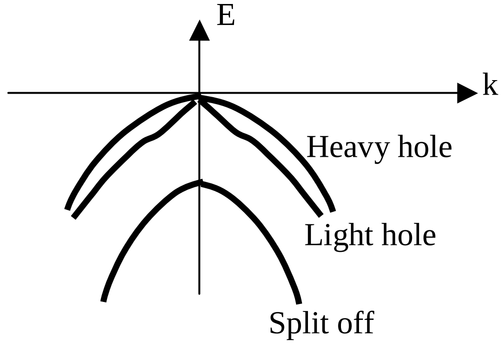
EE 203. HW3 Due 5/12

1. In Si
a. (5 pts) How many equivalent conduction band valleys are there? 6
b. (5 pts)Where are they? At 0.85 $2\pi/a$ in the equivalent <100> directions. These are referred to as the X valleys, because they are near the X point, and also the $\Delta$ valleys since they lie along the $\Delta$ line which is any of the equivalent <100> lines.

2. In Ge,
a. (5 pts)How many equivalent conduction band valleys are there? 4
b. (5 pts)Where are they? Centered at the L point which is at the Brillouin zone edge along the equivalent <111> directions. They are referred to as the L valleys. These valleys are approximately 100 meV below the $\Gamma$ valley of Ge, so Ge is "almost" direct.

3. In GaAs,
a. (5 pts)How many equivalent conduction band valleys are there? 1
b. (5 pts)Where are they? At $\Gamma$.

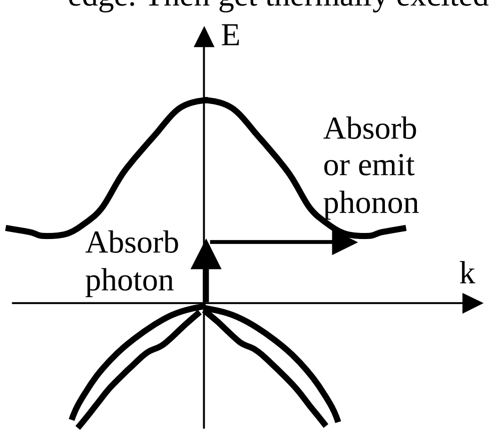
4. Describe the valence band structure of semiconductors.
a. (5 pts)Where does the maximum occur? At $\Gamma$.
b. (5 pts)Sketch the bands.

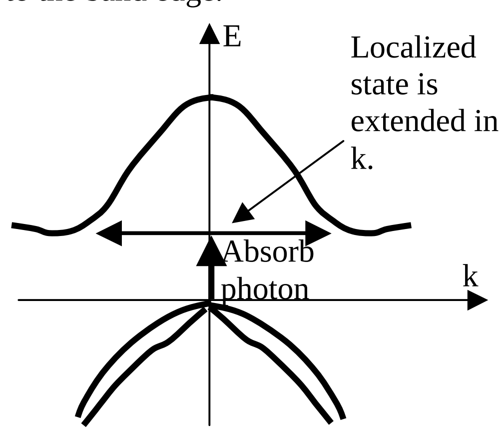



5. (10 pts) Describe the process(es) by which an electron at the valence band maximum in Si could absorb a photon equal to the bandgap. Illustrate the process(es) on a sketch of the E-k diagrams.

Note that neither of these processes correspond to the absorption of a photon of exactly $E_G$, but close.

1. Absorb a photon and then absorb or emit a <100> phonon to pick up the momentum to get out to one of the $\Delta$ valleys at 0.85 $2\pi/a$ <100>.
2. Absorb a photon to end up in a localized state close in energy to the conduction band edge. Then get thermally excited or relaxed to the band edge.



6. (10 pts)Derive the following expressions for **degenerate** statistics. You will need to use the

$$J_p = \mu_p p \vec{\nabla} F_p$$
$$J_n = \mu_n n \vec{\nabla} F_n$$

properties of the $\Im(\eta)$ function described in class.

First, derive the Einstein relation for degenerate statistics starting with the current equation for electrons in equilibrium

$$J_n = q\mu_n nE + qD_n \nabla n = 0 \tag{1}$$

and the equation for n

$$n = N_c \Im_{1/2}(\eta). \tag{2}$$

to give

$$\mu_n EN_c \Im_{1/2}(\eta) = -D_n \nabla N_c \Im_{1/2}(\eta) \tag{3}$$

where

$$\eta = \frac{E_F - E_c}{k_B T}. \tag{4}$$

The $N_c$'s in (3) cancel giving

$$\mu_n E \Im_{1/2}(\eta) = -D_n \Im_{-1/2}(\eta)\left(\frac{1}{k_B T}\right)(\nabla E_F - \nabla E_c) \tag{5}$$

$\nabla E_F = 0$ since $E_F$ is a constant in equilibrium and $\nabla E_c = qE$ where $E$ is the electric field. This cancels the electric field on the left hand side of (5) giving

$$\mu_n \Im_{1/2}(\eta) = D_n \Im_{-1/2}(\eta)\left(\frac{q}{k_B T}\right) \tag{6}$$

which gives the Einstein relation for degenerate statistics

$$D_n = \left(\frac{k_B T}{q}\right)\mu_n \frac{\Im_{1/2}(\eta)}{\Im_{-1/2}(\eta)} \tag{7}$$

Now we go back to the current equation (1) using (2) and (7) to get

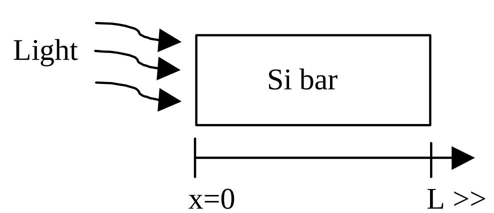
$$\begin{aligned} J_n &= q\mu_n EN_c \Im_{1/2}(\eta) + qD_n N_c \Im_{-1/2}(\eta)\left(\frac{1}{k_B T}\right)(\nabla F_n - \nabla E_c) \\ J_n &= q\mu_n EN_c \Im_{1/2}(\eta) + q\left(\frac{k_B T}{q}\right)\mu_n \frac{\Im_{1/2}(\eta)}{\Im_{-1/2}(\eta)} N_c \Im_{-1/2}(\eta)\left(\frac{1}{k_B T}\right)(\nabla F_n - \nabla E_c) \end{aligned} \tag{8}$$

As before, the $\nabla E_c$ term on the right cancels the electric field term on the left leaving

$$J_n = \mu_n n \nabla F_n. \tag{9}$$

The equation for holes is proved similarly.

7. (10 pts) A Si bar has the following properties: $N_A = 10^{15}/cm^3$, $\mu_n = 1350\ cm^2/Vs$, $\mu_p = 500\ cm^2/Vs$, $\tau_n = \tau_p = 10^{-6}s$. The left end of the bar is illuminated so as to create $10^{10}/cm^3$ excess electron hole pairs at x=0. Assuming none of the light penetrates into the interior of the bar (x>0),
Determine the excess minority carrier profile.



This is the case of steady-state with no light (since no light penetrates beyond x=0). This is case (1) of the notes:

$$0 = D_n \frac{\partial^2 \Delta n_p}{\partial x^2} - \frac{\Delta n_p}{\tau_n} \tag{1}$$

The general solution is

$$\Delta n_p(x) = Ae^{-x/L_n} + Be^{x/L_n} \tag{2}$$

where

$$L_n = \sqrt{D_n \tau_n} \tag{3}$$

At $x = \infty$, $\Delta n_p = 0$ ➔ $B = 0$. At $x$=0, $\Delta n_p = 10^{10}$ ➔ $A = 10^{10}$. Therefore,

$$\Delta n_p(x) = 10^{10} e^{-x/L_n} \text{ (cm}^{-3}\text{)} \tag{4}$$

where $L_n = \sqrt{D_n \tau_n} = \sqrt{\frac{k_B T}{q} \mu_n 10^{-6}} = 59.2\,\mu\text{m}$.

8. (10 pts) Assume that now, L < $L_n$, and, that at the right surface, the surface recombination rate is so large that Δn = Δp = 0. Determine the excess minority carrier profile.

The general form of the solution is the same as above

$$\Delta n_p(x) = Ae^{-x/L_n} + Be^{x/L_n}\,. \tag{1}$$

But now, the boundary conditions give at $x$=0

$$A + B = 10^{10} \tag{2}$$

and at $x=L$

$$Ae^{-L/L_n} + Be^{L/L_n} = 0\,. \tag{3}$$

Using (2) in (3) gives

$$A = \frac{10^{10} e^{L/L_n}}{e^{L/L_n} - e^{-L/L_n}}\,, \tag{4}$$

and putting (4) back into (2) gives

$$B = \frac{-10^{10} e^{-L/L_n}}{e^{L/L_n} - e^{-L/L_n}}\,. \tag{5}$$

Thus, the solution (1) is

$$\Delta n_p(x) = 10^{10} \frac{\sinh\left(\frac{L - x}{L_n}\right)}{\sinh\left(\frac{L}{L_n}\right)} \left(\text{cm}^{-3}\right). \tag{6}$$

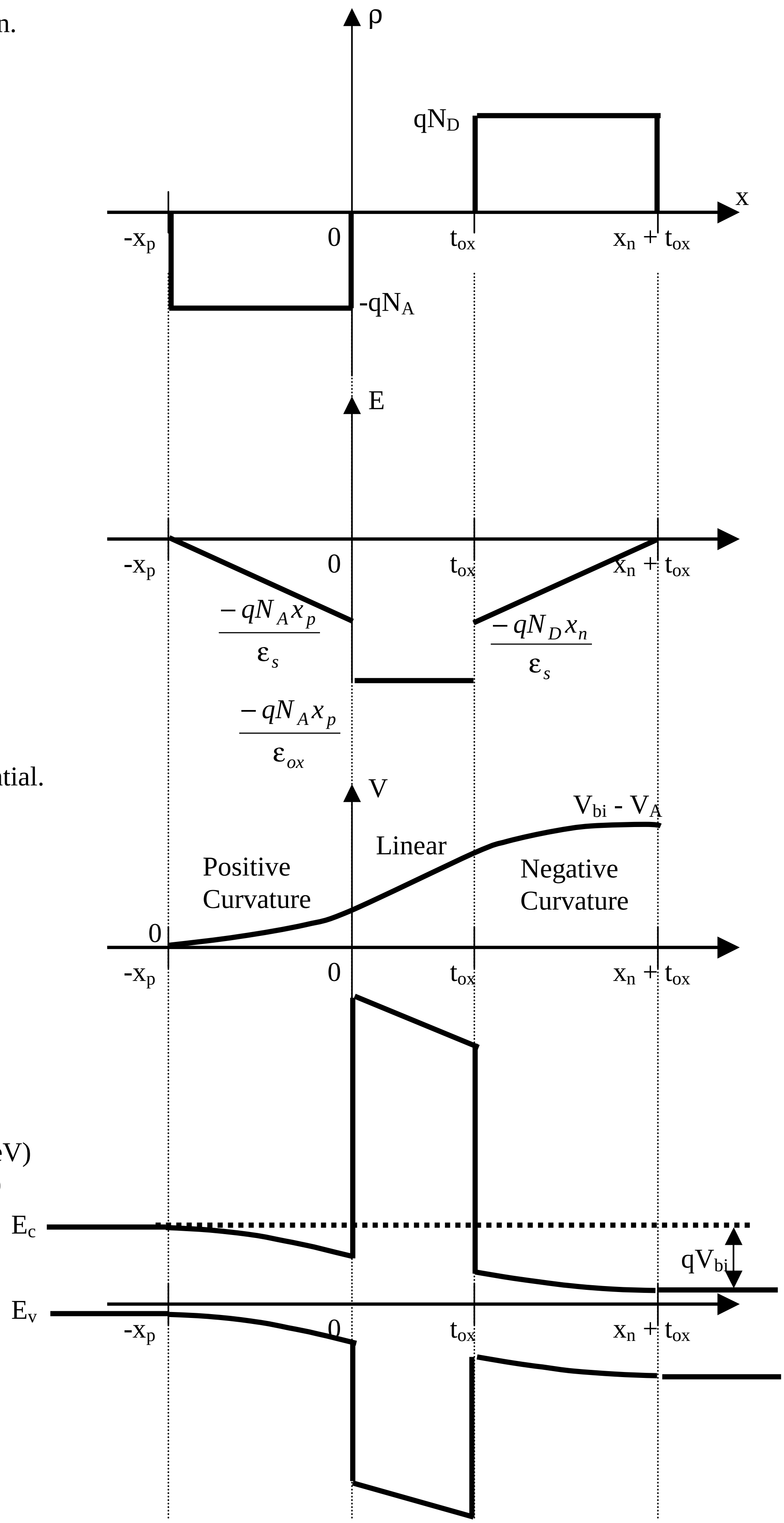
9. Consider a p-type ($N_A = 10^{18}$ cm$^{-3}$) Si / 10 nm $SiO_2$ / n-type Si ($N_D = 10^{18}$ cm$^{-3}$) structure at 0 bias and T=300K, i.e. a pn junction with a 10 nm oxide between the n and p regions. Using the depletion approximation as we did for the pn diode.

a. (10 pts) Sketch the charge distribution.
Label maximum and minimum points.

b. (10 pts) Sketch the electric field.
($\varepsilon_{Si} = 11.9$ & $\varepsilon_{SiO_2} = 3.9$)
Label max and min points.

c. (10 pts) Sketch the electrostatic potential.
Label max and min points.

d. (10 pts) Sketch the band diagram.
($E_G$ $SiO_2$ = 9 eV. Ec($SiO_2$) – Ec(Si) = 3eV)
(you will not be able to draw it to scale)



e. (10 pts) Calculate the built in voltage, $V_{bi}$.(A number in volts).
$V_{bi}$ is given by the usual expression

$$V_{bi} = \frac{k_B T}{q} \ln\left(\frac{N_D N_A}{n_i^2}\right) = 938\,\text{mV} \tag{1}$$

f. (50 pts) Calculate the total depletion width, $W = x_p + t_{ox} + x_n$. (a number in nm). You first have to derive the appropriate expression for W. No credit will be given for simply taking the W derived for a pn or pin junction. Pay attention to the 2 different dielectric constants.

(g) (30 pts) Calculate the capacitance in (F/cm$^2$).

Questions (f) and (g) are answered below.

Evaluating $V(t_{ox})$ by integrating the electric field from $-x_p$ to $t_{ox}$ we get

$$V(t_{ox}) = -\int_{-x_p}^{t_{ox}} dx E(x) = \frac{qN_A x_p^2}{2\varepsilon_s} + \frac{qN_A x_p t_{ox}}{\varepsilon_{ox}} \tag{1}$$

Evaluating $V(t_{ox})$ by integrating from $t_{ox}$ to $x_n$, we get

$$-\int_{t_{ox}}^{x_n} dx E(x) = (V_{bi} - V_A) - V(t_{ox}) = \frac{qN_D x_n^2}{2\varepsilon_s} \tag{2}$$

or

$$V(t_{ox}) = (V_{bi} - V_A) - \frac{qN_D x_n^2}{2\varepsilon_s} \tag{3}$$

Charge neutrality also gives the usual relation

$$N_A x_p = N_D x_n \tag{4}$$

Setting the two expressions (1) and (3) for $V(t_{ox})$ equal and using (4) gives

$$\frac{qN_A}{2\varepsilon_s}\left(1 + \frac{N_A}{N_D}\right)x_p^2 + \frac{qN_A t_{ox}}{\varepsilon_{ox}} x_p - (V_{bi} - V_A) = 0 \tag{5}$$

or upon rearranging

$$x_p^2 + \frac{2\varepsilon_s t_{ox}}{\varepsilon_{ox}\left(1 + \frac{N_A}{N_D}\right)} x_p - \frac{2\varepsilon_s (V_{bi} - V_A)}{qN_A\left(1 + \frac{N_A}{N_D}\right)} = 0 \tag{6}$$

Solving the quadratic equation gives

$$x_p = \frac{\varepsilon_s t_{ox} N_D}{\varepsilon_{ox}(N_A + N_D)}\left[-1 + \sqrt{1 + \frac{2\varepsilon_{ox}^2 (V_{bi} - V_A)(N_A + N_D)}{q t_{ox}^2 \varepsilon_s N_A N_D}}\right] \tag{7}$$

To obtain $x_n$, we switch everywhere $N_D$ and $N_A$. The sum $x_n + x_p$ is

$$x_n + x_p = -\frac{\varepsilon_s t_{ox}}{\varepsilon_{ox}} + \sqrt{\left(\frac{\varepsilon_s t_{ox}}{\varepsilon_{ox}}\right)^2 + \frac{2\varepsilon_s (V_{bi} - V_A)(N_A + N_D)}{qN_A N_D}} \tag{8}$$

The term under the radical on the right is the expression for the depletion width squared for a regular abrupt pn junction, $W^2$. If we move the first term on the right of (8) to the left and divide through by $\varepsilon_s$, we get

$$\frac{x_n + x_p}{\varepsilon_s} + \frac{t_{ox}}{\varepsilon_{ox}} = \sqrt{\left(\frac{t_{ox}}{\varepsilon_{ox}}\right)^2 + \left(\frac{W}{\varepsilon_s}\right)^2} \tag{9}$$

We would expect this to be one over the capacitance of the junction since we would guess the capacitance to be

$$C_j = \left(\frac{1}{C_s} + \frac{1}{C_{ox}}\right)^{-1} \tag{10}$$

where $C_s = \varepsilon_s / (x_n + x_p)$ and $C_{ox} = \varepsilon_{ox} / t_{ox}$. We will now prove that this is true.

We start from the definition of the capacitance that we used in the lectures.

$$C_j = \left|\frac{\partial Q}{\partial V_A}\right| = qN_A \left|\frac{\partial x_p}{\partial V_A}\right| \tag{11}$$

Multiplying (7) by $qN_A$ and taking the derivative, we get

$$C_j = \frac{q\varepsilon_s t_{ox} N_A N_D}{\varepsilon_{ox}(N_A + N_D)} \frac{1}{2} \frac{1}{\sqrt{1 + \frac{2\varepsilon_{ox}^2 (V_{bi} - V_A)(N_A + N_D)}{q t_{ox}^2 \varepsilon_s N_A N_D}}} \frac{2\varepsilon_{ox}^2 (N_A + N_D)}{q t_{ox}^2 \varepsilon_s N_A N_D}$$

which after canceling most of the terms outside the radical becomes

$$C_j = \frac{1}{\sqrt{\left(\frac{t_{ox}}{\varepsilon_{ox}}\right)^2 + \frac{2(V_{bi} - V_A)(N_A + N_D)}{q\varepsilon_s N_A N_D}}} = \frac{1}{\sqrt{\left(\frac{t_{ox}}{\varepsilon_{ox}}\right)^2 + \left(\frac{W}{\varepsilon_s}\right)^2}} \tag{12}$$

as claimed in (9) and (10).

Finally, we are asked to evaluate $x_n + x_p + t_{ox}$. From Eq. (8), I get $x_n + x_p + t_{ox}$ = 37.8 nm. For the capacitance, from (9), (10), and the value that I just calculated for $x_n + x_p$, I get $C_j$ = 0.181 μF/cm$^2$.