ECE 230B: Winter 2003

Solid-State Electronic Devices

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Prerequist: ECE 135A, B, or equivalent and ECE 230A

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Office hour: Open

This course covers the physics of solid-state electronic devices, including p-n junctions, MOS devices, field-effect transistors, bipolar transistors, etc. Principles of CMOS and bipolar scaling to nanometer dimensions and their high frequency performance in digital and analog circuits will be taught.

Topics to be Covered

1) Band diagram, Fermi level, Poisson's eq., Carrier transport (1.5 wks)
2) P-n junction (1 wk)
3) MOS device (1 wk)
4) MOSFETs (1.5 wks)
5) CMOS scaling and design (1.5 wks)
6) CMOS performance factors (1 wk)
7) Bipolar transistors (1.5 wks)
8) SiGe bipolar device (1 wk)

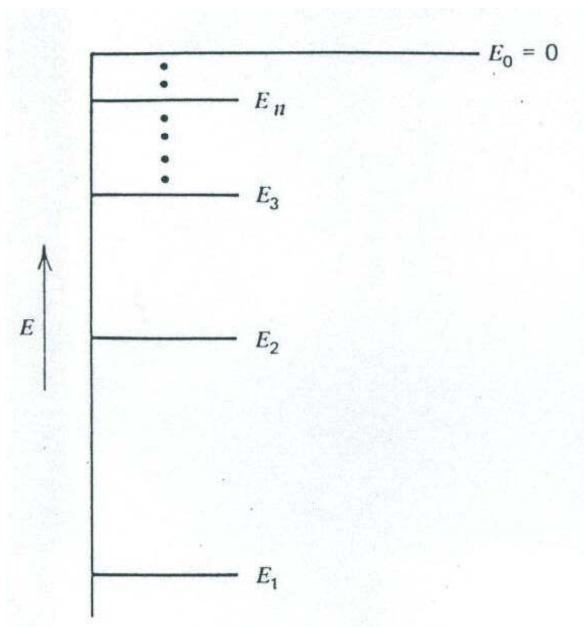
Textbook: "Fundamentals of Modern VLSI Devices"
Y. Taur and T. H. Ning, Cambridge Univ. Press (1998)

Homework and Grading Policy

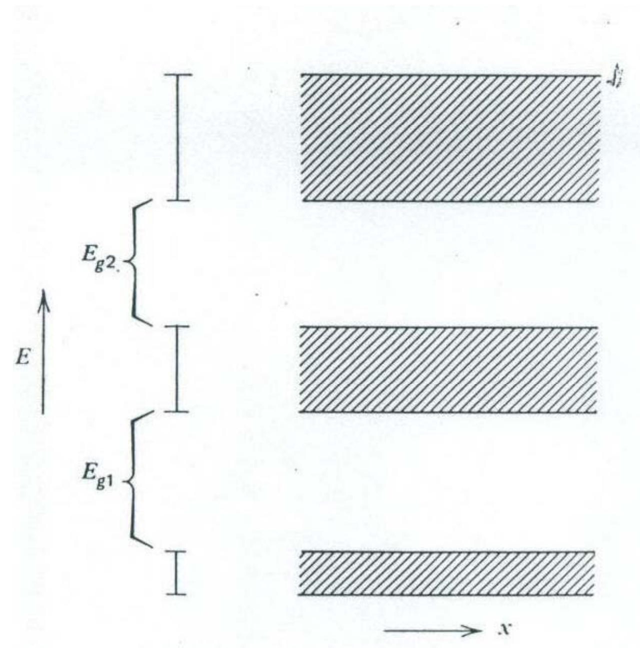
- Five open book quizzes (bring your calculator)
- Five homework assignments
- Equal proportion of credits among above

Electron Energy Levels and Bands

Discrete electron energy levels in an atom:



Broadening into electron energy bands in a solid:



There are forbidden energy gaps between allowed electron energy bands.

The Origin of Energy Gap in a Crystalline Solid

Free electron: $K.E. = \hbar^2 k^2 / 2m$

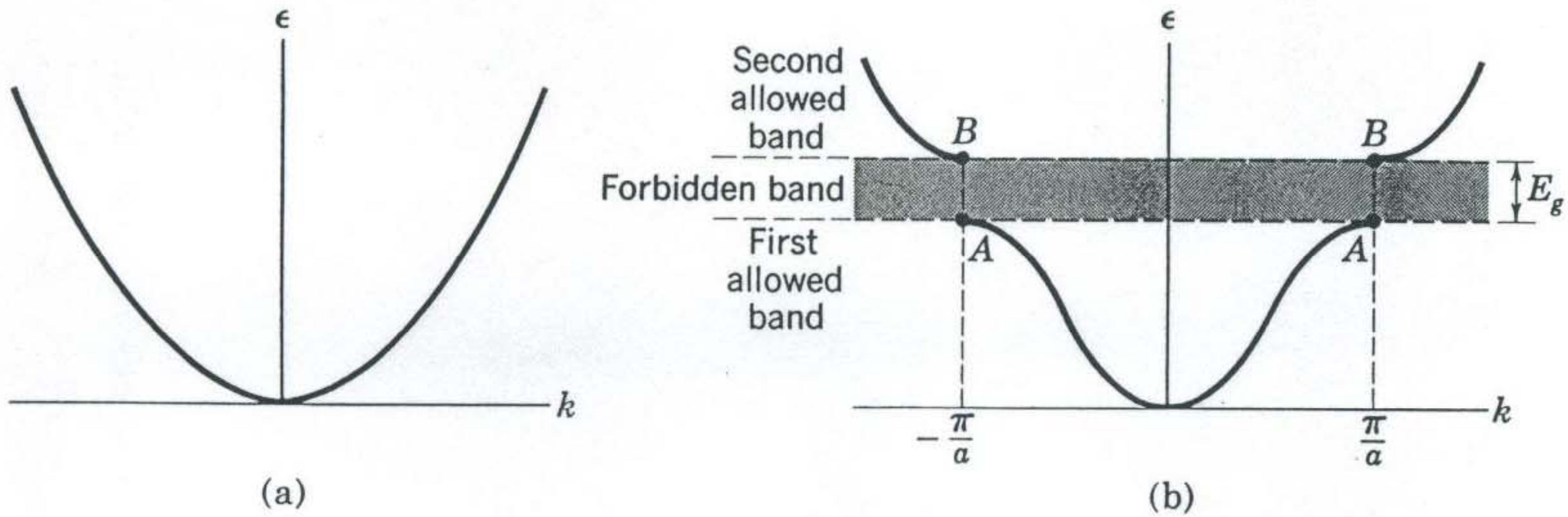


Figure 2 (a) Plot of energy ϵ versus wavevector k for a free electron. (b) Plot of energy versus wavevector for an electron in a monatomic linear lattice of lattice constant a. The energy gap E_g shown is associated with the first Bragg reflection at $k = \pm\pi/a$; other gaps are found at $\pm n\pi/a$, for integral values of n.

The Origin of Energy Gap in a Crystalline Solid

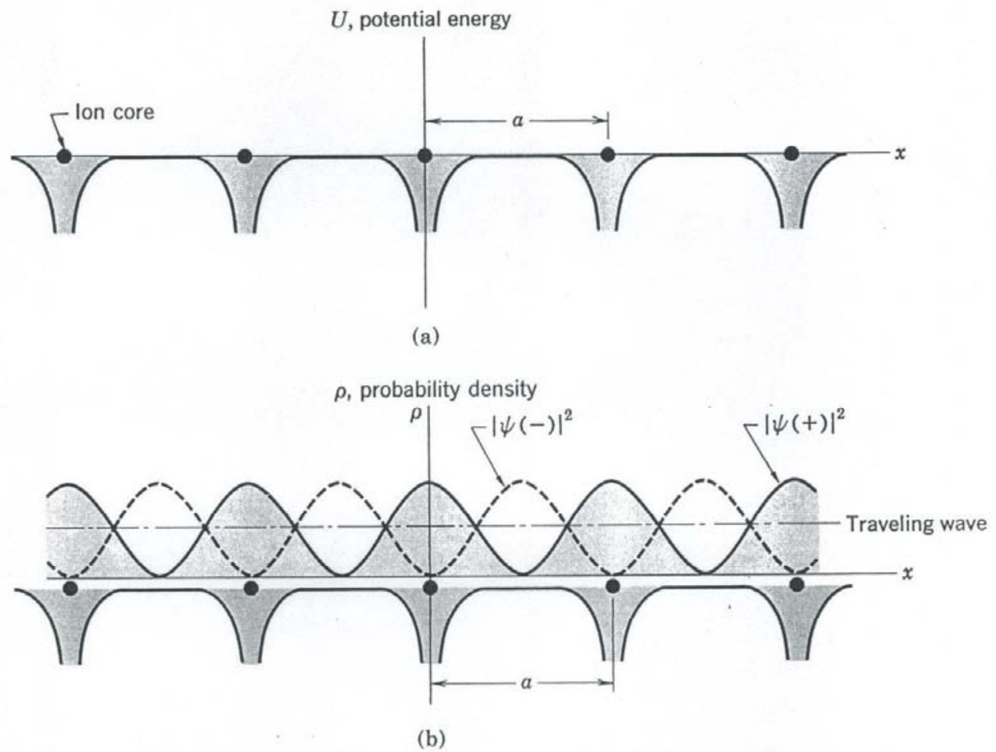
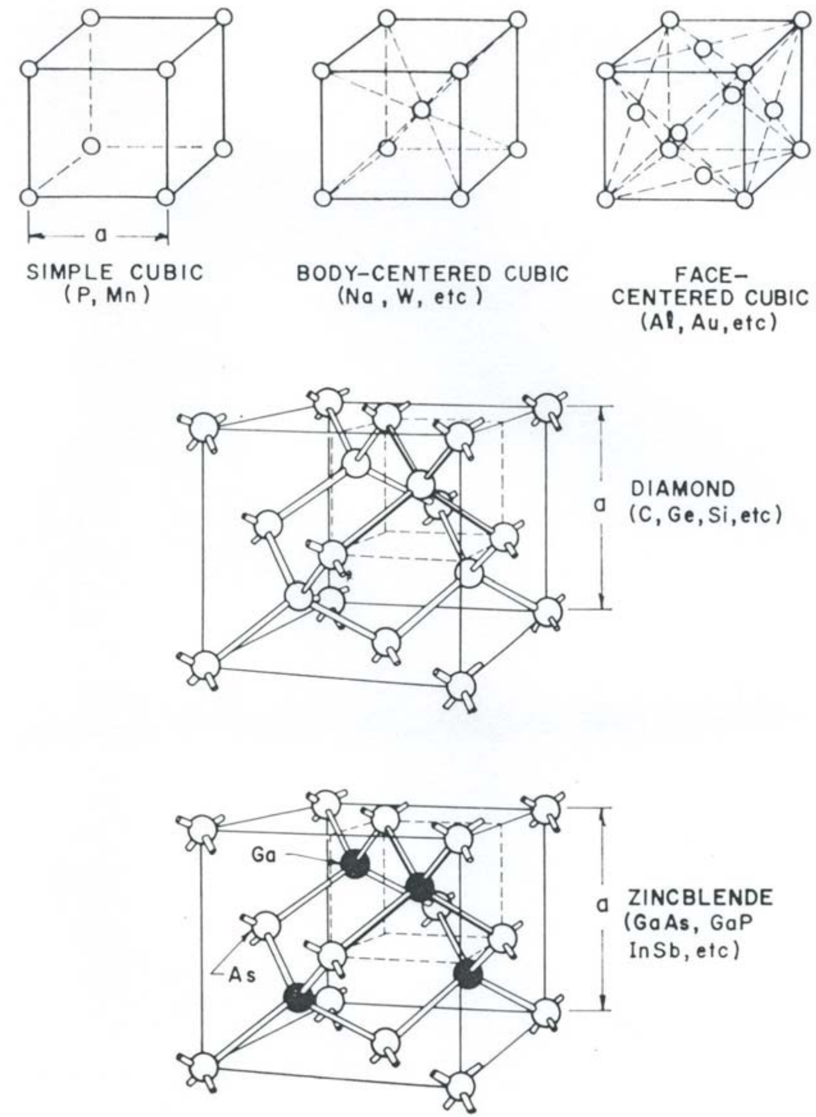


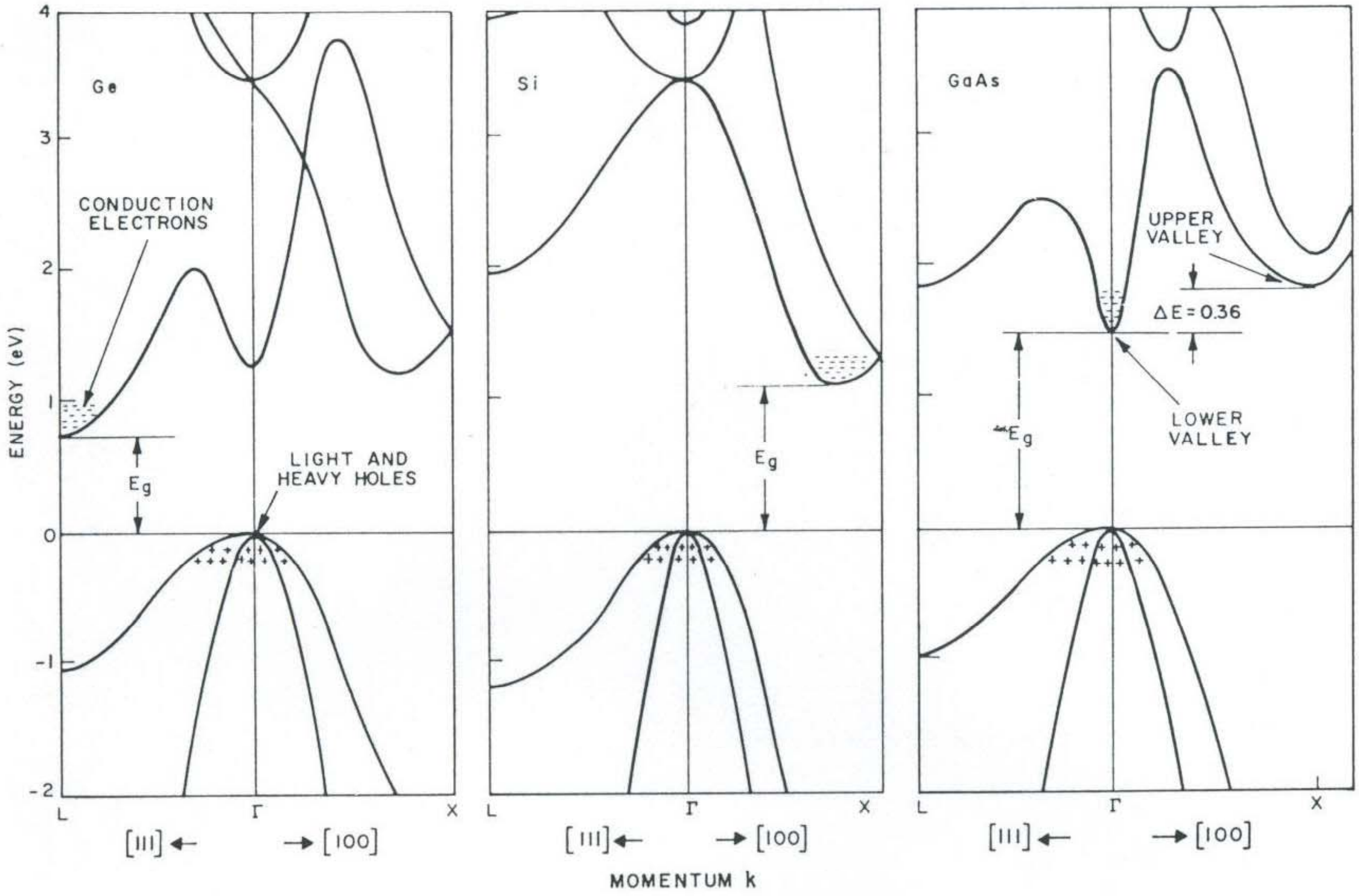
Figure 3 (a) Variation of potential energy of a conduction electron in the field of the ion cores of a linear lattice. (b) Distribution of probability density ρ in the lattice for $|\psi(-)|^2 \propto \sin^2 \pi x/a$; $|\psi(+)|^2 \propto \cos^2 \pi x/a$; and for a traveling wave. The wavefunction $\psi(+)$ piles up electronic charge on the cores of the positive ions, thereby lowering the potential energy in comparison with the average potential energy seen by a traveling wave. The wavefunction $\psi(-)$ piles up charge in the region between the ions and removes it from the ion cores; thereby raising the potential energy in comparison with that seen by a traveling wave. This figure is the key to understanding the origin of the energy gap.

(From Kittel)

Crystalline Lattice



Energy Band Structures



Metals, Insulators, and Semiconductors

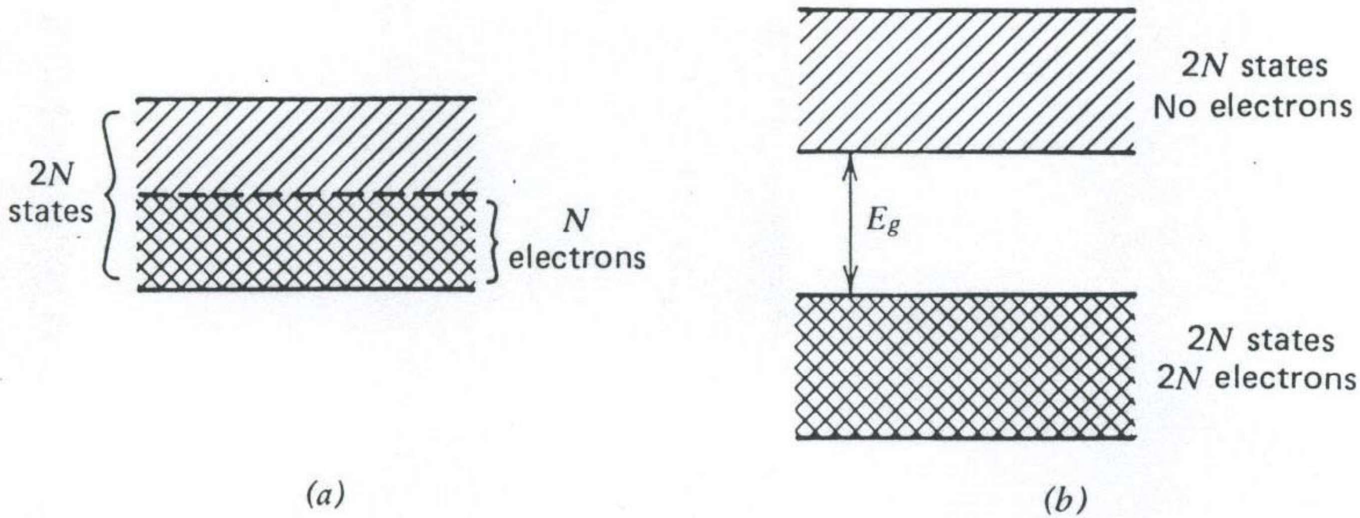


Figure 1.3 Energy-band diagrams: (*a*) *N* electrons filling half of the 2*N* allowed states, as might occur in a metal. (*b*) A completely empty band separated by an energy gap E_g from a band whose 2*N* states are completely filled by 2*N* electrons, representative of an insulator.

Insulators: E_g > 4-5 eV

Semiconductors: E_g < 4-5 eV

Metals, Insulators, and Semiconductors

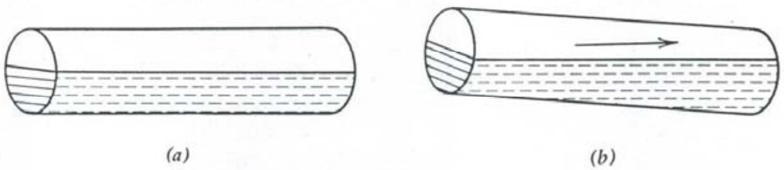


Figure 1.4 Electron motion in an allowed band is analogous to fluid

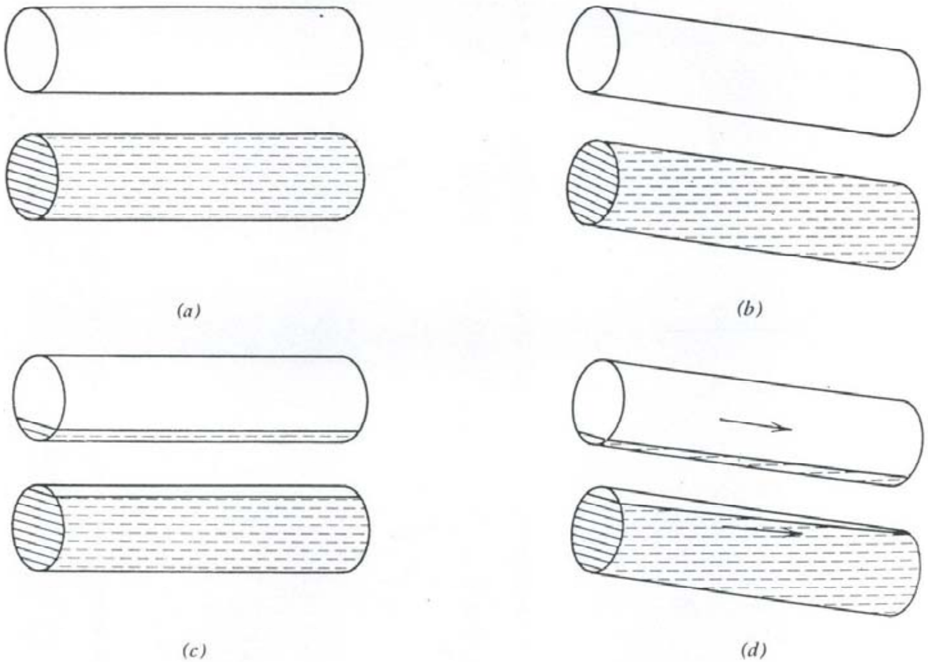
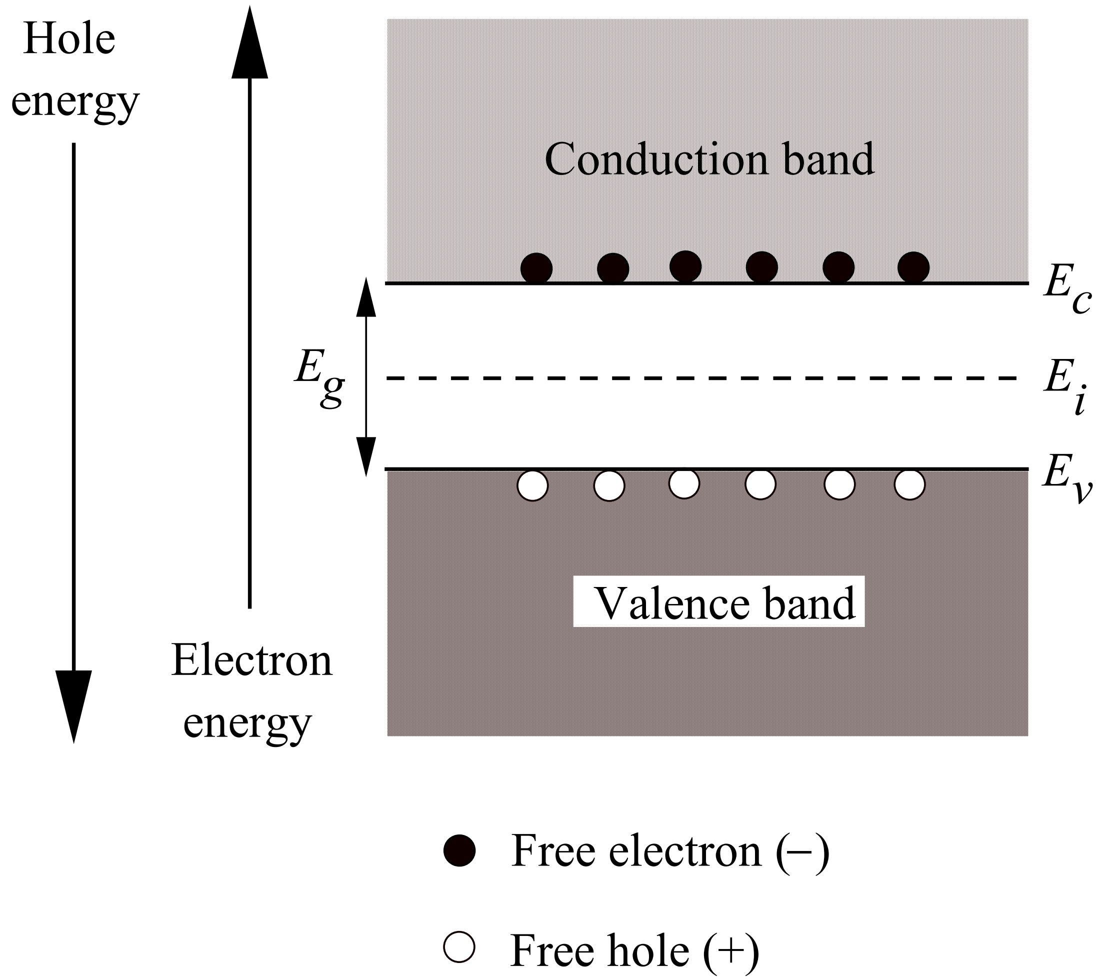


Figure 1.7 Fluid analogy for a semiconductor. (*a*) and (*b*) No flow can occur in either the completely filled or completely empty tube. (*c*) and (*d*) Fluid can move in both tubes if some of it is transferred from the filled tube to the empty one, leaving unfilled volume in the lower tube.

(From Muller and Kamins)

Energy Band Diagram of Silicon



E_g = 1.12 eV

kT/q = 0.026 V @ 300 K

Fermi-Dirac Statistical Distribution

$$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

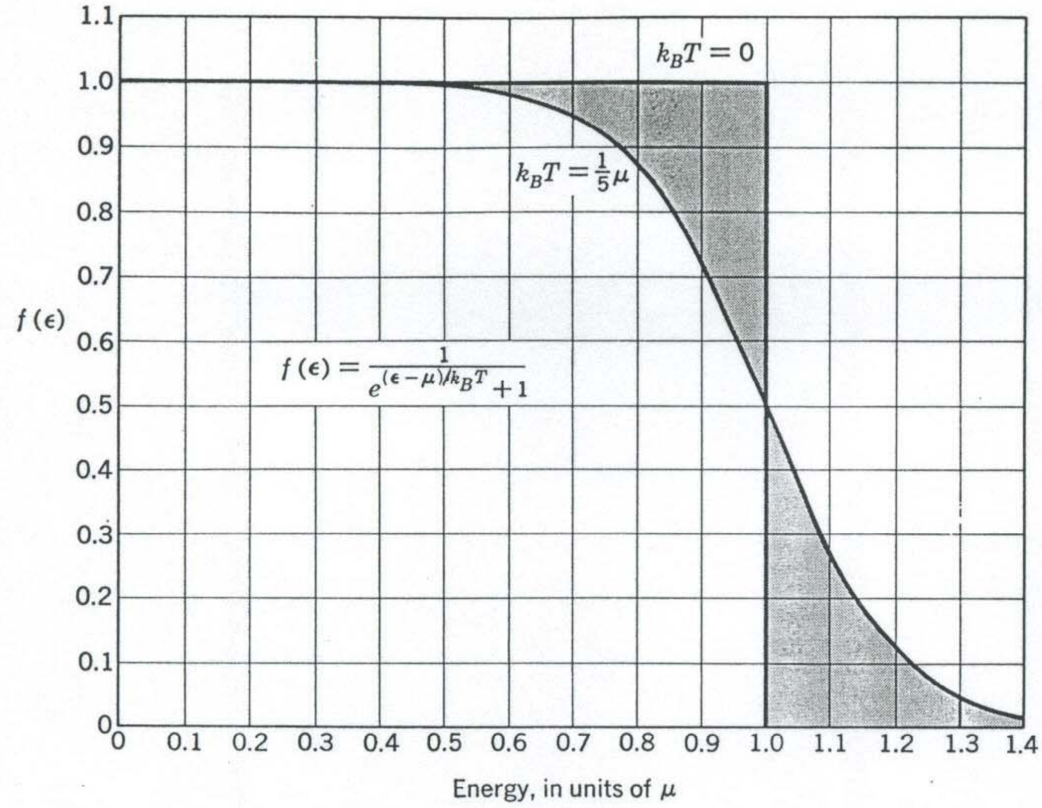
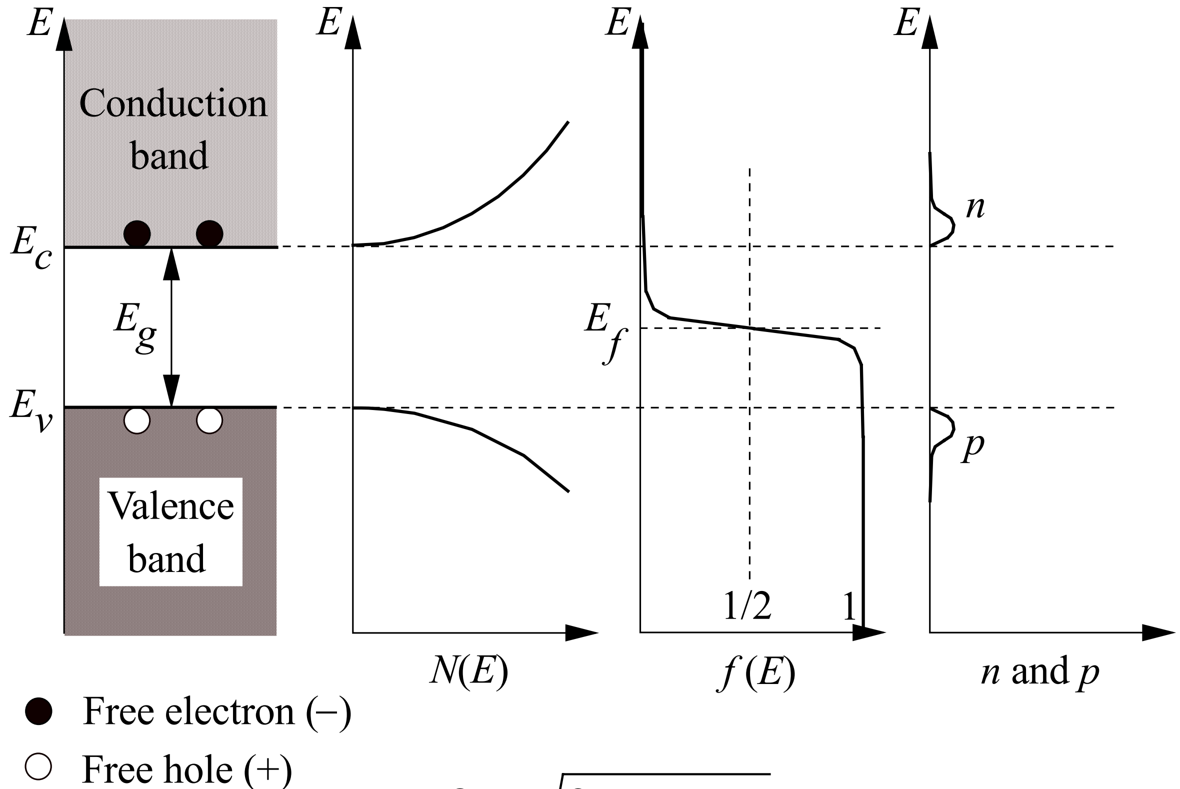


Figure 5a Plot of the Fermi-Dirac distribution function $f(\epsilon)$ versus ϵ/μ, for zero temperature and for a temperature $k_BT = \frac{1}{5}\mu$. The value of $f(\epsilon)$ gives the fraction of levels at a given energy which are occupied when the system is in thermal equilibrium. When the system is heated from absolute zero, electrons are transferred from the shaded region at $\epsilon/\mu < 1$ to the shaded region at $\epsilon/\mu > 1$. For a metal μ might correspond to 50,000°K.

Intrinsic Silicon



$$N(E) = \frac{8\pi g\sqrt{2m_x m_y m_z}}{h^3}\sqrt{E - E_c} \text{ (Appendix C)}$$

$$n = \int_{E_c}^{\infty} N(E) f(E) dE \qquad p = \int_{-\infty}^{E_v} N(E)[1 - f(E)] dE$$

Fermi Level in Intrinsic Silicon

Fermi-Dirac distribution: $$f(E) = \frac{1}{1 + e^{(E - E_f)/kT}}$$

For $(E - E_f)/kT >> 1$, $$f(E) \approx e^{-(E - E_f)/kT}$$

For $(E - E_f)/kT << -1$, $$f(E) \approx 1 - e^{-(E_f - E)/kT}$$

Carrying out the integrations,

$$n = N_c e^{-(E_c - E_f)/kT} \qquad p = N_v e^{-(E_f - E_v)/kT}$$

N_c, N_v: Effective density of states, $$N_c = \frac{2g\sqrt{m_x m_y m_z}}{h^3}(2\pi kT)^{3/2}$$

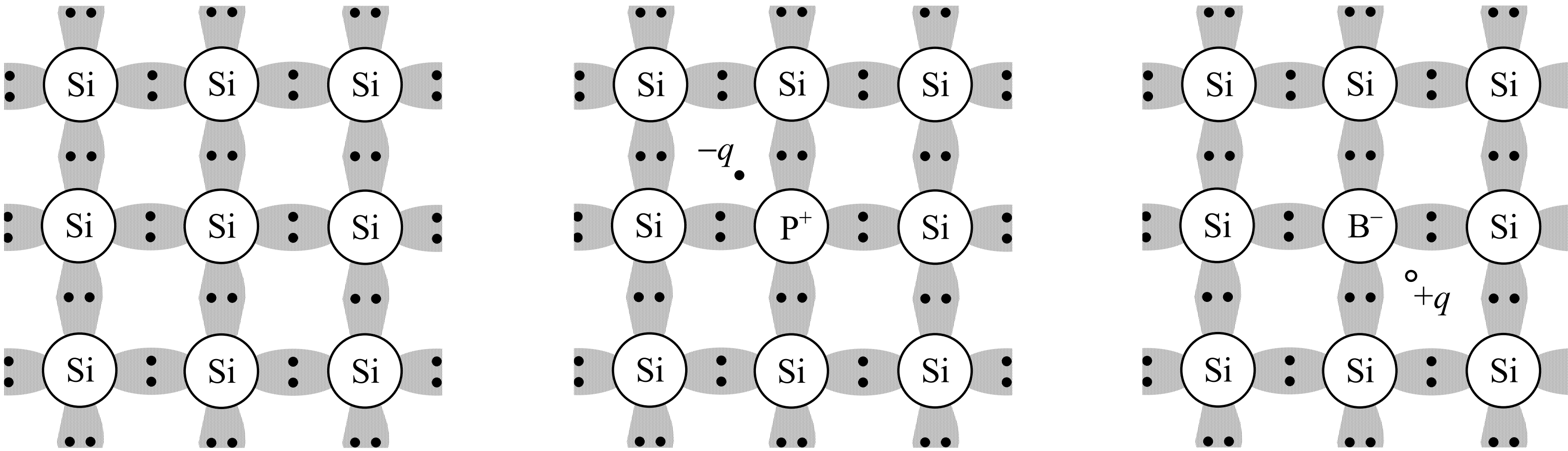
Intrinsic silicon:
charge neutrality
requires $n = p = n_i$,

$$E_i = E_f = \frac{E_c + E_v}{2} - \frac{kT}{2}\ln\left(\frac{N_c}{N_v}\right)$$

$$n_i = \sqrt{N_c N_v} e^{-(E_c - E_v)/2kT} = \sqrt{N_c N_v} e^{-E_g/2kT}$$

Physical Properties	Si	SiO_2
Atomic/molecular weight	28.09	60.08
Atoms or molecules/cm^3	5.0×10^{22}	2.3×10^{22}
Density (g/cm^3)	2.33	2.27
Crystal structure	Diamond	Amorphous
Lattice constant (Å)	5.43	---
Energy gap (eV)	1.12	8-9
Dielectric constant	11.7	3.9
Intrinsic carrier concentration (cm^{-3})	1.4×10^{10}	---
Carrier mobility (cm^2/V-s)	Electron: 1430	---
	Hole: 470	---
Effective density of states (cm^{-3})	Conduction band N_c: 3.2×10^{19}	---
	Valence band N_v: 1.8×10^{19}	---
Breakdown field (V/cm)	3×10^5	$>10^7$
Melting point (°C)	1415	1600-1700
Thermal conductivity (W/cm-°C)	1.5	0.014
Specific heat (J/g-°C)	0.7	1.0
Thermal diffusivity (cm^2/s)	0.9	0.006
Thermal expansion coefficient (°C^{-1})	2.5×10^{-6}	0.5×10^{-6}

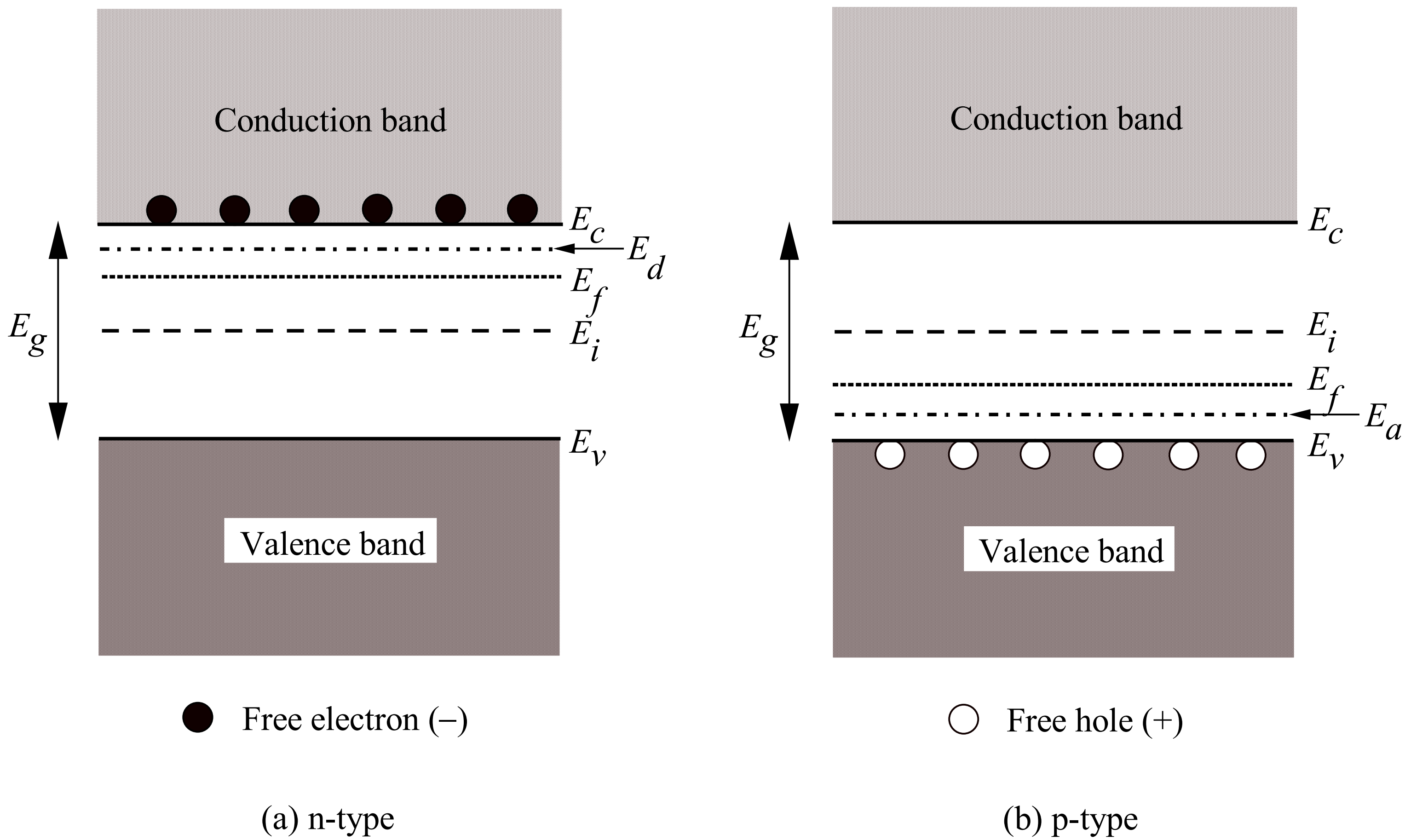
Extrinsic (n-type and p-type) Silicon



(a) (b) (c)

Intrinsic n-type p-type

Band Diagrams of n-type and p-type Silicon



Fermi Level in Extrinsic Silicon

$$n = N_c e^{-(E_c - E_f)/kT} \qquad p = N_v e^{-(E_f - E_v)/kT}$$

For n-type silicon with donor concentration N_d, charge neutrality requires $n = p + N_d^+ \approx N_d$ (assuming complete ionization). Therefore,

$$E_c - E_f = kT \ln\left(\frac{N_c}{N_d}\right)$$

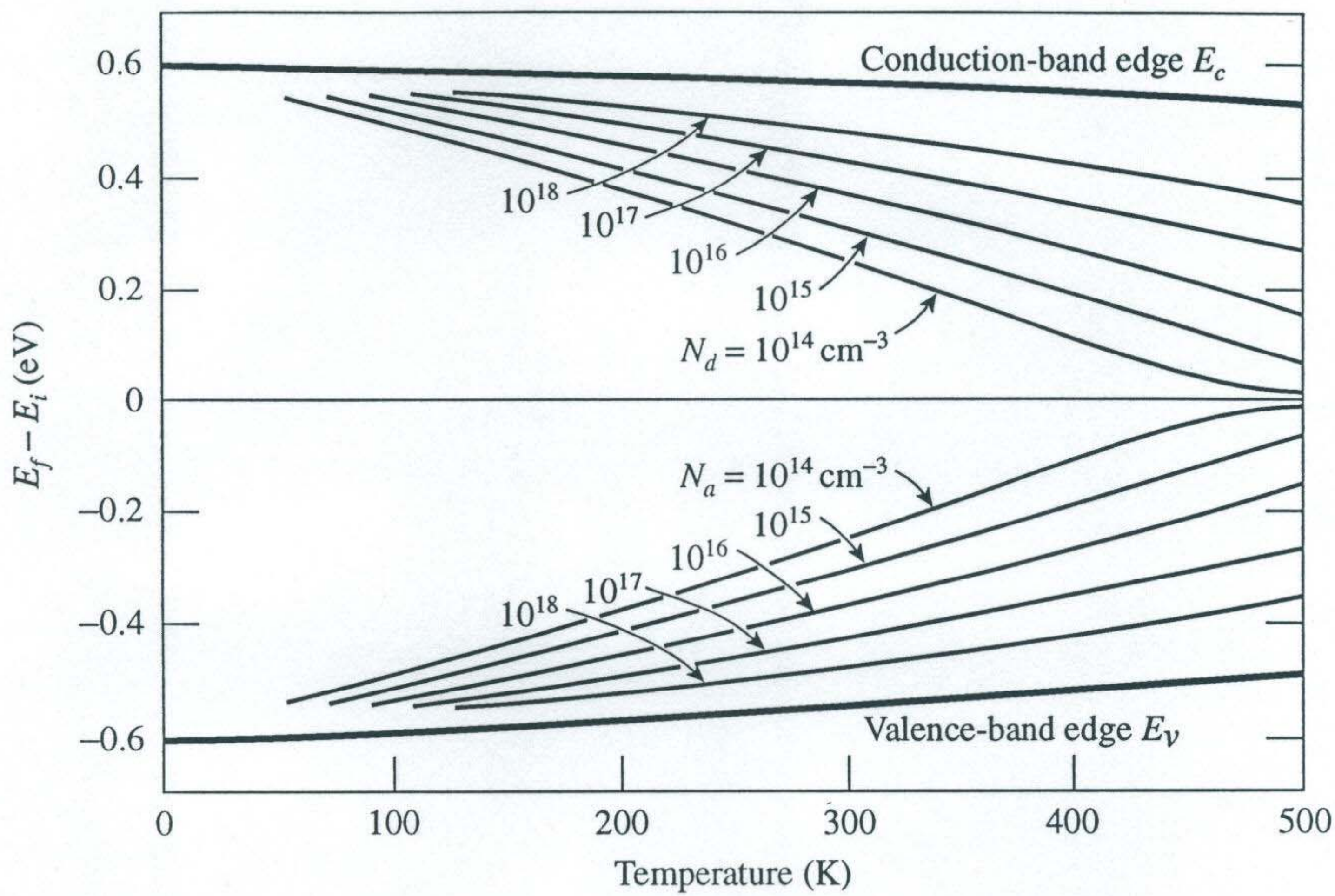
Or, in terms of n_i and E_i,

$$E_f - E_i = kT \ln\left(\frac{N_d}{n_i}\right)$$

For $N_d > N_c$, $E_f > E_c$ → degenerate n^+ silicon. Need full F-D eq.

Note that $np = n_i^2$, independent of E_f (n- or p-type).

Fermi Level vs. Temperature



Fermi Level in Extrinsic Silicon

If incomplete ionization, $N_d^+ < N_d$ where

$$N_d^{\;+} = N_d\left[1 - f(E_d)\right] = N_d\left[1 - \frac{1}{1 + (1/2)e^{(E_d - E_f)/kT}}\right]$$

E_f is then solved from

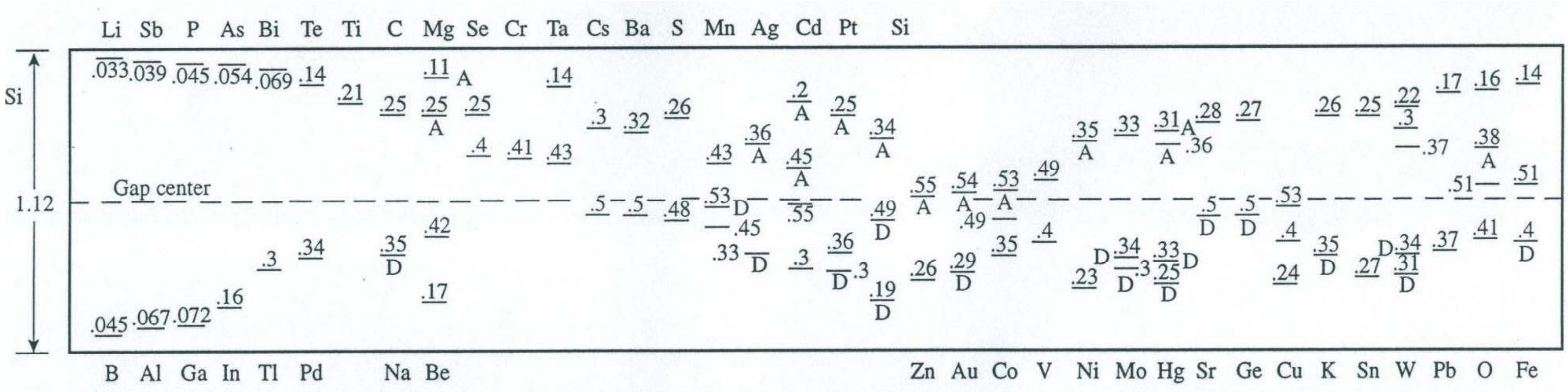
$$N_c e^{-(E_c - E_f)/kT} = \frac{N_d}{1 + 2e^{-(E_d - E_f)/kT}} + N_v e^{-(E_f - E_v)/kT}$$

→ Condition for complete ionization:

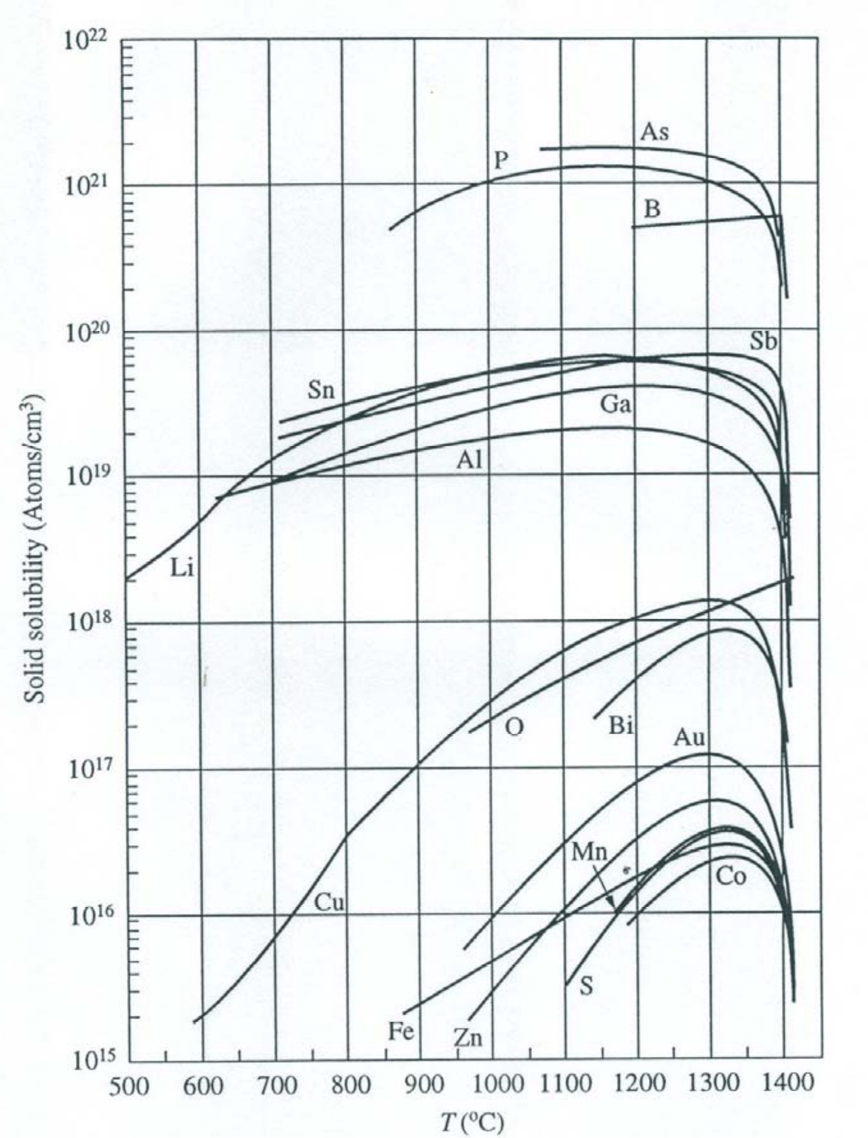
$$\frac{N_d}{N_c} e^{(E_c - E_d)/kT} << 1$$

Requires the donor energy level to be within a few kT of the bottom of the conduction band.

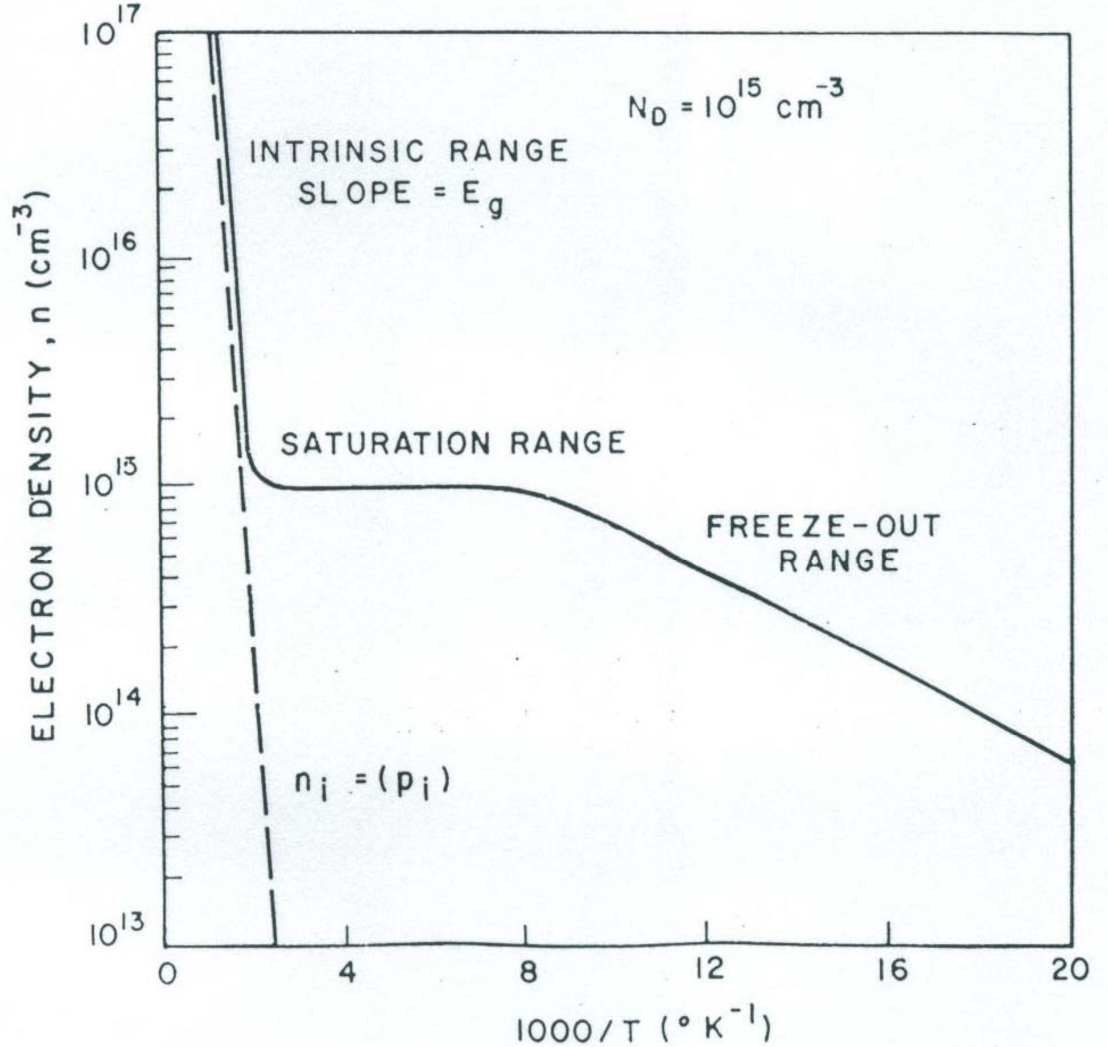
Donor and Acceptor Levels in Silicon



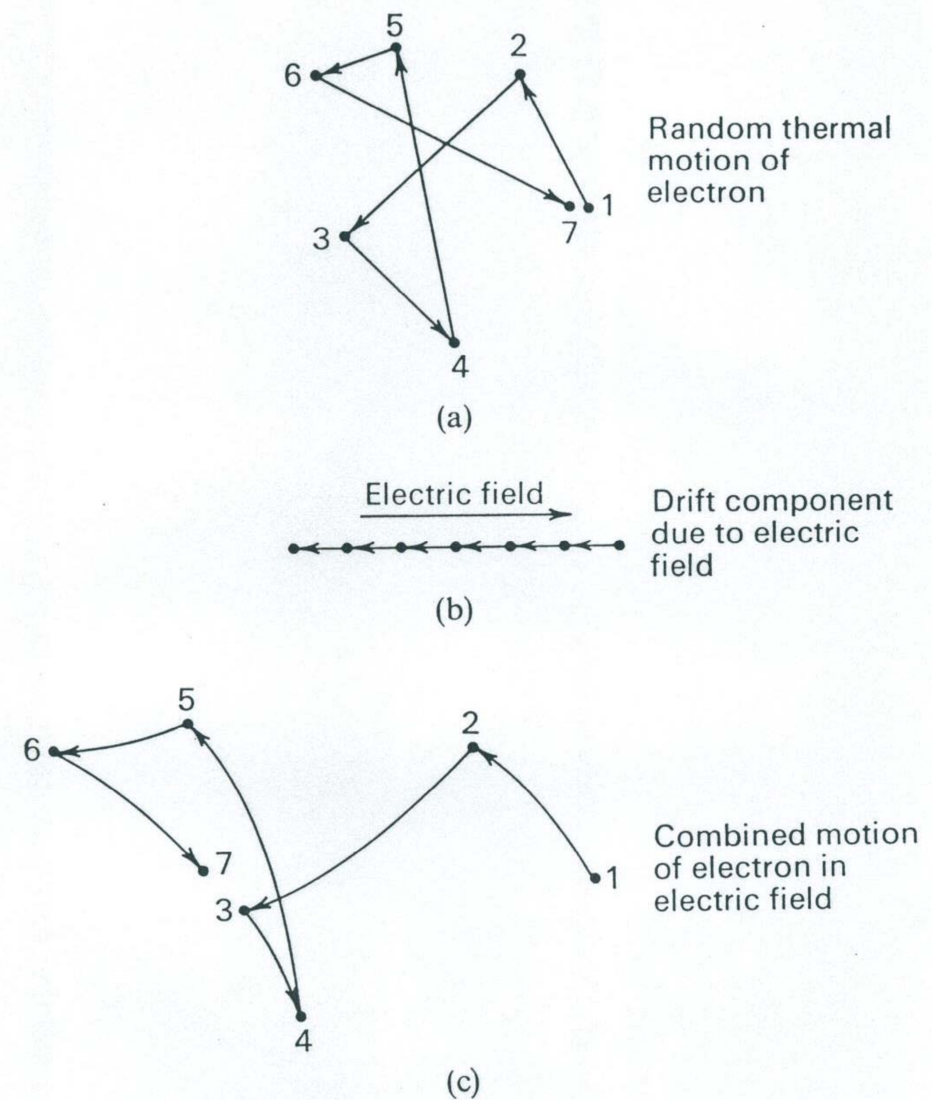
Dopant Solubility in Silicon



Carrier Concentration vs. Temperature



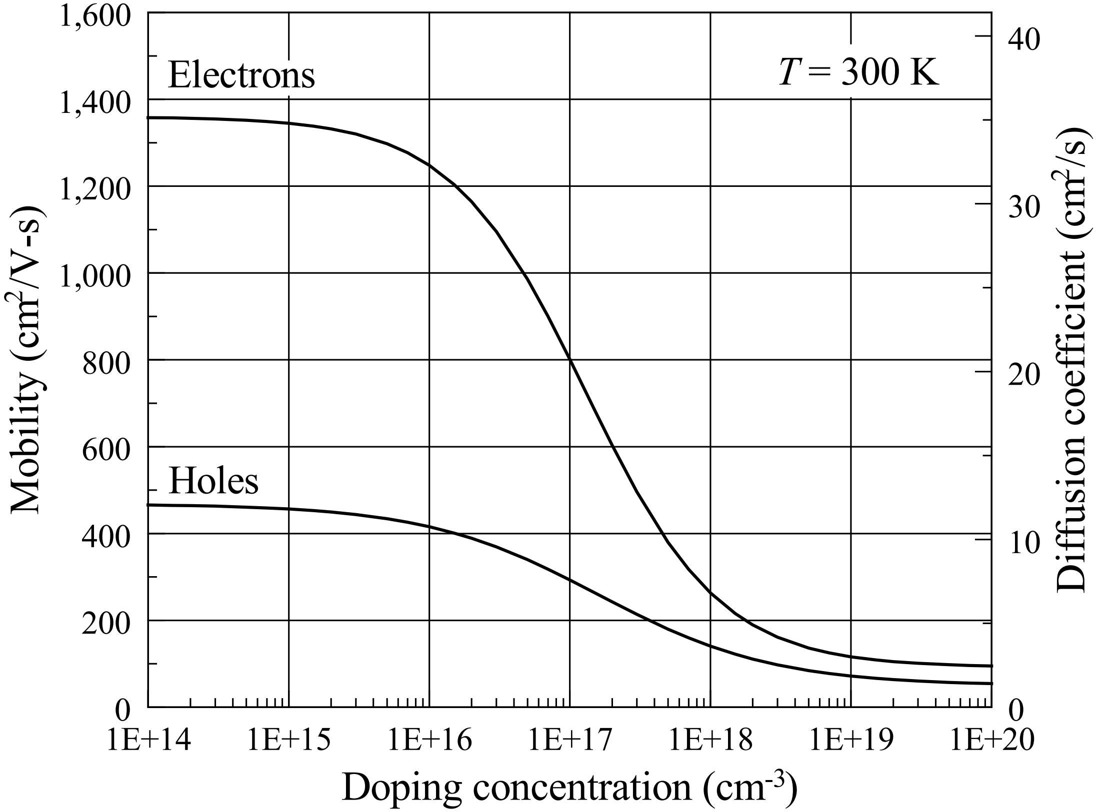
Carrier Transport: Drift



$$v_d = -q\mathcal{E}\tau / m^*$$

Mobility

$$v_d = \mu \mathcal{E}$$

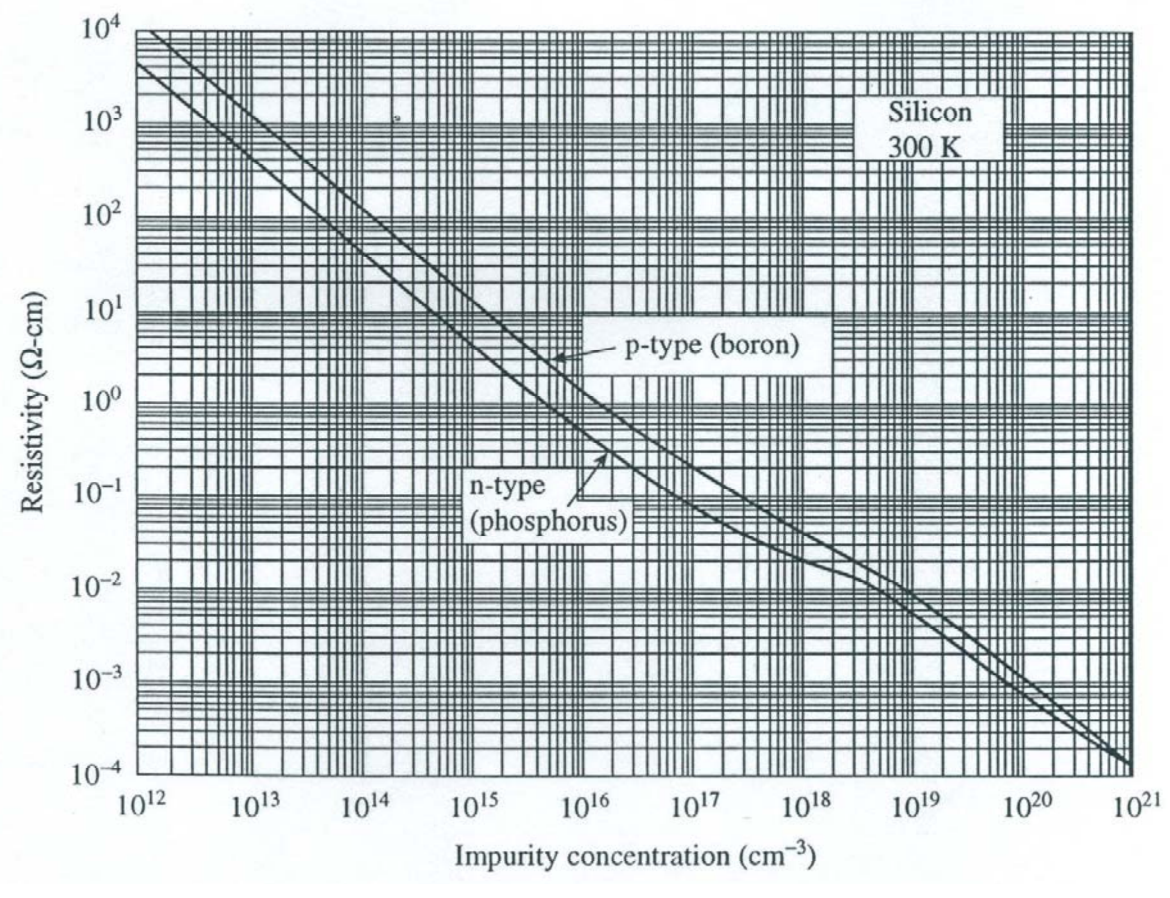


$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_I} + \cdots$$

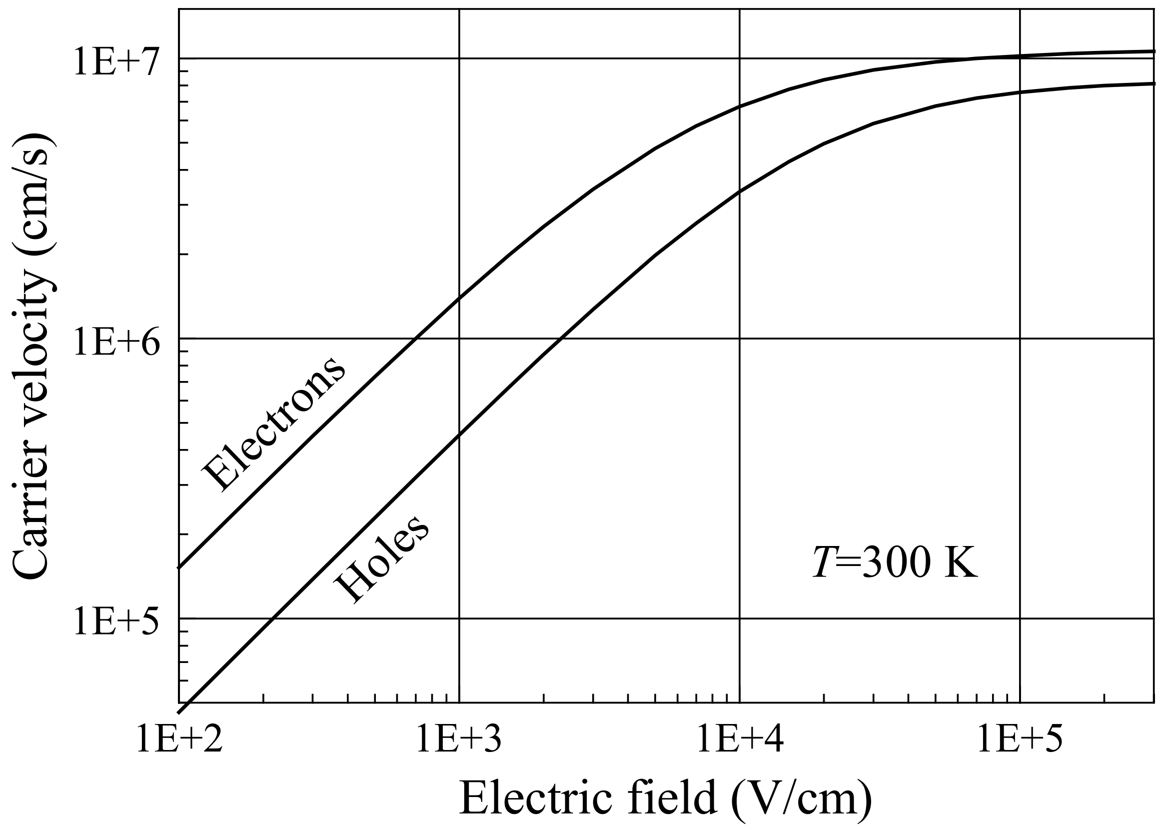
Resistivity of Silicon

$$J_{n,drift} = qnv_d = qn\mu_n\mathcal{E}$$

$$\rho_n = \frac{1}{qn\mu_n}$$

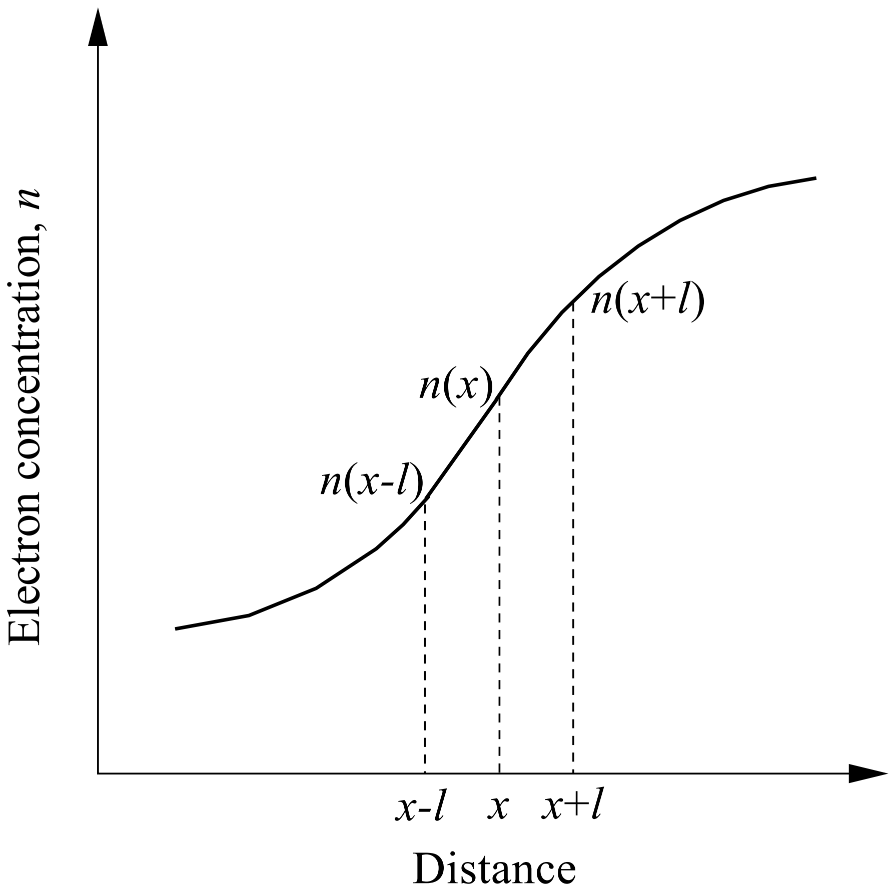


Velocity Saturation



$$v_{sat} \approx 10^7 \text{ cm/s}$$

Carrier Transport: Diffusion



$$J_{n,diff} = qD_n \frac{dn}{dx}$$

$$D_n = \frac{kT}{q}\mu_n$$

$$D_p = \frac{kT}{q}\mu_p$$

Poisson's Equation

Define intrinsic potential: $$\psi_i = -\frac{E_i}{q}$$

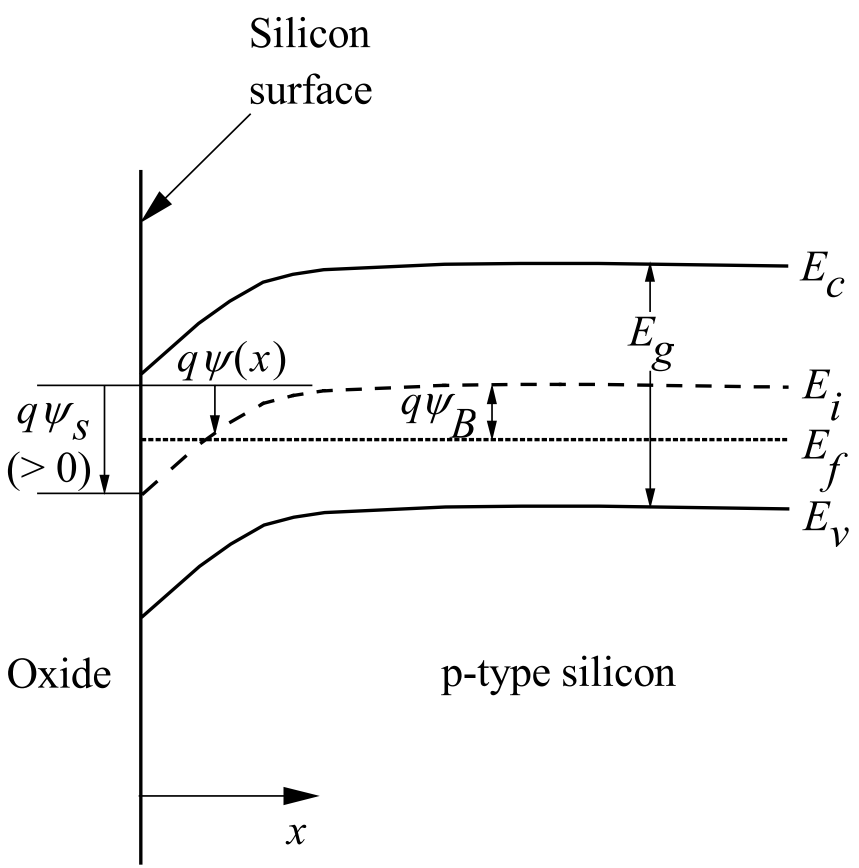
Electric field: $$\mathcal{E} = -\frac{d\psi}{dx}$$

Poisson's eq.: $$\frac{d^2\psi}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{\rho(x)}{\varepsilon_{si}}$$

Or, $$\boxed{\frac{d^2\psi}{dx^2} = -\frac{d\mathcal{E}}{dx} = -\frac{q}{\varepsilon_{si}}\left[p(x) - n(x) + N_d(x) - N_a(x)\right]}$$

Gauss's law: $$\mathcal{E} = \frac{1}{\varepsilon_{si}}\int \rho(x)dx = \frac{Q_s}{\varepsilon_{si}}$$

Carrier Concentration in terms of Electrostatic Potential

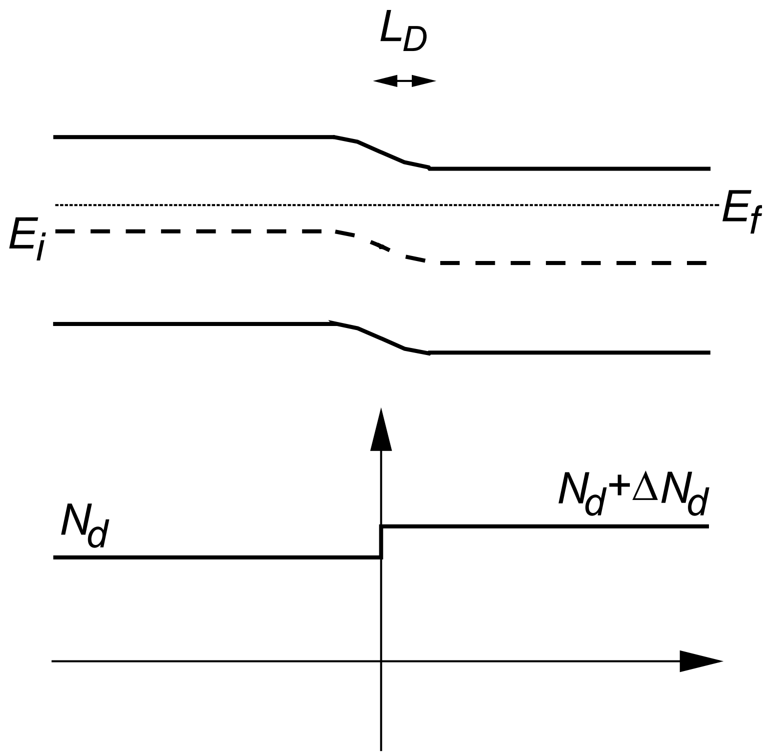


$$\psi_B \equiv \left|\psi_f - \psi_i\right| = \frac{kT}{q}\ln\left(\frac{N_b}{n_i}\right)$$

$$n = n_i e^{(E_f - E_i)/kT} = n_i e^{q(\psi_i - \psi_f)/kT}$$

$$p = n_i e^{(E_i - E_f)/kT} = n_i e^{q(\psi_f - \psi_i)/kT}$$

Debye Length



$$\frac{d^2\psi_i}{dx^2} = -\frac{q}{\varepsilon_{si}}\left[N_d(x) - n_i e^{q(\psi_i-\psi_f)/kT}\right]$$

$$\frac{d^2(\Delta\psi_i)}{dx^2} - \frac{q^2 N_d}{\varepsilon_{si} kT}(\Delta\psi_i) = -\frac{q}{\varepsilon_{si}}\Delta N_d(x)$$

Solution: $\Delta\psi_i \sim \exp(-x/L_D)$
where

$$L_D \equiv \sqrt{\frac{\varepsilon_{si} kT}{q^2 N_d}}$$

Current Density Equations

Combining drift and diffusion components,

$$J_n = qn\mu_n\mathcal{E} + qD_n\frac{dn}{dx}$$

$$J_p = qp\mu_p\mathcal{E} - qD_p\frac{dp}{dx}$$

Or,

$$J_n = -qn\mu_n\left(\frac{d\psi_i}{dx} - \frac{kT}{qn}\frac{dn}{dx}\right) = -qn\mu_n\frac{d\phi_n}{dx}$$

$$J_p = -qp\mu_p\left(\frac{d\psi_i}{dx} + \frac{kT}{qp}\frac{dp}{dx}\right) = -qp\mu_p\frac{d\phi_p}{dx}$$

where

$$\phi_n \equiv \psi_i - \frac{kT}{q}\ln\left(\frac{n}{n_i}\right)$$

$$\phi_p \equiv \psi_i + \frac{kT}{q}\ln\left(\frac{p}{n_i}\right)$$

are the quasi-Fermi potentials.

Current Continuity Equations

From conservation of mobile charge,

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x} - R_n + G_n$$

$$\frac{\partial p}{\partial t} = -\frac{1}{q}\frac{\partial J_p}{\partial x} - R_p + G_p$$

where G_n, G_p, R_n, R_p are electron-hole generation and recombination rates.

In the steady state with negligible electron-hole generation and recombination rates,

$$dJ_n/dx = 0$$
$$dJ_p/dx = 0$$

Simply continuity of electron and hole currents.

Dielectric Relaxation Time

Neglect R_n, G_n in the current continuity equation,

$$\frac{\partial n}{\partial t} = \frac{1}{q}\frac{\partial J_n}{\partial x}$$

From Ohm's law, $J_n = \mathcal{E}/\rho_n$

And from Poisson's equation with majority carrier density n, $\partial\mathcal{E}/\partial x = -qn/\varepsilon_{si}$

Substituting into current continuity equation:

$$\frac{\partial n}{\partial t} = -\frac{n}{\rho_n \varepsilon_{si}}$$

The solution is of the form $n(t) \propto \exp(-t/\rho_n\varepsilon_{si})$ where $\rho_n\varepsilon_{si} \approx 10^{-12}$ s is the majority carrier response time.