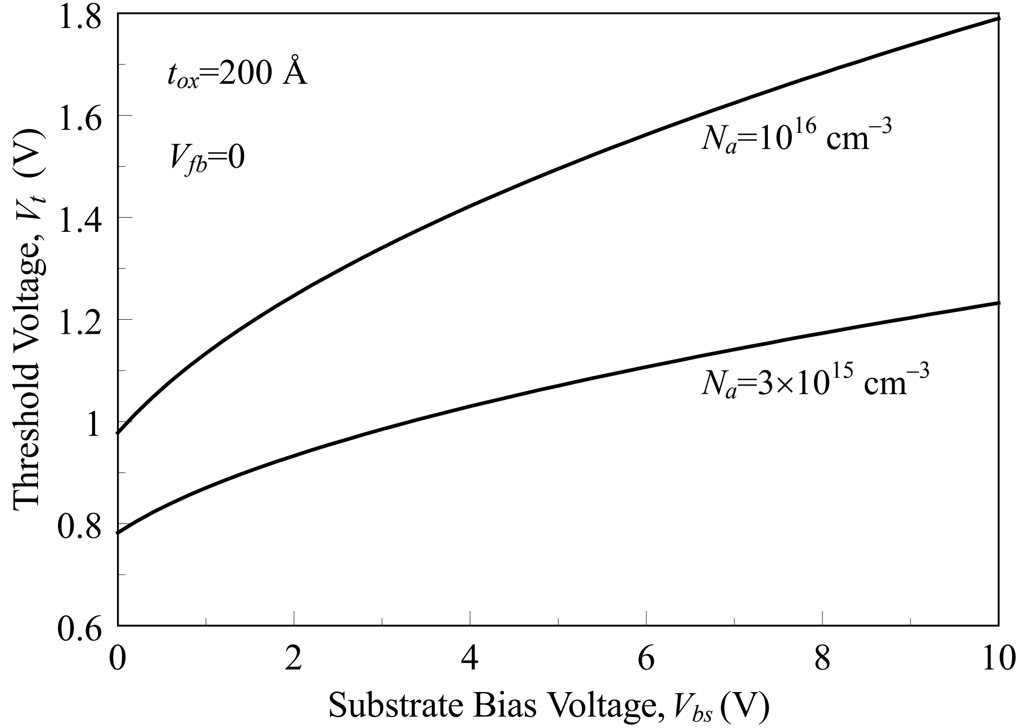
$$I_{ds} = \mu_{eff} C_{ox} \frac{W}{L} \left\{ \left(V_g - V_{fb} - 2\psi_B - \frac{V_{ds}}{2} \right) V_{ds} - \frac{2\sqrt{2\varepsilon_{si} q N_a}}{3C_{ox}} \left[(2\psi_B + V_{bs} + V_{ds})^{3/2} - (2\psi_B + V_{bs})^{3/2} \right] \right\}$$

$$\Downarrow$$

$$V_t = V_{fb} + 2\psi_B + \frac{\sqrt{2\varepsilon_{si} q N_a (2\psi_B + V_{bs})}}{C_{ox}}$$

$$\Downarrow$$

$$\frac{dV_t}{dV_{bs}} = \frac{\sqrt{\varepsilon_{si} q N_a \; / \; 2(2\psi_B + V_{bs})}}{C_{ox}}$$



Dependence of Threshold Voltage on Temperature

For n^+ poly gated nMOSFET, $V_{fb} = -(E_g/2q) - \psi_B$

$$\Rightarrow \qquad V_t = -\frac{E_g}{2q} + \psi_B + \frac{\sqrt{4\varepsilon_{si} q N_a \psi_B}}{C_{ox}}$$

$$\frac{dV_t}{dT} = -\frac{1}{2q}\frac{dE_g}{dT} + \left(1 + \frac{\sqrt{\varepsilon_{si} q N_a / \psi_B}}{C_{ox}}\right)\frac{d\psi_B}{dT} = -\frac{1}{2q}\frac{dE_g}{dT} + (2m-1)\frac{d\psi_B}{dT}$$

$$\Rightarrow \qquad \frac{dV_t}{dT} = -(2m-1)\frac{k}{q}\left[\ln\left(\frac{\sqrt{N_c N_v}}{N_a}\right) + \frac{3}{2}\right] + \frac{m-1}{q}\frac{dE_g}{dT}$$

From Table 2.1, $dE_g/dT \approx -2.7\times10^{-4}$ eV/K and $(N_c N_v)^{1/2} \approx 2.4\times10^{19}$ cm^{-3}.

For $N_a \sim 10^{16}$ cm^{-3} and $m \sim 1.1$,
dV_t/dT is typically –1 mV/K.

MOSFET Channel Mobility

$$\mu_{eff} = \frac{\int_0^{x_i} \mu_n n(x)dx}{\int_0^{x_i} n(x)dx}$$

It was empirically found that when μ_{eff} is plotted against an effective normal field $\mathcal{E}_{eff}$, there exists a "universal relationship" independent of the substrate bias, doping concentration, and gate oxide thickness (Sabnis and Clemens, 1979).

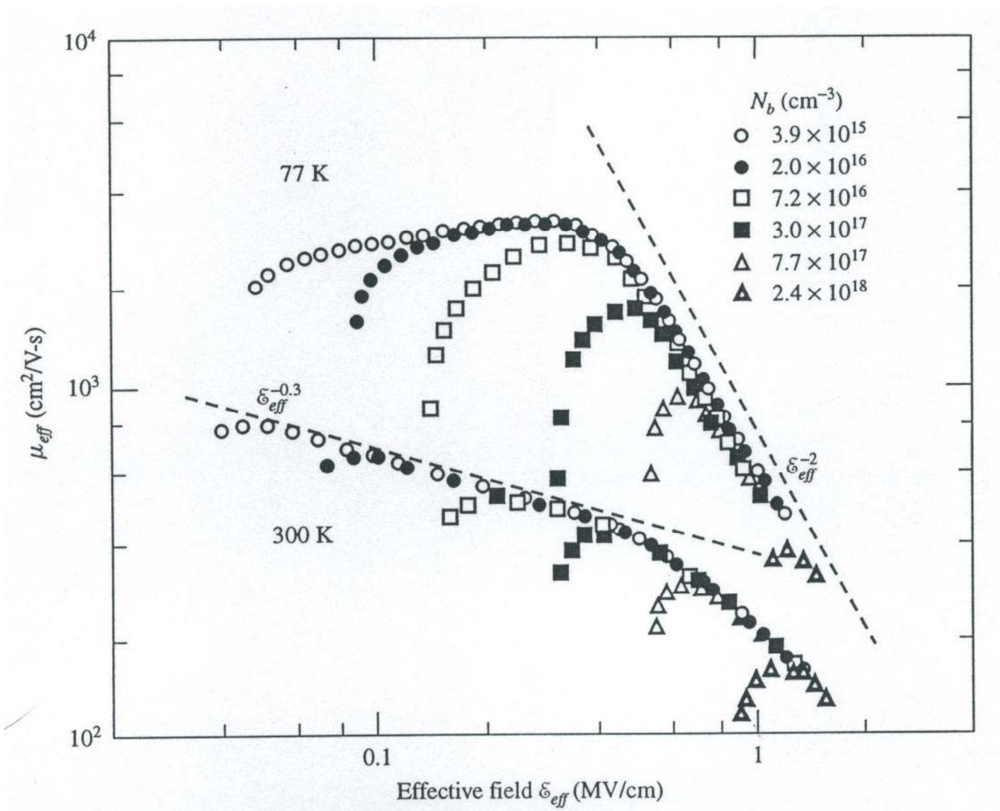
Here $$\mathcal{E}_{eff} = \frac{1}{\varepsilon_{si}}\left(|Q_d| + \frac{1}{2}|Q_i|\right)$$

Since $|Q_d| = \sqrt{4\varepsilon_{si} q N_a \psi_B} = C_{ox}(V_t - V_{fb} - 2\psi_B)$ and $|Q_i| \approx C_{ox}(V_g - V_t)$,

$$\Rightarrow \qquad \mathcal{E}_{eff} = \frac{V_t - V_{fb} - 2\psi_B}{3t_{ox}} + \frac{V_g - V_t}{6t_{ox}}$$

For n^+ poly gated nMOSFET, $$\mathcal{E}_{eff} = \frac{V_t + 0.2}{3t_{ox}} + \frac{V_g - V_t}{6t_{ox}}$$

N-channel MOSFET Mobility



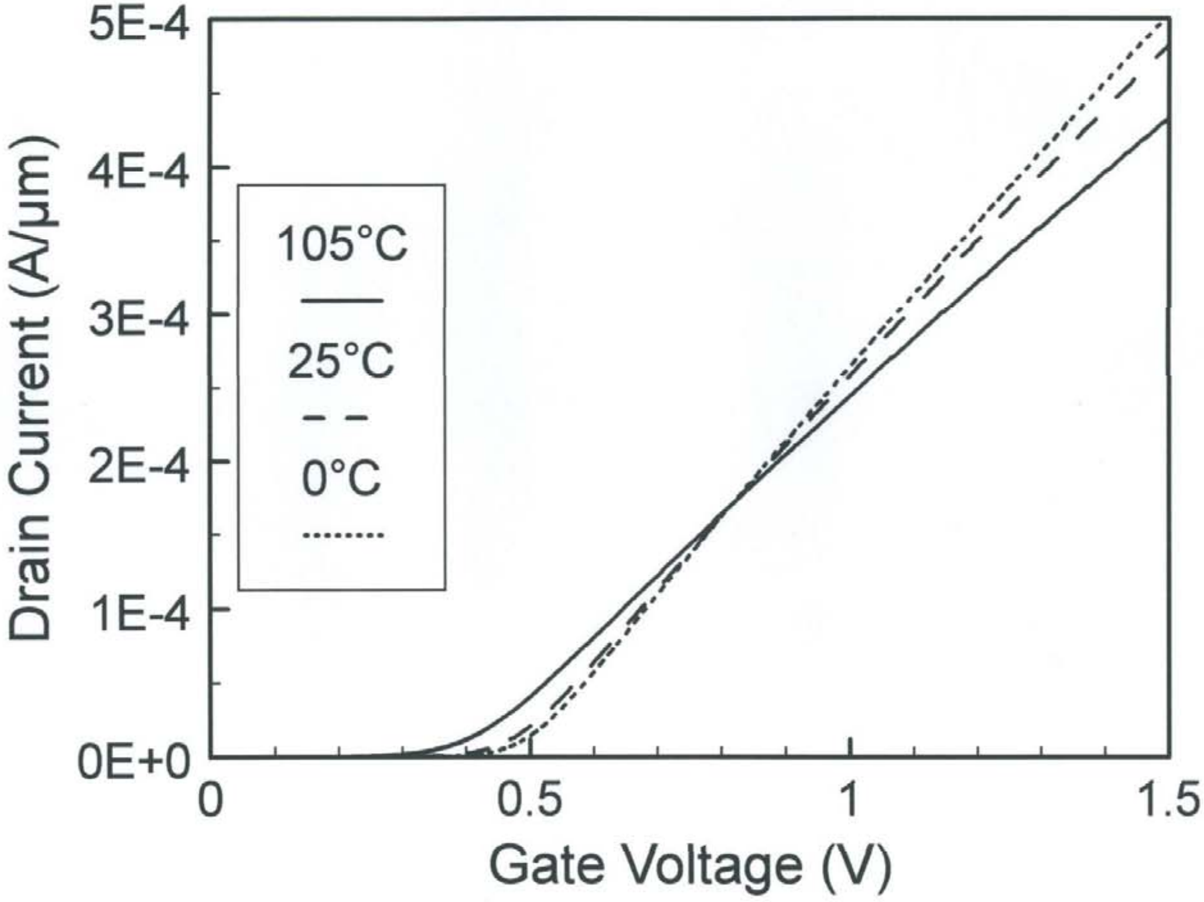
- Low field region (low electron density): Limited by impurity or Coulomb scattering (screened at high electron densities).

- Intermediate field region: Limited by phonon scattering,

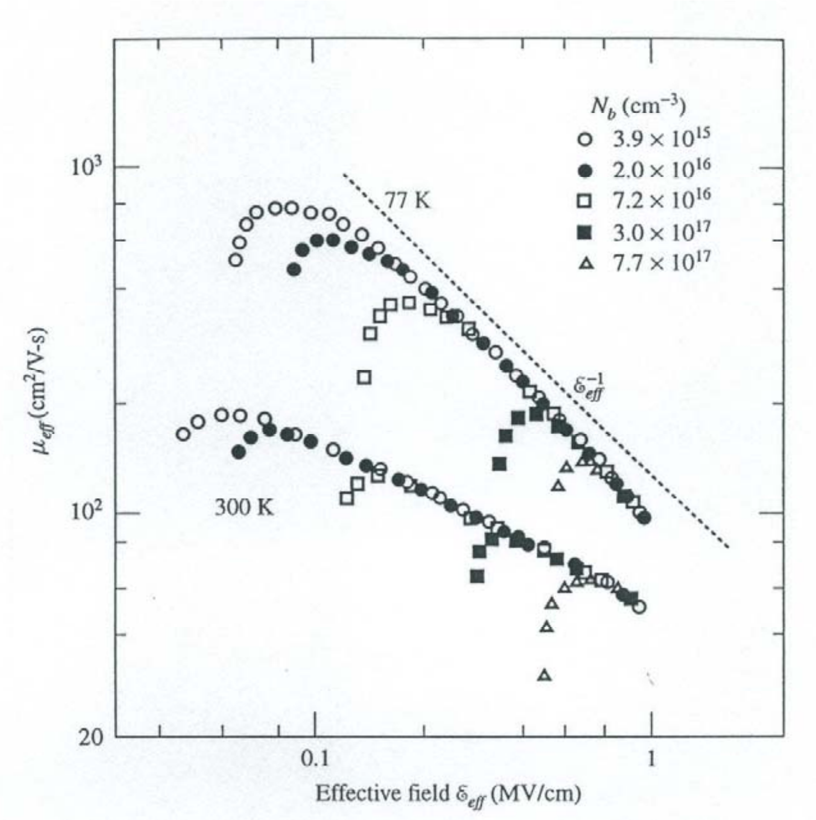
$$\mu_{eff} \approx 32500 \times \mathcal{E}^{-1/3}$$

- High field region (> 1 MV/cm): Limited by surface roughness scattering (less temp. dependence).

Temperature Dependence of MOSFET Current



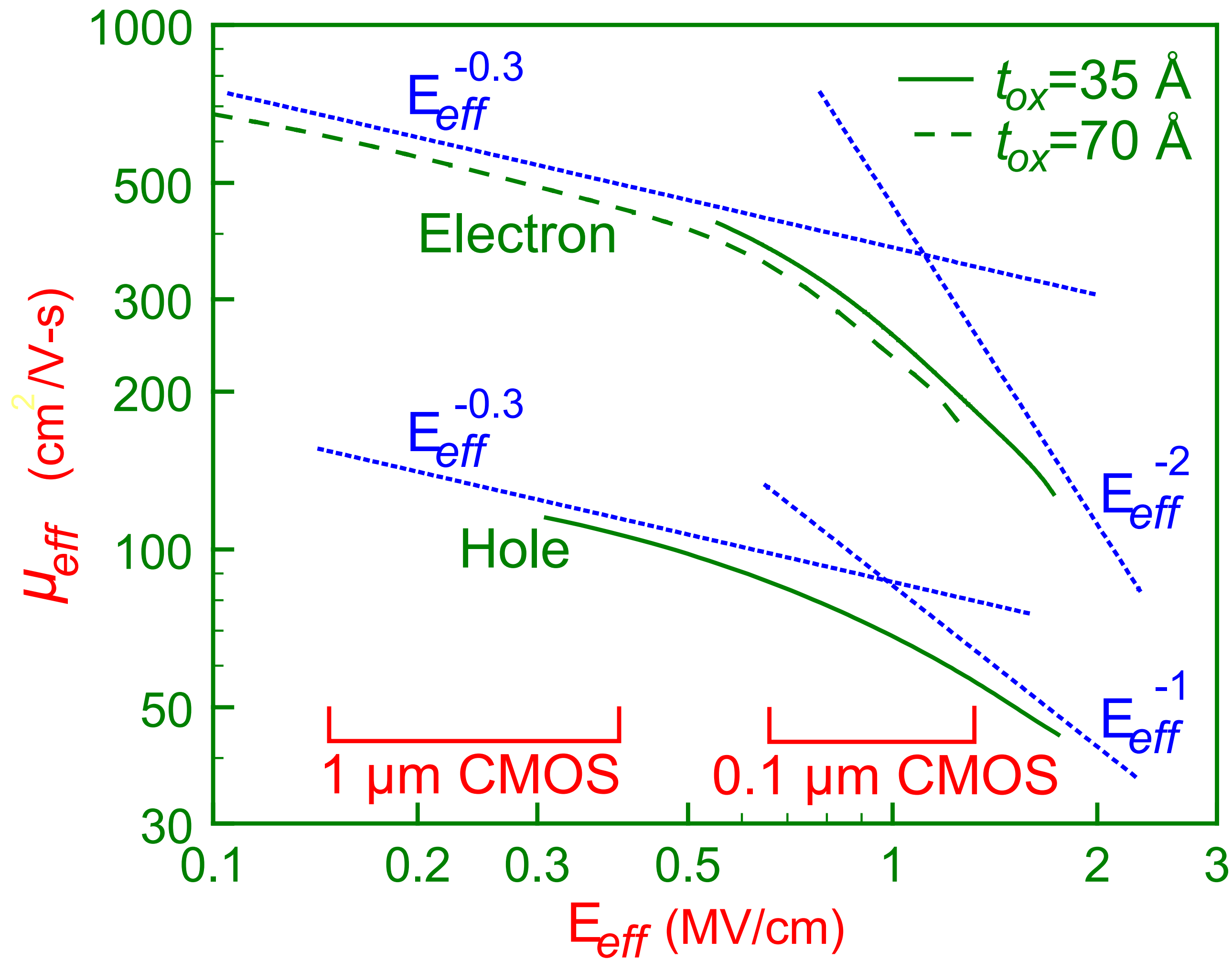
P-channel MOSFET Mobility



$$\mathcal{E}_{eff} = \frac{1}{\varepsilon_{si}}\left(|Q_d| + \frac{1}{3}|Q_i|\right)$$

In general, pMOSFET mobility does not exhibit as "universal" behavior as nMOSFET.

Electron and Hole Mobilities vs. Field

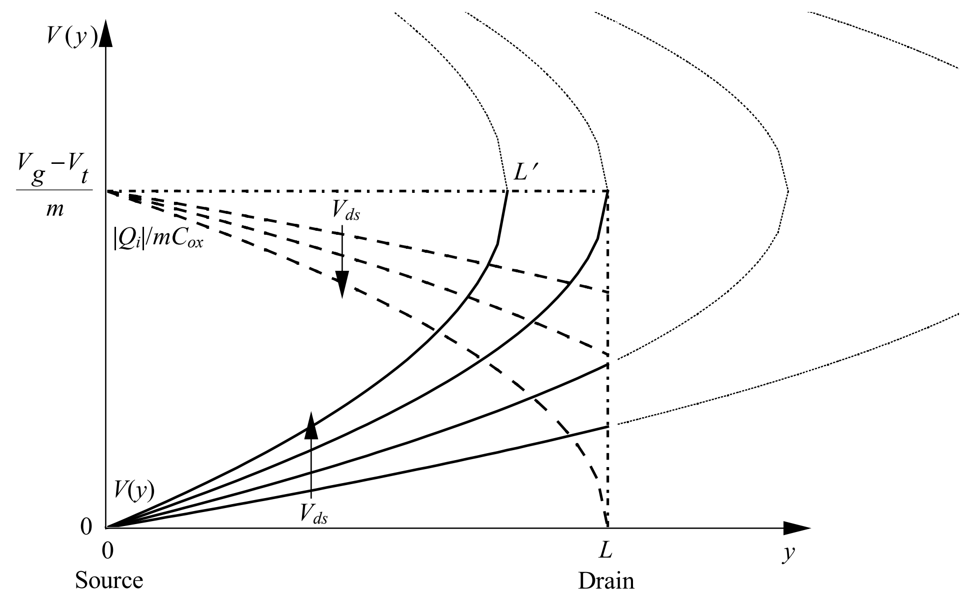


Intrinsic MOSFET Capacitance

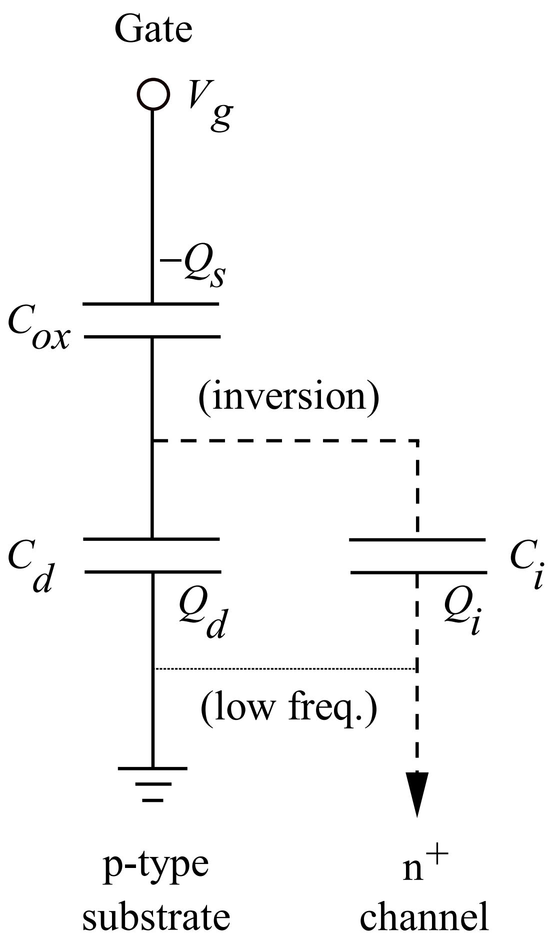
- Subthreshold region: $C_g = WL\left(\frac{1}{C_{ox}} + \frac{1}{C_d}\right)^{-1} \approx WLC_d$
- Linear region: $C_g = WLC_{ox}$
- Saturation region:

$$Q_i(y) = -C_{ox}(V_g - V_t)\sqrt{1 - \frac{y}{L}}$$

$$\Rightarrow \quad C_g = \frac{2}{3}WLC_{ox}$$



Inversion Layer Capacitance



In the charge-sheet model, $C_i = \infty$ and $Q_i = C_{ox}(V_g - V_t)$.

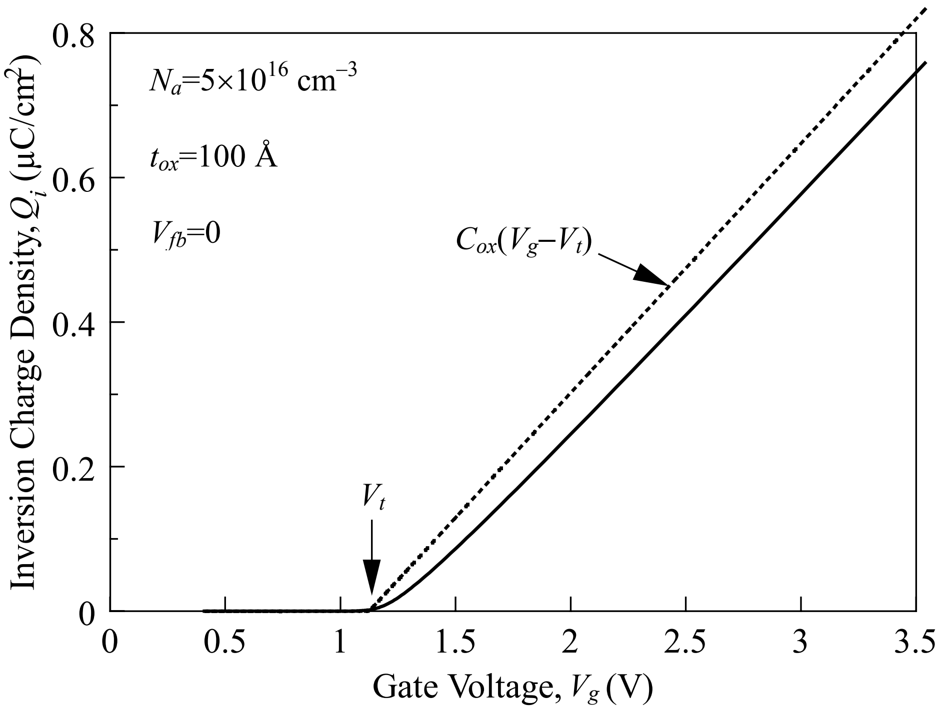
In reality, inversion layer has a finite thickness and finite capacitance.

$$\frac{d(-Q_i)}{dV_g} = \frac{C_{ox} C_i}{C_{ox} + C_i + C_d} \approx C_{ox}\left(1 - \frac{1}{1 + C_i / C_{ox}}\right)$$

Inversion Layer Capacitance

1st order approximation, $C_i \approx |Q_i|/(2kT/q)$ and $|Q_i| \approx C_{ox}(V_g - V_t)$, therefore, $C_i/C_{ox} = (V_g - V_t)/(2kT/q)$.

$$-Q_i = C_{ox}\left[(V_g - V_t) - \frac{2kT}{q}\ln\left(1 + \frac{q(V_g - V_t)}{2kT}\right)\right]$$

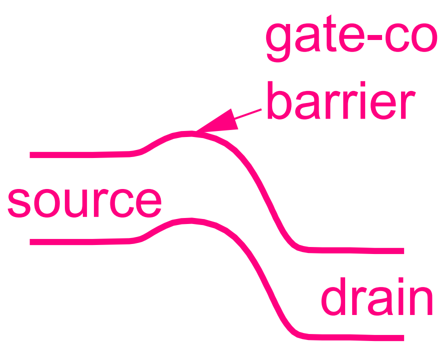


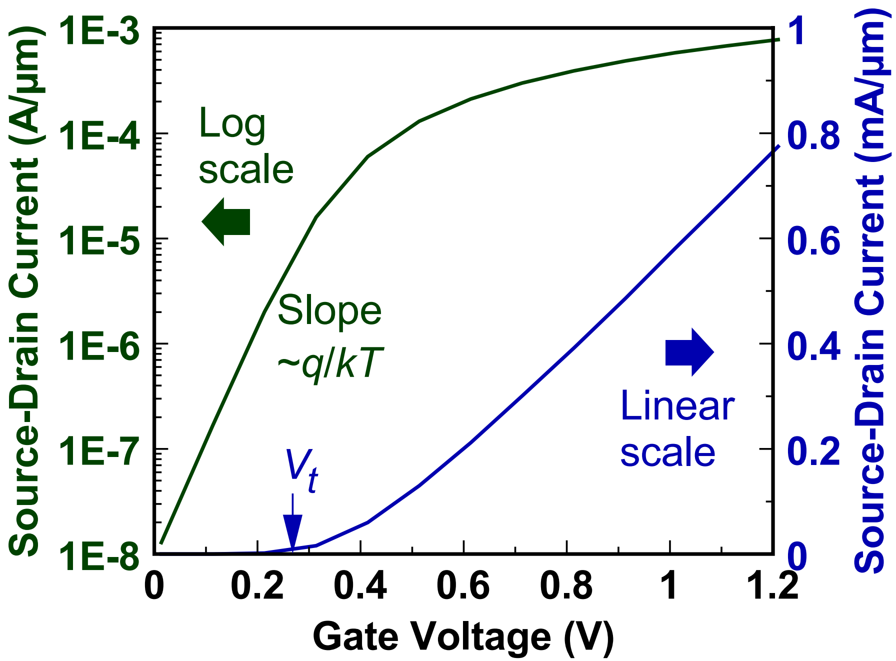
Note:
Linearly extrapolated threshold voltage is typically (2-4)kT/q higher than the threshold voltage V_t at ψ_s(inv.) = $2\psi_B$.

Short-Channel Effect

If $L \downarrow$, $I_{ds} = I_{dsat} = \mu_{eff} C_{ox} \dfrac{W}{L} \dfrac{(V_g - V_t)^2}{2m} \uparrow$

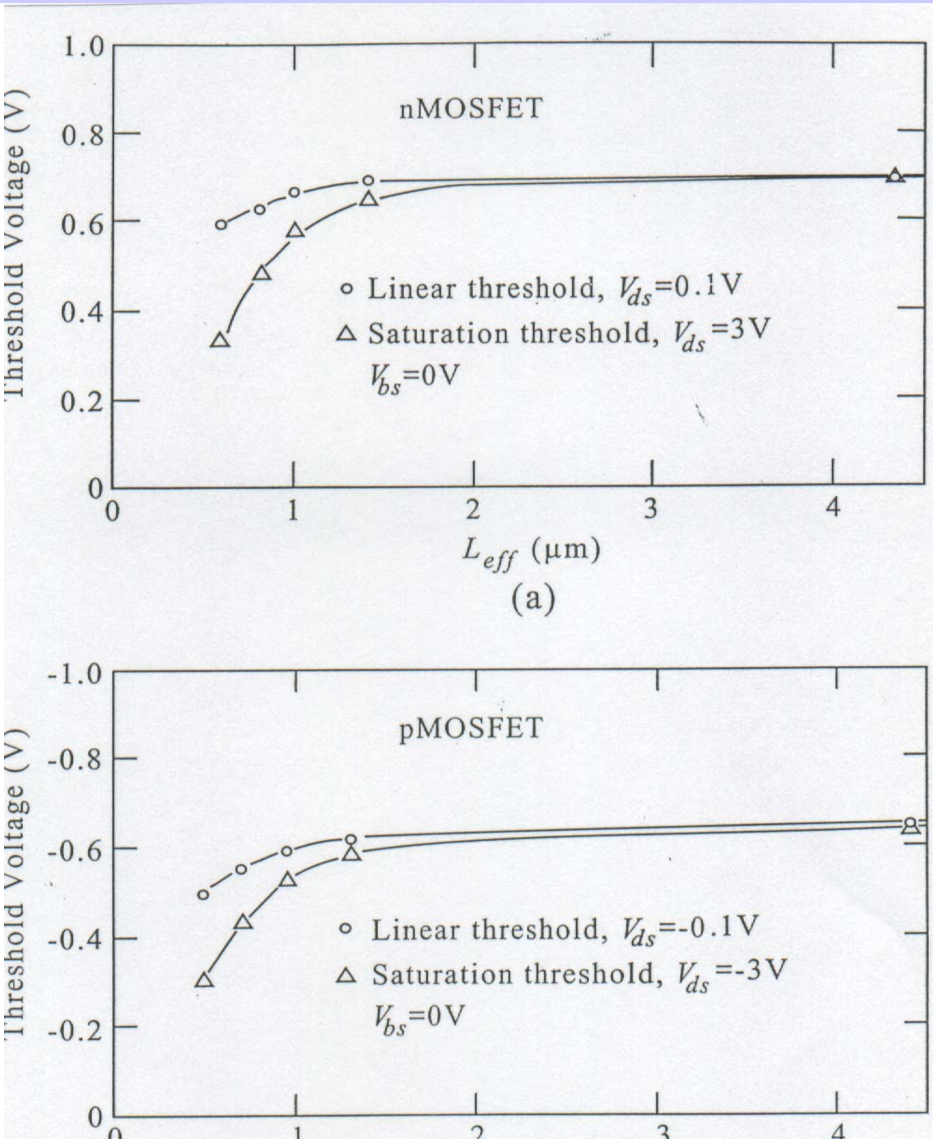
And $C_g = \dfrac{2}{3} W L C_{ox} \downarrow$. But ……





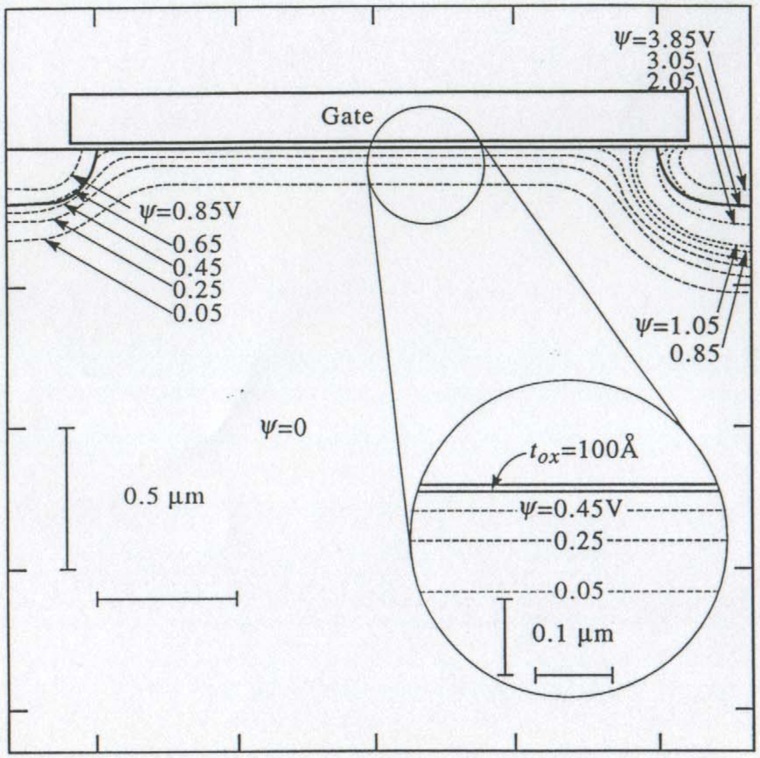
Threshold voltage becomes sensitive to channel length and drain bias.

Short-Channel V_t Roll-off

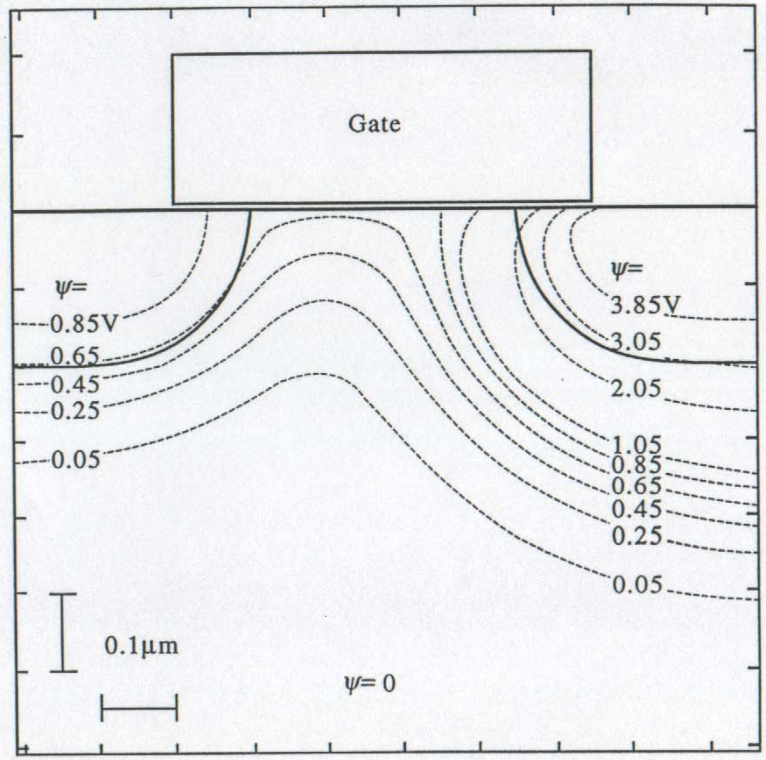


2-D Potential Contours

(Same gate voltage)

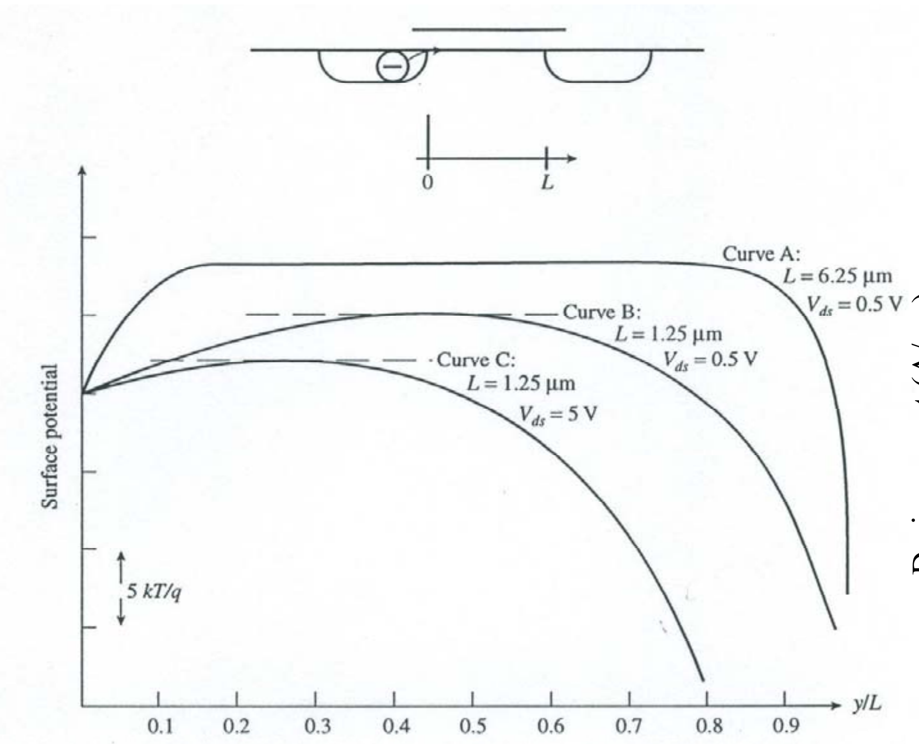


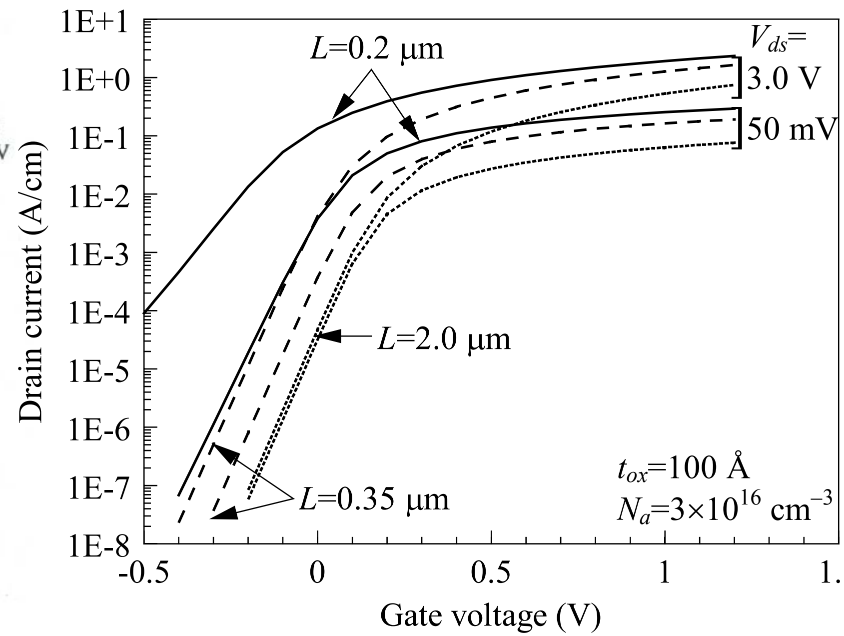
Long channel



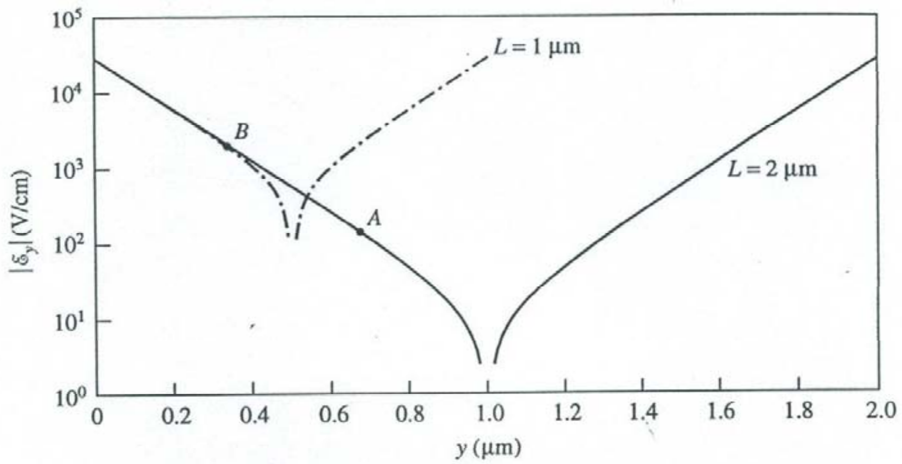
Short channel

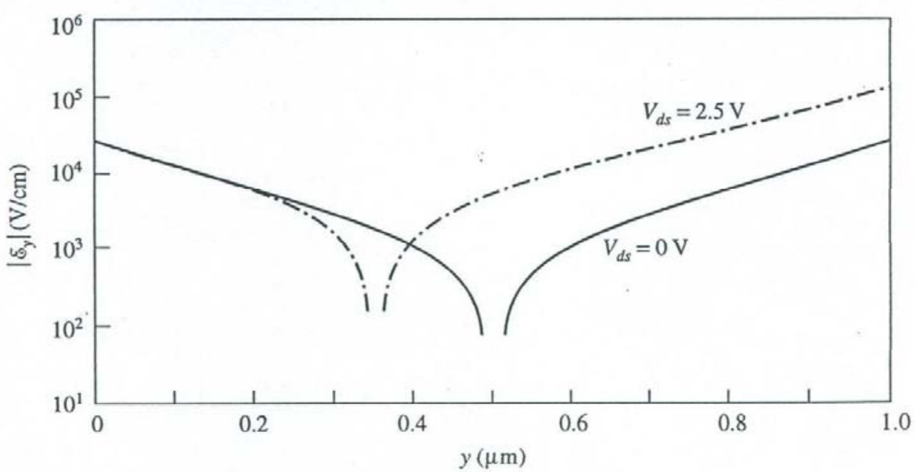
Drain-Induced Barrier Lowering





Lateral Field Penetration





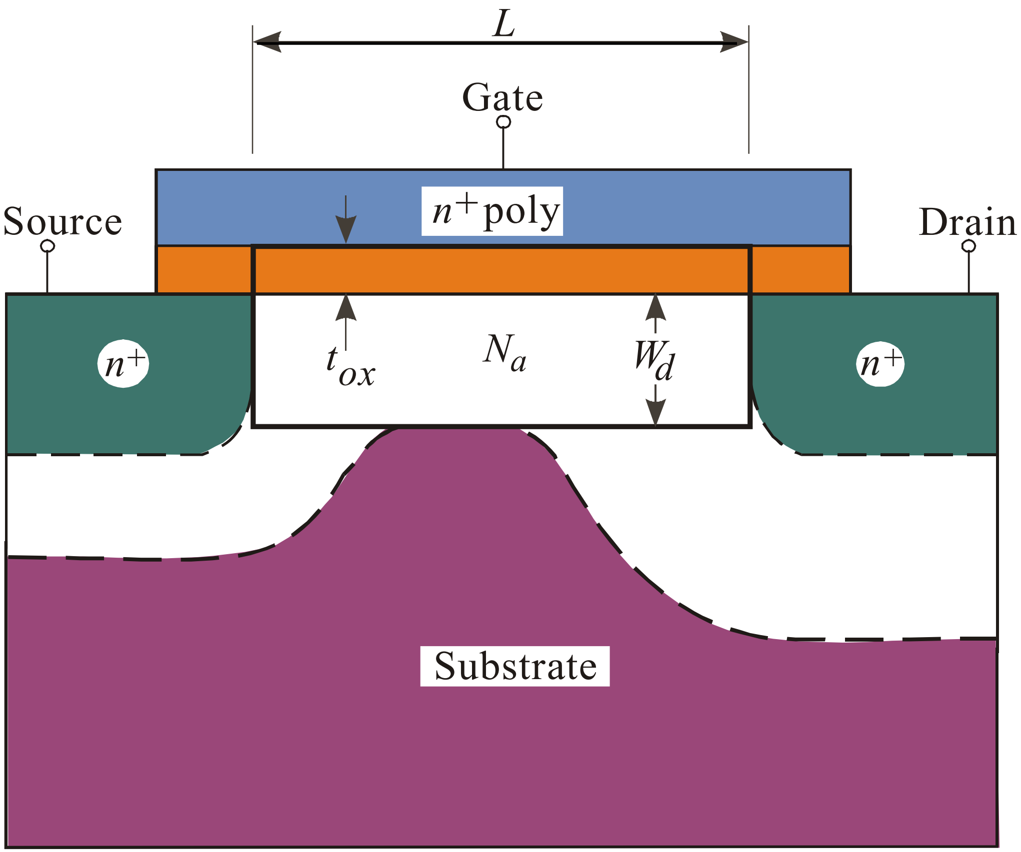
2-D Poisson's Eq.:

$$\frac{\partial \mathcal{E}_x}{\partial x} + \frac{\partial \mathcal{E}_y}{\partial y} = \frac{\rho}{\varepsilon_{si}} = -\frac{qN_a}{\varepsilon_{si}}$$

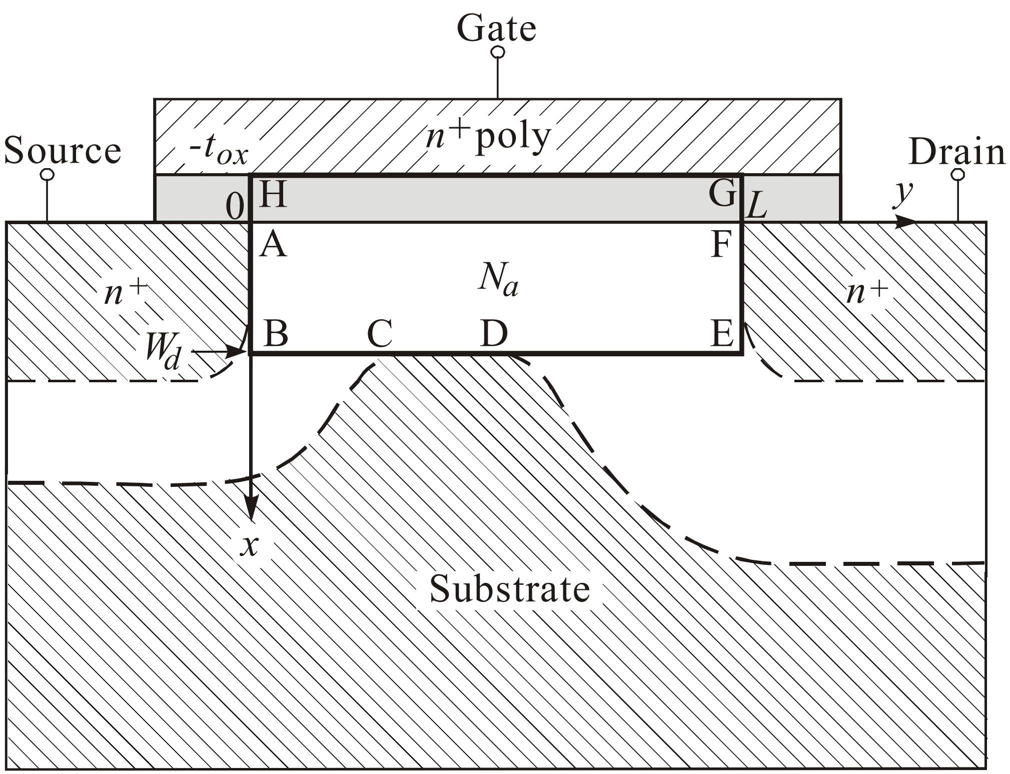
- $\varepsilon_{si}\partial\mathcal{E}_x/\partial x$: gate controlled depletion charge.
- $\varepsilon_{si}\partial\mathcal{E}_y/\partial y$: S/D controlled depletion charge.

Note that the characteristic length of exponential decay is independent of channel length.

2-D Analysis in a Simplified MOSFET Geometry



A 2-D boundary-value problem
with Poisson's equation



Subthreshold region:

In AFGH (oxide),
$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=0$$

In ABEF (silicon),
$$\frac{\partial^2\psi}{\partial x^2}+\frac{\partial^2\psi}{\partial y^2}=\frac{qN_a}{\varepsilon_{si}}$$

To eliminate the boundary condition at the Si/oxide interface, the oxide region is replaced by an equivalent Si region $(\varepsilon_{si}/\varepsilon_{ox})t_{ox} \approx 3t_{ox}$ thick.

Boundary conditions:

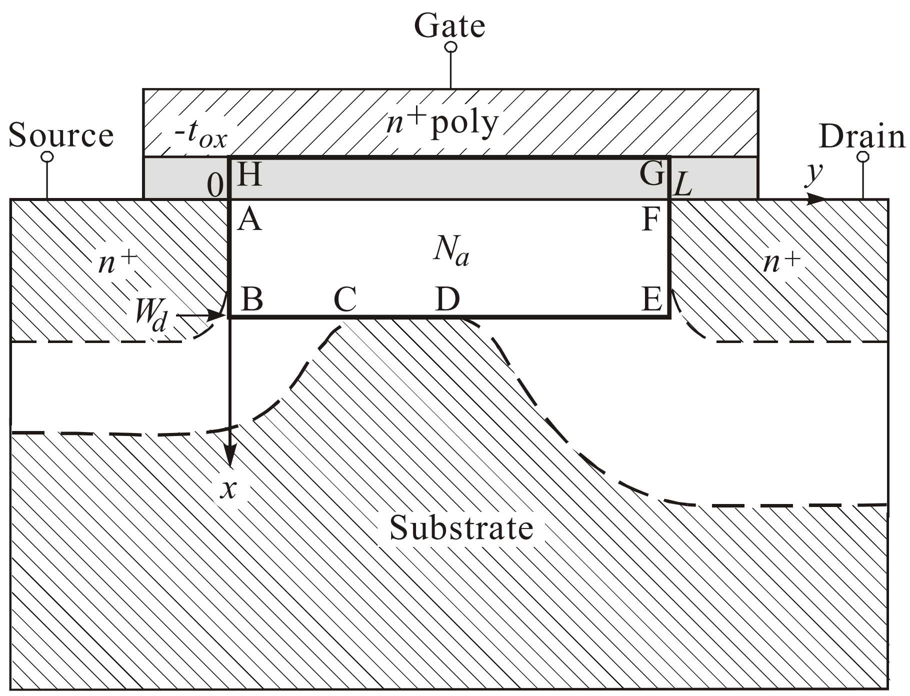
$\psi(-3t_{ox}, y) = V_g - V_{fb}$ along GH,

$\psi(x,0) = \psi_{bi}$ along AB,

$\psi(x, L) = \psi_{bi} + V_{ds}$ along EF,

$\psi(W_d, y) = 0$ along CD.

General approach to a 2-D boundary value problem



Let: $$\psi(x,y) = v(x,y) + u_L(x,y) + u_R(x,y) + u_B(x,y)$$

$v(x,y)$ is a solution to the inhomogeneous equation and satisfies the top boundary condition.

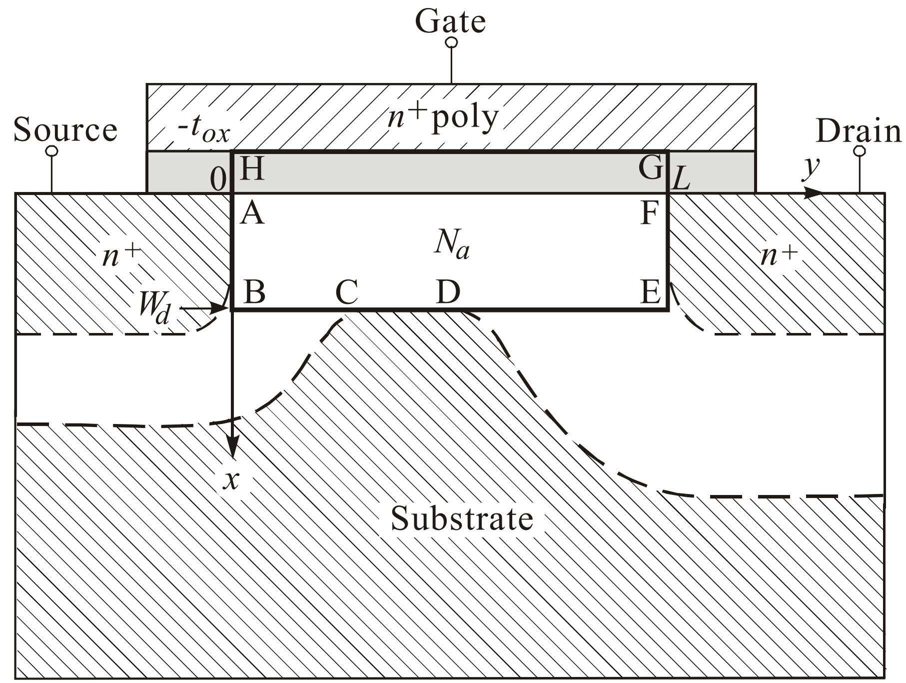
u_L, u_R, u_B are solutions to the homogeneous equation such that $\psi(x,y)$ satisfies the other B.C.'s.

For satisfying the Boundary conditions:

$$u_L(x,y)=\sum_{n=1}^{\infty} b_n^{*}\frac{\sinh\left(\frac{n\pi(L-y)}{W_d+3t_{ox}}\right)}{\sinh\left(\frac{n\pi L}{W_d+3t_{ox}}\right)}\sin\left(\frac{n\pi(x+3t_{ox})}{W_d+3t_{ox}}\right)$$

$$u_R(x,y)=\sum_{n=1}^{\infty} c_n^{*}\frac{\sinh\left(\frac{n\pi y}{W_d+3t_{ox}}\right)}{\sinh\left(\frac{n\pi L}{W_d+3t_{ox}}\right)}\sin\left(\frac{n\pi(x+3t_{ox})}{W_d+3t_{ox}}\right)$$

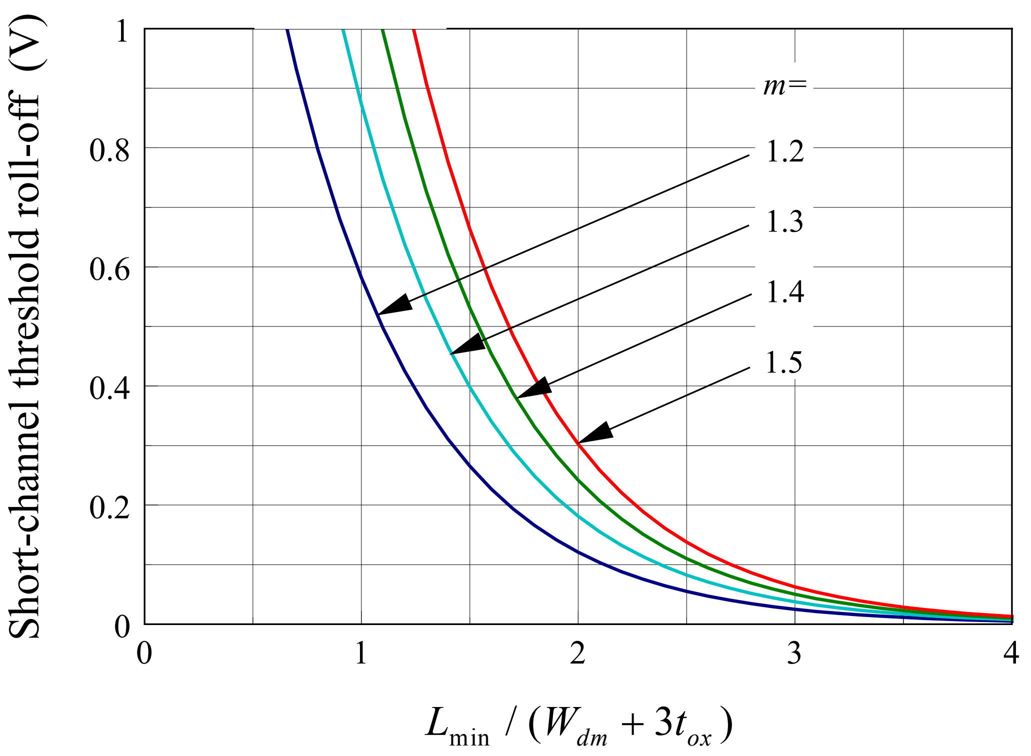
$$u_B(x,y)=\sum_{n=1}^{\infty} d_n^{*}\frac{\sinh\left(\frac{n\pi(x+3t_{ox})}{L}\right)}{\sinh\left(\frac{n\pi(W_d+3t_{ox})}{L}\right)}\sin\left(\frac{n\pi y}{L}\right)$$



Note that for u=sin(kx), $d^2u/dx^2=-k^2u$;
And that for u=sinh(ky), $d^2u/dy^2=k^2u$.

MOSFET Scale Length

$$\Delta V_t = \frac{24 t_{ox}}{W_{dm}} \sqrt{\psi_{bi}(\psi_{bi} + V_{ds})}\, e^{-\frac{\pi L/2}{W_{dm}+3t_{ox}}}$$



Define scale length,

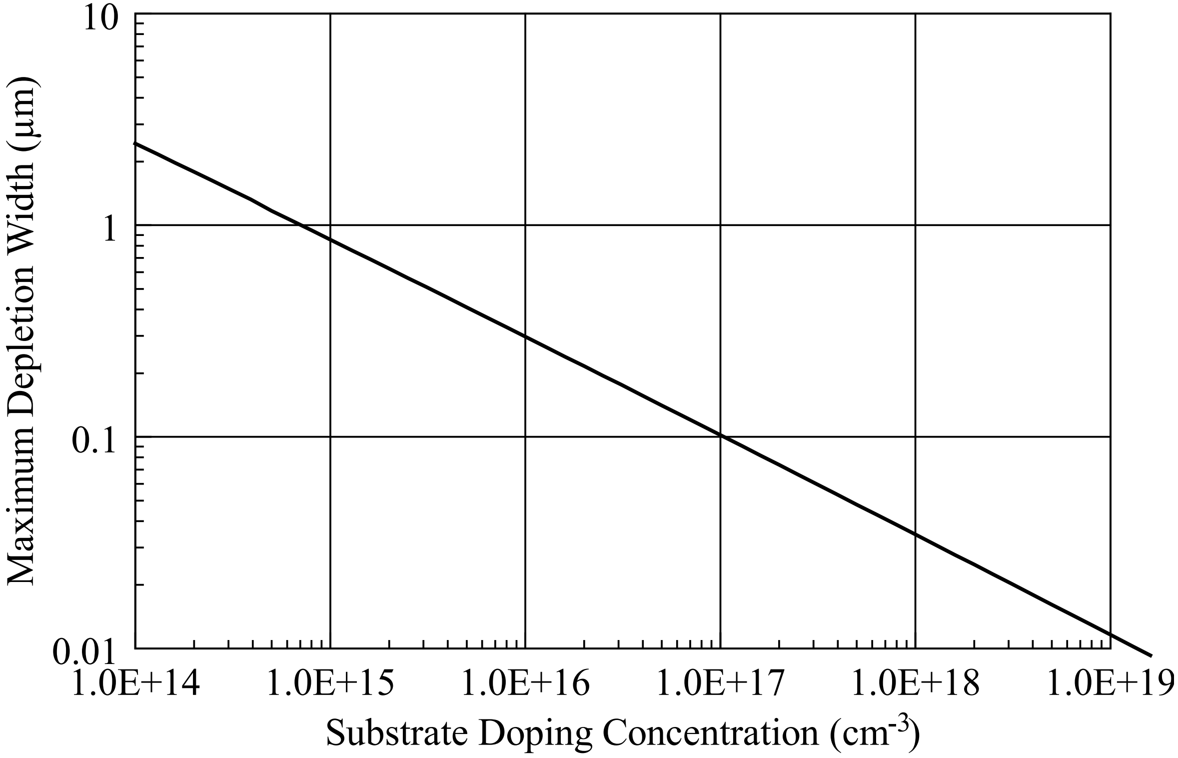
$\lambda \equiv W_{dm} + (\varepsilon_{si}/\varepsilon_{ox})t_{ox}$

To keep short-channel effect under control, L_{min} should be kept larger than about 2λ.

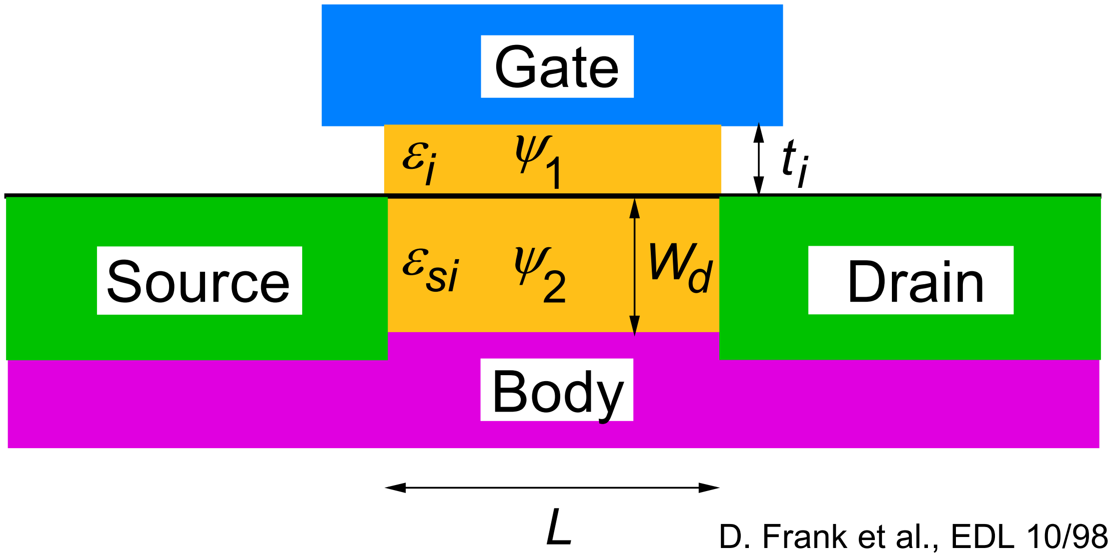
$m = \Delta V_g/\Delta\psi_s = 1 + 3t_{ox}/W_{dm}$

Depletion Width Scaling

$$W_{dm}{}^{0} = \sqrt{\frac{4\varepsilon_{si} kT \ln(N_a / n_i)}{q^2 N_a}}$$



Generalized Scale Length



D. Frank et al., EDL 10/98

In the one-region model, the eigenvalues are:

$$k_n = \frac{n\pi}{W_d + 3t_{ox}}$$

For two regions, assume eigenvalues:

$$k_n = \frac{\pi}{\lambda_n}$$

$$u_{L1}(x,y) = \sum_{n=1}^{\infty} b_{n1} \sinh\left(\frac{\pi(L-y)}{\lambda_n}\right)\sin\left(\frac{\pi(x+t_i)}{\lambda_n}\right)$$

$$u_{L2}(x,y) = \sum_{n=1}^{\infty} b_{n2} \sinh\left(\frac{\pi(L-y)}{\lambda_n}\right)\sin\left(\frac{\pi(x-W_d)}{\lambda_n}\right)$$

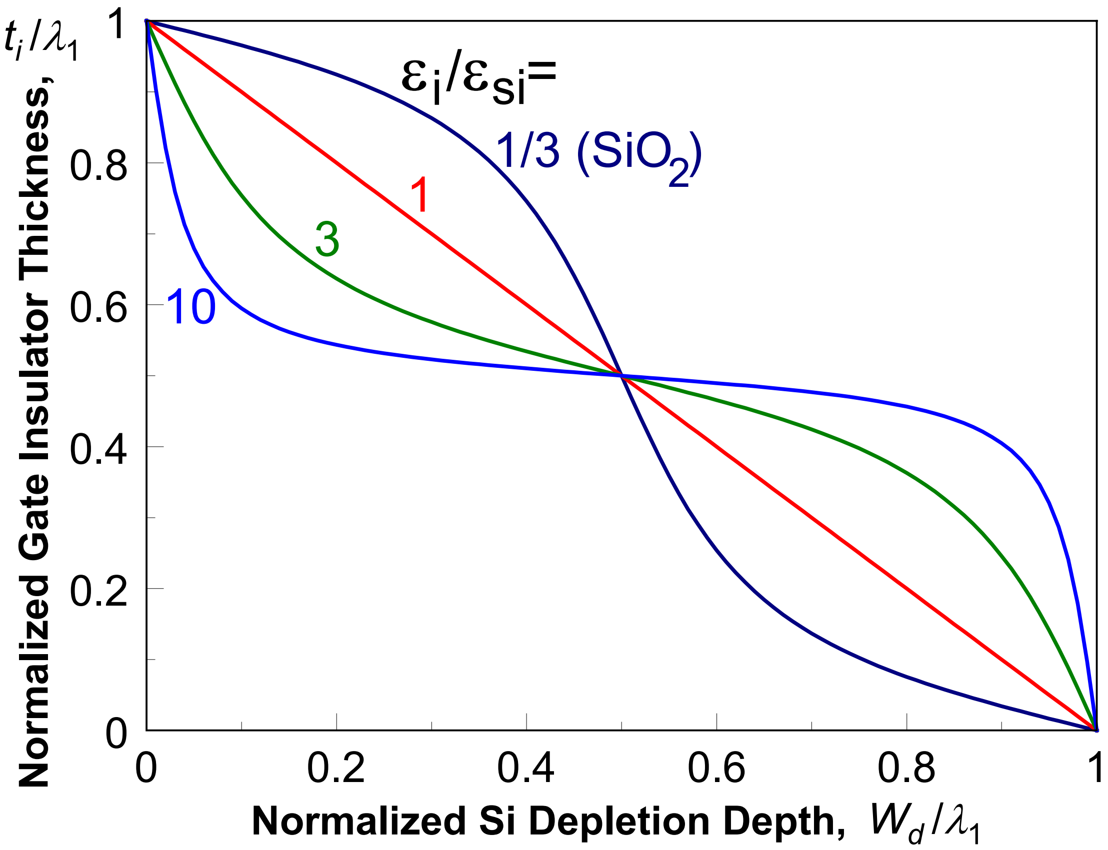
B.C. at $x = 0$: $u_{L1} = u_{L2}$
$(du_{L1}/dy = du_{L2}/dy)$

and $\varepsilon_i du_{L1}/dx = \varepsilon_{si} du_{L2}/dx$

$$\Rightarrow \quad \varepsilon_{si} \tan(\pi t_i/\lambda_n) + \varepsilon_i \tan(\pi W_d/\lambda_n) = 0$$

Generalized Scale Length

Lowest eigenvalue: $\varepsilon_{si} \tan(\pi t_i / \lambda_1) + \varepsilon_i \tan(\pi W_d / \lambda_1) = 0$

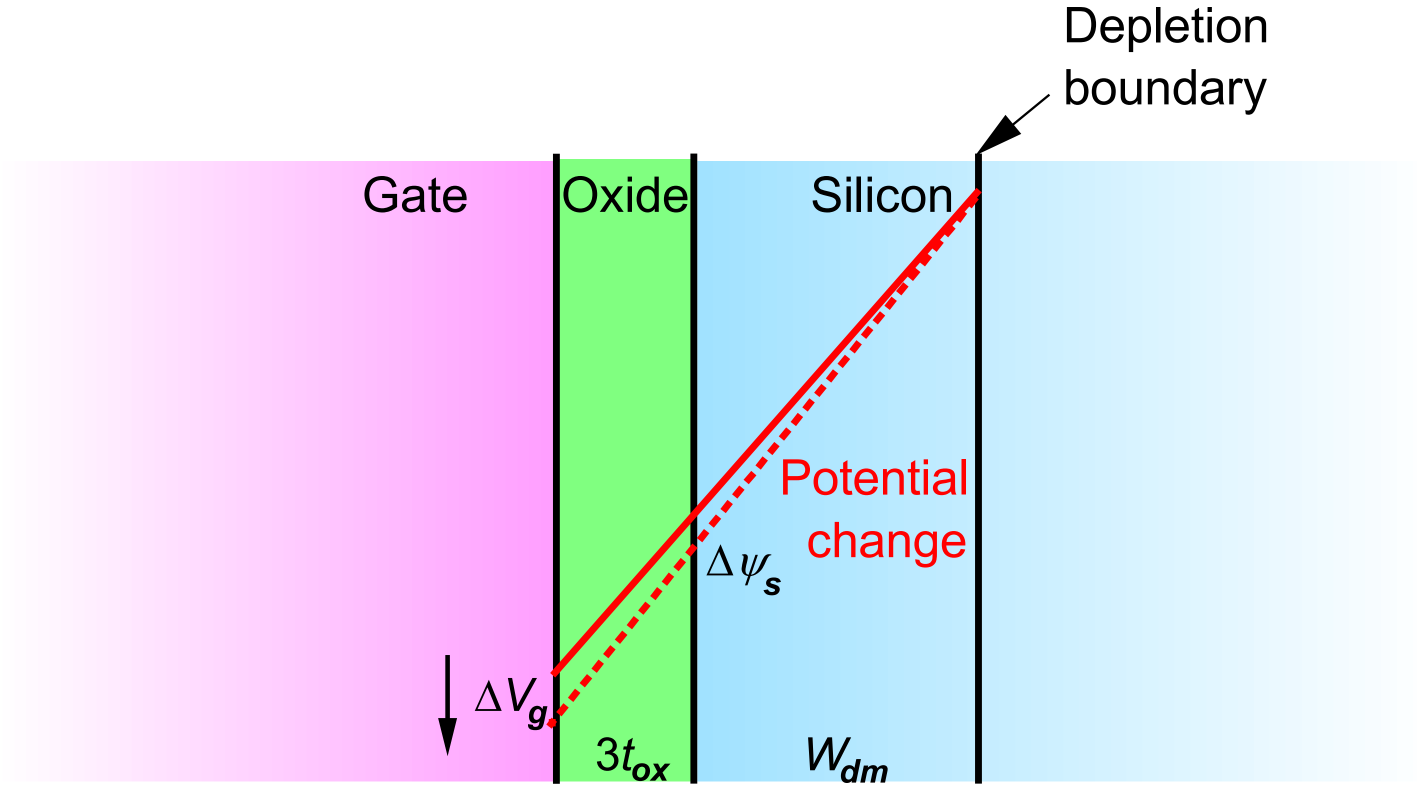


$\Delta\psi_{SCE} \propto \exp(-\pi L/2\lambda_1)$
$L_{min} \sim 2\lambda_1$

Note that:

- $\lambda_1 > W_d$, and $\lambda_1 > t_i$
- $\lambda_1 = 2W_d = 2t_i$ is always a point of symmetry regardless of ε_i, ε_{si}.
- If $\varepsilon_i = \varepsilon_{si}$, $\lambda_1 = W_d + t_i$

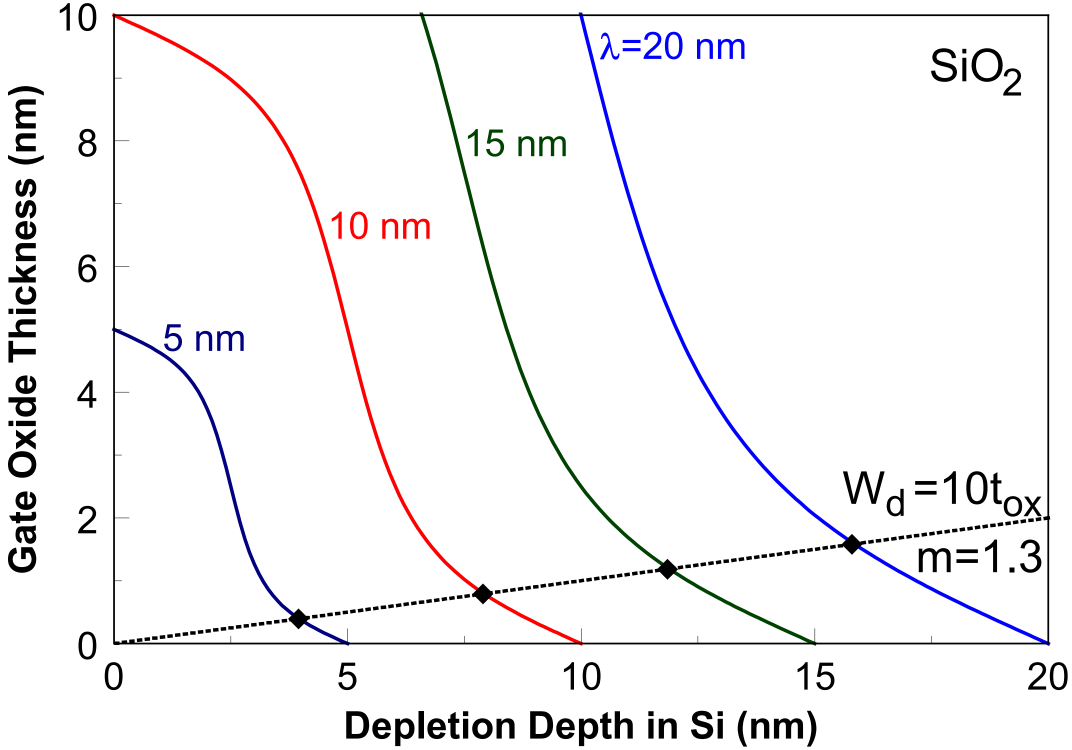
MOSFET Body Effect



$(\varepsilon_{si}/\varepsilon_{ox}=3)$

$$m = \Delta V_g/\Delta\psi_s = 1 + 3t_{ox}/W_{dm}$$

MOSFET Design Space with SiO_2



To obtain a good subthreshold slope, the body-effect coefficient,

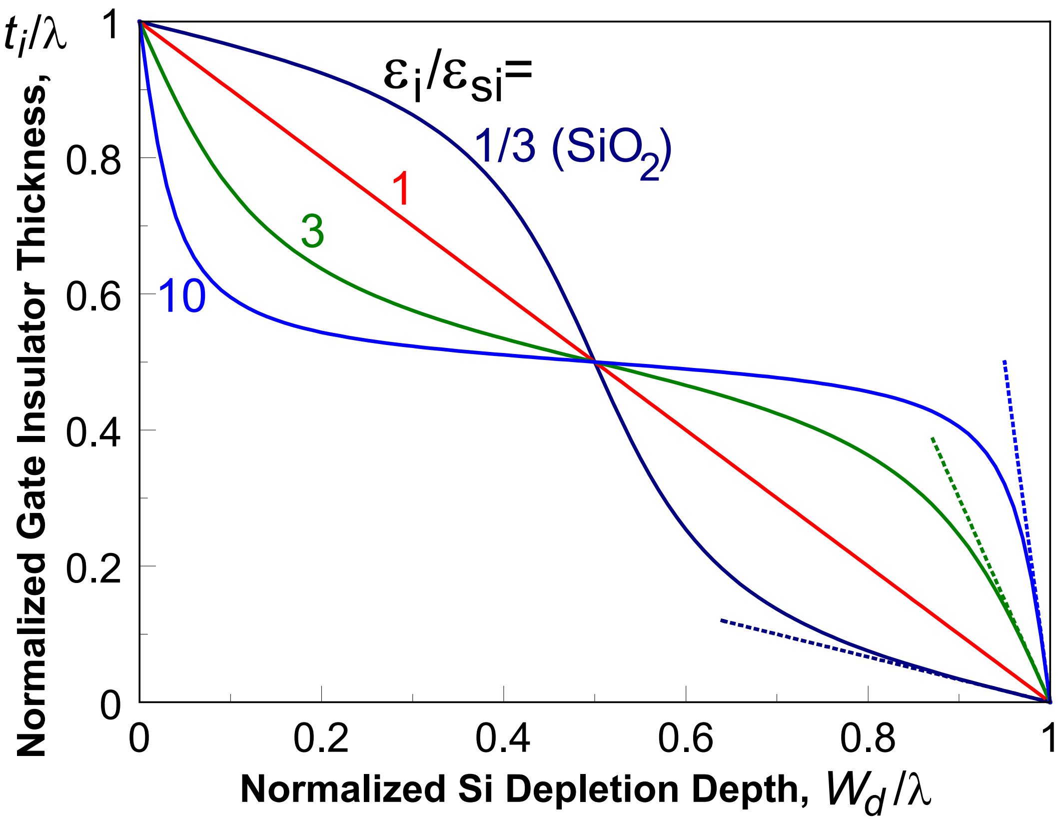
$m = \Delta V_g / \Delta \psi_s = 1 + 3t_{ox}/W_{dm}$

should be kept close to unity.

In the intercept region,
$\lambda = W_d + 3t_{ox}$
is a good approximation.

➡ $L_{min} \sim 1.5\lambda \sim 20t_{ox}$

High-k Gate Insulator

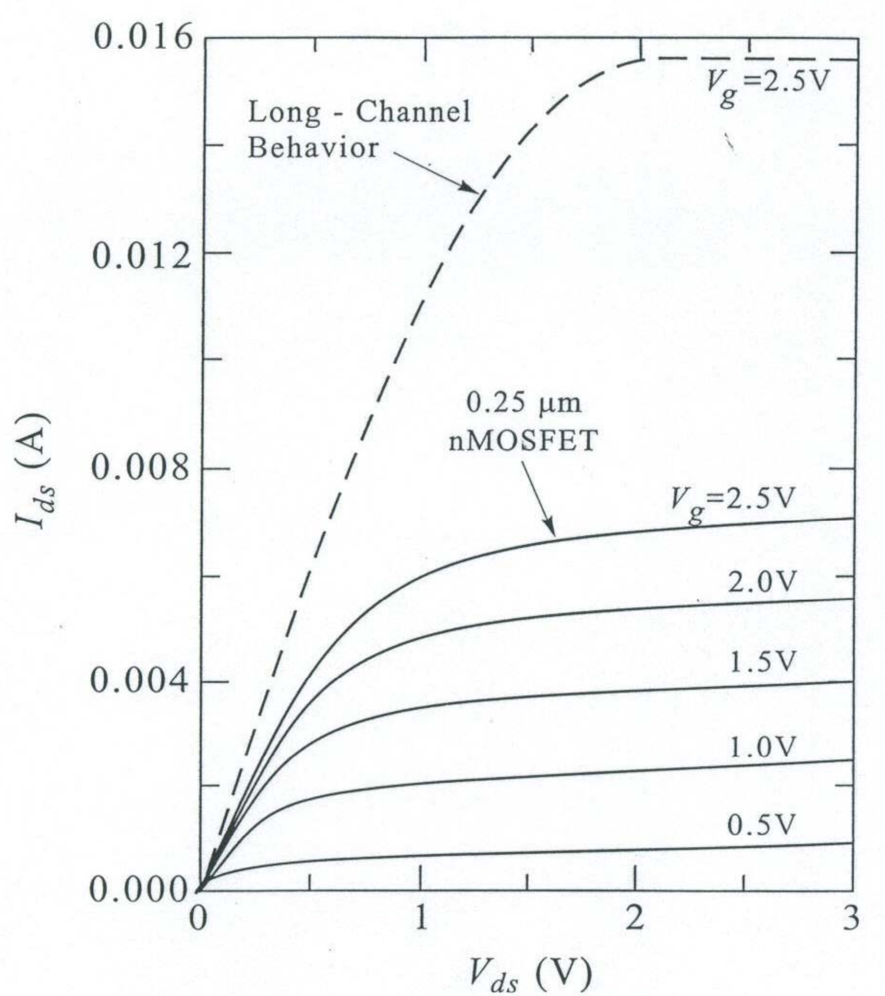


High-k gate insulator is an active area of Si research because it may replace SiO_2 thereby circumventing the tunneling problem.

But
$\lambda \sim W_d + (\varepsilon_{si}/\varepsilon_i)t_i$
is valid only when
$t_i << \lambda$.

In general, requires $t_i < \lambda/2$, regardless of ε_i.

Velocity Saturation



- Because of velocity saturation, the saturation of drain current in a short-channel device occurs at a much lower voltage than $V_{dsat} = (V_g - V_t)/m$ for long channel devices.

- This causes the saturation current, I_{dsat}, to deviate from the $\propto (V_g - V_t)^2$ behavior and from the $1/L$ dependence.

Velocity-Field Relationship

$$v = \frac{\mu_{eff}\mathcal{E}}{\left[1+\left(\frac{\mathcal{E}}{\mathcal{E}_c}\right)^n\right]^{1/n}}$$

- At low fields, $v = \mu_{eff}\mathcal{E}$: Ohm's law.
- As $\mathcal{E} \to \infty$, $v = v_{sat} = \mu_{eff}\mathcal{E}_c$.

Critical Field: $\mathcal{E}_c = \frac{v_{sat}}{\mu_{eff}}$

It is commonly believed that:

- n = 2 for electrons, n = 1 for holes.
- v_{sat} is independent of μ_{eff} (vertical field), but $\mathcal{E}_c$ depends on μ_{eff}.

Only the n = 1 case can be solved analytically.

Analytical Solution for n=1

$$I_{ds} = -WQ_i v = -WQ_i(V)\frac{\mu_{eff}(dV/dy)}{1+(\mu_{eff}/v_{sat})(dV/dy)}$$

Current continuity requires that I_{ds} be a constant, independent of *y*.

$$\Rightarrow \quad I_{ds} = -\left(\mu_{eff}WQ_i(V) + \frac{\mu_{eff}I_{ds}}{v_{sat}}\right)\frac{dV}{dy}$$

Multiplying *dy* on both sides and integrating from *y* = 0 to *L* and from *V* = 0 to V_{ds}, one solves for I_{ds}:

$$I_{ds} = \frac{-\mu_{eff}(W/L)\int_0^{V_{ds}} Q_i(V)dV}{1+(\mu_{eff}V_{ds}/v_{sat}L)}$$

Charge-sheet model: $Q_i(V) = -C_{ox}(V_g - V_t - mV)$

Therefore, $$I_{ds} = \frac{\mu_{eff}C_{ox}(W/L)\left[(V_g - V_t)V_{ds} - (m/2)V_{ds}^{\,2}\right]}{1+(\mu_{eff}V_{ds}/v_{sat}L)}$$

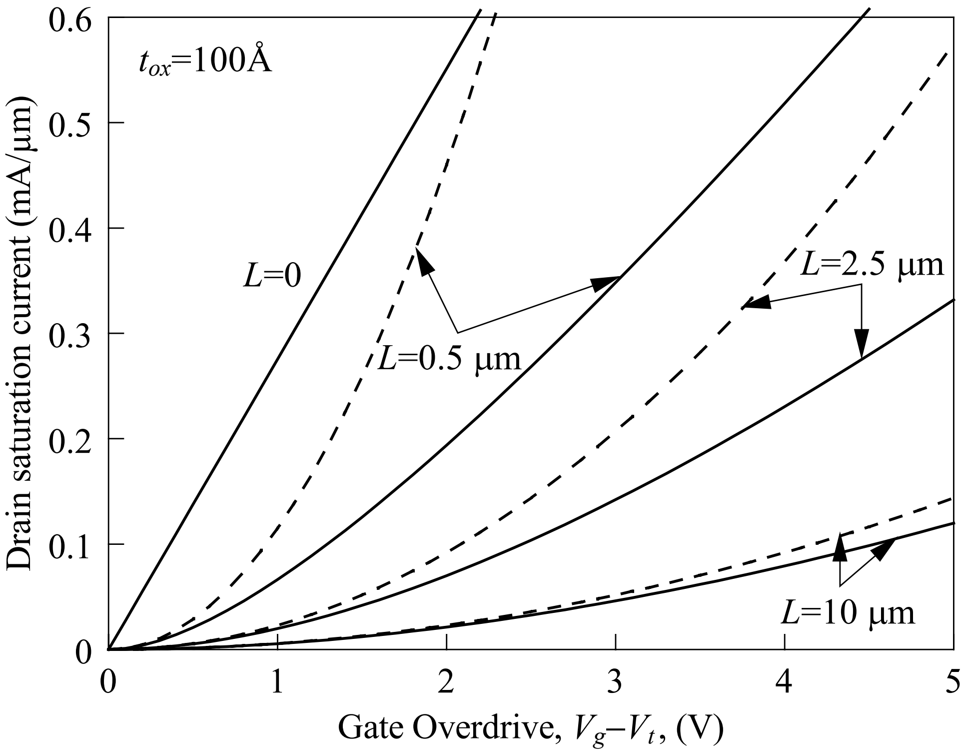
Saturation Drain Voltage and Current

The saturation voltage, V_{dsat}, can be found by solving $dI_{ds}/dV_{ds} = 0$:

$$V_{dsat} = \frac{2(V_g - V_t)/m}{1 + \sqrt{1 + 2\mu_{eff}(V_g - V_t)/(mv_{sat}L)}}$$

And the saturation current is:

$$I_{dsat} = C_{ox}Wv_{sat}(V_g - V_t)\frac{\sqrt{1 + 2\mu_{eff}(V_g - V_t)/(mv_{sat}L)} - 1}{\sqrt{1 + 2\mu_{eff}(V_g - V_t)/(mv_{sat}L)} + 1}$$



(Dashed: long-ch. model, solid: velocity sat. model)

Velocity-Saturation-Limited Current

At the drain end of the channel when $V_{ds} = V_{dsat}$,

$$Q_i(y = L) = -C_{ox}(V_g - V_t - mV_{dsat})$$

and $I_{dsat} = -Wv_{sat}Q_i(y = L)$,
i.e., carriers move at the saturation velocity.
This implies that $dV/dy \to \infty$ at the drain.

Therefore, the gradual channel approximation breaks down and the carriers are no longer confined to the surface channel.

$$I_{dsat} = C_{ox}Wv_{sat}(V_g - V_t)\frac{\sqrt{1 + 2\mu_{eff}(V_g - V_t)/(mv_{sat}L)} - 1}{\sqrt{1 + 2\mu_{eff}(V_g - V_t)/(mv_{sat}L)} + 1}$$

When $(V_g - V_t) << mv_{sat}L/2\mu_{eff}$,

$$I_{dsat} = \mu_{eff}C_{ox}\frac{W}{L}\frac{(V_g - V_t)^2}{2m}$$

Long channel limit.

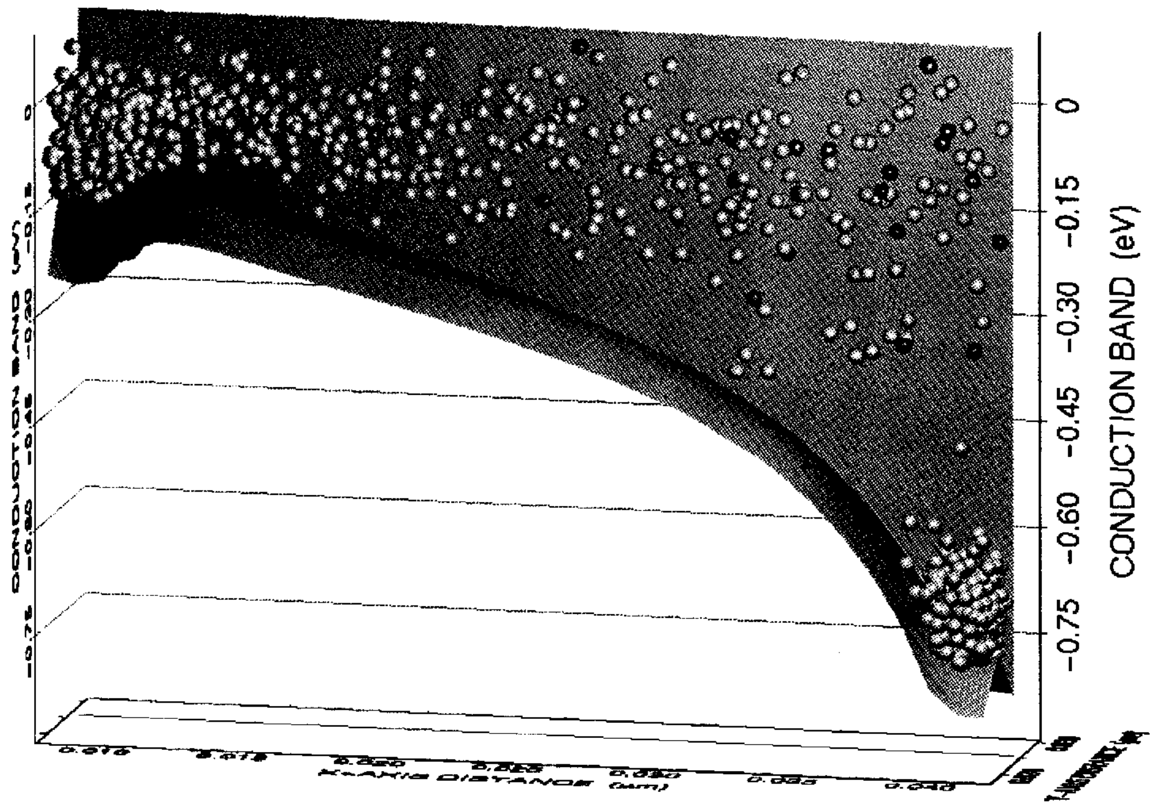
In the limit of $L \to 0$,

$$I_{dsat} = C_{ox}Wv_{sat}(V_g - V_t)$$

Velocity saturation limited current.

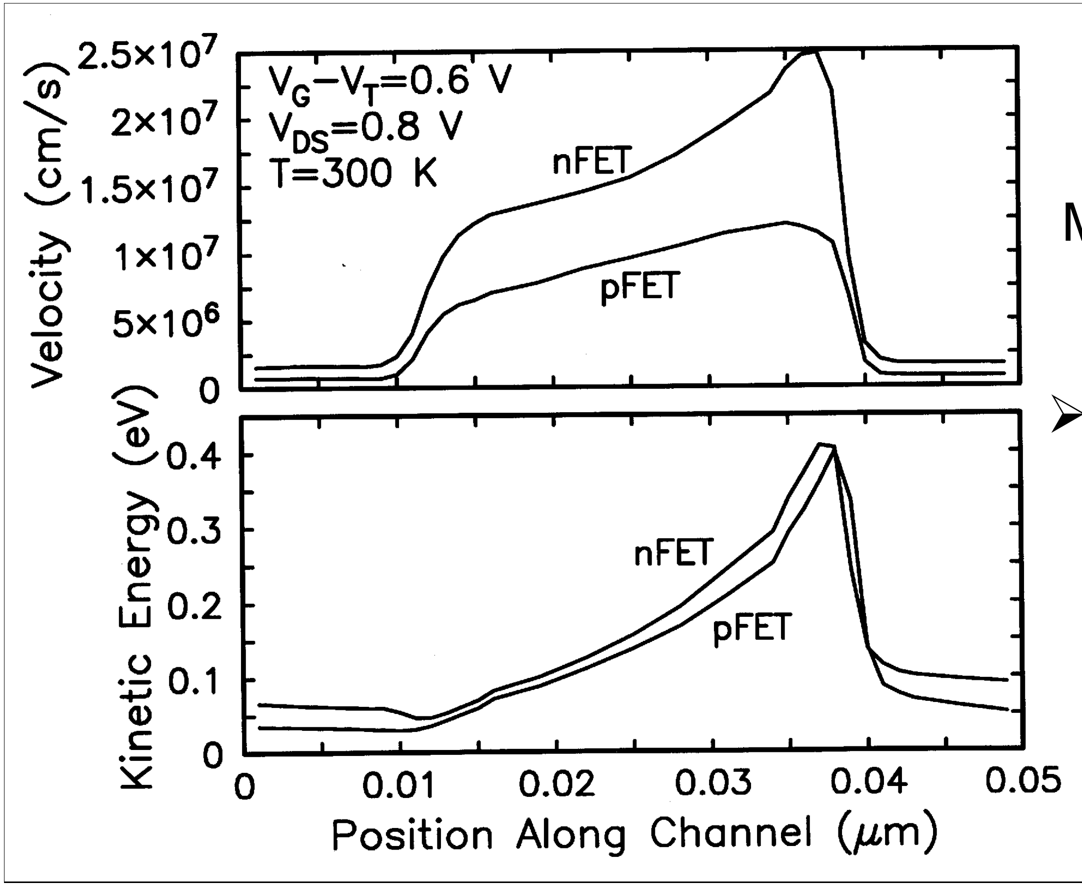
Velocity Overshoot: Monte Carlo Simulation

Velocity saturation is derived from the *drift and diffusion* model which assumes that carriers are always in thermal equilibrium with the silicon lattice.



But if the MOSFET is only a few *mean free path* (~10 nm) long, carriers do not travel enough distance to establish equilibrium ⇒ ***velocity overshoot,*** i.e., carrier velocity at the high field region near the drain can exceed the saturation velocity.

Velocity Overshoot



Monte-Carlo simulation:

- Even in a 30 nm device, nFET/pFET velocity and therefore current ratio is still ≈ 2 because of the difference in effective masses.

- The velocity at the source does not greatly exceed 10^7 cm/s; therefore, current does not greatly exceed $I_{dsat} = C_{ox} W v_{sat} (V_g - V_t)$

Distribution Function

Fermi-Dirac distribution under equilibrium:

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

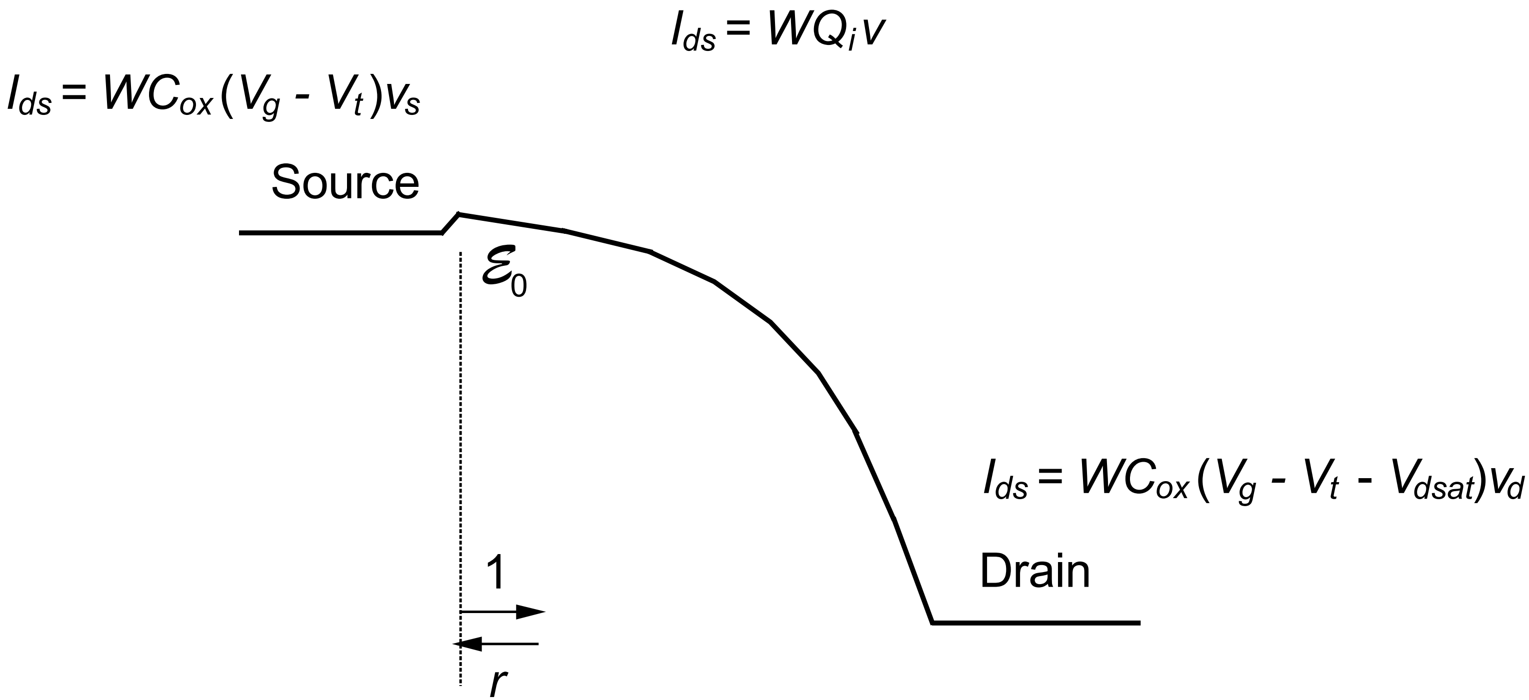
The standard semi-classical transport theory is based on the Boltzmann transport equation (BTE):

$$\frac{\partial f}{\partial t} + \boldsymbol{v} \cdot \nabla_r f + \frac{e\mathcal{E}}{\hbar} \cdot \nabla_k f = \sum_{k'} \{ S(\boldsymbol{k}', \boldsymbol{k}) f(\boldsymbol{r}, \boldsymbol{k}', t)[1 - f(\boldsymbol{r}, \boldsymbol{k}, t)] - S(\boldsymbol{k}, \boldsymbol{k}') f(\boldsymbol{r}, \boldsymbol{k}, t)[1 - f(\boldsymbol{r}, \boldsymbol{k}', t)] \}$$

where ***r*** is the position, ***k*** is the momentum, *f(**r,k,**t)* is the distribution function, ***v*** is the group velocity, $\mathcal{E}$ is the electric field, *S*(***k***,***k'***) is the transition probability between the momentum states ***k*** and ***k'***.

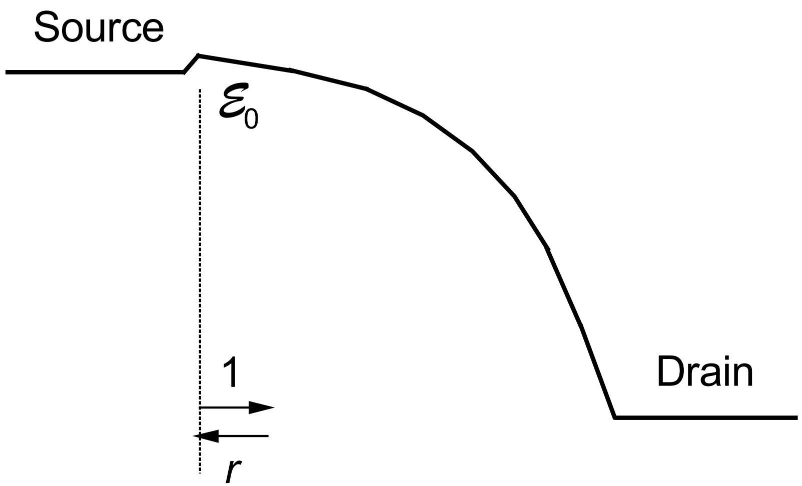
The summation on the right hand side is the collison term, which accounts for all the scattering events. The terms on the left hand side indicate, respectively, the dependence of the distribution function on time, space (explicitly related to velocity), and momentum (explicitly related to electric field).

Velocities at the Source and at the Drain



- Inversion charge density at the source is given by $C_{ox}(V_g-V_t)$.
- Inversion charge density at the drain is much lower because of the drain bias.
- Current continuity is maintained consistent with band bending.

Scattering theory



Ref. Lundstrom*, EDL, p.361, 1997*

At high drain bias, T'=0,

$$I_{ds} = TI^{+}$$

Let $r = n_s^- / n_s^+$, the backscattering coefficient.

Then

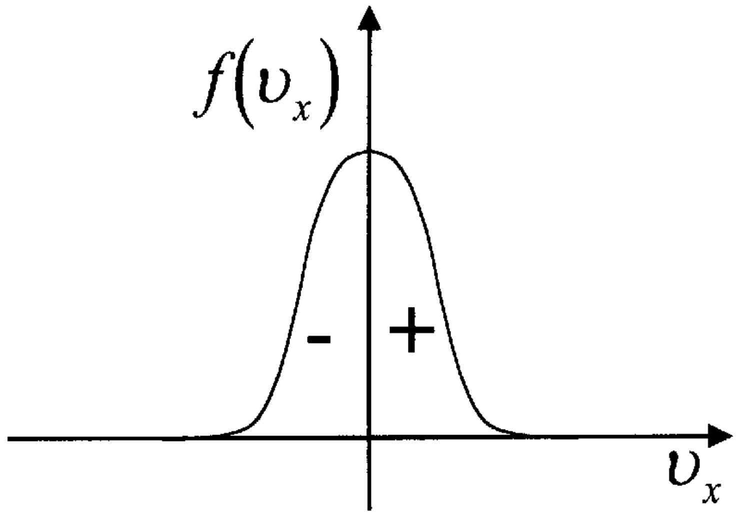
$$I_{ds} / W = C_{ox}(V_g - V_t) v_T \left(\frac{1-r}{1+r} \right)$$

Note that *r* depends on the low-field mobility near the source.

In the ballistic limit, no collisions in the channel, i.e., *r* = 0, and

$$I_{ds} / W = C_{ox}(V_g - V_t) v_T$$

Carrier Thermal Injection Velocity

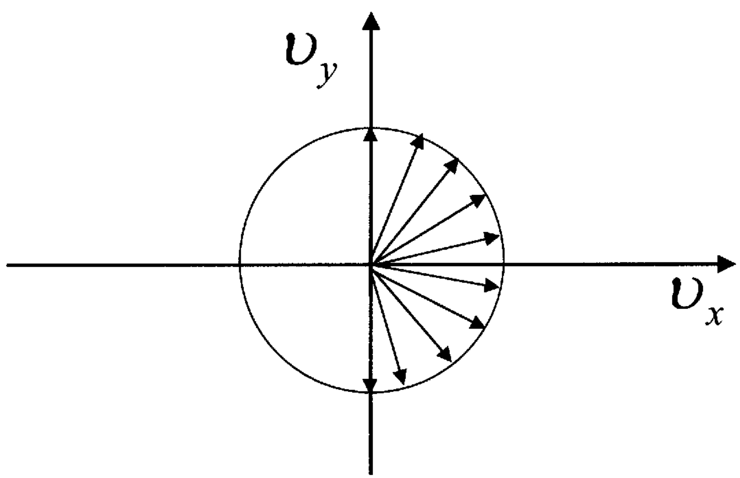


For 2-D nondegenerate carriers,

$$\langle E \rangle = \frac{\int_0^\infty EN(E) f(E) dE}{\int_0^\infty N(E) f(E) dE} = kT$$

so

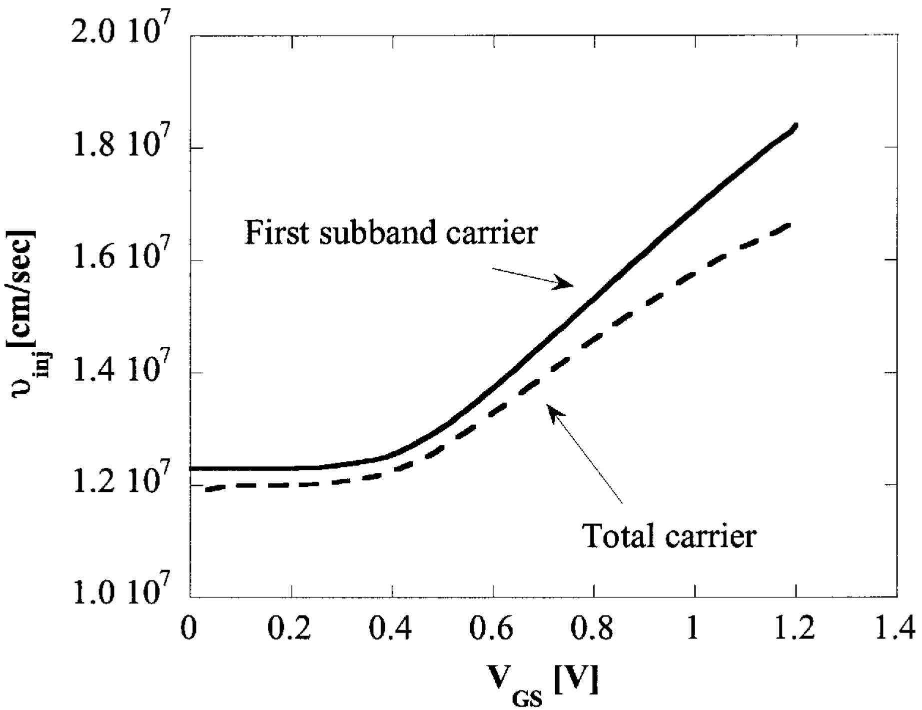
$$v_{rms} = \sqrt{\frac{2kT}{m}}$$



For uni-directional injection,

$$v_T = \frac{\int_0^\infty v_x \exp(-mv_x^2 / 2kT) dv_x}{\int_0^\infty \exp(-mv_x^2 / 2kT) dv_x} = \sqrt{\frac{2kT}{\pi m}}$$

Injection Velocity in the Degenerate Case



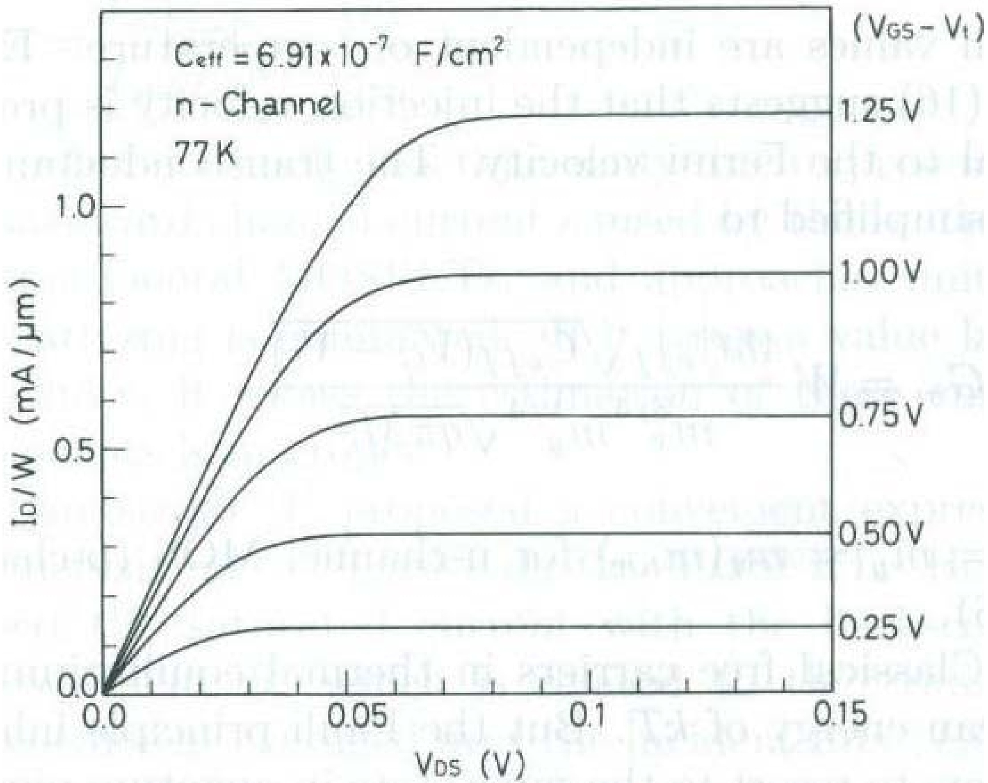
At 0 K, all states below the Fermi energy are filled. In 2-D, define a Fermi circle with velocity v_F.

$$\langle v_x \rangle = \frac{\iint\limits_{v_x>0} v_x \, dv_x \, dv_y}{\iint\limits_{v_x>0} dv_x \, dv_y} = \frac{4}{3\pi} v_F$$

Since $$\frac{1}{2} N(E) E_F = \frac{m}{\pi\hbar^2} \frac{1}{2} m v_F{}^2 = n_s = \frac{C_{ox}(V_g - V_t)}{q}$$

$$\Rightarrow \quad v_T = \frac{4\hbar}{3m} \sqrt{\frac{2C_{ox}(V_g - V_t)}{q\pi}}$$

I-V Curves of a Ballistic MOSFET



1) $n_S(V_{GS})$ from a 1D S-P solver

2) Add 2D effects:

   V_{GS} --> $V_{GS} + \alpha V_{DS}$

3) determine Fermi level:

$$n_S = \frac{m^* k_B T}{2\pi\hbar^2}\left[\ln\left(1+e^{\eta}\right)+\ln\left(1+e^{\eta-\eta_D}\right)\right]$$

4) determine injection velocity:

$$\upsilon_T = \sqrt{2k_B T/\pi m}\left(\frac{\mathcal{F}_{1/2}(\eta)}{\ln(1+e^{\eta})}\right)$$

5) evaluate drain current:

$$I_{DS} = Wq\left(n_s^+ \upsilon_T^+ - n_s^- \upsilon_T^-\right)$$

Natori, JAP, p. 4879, 1994. $I_{ds}/W = \frac{4\hbar}{3m}\sqrt{\frac{2C_{ox}}{q\pi}}C_{ox}(V_g - V_t)^{3/2}$ (T=0 K)

Independent of L!