

Rendezvous Problem in Multi-vehicle Systems: Information Relay and Local Information Based Strategies

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Abstract—Cooperative control in the presence of a limited communication range poses significant theoretical and practical challenges. Some control strategies in this area rely on information relay where all information received by any vehicle is passed to all other vehicles that it communicates with. Some other control strategies in this area are based on local neighbor-to-neighbor information exchange where each vehicle only communicates directly with all other vehicles in its communication range without information relay involved. Both information relay and local information based strategies have their strengths and limitations. In this paper, we investigate the two control strategies in the context of a multi-vehicle rendezvous application. Various aspects of the two strategies are compared in relation to convergence time and overall system performance. Strengths and limitations of both control strategies will be discussed and approaches for overcoming these limitations will be proposed.

I. INTRODUCTION

Cooperative control of multiple vehicles has received significant attention in the control and robotics communities in recent years regarding the benefits of using many inexpensive, simple systems to replace a single monolithic, expensive, and complicated system.

Much work in cooperative control assumes a static fully connected communication network, where each vehicle can communicate with any other vehicle in the team. However, real-world communication topologies are usually not fully connected. In many cases they depend on the relative position of the vehicles and on other environmental factors. For example, vehicles may move in or out of each other's communication range and the communication links between the vehicles may be established or broken randomly. Therefore, cooperative control in the presence of a limited communication range poses significant theoretical and practical challenges.

Some cooperative control strategies rely on information relay where all information received by any vehicle is passed to all other vehicles that it communicates with. Some other cooperative control strategies are based on local neighbor-to-neighbor information exchange where each vehicle only communicates directly with all other vehicles in its communication range without information relay involved. Both information relay and local information based strategies have their strengths and limitations.

The main purpose of this paper is to investigate an information relay based strategy and a local information based strategy in the context of a multi-vehicle rendezvous problem. A rendezvous problem requires multiple vehicles to arrive at a target location simultaneously, which can be thought of as a special case of the consensus problem (e.g., [1], [2], [3], [4], [5], [6], [7]) where the final consensus value is a common destination. We will compare various aspects of the two strategies in relation to convergence time and overall system performance. We will also discuss strengths and limitations of the two control strategies and propose approaches to overcome these limitations.

II. BACKGROUND AND PRELIMINARIES

It is natural to model information exchange between vehicles by directed/undirected graphs. A digraph (directed graph) consists of a pair $(\mathcal{N}, \mathcal{E})$, where \mathcal{N} is a finite nonempty set of nodes and $\mathcal{E} \in \mathcal{N}^2$ is a set of ordered pairs of nodes, called edges. As a comparison, the pairs of nodes in an undirected graph are unordered. If there is a directed edge from node v_i to node v_j , then v_i is defined as the parent node and v_j is defined as the child node. A directed path is a sequence of ordered edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \dots$, where $v_{i_j} \in \mathcal{N}$, in a digraph. An undirected path in an undirected graph is defined accordingly. A digraph is called strongly connected if there is a directed path from every node to every other nodes. An undirected graph is called connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node, except the root, has exactly one parent. A directed spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph has (or contains) a directed spanning tree if there exists a directed spanning tree being a subset of the graph. Note that the condition that a digraph has a directed spanning tree is equivalent to the case that there exists at least one node having a directed path to all the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected. The union of a group of digraphs is a digraph with nodes given by the union of the node sets and edges given by the union of the edge sets of

those digraphs.

Fig. 1 shows a directed graph with more than one possible spanning trees, but is not strongly connected. The double arrows denote one possible spanning tree with A_5 as the parent. Spanning trees with A_1 and A_4 as the parent, are also possible.

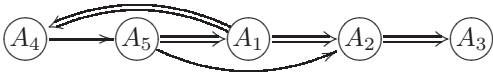


Fig. 1. A directed graph that has more than one possible spanning trees, but is not strongly connected. One possible spanning tree is denoted with double arrows.

III. PROBLEM STATEMENT

Suppose that each vehicle is described by the following dynamics:

$$\dot{\xi}_i = u_i, \quad i = 1, \dots, n, \quad (1)$$

where $\xi_i \in \mathbb{R}^2$ denotes the position of the i^{th} vehicle and $u_i \in \mathbb{R}^m$ is the control input.

We will apply the following consensus algorithm for the rendezvous problem:

$$u_i = - \sum_{j=1}^n g_{ij} k_{ij} (\xi_i - \xi_j), \quad (2)$$

where $k_{ij} > 0$, $g_{ii} \triangleq 0$, and g_{ij} is 1 if information flows from vehicle j to vehicle i and 0 otherwise, $\forall i \neq j$. Note that the parameter g_{ij} specifies the information exchange links between the vehicles and parameter k_{ij} represents a weighting factor. In this paper, we assume bidirectional communication between vehicles, which implies that $g_{ij} = g_{ji}$.

The rendezvous problem is said to be solved among the n vehicles if $\xi_i(t) \rightarrow \xi_j(t)$, $\forall i \neq j$, as $t \rightarrow \infty$. With consensus algorithm (2), the final rendezvous destination is a weighted average of the vehicles' initial positions. Note that the final rendezvous destination may be *a priori* unknown and will depend on the information exchange topologies as well as weighting factors k_{ij} .

Under a fixed information exchange topology, algorithm (2) guarantees that the n vehicles reach a rendezvous destination asymptotically if and only if the information exchange topology has a (directed) spanning tree in the case of directed information exchange [8]. In the case of undirected information exchange, the necessary and sufficient condition becomes being connected. Under switching information exchange topologies, algorithm (2) reaches consensus asymptotically if there exist infinitely many consecutive uniformly bounded time intervals such that the union of the information exchange graph across each interval has a directed spanning tree [6]. As a special case, if the communication links between the vehicles ensure that the initial communication topology is connected and the (possibly switching) communication topologies stay connected for all time, convergence to a rendezvous destination is guaranteed.

IV. RENDEZVOUS MODELS

In this section, we introduce the information relay and local information based rendezvous model.

A. Information Relay Based Rendezvous Model

An information relay based rendezvous model as described here is one where information received by one vehicle is passed by that vehicle to all other vehicles that it communicates with. This process allows each vehicle to obtain information about the position of every other vehicle in the group if the initial communication topology is connected. Conceptually, the use of all the team members' information allows for better overall decision making by each vehicle in the group.

In order to compare an information relay based rendezvous model to a local information based rendezvous model, certain parameters need to be established. The concept of communication delay for information transferal between vehicles along with the relay time for each information transfer needs to be addressed. Here we assume a limited communication range and that each vehicle would transfer as up to date information as it has gathered to every other vehicle in the group. This means that each vehicle's information would have to be relayed from one vehicle to another a number of times before all vehicles in the group have up to date information about all the other vehicles in the group. In actuality, in a group of n vehicles, information can be passed up to $n - 1$ times before the entire group has that vehicle's information considering a line configuration with each vehicle only being able to communicate with adjacent neighbors.

Understanding that each vehicle would get various sets of data, we assume that each vehicle only takes the most up to date information presented to it from another vehicle in the group. In a real world, this could be accomplished by time tagging sent data so that it could be sorted on reception. In order to make the computation more efficient, in simulation this sorting process is simply assigned an arbitrary communication time representing passing all the group information during a fixed communication interval. The shortest route for the information to be obtained is dynamically generated and the most up to date information available is used.

Another crucial assumption is that there would be no interpolation of other vehicles current position based on received information. This assumption can be easily justified by considering the inherent difficulty in projecting other vehicles movement with old information and the computational limitations applied to each vehicle. The data being passed from vehicle to vehicle over a specified time interval indicates that information passed n times will be n times the communication time old. As information is passed between vehicles, the vehicle to whom the information is associated will have moved based on information it has received from other vehicles as well. Interpolating where a vehicle would be based on arbitrarily old information in a dynamic environment does not appear to offer any real value and so the old information is applied as it is received with no reference given to its origination time. A weight could be applied to old data to fade it, but this would tend to lead the information to be more like the local information based model which ignores any information that can not be obtained directly. In light of the fact that the purpose here is to compare these two cases, the information relay case simply applies the old information in the consensus algorithm weighted equally with

the more up to date information from closer neighbors.

B. Local Information Based Rendezvous Model

The local information based rendezvous model relies on the idea of local “neighbor-to-neighbor” information exchange only. Instead of taking the time and effort to pass all the information received from one vehicle to another, each vehicle only transmits its own data to and receives data from vehicles that it directly communicates with. This direct communication reduces computational time and because of less data to be transferred during each communication exchange, speeds up communication time for faster control update times.

V. WEIGHTING FACTORS DEVELOPMENT

In the rendezvous problem, vehicles may move in or out of each other’s communication range. As a result, the communication links between the vehicles may be established or broken randomly. It is relevant to study how given connectivity patterns between the vehicles can be maintained. The problem of preserving connectivity constraints has been discussed in [9], [10] recently. As a preliminary study, we will show how weighting factors k_{ij} can be adjusted dynamically to guarantee that if the initial communication topology is connected, the (possibly switching) communication topologies stay connected for all time in this section. For most work in consensus algorithms, weighting factors have been either assumed to be constant or given consideration only so far as to identify that it may be required without actually developing a weighting algorithm for application.

To understand why communication links between the vehicles may be established or broken, the consensus algorithm must be explained. The simplest explanation can be made for the one dimensional case with a fixed communication range. Comparing this algorithm to a tug of war where the strength of a person is how far they are away from the middle of the rope, naturally the side farther away will pull the vehicle that way. With only one vehicle on each side, this works well without any handicap placed on either side because the maximum pull that can be exerted by either side is at the limit of the communication range of the vehicle. This guarantees that communication links, once established, will never be broken.

The problem that arises with this simple idea comes when the number of vehicles to one side increases more than the number of vehicles on the other. Given a situation where n and m represent the number of vehicles on either side and $n < m$, if every vehicle exists a distance r from the center, then the pull on the m is m/n times greater than the pull from the n side. Given the situation of a “weak” link where only one vehicle exists on one side and near the boundary and multiple vehicles exist on the far side of that vehicle, the pull from the opposite sides can break this link and cause the group to fail to rendezvous. Rendezvous is achievable in isolated groups, but overall rendezvous of the group cannot be achieved.

Understanding that the number of vehicles to each side was the root cause of this link breakage, a weighting factor adjustment could be formulated to compensate. The key issue is that the pull from the side with more vehicles is, in fact, heavily

weighted by the number of vehicles. Once this is realized, the purpose of the weighting factor becomes to even the weighting on the pull to the side with a larger number of vehicles to that of the lesser side. By scaling the distances to vehicles on the heavy side by the ratio n/m with $n < m$ as defined earlier, the maximum pull to the heavy side is limited to the maximum pull to the weak side. This is shown in the following:

Let m and n be the number of vehicles on either side of some center vehicle within some communication range r with $m \geq n$. Also let $x_{mi}, x_{ni} \leq r$ be the distances of the center vehicle to vehicles on either side. The following two equations hold:

$$\begin{aligned} \max_{x_{ni}} \sum_{i=1}^n x_{ni} &= nr \\ \max_{x_{mi}} \sum_{i=1}^n (n/m)x_{mi} &= (n/m)mr = nr. \end{aligned}$$

The scaling of the weighting factors prevents this “weak” link breakage and ensures rendezvous for the algorithm given any number of vehicles in any initial configuration where the communication topology is connected initially.

VI. RESULTS

In order to test the overall effectiveness of the two models, a simulation was devised to contain an arbitrary number of vehicles. This simulation spaced the vehicles in a mainly linear separation to test the overall convergence characteristic of each model.

Intuitively, in real world applications, the local information based model will have faster update times due to shorter communication times and lower processing costs. However, to simply compare the worth of the group knowledge in rendezvous with local information based rendezvous, both models are assigned the same update and communication times. This allows for comparison of the models without argument based on the technology employed to get the information or process the obtained information.

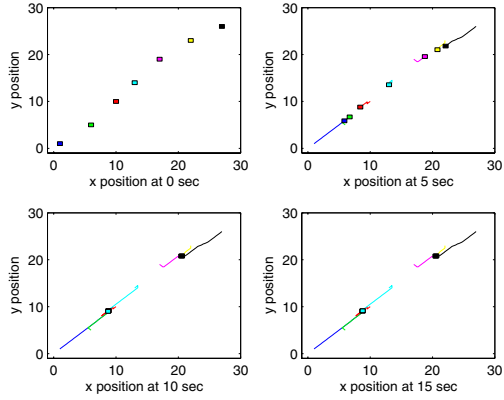
In order to accurately compare the two models, we will consider three scenarios where 7 vehicles are required to reach a rendezvous location. For each scenario, we will compare the performance of the two models under three different communication times of 0.1, 1, and 3 seconds respectively. It is assumed that the initial communication topology is connected.

In the first scenario, the 7 vehicles are spaced out to be nearly at the far reaches of the communication range. The simulation is then run at various communication times to represent the time it takes to accumulate information from neighbors with various communication protocols. Table I shows the convergence time of the two models in this scenario, where NC denotes that convergence is not achieved. Note that the local information based model converges faster than the information relay based model with communication times of 0.1 and 1 sec respectively. The longer the communication delay however, the local information based model loses its advantage quickly to the information relay based model. Finally the communication topology breaks down completely (i.e., not connected) and the

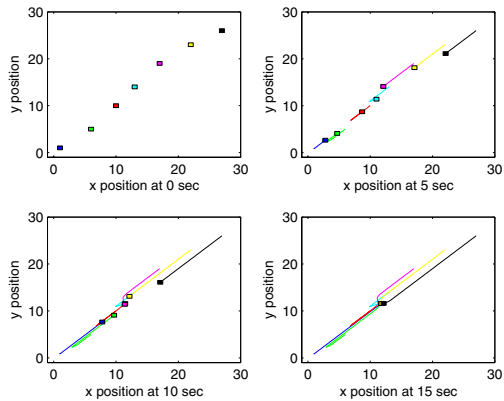
TABLE I
CONVERGENCE TIMES FOR TWO MODELS UNDER NEARLY EVENLY SPACED
SEPARATION.

models / communication times (sec)	0.1	1	3
Local Information Based Model	13.1	14.5	NC
Information Relay Based Model	14.1	14.6	14.8

team forms two distinct subgroups and each subgroup reaches a different rendezvous location with communication time of 3 sec as shown in Fig. 2.



(a) Local Information Model



(b) Information Relay Model

Fig. 2. Comparison of the two models with communication time of 3 sec under a linear spread.

The convergence failure with communication time of 3 sec for the local information based model is due to the lack of properly formulated weighting factors to preserve communication links. This problem becomes more obvious as the number of vehicles is increased. The consensus algorithm essentially works by pulling the very outside vehicles in quickly and slowly collapsing on the center as the ends are drawn in. This creates a bunching of vehicles on the ends in the one dimensional case and as numbers increase, allows for large numbers of vehicles to enter the communication range of interior vehicles before the vehicles from the other end converge to

the same location. This situation creates a “weak” link in the communication topology. Interestingly, the information relay based model does not exhibit this “weak” link tendency as readily, because each vehicle has a knowledge of the entire group and will seek to always center itself in the entire group instead of just locally. The “weak” link only occurs when a vehicle is drawn away to a local center by near neighbors instead of the constant progression toward the group center. This is not evident when information about the entire group is obtainable but can be achieved during the initial formation of the communication topology for the group.

The second scenario is devised to test the “weak” link tendencies of the consensus algorithm. Poor initial conditions are used to create a weak link in the communication topology at initialization of the group. This also allows for a better demonstration of the importance and effectiveness of the weighting algorithm. This situation creates the possibility of breakdown in the communication topology for the information relay based model and the local information based model. As the communication delay increases, the information relay based model’s robustness to communication topology breakdown is tested and the local information based model is shown to fail even with small communication delay of 0.1 seconds as shown in Fig. 3. This demonstrates just how fragile the communication links in the group can be as the number of vehicles in the group are increased. The “weak” link tendency of the consensus algorithm finally arises in the information relay based model when the relay time takes too long and the initial conditions break a communication link before the entire group’s information can be obtained as shown in Fig. 4.

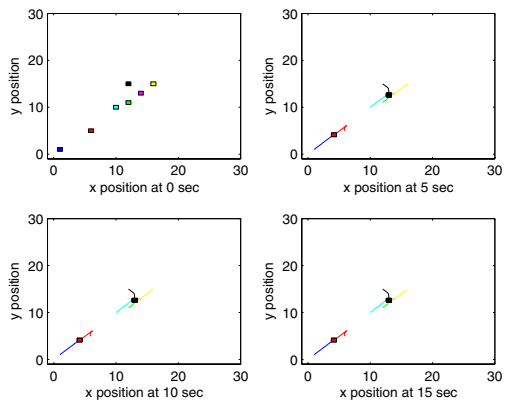
In the third scenario, we introduce dynamically changing weighting factors developed in Section V. As shown in Fig. 5, rendezvous is achieved under the same condition as in Fig. 3.

Table II compares the convergence time of the two models in the second and third scenario. Note that the introduction of dynamically changing weighting factors slows down the rendezvous for both models.

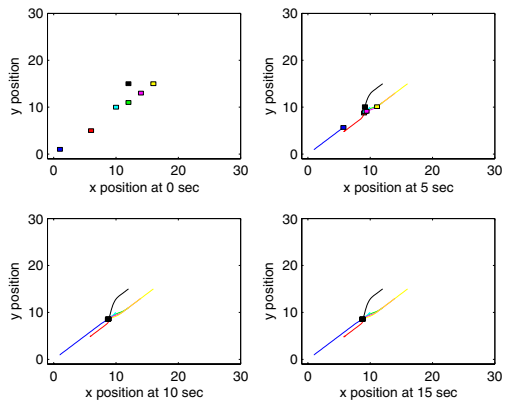
VII. CONCLUSIONS AND FURTHER INVESTIGATION

Given the same initial conditions, rendezvous of the local information based model is generally faster than the information relay based model in cases where rendezvous is achieved. Rendezvous becomes is %20 – %30 faster for the local information case depending on the communication update time with the weighting factors dynamically adjusted. This is most likely due to the use of old information from vehicles far away in the information relay based model. This old information leads to an inaccurate estimate of the dynamic center of the group.

The consensus algorithm employed here is also shown to converge with the use of properly structured weighting factors. By structuring the weighting factors to compensate for the number of vehicles, the consensus algorithm is able to achieve rendezvous in all cases where the communication topology is initially connected. This does slow the overall convergence time of the group because it limits the pull on vehicles located closer to the center of the group, but is required to guarantee convergence.



(a) Local Information Model



(b) Information Relay Model

Fig. 3. Comparison of the two models with communication time of 0.1 sec under poor initial conditions.

TABLE II

CONVERGENCE TIMES UNDER POOR INITIAL CONDITIONS

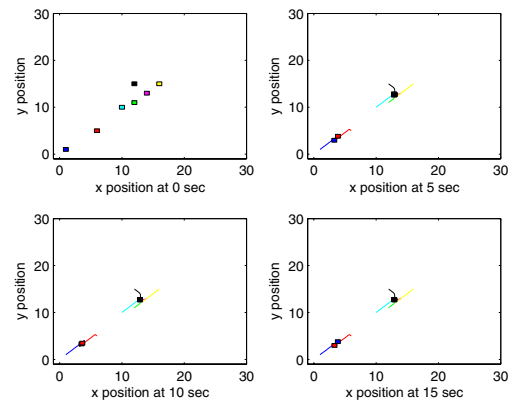
models / communication time (sec)	0.1	1	3
Local Information Based Model	NC	NC	NC
Information Relay Based Model	9.4	9.7	NC
Local Model with Dynamic Weighting	12.8	13.9	14.9
Information Relay Model with Dynamic Weighting	18.9	18.1	19.1

The speed and reliability of a given communication system has also been shown to be an important factor on the speed of convergence. Faster communication times allow for faster, more accurate control updates with increased robustness to communication tree breakdown.

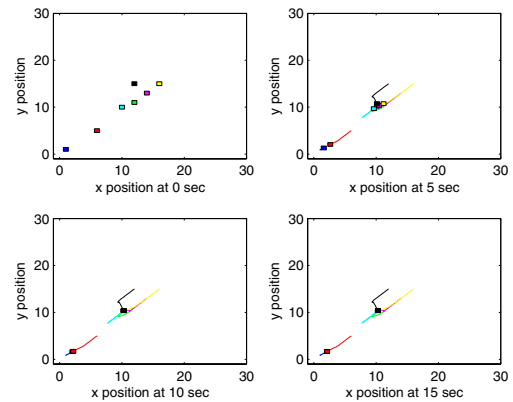
Further research in this area will be to extend the results shown here in simulation to an application on a group of actual vehicles to test cases of communication delay and packet loss. The computational constraints discussed for the router case will thereby be more systematically quantified.

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(a) Local Information Model



(b) Information Relay Model

Fig. 4. Comparison of the two models with communication time of 3 sec under poor initial conditions.

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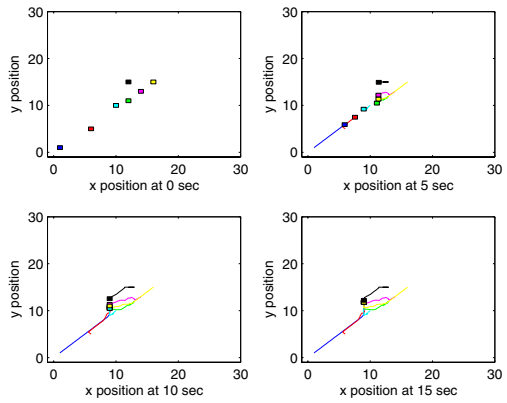
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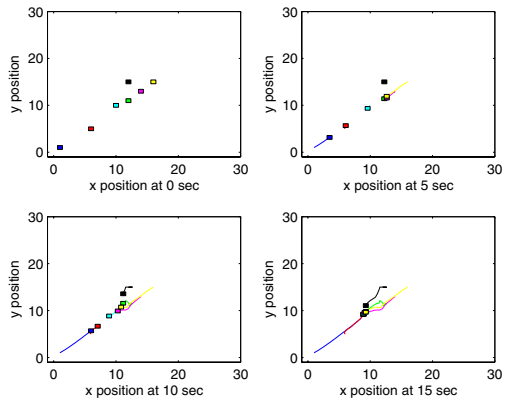
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(a) Local Information Model



(b) Information Relay Model

Fig. 5. Comparison of the two models with communication time of 3 sec under poor initial conditions, where the weighting factors are dynamically adjusted.