1a) From fig. 21 p. 32 Sze, $10\,\Omega\,cm$ p-type $\Rightarrow N_A \approx 1.3e15/cm^3$.

$$q\phi_{ms} = \Phi_m - \chi - (E_c - E_f)_\infty$$

fig. 4 p251 Sze $\quad \Phi_m(Al) = 4.3\,eV$

app H, Sze $\quad \chi(Si) = 4.05\,eV$

$$\cancel{(E_c - E_f)_\infty = kT\ln\left(\frac{N_c}{N_A}\right)}$$

$$p = N_A = N_V\, e^{-(E_F - E_V)/kT}$$

$$E_F - E_V = kT\ln\left(\frac{N_V}{N_A}\right) = 0.2337\,eV$$

$E_c$ ——

$$E_c - E_F = E_G - 0.2337 = 0.8863\,eV$$

$E_F$ ——

$E_V$ ——

$$\boxed{q\phi_{ms} = -0.636\,eV}$$

b) For flat band in Si, we need $\phi_s = 0$.

$$\phi_s = 0 \Rightarrow W_d = 0$$

$$V(-t_{ox}) = -\frac{Q_{ss}}{C_{ox}}$$

$$V_G = V(-t_{ox}) - V_{eq}(-t_{ox})$$

$$V_G = \boxed{\phi_{ms} - \frac{Q_{ss}}{C_{ox}} = V_{FB}}$$

This is the required gate voltage, referred to as the "flat band voltage" required to make the Si bands flat.

$$V_{FB} = -0.636 - \frac{q(10^{11})(10\times10^{-7}\,cm)}{3.9\epsilon_0}$$

$$= -0.636 - 0.0464$$

$$\boxed{V_{FB} = -0.682\ V}$$

c) $\phi_s = 2\phi_F = 2\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) = 0.593\text{ V}$

d) $V_t = \phi_{ms} + 2\phi_F - \frac{Q_{ss}}{C_{ox}} + \frac{\sqrt{2qN_A\epsilon_s 2\phi_F}}{C_{ox}}$

$C_{ox} = \cancel{\frac{\epsilon_{ox}}{3.9\epsilon_0}} = \frac{3.9\,\epsilon_0}{t_{ox}} = 3.453\text{e-}7\ \frac{F}{cm^2}$

$\sqrt{4qN_A(11.9)\epsilon_0\phi_F} = 1.613\text{e-}8\ \frac{c}{cm^2}$

$V_t = -0.636 + 0.593 - 0.0464 + 0.0467$

$\boxed{V_t = -40\text{ mV}}$

e) $W_T = \sqrt{\frac{2\epsilon_s 2\phi_F}{qN_A}} = \frac{1.613\text{e-}8}{qN_A}$

$= \boxed{0.7745\ \mu m}$

f) $\underline{\phi_s = 0}$

$p = N_A = 1.3\text{e}15/cm^3$

$n = \frac{n_i^2}{N_A} = 1.617\text{e}5/cm^3$

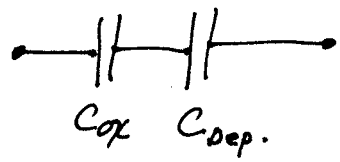
$\underline{\phi_s = \phi_f} \Rightarrow E_f = E_i$ at surface

$n = p = n_i = 1.45\text{e}10/cm^3$

$\underline{\phi_s = 2\phi_f}$

$n = N_A,\ p = \frac{n_i^2}{N_A} = 1.617\text{e}5/cm^3$

(g)




$$C_{ox} = \frac{\epsilon_{ox}}{t_{ox}} \qquad C_{Dep} = \frac{\epsilon_s}{W_D}$$

$$\frac{1}{C} = \frac{1}{C_{ox}} + \frac{1}{C_{Dep}}$$

$C_{min}$ occurs at inversion when $\phi_s = 2\phi_F$ & $W_D = W_T$.

$$C_{min} = \left[ \frac{t_{ox}}{\epsilon_{ox}} + \frac{W_T}{\epsilon_s} \right]^{-1}$$

$\epsilon_{ox} = 3.9\,\epsilon_0$, $\epsilon_s = 11.9\,\epsilon_0$, $t_{ox} = 10 \times 10^{-7}$ cm

$W_T = 0.7745 \times 10^{-4}$ cm

$$C_{min} = \left[ \frac{10^{-6}}{(3.9)} + \frac{0.7745e-4}{11.9} \right]^{-1} (8.854e-14)$$

$$= 1.309 \times 10^{-8} \frac{F}{cm^2}$$

$$= \boxed{13.1 \frac{nF}{cm^2}}$$

$\phi_m - \chi_{Si} - (E_C - E_F)_\infty$

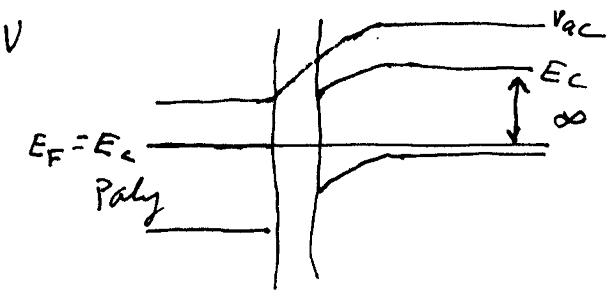
10. Consider an NMOS FET with a polysilicon (poly) gate (instead of metal). The poly is heavily doped poly-crystalline Si. For p-type poly, assume that in the poly, $E_f = E_v$, and for n-type poly, assume that $E_f = E_c$. For the electrostatic calculations, you can treat the poly as a metal, i.e. there is no voltage drop in the poly. The thickness of the $SiO_2$ is 10 nm and it is grown on a 10 Ω cm p-type Si wafer. At the Si / $SiO_2$ interface there is a surface state charge of $10^{11}$ charges / cm$^2$.
(a) Calculate the threshold voltage for an n-type poly gate.

$q\phi_m = \chi_{Si}$ $\qquad$ $10\,\Omega\,cm$ p-type $\Rightarrow N_A = 1.3e15\ /cm^3$

$q\phi_{ms} = -(E_c - E_F)_\infty = -0.8863eV$

$E_F - E_V = kT\ln\left(\frac{N_V}{N_A}\right) = 0.2337eV$

$E_c - E_F = E_G - 0.2337 = 0.8863eV$




$V_t = \phi_{MS} + 2\phi_F - \frac{Q_{SS}}{C_{ox}} - \frac{Q_D}{C_{ox}}$

$2\phi_F = 2\frac{kT}{q}\ln\left(\frac{N_A}{n_i}\right) = 0.593\ V$

$C_{ox} = \frac{3.9\epsilon_0}{t_{ox}} = 3.453e-7\ F/cm^2$

$Q_D = -\sqrt{2qN_A(11.9)\epsilon_0(2\phi_F)} = -1.613e-8\ C/cm^2$

$Q_{SS} = 1.602e-8\ C/cm^2$

$V_t = -0.8863 + 0.593 - \frac{1.1e-10}{3.453e-7} = \boxed{-0.294\ V}$

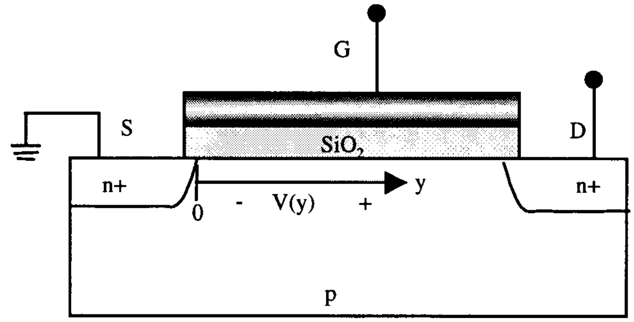
(b) Calculate the threshold voltage for a p-type poly gate.

$q\phi_m = \chi_{Si} + E_G$

$q\phi_{ms} = E_G - (E_c - E_F)_\infty = 0.2337$

$V_{t_p} = V_{t_n} - \phi_{ms_n} + \phi_{ms_p} = V_{t_n} + \frac{E_G}{q} = \boxed{0.826\ V}$

11. For an NMOS FET we write that the electron charge per unit area under the gate is $Q_n(y) = C_{ox}[V_{GS} - V(y) - V_t]$. Using the same approach that we used to derive the "square law" for $I_D$, derive an expression for the potential $V(y)$ and the electric field component $E_y(y)$ in saturation.




$$\int_0^{V(y)} dV\, W \mu_n C_{ox}\, Q_n(V) = \int_0^y dy\, I_D$$

$$W \mu_n C_{ox} \int_0^{V(y)} dV\, (V_{GS} - V_t - V) = I_D\, y$$

$$-\frac{W \mu_n C_{ox}}{2}\Big[(V_{GS} - V_t - V(y))^2 - \underbrace{(V_{GS} - V_t)^2}_{I_D L}\Big] = I_D\, y$$

$$\frac{2 I_D (L-y)}{W \mu_n C_{ox}} = (V_{GS} - V_t - V(y))^2$$

$$V(y) = V_{GS} - V_t - \sqrt{\frac{2 I_D (L-y)}{W \mu_n C_{ox}}}$$

$$\boxed{V(y) = (V_{GS} - V_t)\left[1 - \sqrt{1 - \frac{y}{L}}\right]}$$

$$\boxed{\mathcal{E}_y(y) = \frac{-dV}{dy} = \frac{V_{GS} - V_t}{2L} \frac{(-1)}{\sqrt{1 - \frac{y}{L}}}}$$

$$I_D = \frac{W}{L} \frac{\mu_n C_{ox}}{2} (V_{GS} - V_t)^2$$

$\rho = \nabla \cdot D$

$\nabla \cdot \mathcal{E} = \rho/\epsilon$

$$\cancel{\rho = \frac{\epsilon (V_{GS} - V_t)}{2L} \frac{1/2 \left(\frac{-1}{L}\right)}{(1 - y/L)^{3/2}} = \frac{-\epsilon (V_{GS} - V_t)}{4L^2} \frac{1}{\left(1 - \frac{y}{L}\right)^{3/2}}}$$

8. For an npn BJT at T=300K with a base width of 0.05 μm and a minority electron mobility in the base of 800 $cm^2/Vs$, what is the maximum value for the transition frequency, $f_T$?

$$f_{T_{max}} = 2 \frac{\mu_n \left(\frac{k_B T}{q}\right)}{2\pi W_B^2} = \frac{800\ (0.026)}{\pi\left[(0.05)(1e-4)\right]^2}$$

$$= \boxed{265\ GHz}$$

9. For an NMOS FET with $\mu_n$ in the channel equal to 800 $cm^2/Vs$, a gate length of 0.18 μm, and $(V_{GS} - V_t) = 1V$ what is the maximum value for the transition frequency, $f_T$, assuming the "square law" relation for $I_D$.

$$f_{T_{max}} = \frac{3}{2} \frac{\mu_n (V_{GS} - V_t)}{2\pi L^2} = 0.75 \frac{800}{\pi (0.18e-4)^2}$$

$$= \boxed{589\ GHz}$$