For 1-4 below, your explanation of why should include an appropriate mathematical equation substantiating your answer.

1. If you want to build a diode that has an ideality factor of 1 at room temperature, would you choose Si, Ge, or GaAs? Why?

Ge.
The forward bias current can be written as the sum of the diffusion current and the recombination current.

$$J_{For} = q\left[\frac{D_p}{L_p}\frac{1}{N_D} + \frac{D_n}{L_n}\frac{1}{N_A}\right]n_i^2 e^{qV_A/kT} + \frac{qn_iW}{2\tau}e^{qV_A/2kT} \tag{1}$$

The diffusion current is proportional to n_i^2 and the recombination current is proportional to n_i. If we want the diffusion current to dominate (since it has an ideality factor of one) we want the material with the largest n_i so that the n_i^2 term dominates. Since $n_i^2 = N_C N_V e^{-E_G/kT}$, we want the material with the smallest bandgap, Ge.

2. For a diode with n=1 biased under forward bias at a fixed voltage, $V_A > 3kT$, will the current increase or decrease as the temperature is increased? Write down an equation that gives the dominant temperature dependence.

For n=1, the current is given by the diffusion current.

$$J_{For} = q\left[\frac{D_p}{L_p}\frac{1}{N_D} + \frac{D_n}{L_n}\frac{1}{N_A}\right]n_i^2 e^{qV_A/kT}$$
$$J_{For} = q\left[\frac{D_p}{L_p}\frac{1}{N_D} + \frac{D_n}{L_n}\frac{1}{N_A}\right]N_C N_V e^{-E_G/kT} e^{qV_A/kT} \tag{2}$$

$N_C N_V \propto T^3$, but the dominant temperature dependence is in the exponential.
$J_{For} \propto e^{-(E_G - qV_A)/kT}$. Therefore, since $E_G > qV_A$, as T increases, the current increases for a fixed V_A.

3. Under low forward bias the ideality factor of a Si diode is n=2. Explain why.

Under low forward bias,the current is dominated by the recombination current, the second term on the right of Eq. (1) above, which has an ideality factor of 2. The exponential dependence of $e^{qV_A/2kT}$ comes from the expression for R_{max} in the depletion region which occurs when n=p. When n=p,

$$n = n_i e^{(F_n - E_i)/kT} = n_i e^{(E_i - F_p)/kT} \tag{3}$$

Multiplying the 2 terms on the right together and taking the square root gives

$$n = p = n_i e^{qV_A/2kT} \tag{4}$$

using F_n-F_p = qV_A. The expression for the recombination rate

$$R = \frac{n_i^2\left(e^{qV_A/kT}-1\right)}{\tau_p\left(n+n_i e^{(E_T-E_i)/kT}\right)+\tau_n\left(p+n_i e^{(E_i-E_T)/kT}\right)} \tag{5}$$

becomes, assuming E_T=E_i, V_A > $3kT/q$, and $\tau_n = \tau_p$,

$$R_{\max} = \frac{n_i}{2\tau} e^{qV_A/2kT} \tag{6}$$

When then approximate the recombination current as

$$J_{rec} = \int_0^W dxqR \approx qR_{\max}W = \frac{qWn_i}{2\tau} e^{qV_A/2kT} \tag{7}$$

giving the ideality factor of 2.

4. Under very high forward bias the ideality factor of a Si diode is also n=2. Explain why.

Under high level injection, one can reach a condition where

$$n_p(x_p) = p_p(x_p) \tag{8}$$

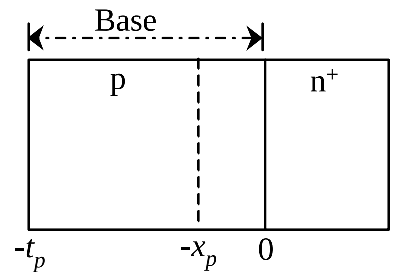
Then, from

$$np = n_i^2 e^{qV_A/kT}$$

$$n_p(x_p) = n_i\, e^{qV_A/2kT}$$

resulting in the ideality factor of 2.

5. For an n^+-p short base diode shown at right, assume that $\Delta n_p(-t_p) = 0$ (see Sec. 2.2.4 of Taur).



(a) Derive $\Delta n_p(x)$.

$$\Delta n_p(x) = Ae^{-x/L_n} + Be^{x/L_n}$$

$$\Delta n_p(-t_p) = 0$$

$$\Delta n_p(-x_p) = n_{po}\left(e^{qV_A/k_BT} - 1\right)$$

$$\Delta n_p(x) = \Delta n_p(-x_p)\frac{\sinh\left[\left(x + t_p\right)/L_n\right]}{\sinh\left[\left(-x_p + t_p\right)/L_n\right]}$$

(b) Assume that t_p-x_p << L_n. Derive a linear expression for $\Delta n_p(x)$.

Using $\sinh(\delta) \approx \delta$ for $\delta << 1$,

$$\Delta n_p(x) = \Delta n_p(-x_p)\frac{\left(x + t_p\right)}{\left(-x_p + t_p\right)}$$

Physically, a linear expression like this means that there is no recombination in the p region. It all occurs at the left edge, $-t_p$.

(c) Define a velocity v(x) by the following expression, $J_n(x) = -q\, v(x)\, \Delta n_p(x)$. Derive an expression for v(x).

$$J_n = qD_n \frac{d}{dx}\Delta n_p(x)\,|_{-x_p} = \frac{qD_n}{t_p - x_p}\Delta n_p(-x_p)$$

$$v(x) = \frac{J_n}{-q\Delta n_p(x)} = \frac{-D_n}{t_p - x_p}\Delta n_p(-x_p)\frac{1}{\Delta n_p(-x_p)}\frac{\left(t_p - x_p\right)}{\left(x + t_p\right)} = \frac{-D_n}{x + t_p}$$

(d) The average time to traverse the base is called the base transit time given by

$$\tau_B = \int_{-t_p}^{-x_p} \frac{dx}{v(x)}$$

Derive τ_B.

$$\tau_B = \int_{-t_p}^{-x_p} dx \frac{x + t_p}{D_n} = \frac{\left(t_p - x_p\right)^2}{2D_n} \approx \frac{\left(t_p - W\right)^2}{2D_n}$$ where I have replaced x_p by the depletion width W since this is an n^+p junction and the depletion region lies largely in the p region.

(e) Show that the current is given by $J_n = q\Delta N_p/\tau_B$ where ΔN_p is given by

$$\Delta N_p = \int_{-t_p}^{-x_p} dx \Delta n_p(x)$$

$$\Delta N_p = \int_{-t_p}^{-W} dx \frac{\Delta n_p(-W)}{t_p - W}(x + t_p) = \frac{\Delta n_p(-W)}{t_p - W}\frac{(t_p - W)^2}{2} = \frac{1}{2}\Delta n_p(-W)(t_p - W)$$

$$\frac{q\Delta N_p}{\tau_B} = \frac{q}{2}\Delta n_p(-W)(t_p - W)\frac{2D_n}{(t_p - W)^2} = \frac{qD_n}{(t_p - W)}\Delta n_p(-W) = J_n$$

(f) For V_A = 0.7V, $N_D = 10^{18}/\text{cm}^3$, $N_A = 10^{16}/\text{cm}^3$, t_p-x_p=0.1 μm, T=300K, and μ_n = 1500 cm^2/Vs, calculate J_n

$$D_n = \frac{k_B T}{q}\mu_n = 0.026(1500) = 39\left(\text{cm}^2/\text{s}\right)$$
.
$$J_n = q\frac{39}{10^{-5}}\frac{n_i^2}{10^{16}}\left(e^{0.7/0.026} - 1\right) = 6.47\left(\frac{\text{kA}}{\text{cm}^2}\right) \approx J$$

for an n^+p junction. $n_i = 1.45\text{e}10/\text{cm}^3$ was used.

(g) Compare this value to the current for the above diode with t_p-x_p >> L_n and τ_n = 1ns.

Replace t_p-W with L_n in (e).
$L_n = \sqrt{D_n \tau_n} = 1.975 \times 10^{-4}$ (cm) .
So
$J_n = J_n$(short base) / 19.75 = 328 A/cm^2.

(h) Considering the linear expression for $\Delta n_p(x)$ from part (b), derive, as we did in class, an expression for the ac conductance and diffusion capacitance under forward bias. What is the frequency dependence?

Using standard linear response approach, we let all physical quantities take the form
$V(t) = V_0 + V_1 e^{j\omega t}$
$J(t) = J_0 + J_1 e^{j\omega t}$
$\Delta n(t) = \Delta n_0 + \Delta n_1 e^{j\omega t}$
etc.

where the amplitudes V_1, J_1, and Δn_1 are complex quantities. We write down the minority carrier diffusion equation, group all of the terms proportional to $e^{j\omega t}$ together and cancel the $e^{j\omega t}$ terms leaving the ac minority carrier diffusion equation for Δn_1.

$$j\omega\Delta n_1 = D_n \frac{\partial \Delta n_1}{\partial x^2} - \frac{\Delta n_1}{\tau_n} \text{ or}$$

$$0 = D_n \frac{\partial \Delta n_1}{\partial x^2} - \frac{\Delta n_1}{\tilde{\tau}_n} \text{ where } \tilde{\tau}_n = \frac{\tau_n}{1 + j\omega\tau_n}.$$

Now, the solution for Δn_1 is the usual expression for Δn with the replacement everywhere of $\tilde{\tau}_n$ for τ_n. However, in the linear approximation of (b), the L_n and thus the $\tilde{\tau}_n$ cancels leaving

$$\Delta n_1(x) = \Delta n_1(-x_p)\frac{(x+t_p)}{(-x_p+t_p)}.$$

$$\Delta n(-x_p) = n_{po}\left(e^{qV(t)/k_BT} - 1\right) = n_{po}\left(e^{q\left(V_0+V_1e^{j\omega t}\right)/k_BT} - 1\right) = n_{po}\left(e^{qV_0/k_BT}e^{qV_1e^{j\omega t}/k_BT} - 1\right)$$

Under forward bias, $e^{V_0/k_BT} >> 1$, so we ignore the 1 in the parenthesis. Also, since this is linear response, $qV_1/k_BT << 1$, so we have

$$\Delta n(-x_p) = n_{po}e^{qV_0/k_BT}e^{qV_1e^{j\omega t}/k_BT} = n_{po}e^{qV_0/k_BT} + n_{po}e^{qV_0/k_BT}\frac{V_1}{k_BT/q}e^{j\omega t}$$

$$\Delta n(-x_p) = \Delta n_{DC} + \Delta n_{DC}\frac{V_1}{k_BT/q}e^{j\omega t}$$

From this we see that

$$\Delta n_1 = \Delta n_{DC}\frac{V_1}{k_BT/q}$$

The ac current amplitude is thus

$$J_1 = qD_n\frac{d}{dx}\Delta n_1(x)|_{-x_p} = \frac{qD_n}{t_p - x_p}\Delta n_1(-x_p) = J_{DC}\frac{V_1}{k_BT/q}.$$

The ac admittance is $J_1/V_1 = J_{DC}/(k_BT/q)$. This quantity is purely real. It is the ac conductance. The susceptance is zero and therefore the diffusion capacitance is zero (at least due to the electrons – there will be some contribution due to the holes which we are ignoring in the n$^+$p juncition). There is no frequency dependence in this approximation.