EE 203. HW1 Due Monday, April 21.

1. Calculate the product of the 1D density of states times the 1D velocity of an electron in the conduction band.

2. Assuming a parabolic dispersion relation, $\varepsilon(\mathbf{k}) = (\hbar k)^2 / 2m^*$, derive the expression for the single-spin density of states *g(E)* for (a) a 1-dimensional crystal, (b) a 2-dimensional crystal, and (c) a three dimensional crystal by explicitly evaluating.

$$g(E) = \frac{1}{L^D} \sum_{\mathbf{k}} \delta[E - \varepsilon(\mathbf{k})]$$

where D is the dimensionality.

3. Show that for an *arbitrary*, smooth dispersion relation of the form $\varepsilon(\mathbf{k}) = \varepsilon_x(k_x) + \varepsilon_y(k_y) + \varepsilon_z(k_z)$, (a) $N_{2D}(E) = N_{1D}(E) \otimes N_{1D}(E)$ and that (b) $N_{3D}(E) = N_{1D}(E) \otimes N_{1D}(E) \otimes N_{1D}(E)$ where N_{1D}, N_{2D}, and N_{3D} are the single spin 1D, 2D, and 3D density of states, respectively, and $\otimes$ means convolution.

4. Assuming a dispersion relation

$$E = E_C + \frac{\hbar^2}{ma^2}[1 - \cos(ka)]$$

(a) Calculate the velocity of the electron at $k = \pi/a$.
(b) If the electric field is applied in the -x direction, derive the time dependence of k for an electron initially at state $k = -\pi/a$ and position $x = 0$; and
(c) Derive the time dependence of the electron velocity, $v(t)$; and
(d) Derive the time dependence of the electron position, $x(t)$.
(e) For $a = 5$ nm, $E = 10^4$V/cm, and $m = 0.2\ m_0$, what are the maximum and minimum values of x that the electron will reach?
(f) What is the period of the oscillation?
(g) For the parameters of part (e), derive an expression for the effective mass as a function of k. Sketch the function.