1) For steady-state, no light:

a)

$$D_n \frac{\partial^2 \Delta n_p}{\partial x^2} = \frac{\Delta n_p}{\tau_n} \qquad (1)$$

Plug solution into (1) & see if satisfied:

$$D_n \frac{\partial^2 \Delta n_p}{\partial x^2} = \frac{D_n}{(L_n)^2} \Delta n_p(x) = \frac{\Delta n_p(x)}{\tau_n}$$

$$L_n = \sqrt{D_n \tau_n}$$

At $x = 0$, $\Delta n(x) = \Delta n_B(0)$

At $x = W$, $\Delta n(x) = \Delta n_B(W)$

The distribution satisfies the minority carrier diffusion equation & the 2 boundary conditions. $\Rightarrow$ it is the unique solution.

b)

$$I_{En} = \left. -q \frac{D_{nB}}{L_{nB}} \Delta n_B(0) \frac{\cosh\left(\frac{W-x}{L_{nB}}\right)}{\sinh(W/L_{nB})} + \Delta n_B(W) q \frac{D_{nB}}{L_{nB}} \frac{\cosh\left(\frac{x}{L_{nB}}\right)}{\sinh(W/L_{nB})} \right|_{x=0}$$

$$\boxed{I_{EN} = -q \frac{D_{nB}}{L_{nB}} \Delta n_B(0) \frac{\cosh\left(\frac{W}{L_{nB}}\right)}{\sinh\left(\frac{W}{L_{nB}}\right)} + q \frac{D_{nB}}{L_{nB}} \Delta n_B(W) \frac{1}{\sinh\left(\frac{W}{L_{nB}}\right)}}$$

$$\Delta n_B(0) = \frac{n_i^2}{N_{AB}} \left(e^{V_{BE}/V_T} - 1\right) \qquad \text{where } V_T = k_B T / q$$

$$\Delta n_B(W) = \frac{n_i^2}{N_{AB}} \left(e^{V_{BC}/V_T} - 1\right)$$

$$\boxed{I_{Cn} = +q \frac{D_{nB}}{L_{nB}} \frac{n_i^2}{N_{AB}} \left[\frac{-\left(e^{V_{BE}/V_T} - 1\right)}{\sinh \frac{W}{L_{nB}}} + \frac{\left(e^{V_{BC}/V_T} - 1\right) \cosh \frac{W}{L_{nB}}}{\sinh \frac{W}{L_{nB}}} \right]}$$

c) Forward active region $\Rightarrow$ only keep terms $\propto e^{V_{BE}/V_T}$

$$\boxed{\alpha_T = \frac{I_{Cn}}{I_{EN}} = \frac{1}{\cosh\left(\frac{W}{L_{nB}}\right)} \approx \frac{1}{1+\frac{1}{2}\left(\frac{W}{L_{nB}}\right)^2}}$$

$$\frac{W}{L_{nB}} \ll 1$$

d) From class notes:

$$I_{Ep} = \frac{-q\,D_{pE}}{L_{pE}} \frac{n_i^2}{N_{DE}} e^{qV_{BE}/kT}$$

$$\frac{I_{EN}}{I_{EN}+I_{Ep}} = \frac{\frac{D_{nB}}{L_{nB}} \frac{n_i^2}{N_{AB}} \frac{\cosh\left(\frac{W}{L_{nB}}\right)}{\sinh\left(\frac{W}{L_{nB}}\right)}}{I_{EN} + \frac{D_{pE}}{L_{pE}} \frac{n_i^2}{N_{DE}}}$$

$$\boxed{\gamma = \frac{1}{1 + \frac{D_{pE}}{L_{pE}} \frac{L_{nB}}{D_{nB}} \frac{N_{AB}}{N_{DE}} \frac{\sinh\left(\frac{W}{L_{nB}}\right)}{\cosh\left(\frac{W}{L_{nB}}\right)}}}$$

Keeping only terms to 1st order in $\frac{W}{L_{nB}}$:

$$\boxed{\gamma \cong \frac{1}{1 + \frac{D_{pE}}{D_{nB}} \frac{\cancel{L_{nB}}}{L_{pE}} \frac{N_{AB}}{N_{DE}} \frac{W}{\cancel{L_{nB}}}}}$$

2) $$I_C = -I_{RO}\left(e^{V_{BC}/V_T} - 1\right) + \alpha_F I_{FO}\left(e^{V_{BE}/V_T} - 1\right)$$

a)

$$V_{CE} = 0 \Rightarrow V_{EB} + V_{BC} = 0 \Rightarrow V_{BE} = V_{BC}$$

so

$$I_C = \left(\alpha_F I_{FO} - I_{RO}\right)\left(e^{V_{BE}/V_T} - 1\right) \quad \text{where}$$

$$I_{FO} = q\left(\frac{D_{pE}}{L_{pE}} \frac{n_i^2}{N_{DE}} + \frac{D_{nB}}{L_{nB}} \frac{n_i^2}{N_{AB}}\right)$$

$$I_{RO} = q\left(\frac{D_{pC}}{L_{pC}} \frac{n_i^2}{N_{DC}} + \frac{D_{nB}}{L_{nB}} \frac{n_i^2}{N_{AB}}\right)$$

b) Assuming $\alpha_F \approx 1$,

$$\alpha_F I_{F0} - I_{R0} \approx q\left(\frac{D_{pE}}{L_{pE}}\frac{n_i^2}{N_{DE}} - \frac{D_{pC}}{L_{pC}}\frac{n_i^2}{N_{DC}}\right)$$

$$< 0 \text{ since } N_{DE} \sim 10^4 N_{DC}$$

Also:

$$I_B = \cancel{I_{F0}(e^{V_{BE}/V_T}-1)}$$

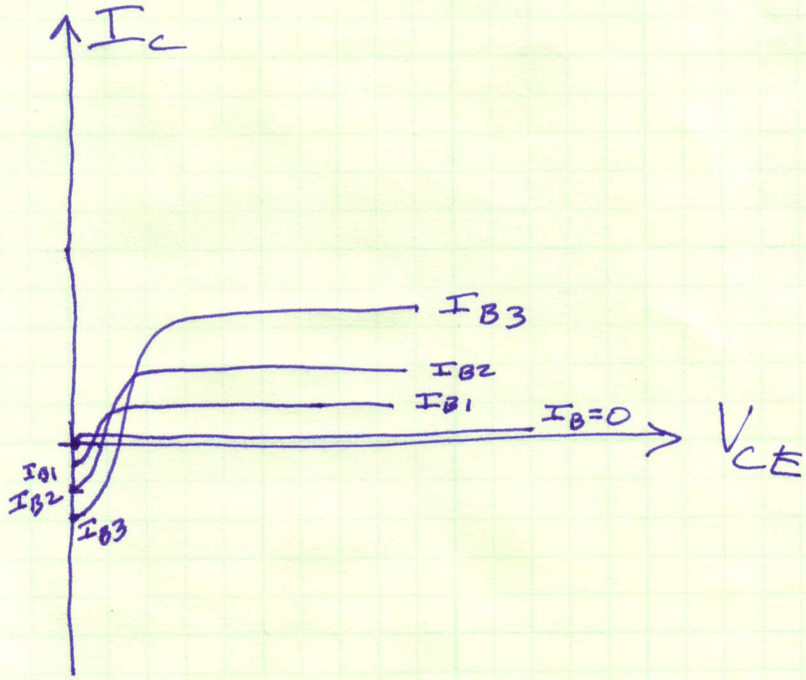
$$= \left[I_{F0}(1-\alpha_F) + I_{R0}(1-\alpha_R)\right](e^{V_{BE}/V_T}-1)$$

$$I_B = \left[\frac{I_{F0}}{\beta_F+1} + \frac{I_{R0}}{\beta_R+1}\right](e^{V_{BE}/V_T}-1)$$

At $V_{CE} = 0$, $I_B = 0 \Rightarrow V_{BE} = V_{BC} = 0$.

$\Rightarrow I_C = 0$

As $I_B \uparrow$, $V_{BE} \uparrow \Rightarrow I_C \downarrow$ at $V_{CE} = 0$.



3) <u>with collector open:</u>

$$\underbrace{I_{RO}\left(e^{V_{BC}/V_T}-1\right)}_{I_R} - \alpha_F I_F = 0$$

$$I_R = \alpha_F I_F$$

$$I_E = I_F - \alpha_R I_R = I_F\left(1-\alpha_R\alpha_F\right)$$

$$I_E = I_{FO}\left(1-\alpha_R\alpha_F\right)\left(e^{V_{BE}/V_T}-1\right) \qquad (1)$$

<u>with Collector shorted to base:</u>

$$V_{BC} = 0 \Rightarrow I_R = 0$$

$$I_E = I_{FO}\left(e^{V_{BE}/V_T}-1\right) \qquad (2)$$

$(2) > (1) \Rightarrow$ does make a difference.

w/ C open

$$I_{RO}\left(e^{V_{BC}/V_T}-1\right) = \alpha_F I_{FO}\left(e^{V_{BC}}\right.$$