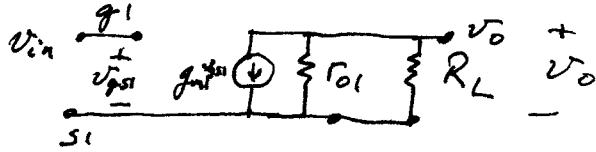
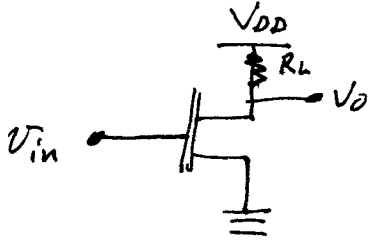


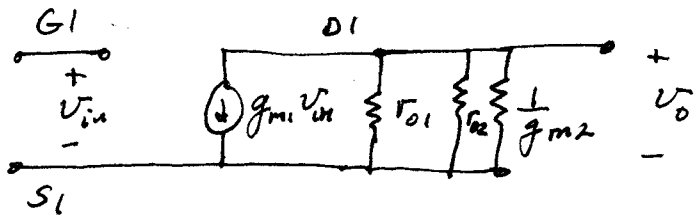
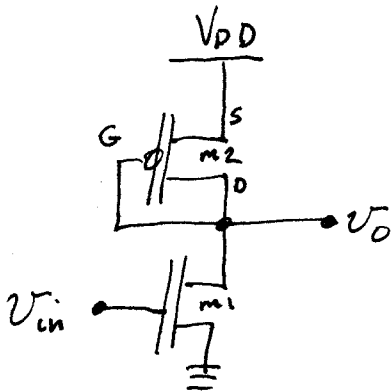
CH 22

Single Stage AmpsCOMMON SOURCE

$$V_{gs1} = V_{in}$$

$$V_o = -g_m V_{in} r_{O1} \parallel R_L$$

$$A_v \equiv \frac{V_o}{V_{in}} = -g_m r_{O1} \parallel R_L$$

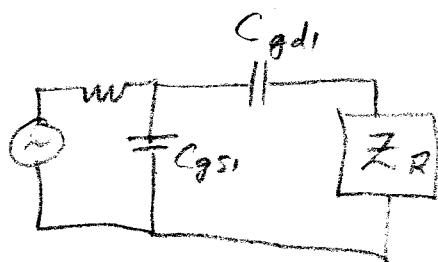


$$A_v = -g_{m1} \left(r_{O1} \parallel \underbrace{r_{O2} \parallel \frac{1}{g_{m2}}}_{R_L} \right)$$

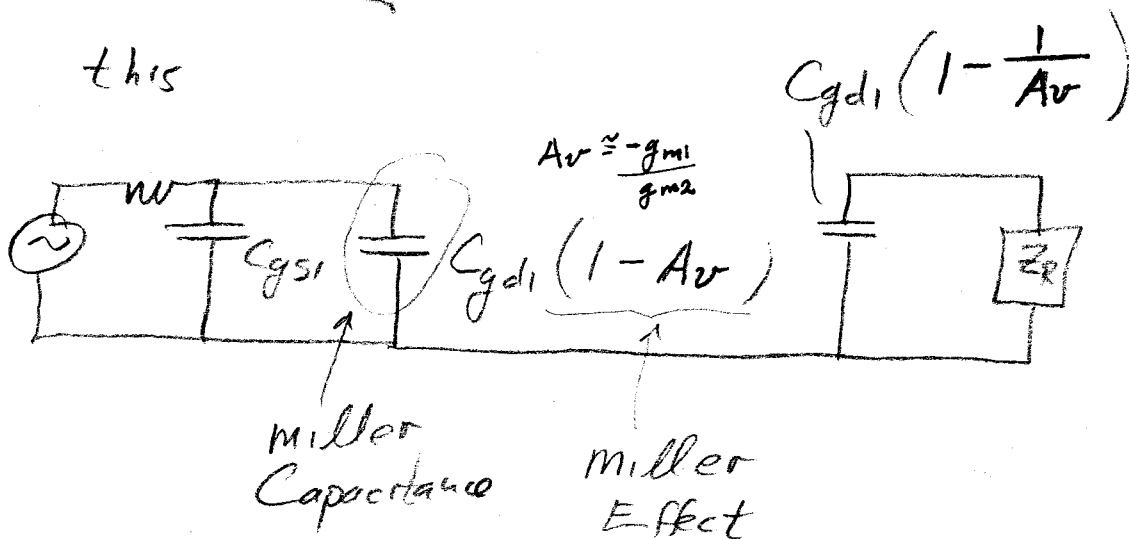
$$\approx -\frac{g_{m1}}{g_{m2}} \quad \left(\frac{1}{g_{m2}} \ll r_{O1}, r_{O2} \right)$$

FREQUENCY RESPONSE

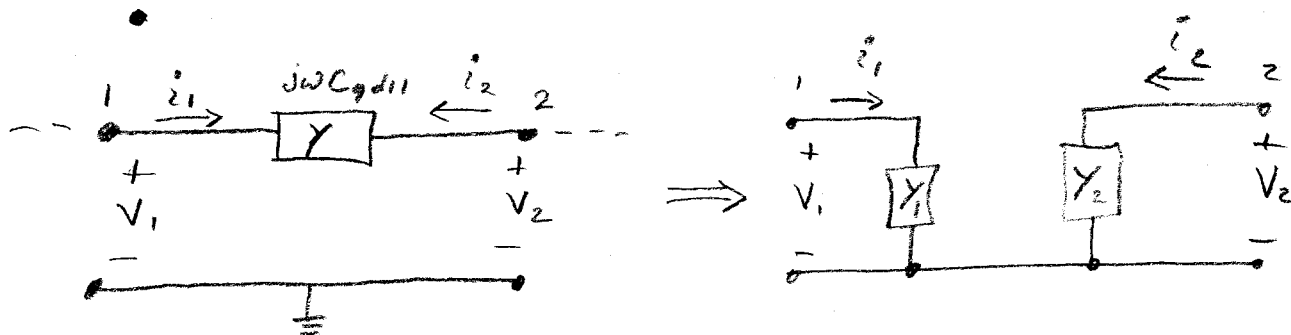
USE MILLERS THM to write this



As this



MILLER'S THM



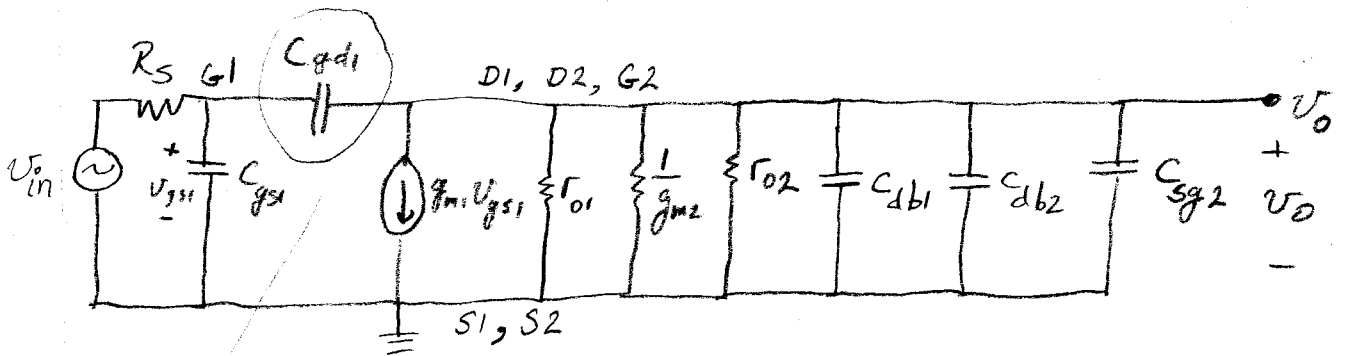
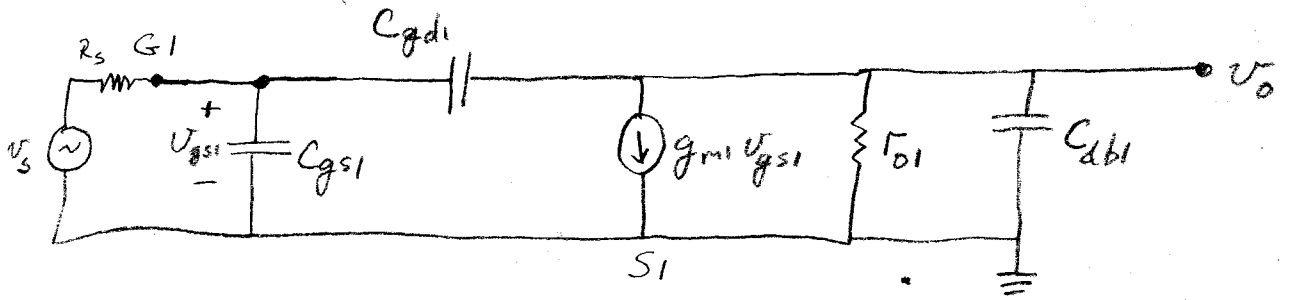
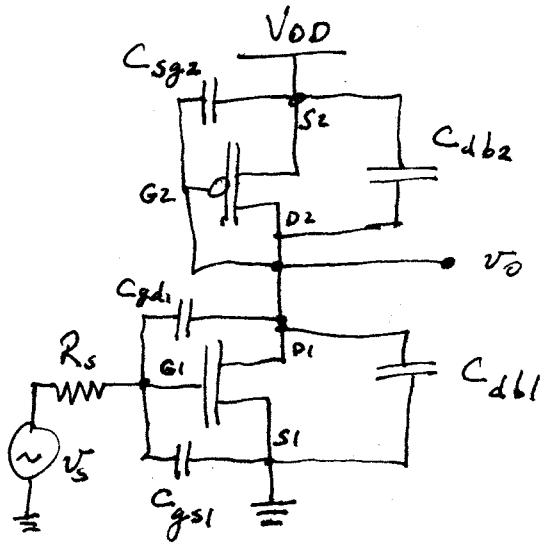
$$i_1 = Y(V_1 - V_2) = YV_1 \left(1 - \frac{V_2}{V_1}\right) = \underbrace{Y(1 - A_v)}_{Y_1} V_1$$

$$i_2 = Y(V_2 - V_1) = \underbrace{Y\left(1 - \frac{V_1}{V_2}\right)}_{Y_2} V_2 = Y \left(1 - \frac{1}{A_v}\right) V_2$$

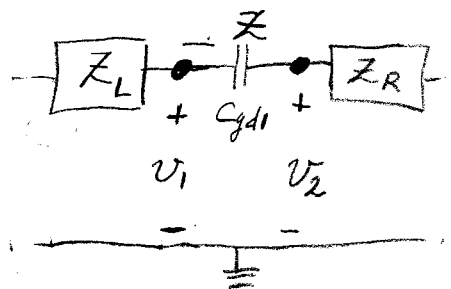
* Get effective large C on chip.

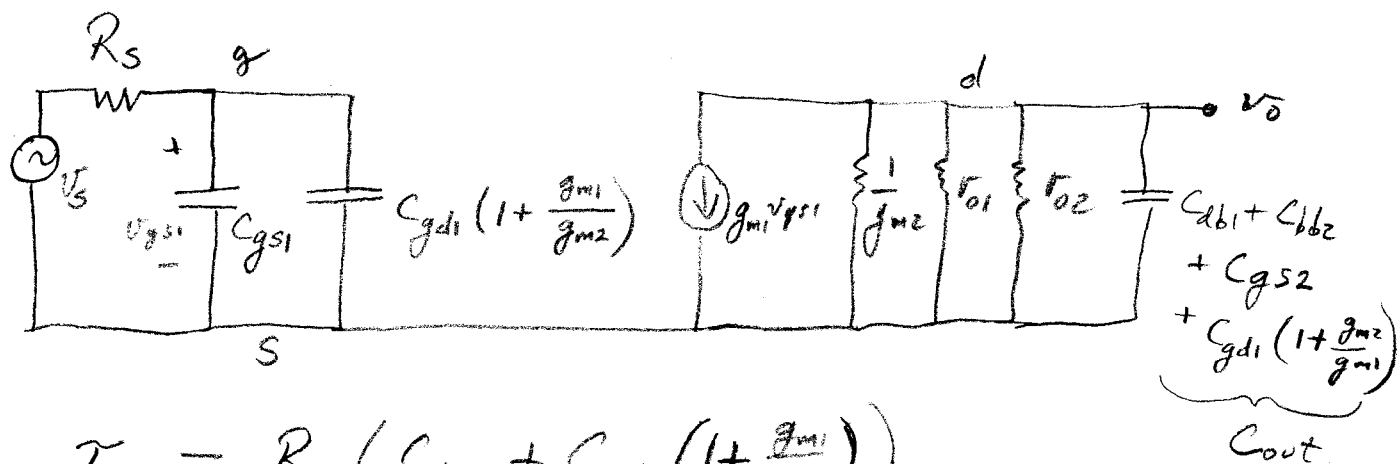
Used in Op-Amps to get large effective C for compensation. $jwC \rightarrow jwC(1 + |A_v|)$

In an Op-Amp, $A_v \sim 1000 - 100,000$.



Makes analysis hard.





$$Z_{in} = R_s \left(C_{gs1} + C_{gd1} \left(1 + \frac{g_{m1}}{g_{m2}} \right) \right)$$

$$Z_{out} \approx \frac{1}{g_{m2}} \left(C_{gs2} + C_{gd1} \left(1 + \frac{g_{m2}}{g_{m1}} \right) + C_{db1} + C_{db2} \right)$$

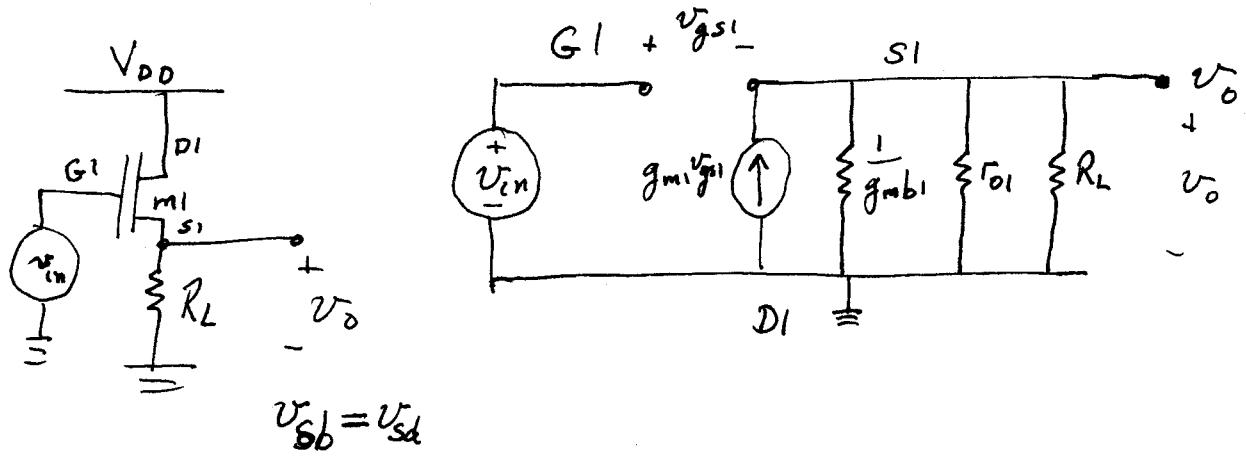
$$v_{gs1} = \frac{v_s \frac{1}{j\omega C}}{R_s + \frac{1}{j\omega \left(\frac{C_{gs1}}{1 + \frac{g_{m1}}{g_{m2}}} \right)}} = v_s \frac{1}{1 + j\omega R_s C}$$

$$v_{gs1} = v_s \frac{1}{1 + j\omega Z_{in}}$$

$$\frac{v_o}{v_s} \approx -g_{m1} v_s \frac{1}{(1 + j\omega Z_{in})} \left(\frac{1}{g_{m2}} \parallel \frac{1}{j\omega C_{out}} \right)$$

$$A_v = \frac{-g_{m1}/g_{m2}}{(1 + j\omega Z_{in})(1 + j\omega Z_{out})} \frac{1/g_{m2}}{1 + j\omega \frac{1}{g_{m2}} C_{out}}$$

Common DRAIN (Source Follower)



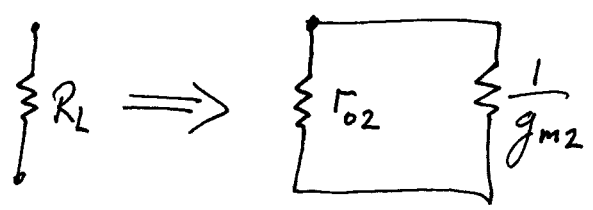
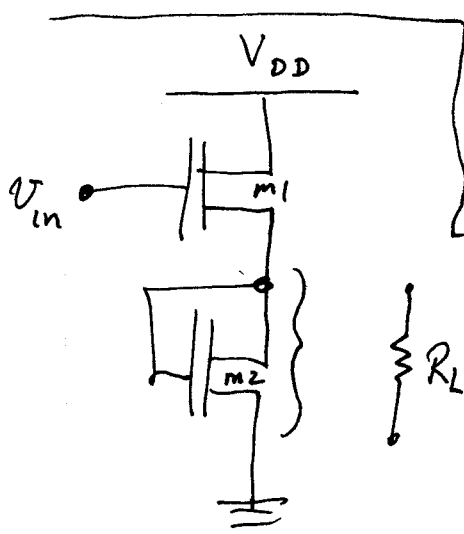
$$v_{in} - v_o = v_{gs1}$$

$$v_o = g_m (v_{in} - v_o) \left(\frac{1}{g_{mb1}} \parallel r_{o1} \parallel R_L \right)$$

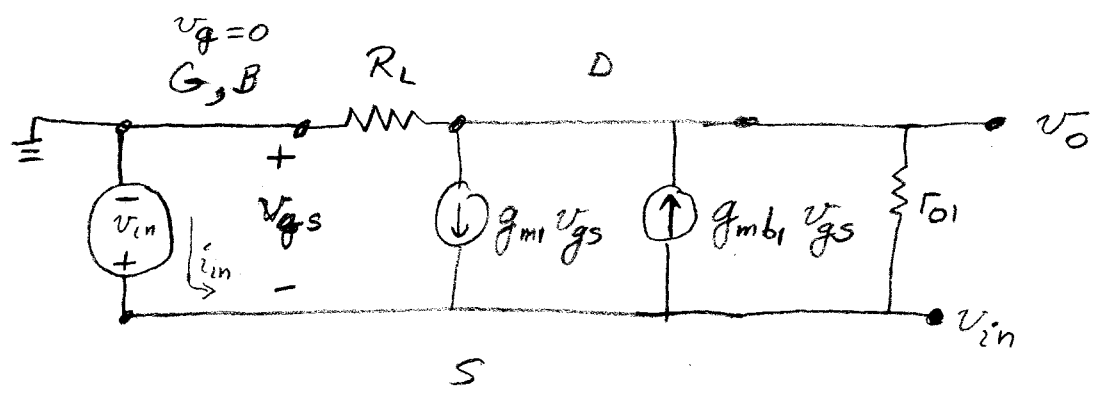
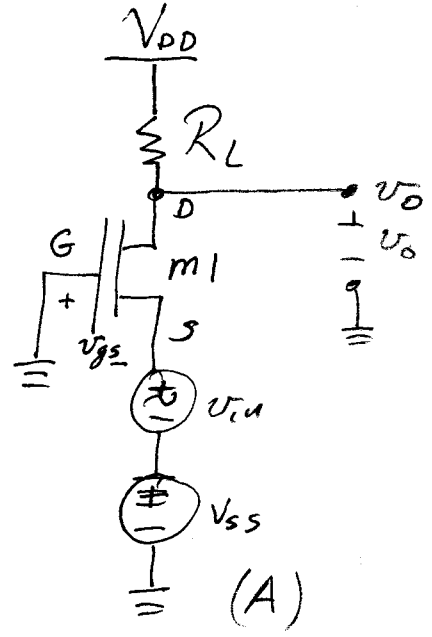
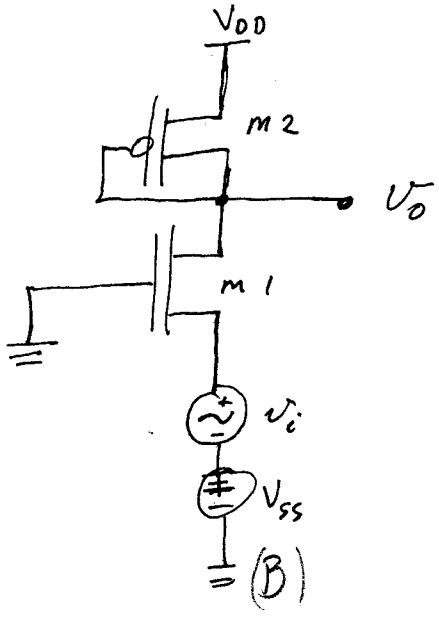
$$v_o \left(1 + g_m \left(\frac{1}{g_{mb1}} \parallel r_{o1} \parallel R_L \right) \right) = g_m v_{in} \left(\frac{1}{g_{mb1}} \parallel r_{o1} \parallel R_L \right)$$

$$\frac{v_o}{v_{in}} = A_v = \frac{g_m \left(\frac{1}{g_{mb1}} \parallel r_{o1} \parallel R_L \right)}{1 + g_m \left(\frac{1}{g_{mb1}} \parallel r_{o1} \parallel R_L \right)} < 1$$

$$R_o = v_o = v_{sd} = v_{sg} = -v_{gs} = \frac{1}{\frac{1}{g_{m1}} \parallel \frac{1}{g_{mb}} \parallel r_{o1}} \parallel R_L \sim \frac{1}{g_m} \text{ (low)}$$



Common Gate



$$v_{gs} = -v_{in} = -v_s$$

$$\frac{v_o}{R_L} + \frac{v_o - v_{in}}{r_{o1}} + g_{m1}(1-\chi)(-v_{in}) = 0$$

$$v_o \left(\frac{1}{R_L} + \frac{1}{r_{o1}} \right) = v_{in} \left(\frac{1}{r_{o1}} + g_{m1}(1-\chi) \right)$$

$$\boxed{\frac{v_o}{v_{in}} \equiv A_v = \frac{\frac{1}{r_{o1}} + g_{m1}(1-\chi)}{\frac{1}{R_L} + \frac{1}{r_{o1}}} \approx R_L g_{m1}(1-\chi) \quad r_{o1} \gg R_L}$$

$$R_{in} = \frac{v_{in}}{i_{in}} = \frac{v_{in}}{v_o/R_L} = \frac{R_L}{A_v} \approx \frac{1}{g_{m1}(1-\chi)}$$

$$R_{out} = r_{o1} \parallel R_L$$

FOR (B) $R_L \Rightarrow \frac{1}{g_{m2}} \parallel r_{o2} \approx \frac{1}{g_{m2}}$

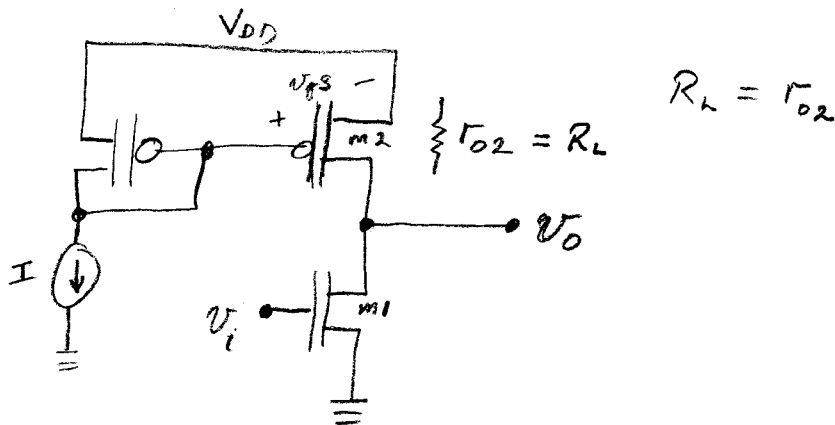
$$A_v = \frac{g_{m1}}{g_{m2}} (1 - \kappa)$$

$$R_{in} = \frac{1}{g_{m2}} \frac{g_{m2}}{g_{m1} (1 - \kappa)} = \frac{1}{g_{m1} (1 - \kappa)}$$

$$R_{out} = r_{o1} \parallel \frac{1}{g_{m2}} = \frac{1}{g_{m2}}$$

CURRENT SOURCE LOADS

COMMON SOURCE

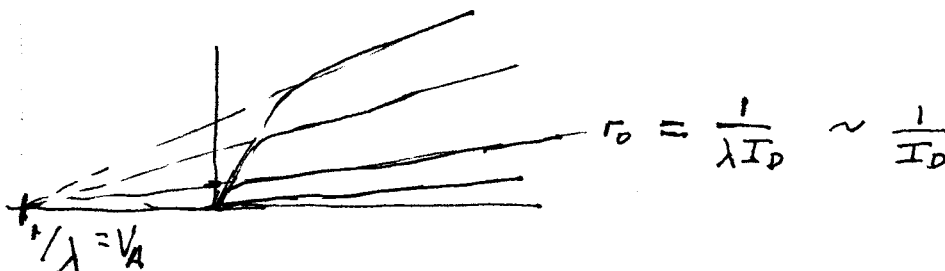


$$A_v = -g_{m1} r_{o1} \parallel r_{o2} \quad (\text{high})$$

$$R_o = r_{o1} \parallel r_{o2} \quad (\text{high})$$

$$A_v = - \frac{\sqrt{2\beta_1 I_D}}{(\lambda_1 + \lambda_2) I_D} \quad (r_o = \frac{1}{\lambda I_D})$$

$$= - \frac{\sqrt{2\beta_1}}{(\lambda_1 + \lambda_2) \sqrt{I_D}} \sim \frac{1}{\sqrt{I_D}}$$



$$\lambda = 0.6$$

$$r_o(5\mu A) = 333 \text{ k}\Omega$$

$$r_o(5\text{ nA}) = 333 \Omega$$

FREQ. RESPONSE

$$R_L \rightarrow r_{o2}$$

$$A_v = -g_{m1} r_{o1} \parallel r_{o2}$$

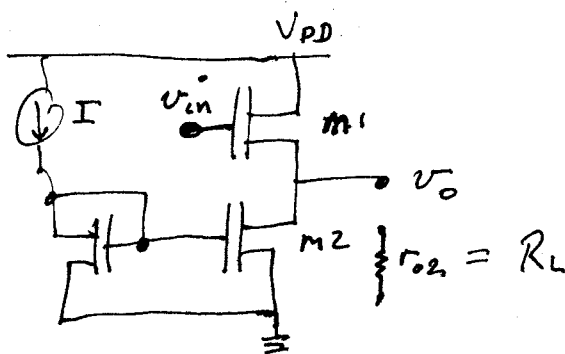
Miller Capacitance

$$C_m = C_{gd1} (1 - A_v) \text{ large} \rightarrow Z_{in} = R_s C_{in} \text{ long}$$

Gain Band Width Product

$$|A_v| \frac{1}{R_s C_{gd1} |A_v|}$$

Source follower (Impedance Transformer)



$$A_v =$$

$$(r_{o1} = r_{o2} = r_o)$$

$$A_v = \frac{g_{m1} \left(\frac{1}{g_{m1} \chi} \parallel r_{o1} \parallel r_{o2} \right)}{1 + g_{m1} \left(\frac{1}{\chi g_{m1}} \parallel r_{o1} \parallel r_{o2} \right)} = \frac{1}{1 + \chi + \frac{2}{g_{m1} r_o}} < 1$$

$$R_o = \frac{1}{g_{m1}} \parallel \frac{1}{\chi g_{m1}} \parallel r_{o1} \parallel r_{o2} = g_{m1} (1 + \chi) \parallel r_{o1} \parallel r_{o2}$$

$$\left(g_{m1} (1 + \chi) + \underbrace{\frac{1}{r_{o1}} + \frac{1}{r_{o2}}}_{\frac{2}{r_o}} \right)^{-1} = \frac{r_o}{g_{m1} (1 + \chi) r_o + 2}$$

$$= \frac{\frac{1}{g_{m1}}}{1 + \chi + \frac{2}{g_{m1} r_o}}$$

$$\left(\frac{g_{m1} \chi r_o + 2}{r_o} \right)^{-1} = \frac{r_o}{g_{m1} r_o \chi + 2}$$

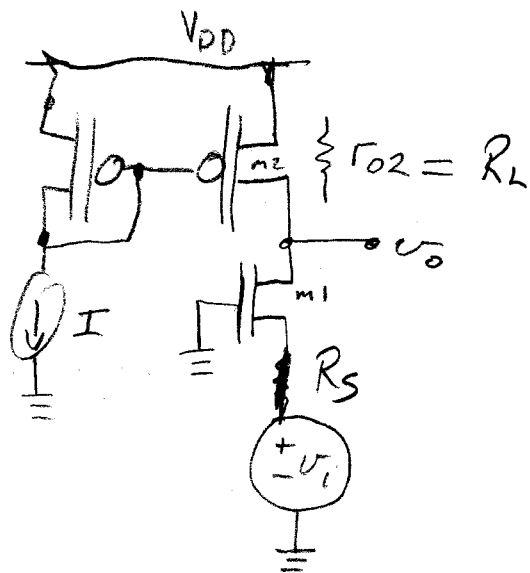
$$\begin{aligned}
 A_v &= \frac{g_{m1} \frac{\Gamma_o}{g_{m1} \Gamma_o \chi + 2}}{1 + g_{m1} \left(\frac{\Gamma_o}{g_{m1} \Gamma_o \chi + 2} \right)} \\
 &= \frac{g_{m1} \Gamma_o}{g_{m1} \Gamma_o (1 + \chi) + 2} \\
 &= \frac{1}{1 + \chi + \frac{2}{g_{m1} \Gamma_o}} < 1
 \end{aligned}$$

Common Gate

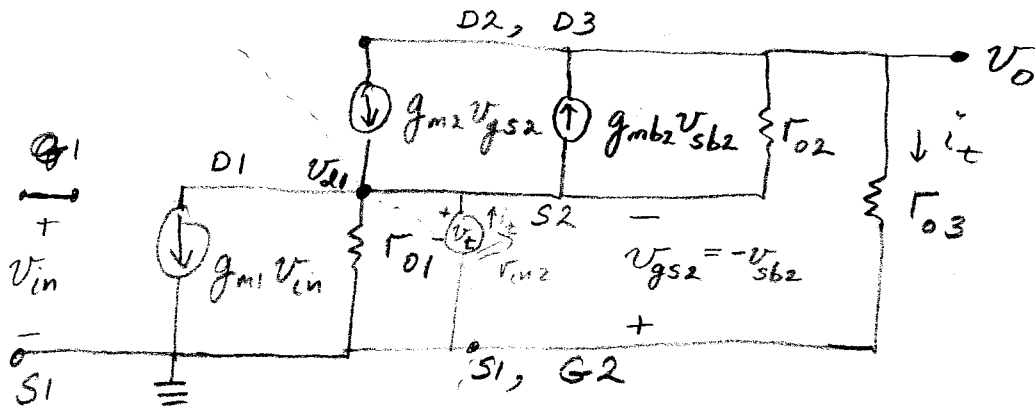
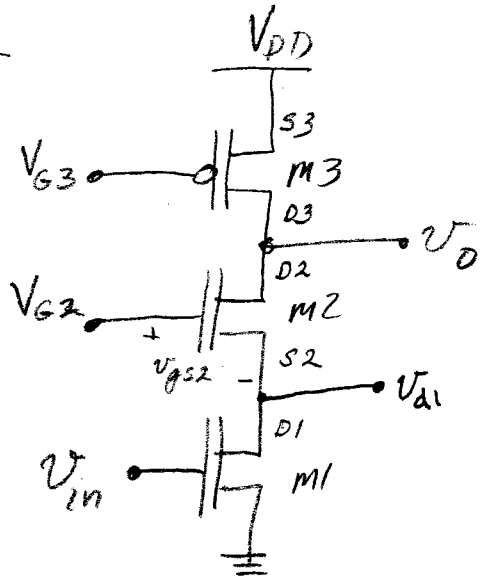
$$\begin{aligned}
 A_v &= \frac{1}{r_{o1}} + g_{m1} (1 - \chi) r_{o1} \parallel r_{o2} \\
 &\approx g_{m1} (1 - \chi) r_{o1} \parallel r_{o2}
 \end{aligned}$$

$$\begin{aligned}
 R_{in} &= \frac{r_{o2}}{A_v} \approx \frac{r_o}{g_{m1} (1 - \chi) \frac{r_o}{2}} \\
 &= \frac{2}{g_{m1} (1 - \chi)}
 \end{aligned}$$

$$R_o = r_{o1} \parallel r_{o2}$$



Cascode Amp



$$v_{d1} = -g_{m1} v_{in} r_{o1} \parallel r_{in2}$$

$$v_t = -v_{gs2} = v_{sb2}$$

$$i_t + g_{m2}(-v_t) - g_{mb2}v_t + \frac{v_o - v_t}{r_{o2}} = 0$$

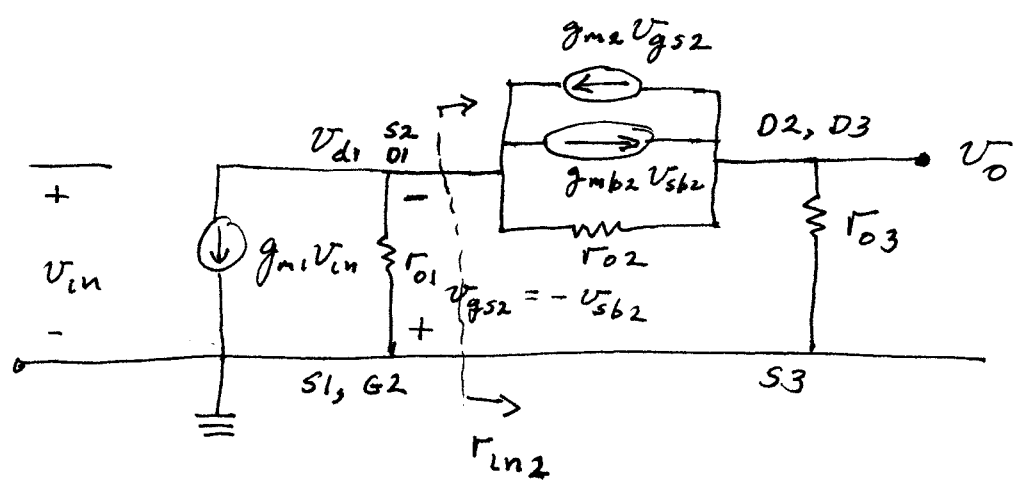
$$-\frac{v_o}{r_{o3}} - \frac{(v_o - v_t)}{r_{o2}} + g_{mb2}v_t + g_{m2}v_t = 0$$

$$v_o = i_t r_{o3}$$

$$i_t \left(1 + \frac{r_{o3}}{r_{o2}}\right) = v_t \left(g_{m2}(1+\chi) + \frac{1}{r_{o2}}\right)$$

$$r_{in2} = \frac{v_t}{i_t} = \frac{1 + \frac{r_{o3}}{r_{o2}}}{g_{m2}(1+\chi) + \frac{1}{r_{o2}}}$$

$$r_{in2} = \frac{r_{o2} + r_{o3}}{g_{m2}r_{o2}(1+\chi) + 1} \approx \frac{1}{g_{m2}(1+\chi)} \left(1 + \frac{r_{o3}}{r_{o2}}\right)$$



$$A_{v1} \equiv \frac{v_{d1}}{v_{in}} = -g_{m1} \Gamma_{o1} \parallel \frac{1}{g_{m2}} \left(1 + \frac{\Gamma_{o3}}{\Gamma_{o2}}\right)$$

$$A_{v1} \approx -\frac{g_{m1}}{g_{m2}} \left(1 + \frac{\Gamma_{o3}}{\Gamma_{o2}}\right) \quad \left(\Gamma_{o1} \gg \frac{1}{g_{m2}}\right)$$

M2 is acting as a common gate amp.

$$A_{v2} = \frac{\frac{1}{\Gamma_{o2}} + g_{m2}(1-\kappa)}{\frac{1}{\Gamma_{o3}} + \frac{1}{\Gamma_{o2}}}$$

$$i_2 = \frac{v_{d1}}{\Gamma_{in2}}; \quad v_o = i_2 \Gamma_{o3} = \frac{v_{d1}}{\Gamma_{in2}} \Gamma_{o3}$$

$$v_{d1} = A_{v1} v_{in}$$

$$v_o = \frac{A_{v1} v_{in} \Gamma_{o3}}{\Gamma_{in2}}$$

$$A_v = A_{v1} \frac{\Gamma_{o3}}{\Gamma_{in2}} =$$

$$A_{v1} = -g_{m1} \Gamma_{o1} \parallel \Gamma_{in2} = -g_{m1} \frac{\Gamma_{o1} \Gamma_{in2}}{\Gamma_{o1} + \Gamma_{in2}}$$

$$A_v = -g_{m1} \frac{\Gamma_{o1} \Gamma_{o3}}{\Gamma_{o1} + \Gamma_{in2}}$$

$$\approx -g_{m1} \frac{\Gamma_{o1} \Gamma_{o3}}{\Gamma_{o1} + \Gamma_{o2} + \Gamma_{o3}} \approx \frac{\Gamma_{o1} \Gamma_{o3}}{\Gamma_{o1} + \frac{1}{g_{m2}(1+\kappa)} (1 + \dots)}$$

$$= -g_{m1} \frac{\Gamma_{o1} \Gamma_{o3}}{\Gamma_{o1} + \frac{\Gamma_{o2} + \Gamma_{o3}}{g_{m2} \Gamma_{o2} (1+\kappa)}}$$

$$= -g_{m1} \frac{g_{m2} \Gamma_{01} \Gamma_{02}^* \Gamma_{03}}{g_{m2} \Gamma_{01} \Gamma_{02} + \frac{\Gamma_{02} + \Gamma_{03}}{(1+x)}}$$

$$\approx -g_{m1} \Gamma_{03}$$

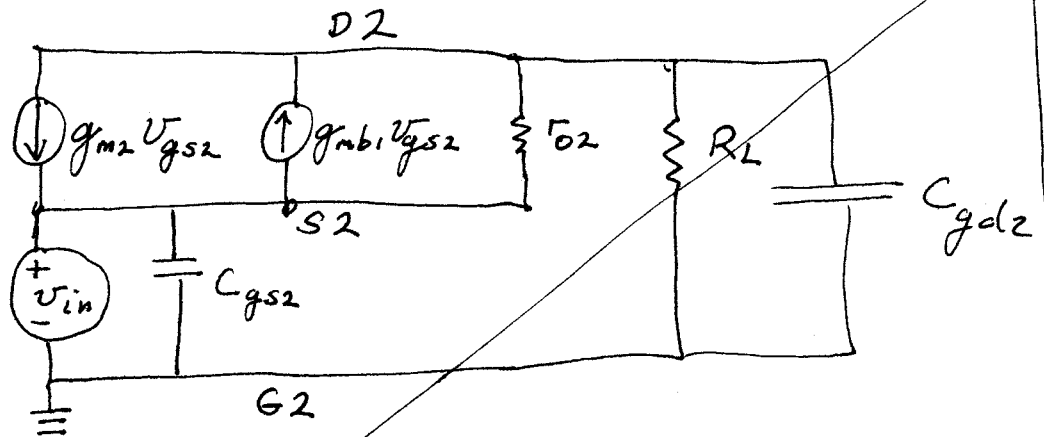
• For Com. Source w/ current mirror load

$$A_v = -g_{m1} \Gamma_{01} \parallel \Gamma_{02} \approx -g_{m1} \frac{\Gamma_0}{2}$$

⇒ The cascode \approx doubles A_v .

FREQUENCY RESPONSE

Common Gate m_2



$$(j\omega C + \frac{1}{R})^{-1} = \frac{R}{1 + j\omega RC}$$

$$Y = \frac{1 + j\omega RC}{R}$$

$$Y_m = \frac{1 + j\omega RC}{R} (1 - g_m R) \approx -g_m - j\omega g_m RC$$

$$= -g_m (1 + j\omega RC)$$

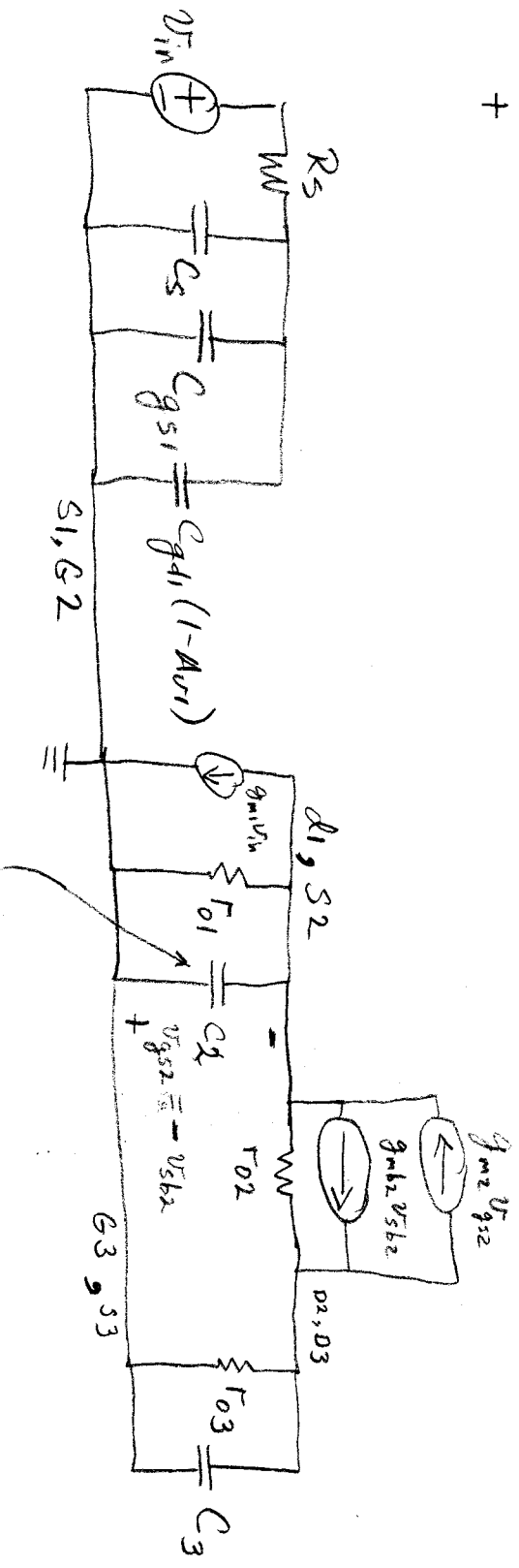
$$R_m = -\frac{1}{g_m} / (1 + j\omega RC)$$

$$Z_{in} = R_S \left(C_S + C_{gs1} + C_{gd1} \frac{g_{m1}}{g_{m2}} \left(1 + \frac{r_{o2}}{r_{o3}} \right) \right)$$

MILLER EFFECT SMALL
SINCE A_{v1} SMALL

$$Z_{r2} = C_2 r_{o1} \parallel r_{in2} = C_2 r_{o1} \parallel \frac{1}{g_{m2} \left(1 + \frac{r_{o2}}{r_{o3}} \right)} \approx C_2 \frac{1}{g_{m2}} \quad (\text{SMALL}) \text{ since } \frac{1}{g_{m2}} \text{ low } R$$

$$Z_{out} \approx r_{o3} C_3 \approx r_{o3} C_L$$



$$C_{db1} + C_{sb2} + C_{gs2} + C_{gd1} \left(1 - \frac{1}{A_{v1}} \right)$$

$$C_3 = C_{gd3} + C_{gd2} + C_{db2} + C_{db3} + C_L$$

Small

$$\frac{V_{S2}}{r_{o1}} + g_{m2} V_{S2}^2 + \frac{V_{S2} - V_{D2}}{r_{o2}} = 0$$

$$V_{S2} \left(\frac{1}{r_{o1}} + g_{m2} (1 + \lambda) + \frac{1}{r_{o2}} \right) = - \frac{V_{D2}}{r_{o2}}$$