HW 2 Solutions

1. $$E_i = \frac{3kT}{4} \ln\left(\frac{m^*_{dh}}{m^*_{de}}\right) + \underbrace{\frac{E_v + E_c}{2}}_{\text{midgap}}$$

$$= \frac{3}{4}(0.02585) \ln\left(\frac{0.5492}{1.084}\right) + \text{midgap}$$

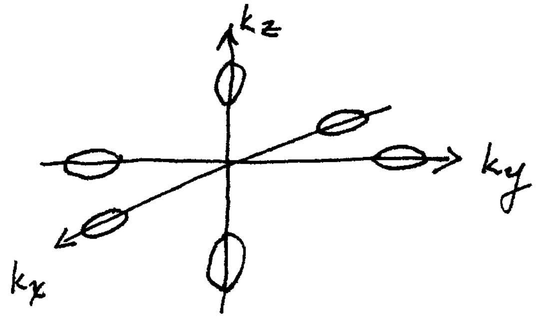
$$E_i = \text{midgap} - 13.18\,\text{meV}$$

2. a) Ga has 3 valence electrons. Si has 4.
   Donor

   b) $E_{d\,Si} = E_c - 5.8\,\text{meV}$.

   c). Acceptor. $E_{A\,Si} = E_v + 38\,\text{meV}$.

3) The constant energy surfaces of the conduction band in Si are ellipsoids of revolution about the 6 principal axes.



The acceleration vector in the 2 k_x valleys is

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 1/m_l & 0 & 0 \\ 0 & 1/m_t & 0 \\ 0 & 0 & 1/m_t \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad (2\ k_x \text{ valleys})$$

Similarly, for the k_y & k_z valleys

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 1/m_t & 0 & 0 \\ 0 & 1/m_l & 0 \\ 0 & 0 & 1/m_t \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad (2\ k_y \text{ valleys})$$

$$\begin{bmatrix} \dot{v}_x \\ \dot{v}_y \\ \dot{v}_z \end{bmatrix} = \begin{bmatrix} 1/m_t & 0 & 0 \\ 0 & 1/m_t & 0 \\ 0 & 0 & 1/m_l \end{bmatrix} \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix} \qquad (2\ k_z \text{ valleys})$$

The average electron acceleration due to the response of all 6 valleys is

$$\langle \dot{v} \rangle = \frac{1}{6} \sum_{i=1}^{6} \dot{v}_i$$

Thus

$$\begin{bmatrix} \langle \dot{v}_x \rangle \\ \langle \dot{v}_y \rangle \\ \langle \dot{v}_z \rangle \end{bmatrix} = \begin{bmatrix} 2/m_t + 1/m_l & 0 & 0 \\ 0 & 2/m_t + 1/m_l & 0 \\ 0 & 0 & 2/m_t + 1/m_l \end{bmatrix} \left(\frac{1}{3}\right) \begin{bmatrix} F_x \\ F_y \\ F_z \end{bmatrix}$$

or $$\langle \dot{\vec{v}} \rangle = \frac{1}{3}\left(\frac{2}{m_t} + \frac{1}{m_l}\right) \vec{F}$$

Thus, the acceleration vector is co-linear with the force vector for an arbitrary force direction. Hence, the electronic conduction is isotropic.

4) (a) The conductivity effective mass is defined by the above force equation, solution to (1).

(b) $m_\ell^* = 0.98$, $m_t^* = 0.19$

$$m_{ce}^* = 3\left(\frac{2}{m_t} + \frac{1}{m_\ell}\right)^{-1} = \boxed{0.260\ m_o}$$

(5) GaAs

$$\boxed{m_{de}^* = m_{ce}^* = 0.067\, m_o}$$

Ge

$$m_\ell^* = 1.64, \quad m_t^* = 0.082$$

$$m_{ce}^* = 3\left(\frac{2}{m_t} + \frac{1}{m_\ell}\right)^{-1} = \boxed{0.12\ m_o}$$

$$m_{de}^* = \left[M^{2/3}\left(m_\ell m_t^2\right)^{1/3}\right] = 4^{2/3}\left[(1.64)(0.082)^2\right]^{1/3}$$

$$\boxed{m_{de}^* = 0.561\ m_o}$$

(6) $$\mu = \frac{e\tau}{m_{ce}^*} = 200\,\frac{cm^2}{VS} = 0.02\,\frac{m^2}{VS}$$

$$\tau = 0.02\,\frac{m_{ce}^*}{e} = \frac{0.02\,(0.260)(9.11e{-31})}{1.602\,e{-19}}$$

$$\tau = 2.957e{-14}\ s$$

$$\boxed{\tau = 29.6\ fs}$$

(7) GaAs $$\mu = \frac{e\tau}{m_{ce}^*} = \frac{e}{0.067\,m_o}\,\tau = \boxed{776\ \frac{cm^2}{Vs}}$$

Ge $$\mu = \frac{e\tau}{0.12\,m_o} = \boxed{433\ \frac{cm^2}{VS}}$$

(8) (a) $p = N_V e^{-(E_F - E_V)/kT} \underset{@300K}{=} N_A = 2e16/cm^3$

$p = p_{LH} + p_{HH}$

$N_V = N_{V_{LH}} + N_{V_{HH}}$

$$p_{LH} = \frac{N_{V_{LH}}}{N_V} N_A = \frac{m_{LH}^{3/2}}{m_{LH}^{3/2} + m_{HH}^{3/2}} N_A$$

$$p_{LH} = \frac{(0.082)^{3/2}}{(0.082)^{3/2} + (0.45)^{3/2}} (2e16)$$

$$\boxed{p_{LH} = 1.44\,e15/cm^3}$$

(b) $E_F - E_V = -kT \ln\left(\frac{N_D}{N_V}\right) = 0.026 \ln\left(\frac{7e18}{2e16}\right)$

$$\boxed{E_F = E_V + 0.152\,eV}$$

9.) In steady state $R = G = 10^{22}/cm^3 s$

$$R = \frac{np - n_i^2}{\tau_n\left(n + n_i e^{(E_T - E_i)/kT}\right) + \tau_n\left(p + n_i e^{(E_i - E_T)/kT}\right)}$$

midgap $\Rightarrow E_i \simeq E_T$

also $\tau_n = \tau_p = \tau$.

NB: Minority carrier diffusion equations are NOT valid here since $n_o = p_o = n_i$ & $\Delta n = \Delta p >> n_i$.

$$R = \frac{np - n_i^2}{\tau(n + p + 2n_i)}$$

$$n = n_o + \Delta n$$

$$p = p_o + \Delta p$$

$$R = \frac{\cancel{n_o p_o} + n_o \Delta p + p_o \Delta n + \Delta n \Delta p \cancel{- n_i^2}}{\tau(n_o + \Delta n + p_o + \Delta p + 2n_i)}$$

$$\Delta n = \Delta p$$

$$R = \frac{(n_o + p_o)\Delta n + (\Delta n)^2}{\tau(n_o + p_o + 2\Delta n + 2n_i)}$$

The doping of $2e6/cm^3$ is negligible compared to n_i, so $n_o = p_o = n_i$.

$$R = \frac{2n_i \Delta n + (\Delta n)^2}{\tau(4n_i + 2\Delta n)} = \frac{\Delta n \cancel{(2n_i + \Delta n)}}{\tau 2 \cancel{(2n_i + \Delta n)}}$$

$$\boxed{\Delta n = 2R\tau = 2e16/cm^3 = \Delta p}$$

10. No. Au is a midgap state in Si – an efficient R-G center.

11. a) $10^{18}/cm^3 \Rightarrow 0.02\ \Omega\, cm$ } From Fig. 21 of Sze p. 32.

b) $2.5e15/cm^3$