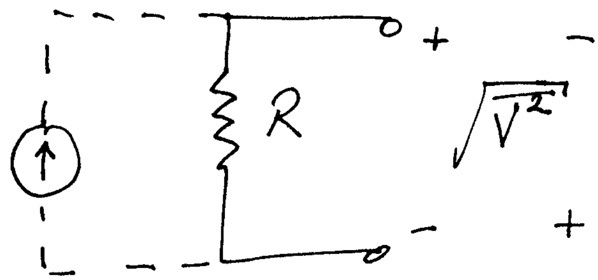
CH. 7 NOISE in resistors (Sec. 7.3)

Thermal or Johnson noise

- Random motion of electrons due to thermal effects.
- Model with a parallel current source



$\sqrt{\overline{i^2}} = \sqrt{\frac{4k_BT}{R}}$ $\left(\frac{A}{\sqrt{Hz}}\right)$

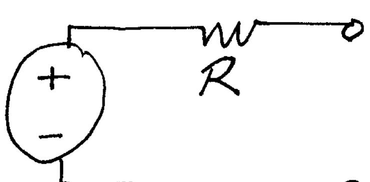
$\sqrt{\overline{V^2}}$ rms noise voltage?

$$\sqrt{\overline{i^2}} = \sqrt{\frac{4k_BT}{R}} \cdot \sqrt{B}$$ $B =$ bandwidth in Hz.

Leave out B until end of calculation.

$$\sqrt{\overline{V^2}} = \sqrt{\overline{i^2}}\,R = \sqrt{4k_BTR} \quad \left(\frac{V}{\sqrt{Hz}}\right)$$

$\sqrt{\overline{V^2}}$



TOTAL RMS Noise Voltage $= \sqrt{\overline{V^2}}\sqrt{B}$

Noise Analysis

1. Add RMS noise sources to circuit
2. Determine RMS circuit output noise from each component using superposition
3. Square each output noise component
4. Add together

Output Noise

5. Integrate over bandwidth
6. Take $\sqrt{}$ of result

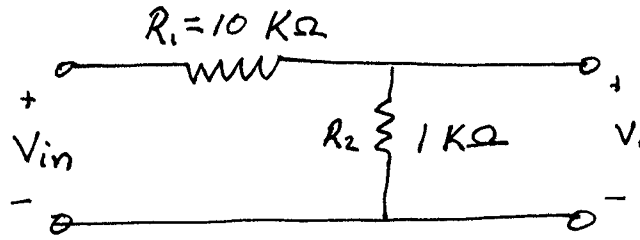
Input Referred Noise

5. Divide sum in (4) by $|H(j\omega)|^2$

$$= \left| \frac{V_o}{V_{in}} \right|^2$$

6. Integrate over bandwidth
7. $\sqrt{}$

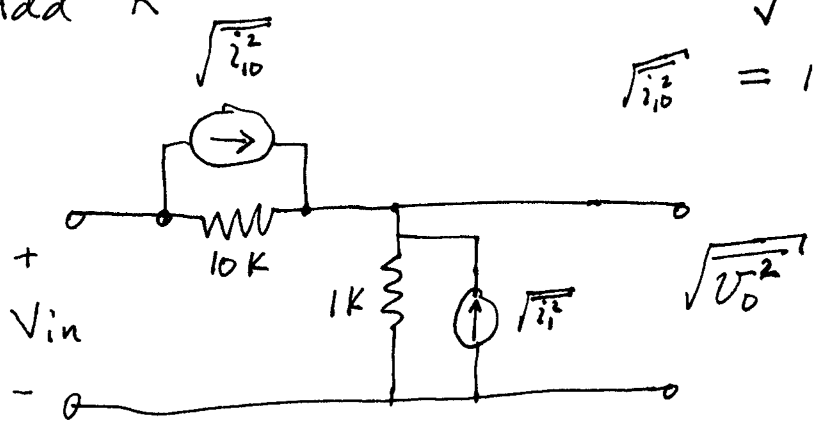
Example



(1) Add RMS noise sources $\sqrt{\overline{i_{10}^2}} = \sqrt{\dfrac{4 \cdot 0.026 \cdot 1.602e{-}19}{10^4}}$

$$\sqrt{\overline{i_{10}^2}} = 1.29e{-}12\ \frac{A}{\sqrt{Hz}}$$

$$\sqrt{\overline{i_1^2}} = 4.08e{-}12\ \frac{A}{\sqrt{Hz}}$$



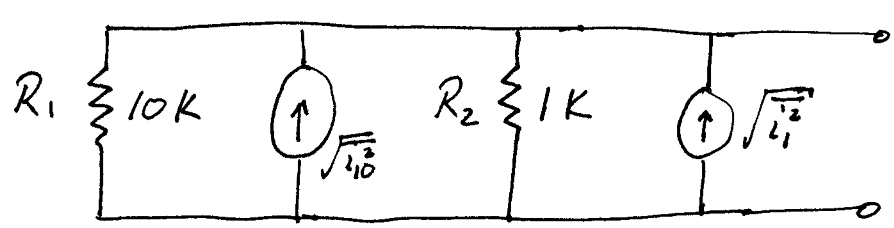
(2) Superposition

"Kill" all sources but one of interest

i.e. short Voltage sources

Open Current sources

$\Rightarrow$ (a) Short Vin



(b) Open $\sqrt{\overline{i_1^2}}$

$$\sqrt{\overline{v_{10}^2}} = \sqrt{\overline{i_{10}^2}}\; R_1 \| R_2 = \sqrt{\overline{i_{10}^2}}\; \frac{10}{11} k\Omega$$

$$\sqrt{\overline{v_{10}^2}} = 1.17 \frac{nV}{\sqrt{Hz}}$$

(c) Open $\sqrt{\overline{i_{10}^2}}$ (close $\sqrt{\overline{i_1^2}}$)

$$\sqrt{\overline{v_1^2}} = \sqrt{\overline{i_1^2}}\; R_1 \| R_2 = 3.71 \frac{nV}{\sqrt{Hz}}$$

(3) Square each output noise component
(4) Add together

$$\overline{v_o^2} = \overline{v_{10}^2} + \overline{v_1^2} = 1.5133e{-}17 \frac{V^2}{Hz}$$

Output Noise

(5) Integrate over Bandwidth (1000 Hz)

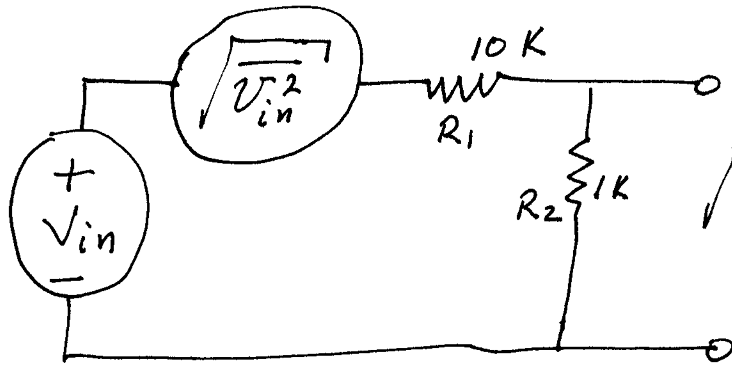
$$\overline{v_o^2} \cdot 1000\,Hz = 1.5133e{-}14\, V^2$$

(6) Take $\sqrt{}$

$$\sqrt{\overline{v_o^2}} = 1.23e{-}7\, V = 0.123\, \mu V$$

$$\boxed{\sqrt{\overline{v_o^2}} = 123\, nV}$$

Effective Input Noise (Input referred noise)



$\sqrt{\overline{v_o^2}} = 123\ nV$

What noise source at input will generate **123 nV** noise at output?

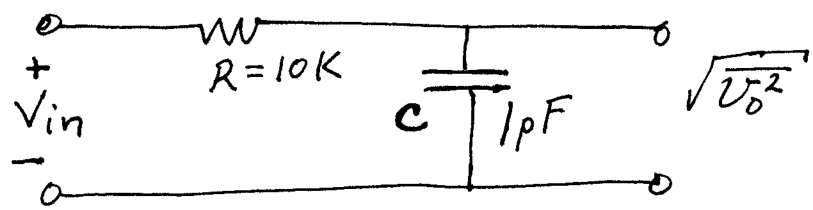
$$\sqrt{\overline{v_{in}^2}} \; \frac{1K}{1K + 10K} = 123\,nV$$

$$\boxed{\sqrt{\overline{v_{in}^2}} = 1.35\,\mu V}$$

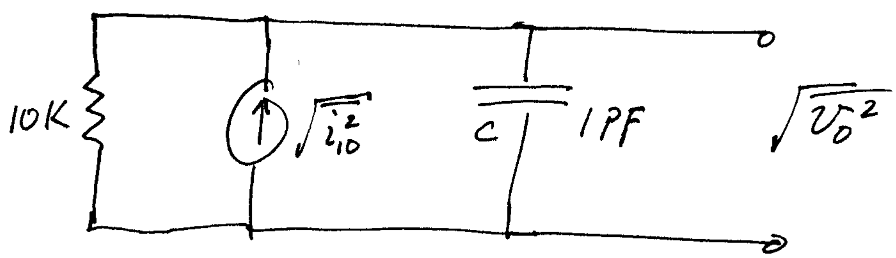
MODEL Noise with SPICE

. noise statement in netlist pp. 143–4

Example



(1) & (2) Add Sources + Superposition ("kill" V_{in})



$$\sqrt{\overline{i_{10}^2}} = 1.29e-12 \frac{A}{\sqrt{Hz}}$$

$$Z = 10\ k\Omega \,||\, 1pF = \frac{R \frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = \frac{R}{1 + j\omega RC}$$

$$\sqrt{\overline{v_o^2}} = \sqrt{\overline{i_{10}^2}}\ |Z| = \sqrt{\overline{i_{10}^2}} \frac{R}{\sqrt{1 + \omega^2 R^2 C^2}}$$

$$\sqrt{\overline{v_o^2}} = \sqrt{\overline{i_{10}^2}} \frac{R}{\sqrt{1 + (2\pi f RC)^2}}$$

(3) Square: $\overline{v_o^2} = \overline{i_{10}^2} \dfrac{R^2}{1 + (2\pi f RC)^2}$

(4) Add all contributions ~~together~~.

Output Noise

(5) Integrate over f.

$$\int_0^{f_H} df \; \overline{i_{10}^2} \; \frac{R^2}{1 + (2\pi f RC)^2}$$

$u = 2\pi f RC \qquad du = 2\pi RC\, df$

$$\overline{v_o^2} = \int_0^{2\pi RC f_H} du \, \frac{R^2}{2\pi RC} \, \overline{i_{10}^2} \, \underbrace{\frac{1}{1 + u^2}}_{\text{atan}(u)}$$

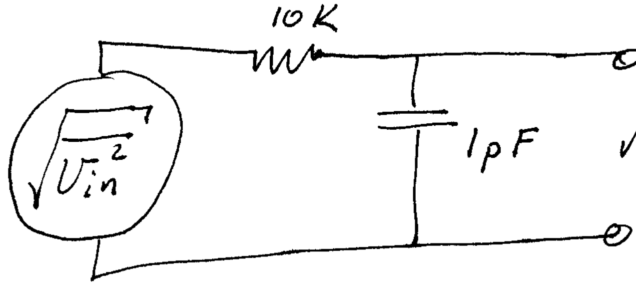
Let $f_H \to \infty$

$$\overline{v_o^2} = \frac{\overline{i_{10}^2} R^2}{2\pi RC} \frac{\pi}{2}$$

$$\overline{v_o^2} = \frac{(1.29e-12)^2 (10^4)^2}{4 (10^4)(10^{-12})} = 4.16\,e-9 \; (V^2)$$

$$\sqrt{\overline{v_o^2}} = 6.45\,e-5 \text{ V} = \boxed{64.5 \; \mu V}$$

Input Referred Noise



$\sqrt{\overline{v_o^2}} = 64.5\ \mu V$

(5) Divide sum in (4) by

$$|H(j\omega)|^2 = \left|\frac{V_o}{V_{in}}\right|^2 = |$$

$$V_o = V_i \frac{\frac{1}{j\omega C}}{R + \frac{1}{j\omega C}} = V_i \frac{1}{1 + j\omega RC}$$

$$|H(j\omega)|^2 = \frac{1}{1 + (\omega RC)^2}$$

$$\frac{\overline{v_o^2}}{|H|^2} = \overline{i_{10}^2} \frac{R^2}{1 + (2\pi f RC)^2} (1 + (\omega RC)^2)$$

$$\overline{v_{in}^2} = \overline{i_{10}^2} R^2$$

(6) Integrate over bandwidth

$$\int_0^{f_H} df\, \overline{i_{10}^2} R^2 = \overline{i_{10}^2} R^2 f_H$$

(7) Take $\sqrt{\ }$: $\sqrt{\overline{v_{in}^2}} = \sqrt{\overline{i_{10}^{\text{[redacted]}}} R^{\text{[redacted]}}} \sqrt{f_H}$