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VIRTUAL STRUCTURE BASED SPACECRAFT FORMATION CONTROL WITH FORMATION FEEDBACK

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Abstract

Formation control for multiple vehicles has become an active research area in recent years. Generally there are three approaches to this problem, namely leader-following, behavioral, and virtual structure approaches. In this paper, formation control ideas for multiple spacecraft using virtual structure approach are presented. If there is no formation feedback from spacecraft to the virtual structure, the spacecraft will get out of formation when the virtual structure moves too fast for the spacecraft to track or the total system must sacrifice convergence speed in order to keep the spacecraft in formation. The spacecraft may also get out of formation when the system is affected by internal or external disturbances. To overcome these drawbacks, a novel way of introducing formation feedback from spacecraft to the virtual structure is illustrated in detail. An application of these ideas to multiple spacecraft interferometers in deep space is given.

1 Introduction

Formation control for multiple vehicles has become an active research area in recent years. Applications in this area include the coordination of multiple robots, UAVs, satellites, aircraft, and spacecraft.[1–5] Many papers have been published to deal with different control strategies, schemes, and applications of multiple vehicle control. While the applications are different, the fundamental approaches to formation control are similar: the common theme being the coordination of multiple vehicles to accomplish an objective.

Generally there are three approaches to multi-vehicle coordination reported in the literature, namely leader-following, behavioral, and virtual structure approaches. In the leader following approach,[1,4,6] one of the agents is designated as the leader, with the rest of the agents designated as followers. The leader tracks a pre-defined trajectory, and the followers track a transformed version of the leader's states. The advantage of leader following is that group behavior is directed by specifying the behavior of a single quantity: the leader. The disadvantage is that there is no explicit feedback to the formation. Another disadvantage is that the leader is a single point of failure for the formation. In the remainder of the paper, we use the following definition:[7]
The group feedback from vehicles to the formation is referred to as formation feedback.

In the behavioral approach,[8–10] several desired behaviors are prescribed for each agent. The basic idea is to make the control action of each agent a weighted average of the control for each behavior. Possible behaviors include collision avoidance, obstacle avoidance, goal seeking, and formation keeping. The advantage of the behavioral approach is that it is natural to derive control strategies when agents have multiple competing objectives. In addition, there is explicit feedback to the formation since each agent reacts according to the position of its neighbors. Another advantage is that the behavioral approach lends itself naturally to a decentralized implementation. The disadvantage is that the group behavior cannot be explicitly defined, rather the group behavior is said to "emerge". In addition, it is difficult to analyze the behavioral approach mathematically and guarantee its group stability.

In the virtual structure approach,[2,7,11] the entire formation is treated as a single structure. The virtual structure can evolve as a rigid body in a given direction with some given orientation and maintain a rigid geometric relationship among multiple vehicles. The advantage of the virtual structure approach is that it is fairly easy to prescribe a coordinated behavior of the group. The disadvantage is that requiring the formation to act as a virtual structure limits the class of potential applications of this approach. Another disadvantage is that its current development lends itself to a centralized control implementation.

In the case of the application of synthesizing multiple spacecraft interferometers in deep space, it is desirable to have a constellation of spacecraft act as a single rigid body in order to image stars in deep space. As a result, it is suitable to choose the virtual structure approach to accomplish formation maneuvers. In the remainder of the paper we use the term *formation* and *virtual structure* interchangeably.

In general, there is a dilemma when there is no feedback applied from spacecraft to the virtual structure. On the one hand, if the virtual structure evolves too fast, the spacecraft cannot track their desired trajectories accurately and they will get out of formation. On the other hand, the virtual structure might be slowed down sufficiently to allow the spacecraft to track their trajectories accurately. However, this results in unreasonably slow formation dynamics. Also, when performing formation maneuvers, the total system is often disturbed by internal or external factors. For example, some spacecraft may fail for a period of time due to mechanical or electrical malfunctions or may disintegrate from the formation due to external disturbances in deep space. If there is no formation feedback from spacecraft to the formation, the failed or disintegrated spacecraft will be left behind while the other spacecraft still keep moving towards their final goals, and the entire system cannot adjust to maintain formation. Formation feedback from spacecraft to the virtual structure provides a good compromise between formation keeping and convergence speed as well as improved group stability and robustness.

In Ref. 11 the authors introduce a coordination architecture for spacecraft formation control which subsumes leader-following, behavioral, and virtual structure approaches to the multi-agent coordination problem. In Ref. 11 formation maneuvers are easily prescribed and group stability is guaranteed, but formation feedback is not included. In Ref. 7 formation feedback is used for the coordinated control problem for multiple robots. This paper is aimed as a further development for Ref. 11 and Ref. 7. The main contribution of this paper is to propose a novel idea of introducing formation feedback from spacecraft to the virtual structure and apply this idea to the spacecraft interferometry problem so that formation keeping is guaranteed throughout the maneuver and the total system robustness is improved. The decentralized control implementation of the virtual structure approach needs to be explored in the future.

The outline of this paper is as follows. In section 2 we introduce spacecraft dynamics. In section 3 we describe virtual structure equations of motion for spacecraft. In section 4 we present formation control strategies with formation feedback. In section 5 we illustrate simulation results for spacecraft formation control. By comparing the results with and without formation feedback, we demonstrate the superiority of the system with formation feedback over the one without formation feedback.

2 Spacecraft Dynamics

In this paper each spacecraft is modeled as a rigid body, with r_i, v_i, q_i and ω_i representing the position, velocity, unit quaternion, and angular velocity of the ith spacecraft, where r_i, v_i, and ω_i are vectors and q_i is a unit quaternion used to represent the attitude of a rigid body. We will represent r_i, v_i, and ω_i in terms of their components in the inertial frame C_O. For simplicity, we use the same symbol to denote a vector and its corresponding coordinate frame representation in the remainder of the paper.

Euler's theorem for rigid body rotations states that "the general displacement of a rigid body with one point fixed is a rotation about some axis." Let z represent a unit vector in the direction of rotation, called the eigenaxis, and let ϕ represent the angle of rotation about z, called the Euler angle. The unit quaternion representing this rotation is given by $q = [z^T sin(\phi/2), cos(\phi/2)]^T = [\vec{q}^T, \bar{q}]^T$, where $\vec{q}$ is a 3×1 vector with its components represented in the given coordinate frame and $\bar{q}$ is a scalar. It is easy to see that q and $\quad q$ represent the same attitude. To simplify our discussion in the remainder of the paper, we assume $\bar{q} \geq 0$.

Given a vector p, the corresponding cross-product operation $p^\times$ is defined as

$$p^\times = \begin{bmatrix} 0 & \quad p_3 & p_2 \\ p_3 & 0 & \quad p_1 \\ \quad p_2 & p_1 & 0 \end{bmatrix},$$

where $p = [p_1, p_2, p_3]^T$ in terms of its components in the given coordinate frame.

If we let $C_{O'O}$ be a rotation matrix that represents the orientation of the frame $C_{O'}$ with respect to C_O, then $r_{O'} = C_{O'O} r_O$, where $r_{O'}$ and r_O represent vector r in terms of its components in the frame $C_{O'}$ and C_O separately. The relationship between the unit quaternion q represented in the frame C_O and the rotation matrix $C_{O'O}$ is defined as[12]

$$C_{O'O} = (2\bar{q}^2 \quad 1)I + 2\vec{q}\vec{q}^T \quad 2\bar{q}\vec{q}^\times.$$

The relationship between two rotation matrices $C_{O'O}$ and $C_{OO'}$ is given as

$$C_{OO'} = C_{O'O}^T = (2\bar{q}^2 \quad 1)I + 2\vec{q}\vec{q}^T + 2\bar{q}\vec{q}^\times.$$

The multiplication of two quaternions is given by the formula $q_a q_b = Q(q_b) q_a$, where $Q(q_b) = \begin{pmatrix} \bar{q}_b I \quad \vec{q}_b^\times & \vec{q}_b \\ \quad \vec{q}_b^T & \bar{q}_b \end{pmatrix}$. Let q^* be the inverse of the quaternion q given by $q^* = \begin{pmatrix} \vec{q} \\ \bar{q} \end{pmatrix}^* = \begin{pmatrix} \quad \vec{q} \\ \bar{q} \end{pmatrix}$. Suppose that the unit quaternions q and q^d represent the actual attitude and the desired attitude of a rigid body respectively, then the attitude error is given by $q_e = q^* q^d = \begin{pmatrix} \vec{q}_e \\ \bar{q}_e \end{pmatrix}$.

The translational dynamics for the spacecraft are

$$\begin{aligned} \dot{r}_i &= v_i \\ M_i \dot{v}_i &= f_i, \end{aligned} \tag{1}$$

where M_i is the mass of the ith spacecraft, and f_i is the control force.

The rotational dynamics for the spacecraft are

$$\begin{aligned} \dot{\vec{q}}_i &= \frac{1}{2}\omega_i \times \vec{q}_i + \frac{1}{2}\bar{q}_i \omega_i \\ \dot{\bar{q}}_i &= \frac{1}{2}\omega_i \cdot \vec{q}_i \\ J_i \dot{\omega}_i &= \omega_i \times J_i \omega_i + \tau_i, \end{aligned} \tag{2}$$

where J_i is the moment of inertia of the ith spacecraft, and τ_i is the control torque on the ith spacecraft.

3 Virtual Structure Equations Of Motion For Spacecraft

In the virtual structure approach, we treat the entire formation as a rigid body with place-holders fixed in the formation to represent the desired position and attitude for each spacecraft. As the virtual structure evolves in time, the place-holders trace out trajectories for each spacecraft to track. The relative orientation of each place-holder within the formation is fixed with respect to each other.[7] The control is derived in four steps: first, the desired dynamics of the virtual structure are defined, second, the motion of the virtual structure is translated into the desired motion for each spacecraft, third, tracking controls for each spacecraft are derived, and finally, formation feedback is introduced from each spacecraft to the virtual structure.

The coordinate frame geometry is shown in Figure 1. Frame C_O is an inertial frame. Since the formation can be thought of as a rigid body with inertial position r_F, velocity v_F, attitude q_F, and angular velocity ω_F, we define the formation reference frame C_F located at r_F with an orientation given by q_F with respect to the inertial frame C_O. We also have one reference frame C_i imbedded in each spacecraft to represent the configuration of each spacecraft. Each spacecraft can be represented either by position r_i, velocity v_i, unit quaternion attitude q_i, and angular velocity ω_i with respect to the inertial frame C_O or by r_{iF}, v_{iF}, q_{iF}, and ω_{iF} with respect to the formation reference frame C_F.

Virtual structure equations of motion for each place-holder, that is, the desired motion for each spacecraft are

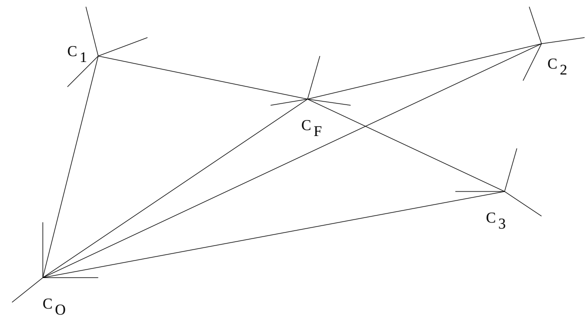

Figure 1: Coordinate frame geometry.

given by

$$\begin{aligned} r_i^d(t) &= r_F(t) + C_{OF}(t) r_{iF}^d(t) \\ v_i^d(t) &= v_F(t) + C_{OF}(t) v_{iF}^d(t) \\ &\quad + \omega_F(t) \times (C_{OF}(t) r_{iF}^d(t)) \\ q_i^d(t) &= q_F(t) q_{iF}^d(t) \\ \omega_i^d(t) &= \omega_F(t) + C_{OF}(t)\omega_{iF}^d(t), \end{aligned} \tag{3}$$

where $C_{OF}(t)$ is the rotation matrix of the frame C_O with respect to C_F, and superscript d above a vector means a desired state for each spacecraft. C_{OF} is given as

$$C_{OF} = C_{FO}^T = (2\bar{q}_F^2 \quad 1)I + 2\vec{q}_F \vec{q}_F^T + 2\bar{q}_F \vec{q}_F^{\times}.$$

Group maneuvers that preserve formation shape can be achieved as a succession of elementary formation maneuvers. Therefore, we will introduce virtual structure equations of motion for spacecraft via elementary formation maneuvers. The elementary formation maneuvers include translations, rotations, and expansions/contractions.

Let $\xi(t) = [\xi_1(t), \xi_2(t), \xi_3(t)]^T$ with its components represent the expansion/contraction rates along each formation reference frame axis. An expansion/contraction matrix is defined as $\Xi(t) = diag(\xi(t))$, which is a diagonal matrix.

Generally all parameters in (3) can vary with time. However, if we are concerned with formation maneuvers that preserve the overall formation shape, r_{iF}^d, v_{iF}^d, q_{iF}^d, and ω_{iF}^d are constant. To realize elementary formation maneuvers, we can vary r_F and v_F to translate the formation, vary q_F (q_F can be transformed to the rotation matrix C_{OF}.) and ω_F to rotate the formation, and replace r_{iF}^d in the first and second equations in (3) by $\Xi(t) r_{iF}^d$ and replace v_{iF}^d in the second equation in (3) by $\dot{\Xi}(t) r_{iF}^d$ to expand or contract the formation. Arbitrary formation maneuvers can be realized by varying $r_F(t)$, $v_F(t)$, $q_F(t)$, $\omega_F(t)$, $\Xi(t)$, and $\dot{\Xi}(t)$ simultaneously. In the case of preserving the overall formation shape during formation maneuvers, the equations of motion are given as

$$\begin{aligned} r_i^d(t) =& r_F(t) + C_{OF}(t)\Xi(t)r_{iF}^d \\ v_i^d(t) =& v_F(t) + C_{OF}(t)\dot{\Xi}(t)r_{iF}^d \\ &+ \omega_F(t) \times (C_{OF}(t)\Xi(t)r_{iF}^d) \\ q_i^d(t) =& q_F(t)q_{iF}^d \\ \omega_i^d(t) =& \omega_F(t). \end{aligned} \tag{4}$$

Note that ω_{iF}^d is zero since q_{iF}^d is constant.

The derivatives of the desired states are given by

$$\begin{aligned} \dot{r}_i^d(t) =& \dot{r}_F(t) + \dot{C}_{OF}(t)\Xi(t)r_{iF}^d + C_{OF}(t)\dot{\Xi}(t)r_{iF}^d \\ \dot{v}_i^d(t) =& \dot{v}_F(t) + \dot{C}_{OF}(t)\dot{\Xi}(t)r_{iF}^d + C_{OF}(t)\ddot{\Xi}(t)r_{iF}^d \\ &+ \dot{\omega}_F(t) \times (C_{OF}(t)\Xi(t)r_{iF}^d) \\ &+ \omega_F(t) \times (\dot{C}_{OF}(t)\Xi(t)r_{iF}^d + C_{OF}\dot{\Xi}(t)r_{iF}^d) \\ \dot{q}_i^d(t) =& \dot{q}_F(t)q_{iF}^d \\ \dot{\omega}_i^d(t) =& \dot{\omega}_F(t). \end{aligned} \tag{5}$$

From (4), we can see that if the velocity of the formation is zero, that is, $v_F(t) = 0$, $\omega_F(t) = 0$, and $\dot{\xi}(t) = 0$, then the desired velocity of each spacecraft is zero, that is, $v_i^d(t) = 0$ and $\omega_i^d(t) = 0$. Also, if both the velocity and acceleration of the formation are zero, that is, $v_F(t) = 0$, $\omega_F(t) = 0$, $\dot{\xi}(t) = 0$, $\dot{v}_F(t) = 0$, $\dot{\omega}_F(t) = 0$, and $\ddot{\xi}(t) = 0$, then both the desired velocity and acceleration of each spacecraft are zero, that is, $v_i^d(t) = 0$, $\omega_i^d(t) = 0$, $\dot{v}_i^d(t) = 0$, and $\dot{\omega}_i^d(t) = 0$.

4 Formation Control Strategies With Formation Feedback

Let $X_i = [r_i^T, v_i^T, q_i^T, \omega_i^T]^T$ and $X_i^d = [{r_i^d}^T, {v_i^d}^T, {q_i^d}^T, {\omega_i^d}^T]^T$ represent the states and desired states for the ith spacecraft with respect to the inertial frame C_O repectively. Let $X_{iF} = [r_{iF}^T, v_{iF}^T, q_{iF}^T, \omega_{iF}^T]^T$ and $X_{iF}^d = [{r_{iF}^d}^T, {v_{iF}^d}^T, {q_{iF}^d}^T, {\omega_{iF}^d}^T]^T$ represent the states and desired states for the ith spacecraft with respect to the formation frame C_F respectively. Let $X_F = [r_F^T, v_F^T, q_F^T, \omega_F^T, \xi_F^T, \dot{\xi}_F^T]^T$ and $X_F^d = [{r_F^d}^T, {v_F^d}^T, {q_F^d}^T, {\omega_F^d}^T, {\xi_F^d}^T, {\dot{\xi}_F^d}^T]^T$ represent respectively the states and desired states for the virtual structure with respect to the inertial frame C_O. We know that X_{iF}^d is constant since we want to preserve the formation shape during the group maneuvers.

The aim of the formation maneuver is to evolve $X_F(t)$ to $X_F^d(t)$ while guaranteeing that $X_i(t)$ tracks $X_i^d(t)$. Accordingly, a formation maneuver is defined as follows: *A formation maneuver is asymptotically achieved if $X_F(t) \to X_F^d(t)$ and $X_i(t) \to X_i^d(t)$ as $t \to \infty$.*

The control law for each spacecraft without formation feedback is given by the following lemma.

Lemma 4.1 *Let $X = [r^T, v^T, q^T, \omega^T]^T$, $X^d = [{r^d}^T, {v^d}^T, {q^d}^T, {\omega^d}^T]^T$, and let*

$$\begin{pmatrix} \dot{r} \\ \dot{v} \\ \dot{q} \\ J\dot{\omega} \end{pmatrix} = \begin{pmatrix} \dot{v} \\ K_r(r \quad r^d) \quad K_v(v \quad v^d) \\ \frac{1}{2}\Omega(\omega)q \\ \omega \times J\omega + K_q\vec{q}_e \quad K_\omega(\omega \quad \omega^d) \end{pmatrix}, \tag{6}$$

where $\Omega(\omega) = \begin{pmatrix} \omega^\times & \omega \\ \omega^T & 0 \end{pmatrix}$ and $q_e = q^* q^d$.

If

1. $K_r = K_r^T > 0$, $K_v = K_v^T > 0$, $K_q = K_q^T > 0$, $K_\omega = K_\omega^T > 0$,
2. $\ddot{r}^d \in L_2[0,\infty) \cap L_\infty[0,\infty)$,
3. $\|\dot{\omega}^d\| + \|\omega^d\| \in L_2[0,\infty) \cap L_\infty[0,\infty)$,

then $\|X \quad X^d\| \to 0$ as $t \to \infty$.

Proof: see Ref. 11 and Ref. 13.

In the case of the control law for the virtual structure, we need to add two equations to (6) since ξ and $\dot{\xi}$ are used to represent the expansion/contraction rate of the formation, and pairs (K_r, K_v), (k_q, K_ω), $(K_\xi, K_{\dot{\xi}})$ correspond to translation, rotation, and expansion/contraction gains for the formation respectively. We also assume that q_F^d and ω_F^d satisfy the rotational dynamics. Note that in the simple case when X_F^d is constant, the rotational dynamics is satisfied obviously. The control law for the virtual structure is given as follows.

Lemma 4.2 *Let*

$$\begin{pmatrix} \dot{r}_F \\ \dot{v}_F \\ \dot{q}_F \\ \dot{\omega}_F \\ \dot{\xi}_F \\ \ddot{\xi}_F \end{pmatrix} = \begin{pmatrix} \dot{v}_F \\ \dot{v}_F^d \quad K_r(r_F \quad r_F^d) \quad K_v(v_F \quad v_F^d) \\ \frac{1}{2}\Omega(\omega_F)q_F \\ \dot{\omega}_F^d + k_q\vec{q}_e \quad K_\omega(\omega_F \quad \omega_F^d) \\ \dot{\xi}_F \\ \ddot{\xi}_F^d \quad K_\xi(\xi_F \quad \xi_F^d) \quad K_{\dot{\xi}}(\dot{\xi}_F \quad \dot{\xi}_F^d) \end{pmatrix}, \tag{7}$$

where $q_e = q_F^* q_F^d$.

If K_r, K_v, K_ω, K_ξ, and $K_{\dot{\xi}}$ are symmetric positive definite matrices, and k_q is a positive scalar, then $\|X_F \quad X_F^d\| \to 0$ as $t \to \infty$.

Proof: We can rewrite the second equation in (7) as

$$\ddot{r}_F \quad \ddot{r}_F^d = \quad K_r(r_F \quad r_F^d) \quad K_v(\dot{r}_F \quad \dot{r}_F^d).$$

Let $\tilde{r}_F = r_F \quad r_F^d$, then $\ddot{\tilde{r}}_F = \quad K_r\tilde{r}_F \quad K_v\dot{\tilde{r}}_F$. Since K_r and K_v are positive definite, it is obvious to see that

$\left\|r_F - r_F^d\right\| \to 0$ and $\left\|v_F - v_F^d\right\| \to 0$ asymptotically as $t \to \infty$.

Rewriting the third equation in (7) as

$$\begin{aligned} \dot{\vec{q}}_F =& \frac{1}{2}(\bar{q}_F \omega_F - \omega_F \times \vec{q}_F) \\ \dot{\bar{q}}_F =& -\frac{1}{2}\omega_F^T \vec{q}_F. \end{aligned} \tag{8}$$

Based on the assumption above, this equation is also true for q_F^d, so we can get

$$\begin{aligned} \dot{\vec{q}}_F^d =& \frac{1}{2}(\bar{q}_F^d \omega_F^d - \omega_F^d \times \vec{q}_F^d) \\ \dot{\bar{q}}_F^d =& -\frac{1}{2}\omega_F^{d^T} \vec{q}_F^d. \end{aligned} \tag{9}$$

Let $\tilde{q}_F = q_F - q_F^d$ and $\tilde{\omega}_F = \omega_F - \omega_F^d$. From (8) and (9), we know that

$$\begin{aligned} \dot{\tilde{\vec{q}}}_F =& \frac{1}{2}(\bar{q}_F \omega_F - \omega_F \times \vec{q}_F - \bar{q}_F^d \omega_F^d + \omega_F^d \times \vec{q}_F^d) \\ \dot{\tilde{\bar{q}}}_F =& -\frac{1}{2}(\omega_F^T \vec{q}_F - \omega_F^{d^T} \vec{q}_F^d). \end{aligned} \tag{10}$$

Let $V_1 = \tilde{q}_F^T \tilde{q}_F$, $V_2 = \frac{1}{2}\tilde{\omega}_F^T \tilde{\omega}_F$, and consider the Lyapunov function candidate:

$$V = k_q V_1 + V_2.$$

Differentiating V_1 , we get

$$\dot{V}_1 = 2\tilde{q}_F^T \dot{\tilde{q}}_F = \tilde{\omega}_F^T (\bar{q}_F^d \vec{q}_F - \vec{q}_F^d \times \vec{q}_F - \bar{q}_F \vec{q}_F^d).$$

After some manipulation, we also know that

$$q_e = q_F^* q_F^d = \begin{pmatrix} -\bar{q}_F^d \vec{q}_F + \vec{q}_F^d \times \vec{q}_F + \bar{q}_F \vec{q}_F^d \\ \vec{q}_F^{d^T} \vec{q}_F + \bar{q}_F^d \bar{q}_F \end{pmatrix},$$

which means that $\vec{q}_e = -\bar{q}_F^d \vec{q}_F + \vec{q}_F^d \times \vec{q}_F + \bar{q}_F \vec{q}_F^d$. Thus, $\dot{V}_1 = -\tilde{\omega}_F^T \vec{q}_e$.

Rewriting the fourth equation in (7) as $\dot{\tilde{\omega}}_F = k_q \vec{q}_e - K_\omega \tilde{\omega}_F$, and differentiating V_2 , we can arrive at $\dot{V}_2 = \tilde{\omega}_F^T \dot{\tilde{\omega}}_F = \tilde{\omega}_F^T (k_q \vec{q}_e - K_\omega \tilde{\omega})$. Therefore, $\dot{V} = k_q \dot{V}_1 + \dot{V}_2 = -\tilde{\omega}_F^T K_\omega \tilde{\omega}_F$.

Let $\Omega = \{(\tilde{q}_F, \tilde{\omega}_F) | \dot{V} = 0\}$, and $\bar{\Omega}$ be the largest invariant set contained in Ω . On $\bar{\Omega}$, $\dot{V} = 0$, which implies that $\tilde{\omega}_F \equiv 0$ since K_ω is positive definite. When we plug $\omega_F = \omega_F^d$ into the fourth equation in (7), we can show that $\vec{q}_e = [0, 0, 0]^T$, which implies that $q_F = q_F^d$. Therefore, by LaSalle's invariance principle, $\left\|q_F - q_F^d\right\| \to 0$ and $\left\|\omega_F - \omega_F^d\right\| \to 0$ asymptotically as $t \to \infty$. Thus $\left\|X_F - X_F^d\right\| \to 0$ as $t \to \infty$.

Therefore, $X_F(t) \to X_F^d(t)$ and $X_i(t) \to X_i^d(t)$ as $t \to \infty$, and formation maneuvers without formation feedback are asymptotically achieved.

For a second order system $s^2 + k_1 s + k_2 = 0$, if we define rise time t_r and damping ratio ζ , then natural frequency ω_n is approximately $1.8/t_r$. Therefore, if we let $k_2 = \omega_n^2 = (1.8/t_r)^2$ and $k_1 = 2\zeta\omega_n = 2\zeta(1.8/t_r)$, the transient specifications for the system are satisfied. We can define K_r, k_q, K_ξ according to k_2, and define K_v, K_ω, $K_{\dot{\xi}}$ according to k_1.

From Lemma 4.1 and 4.2, we can see that the performed maneuver will be achieved and the spacecraft will track their desired states in the end. However, how well the spacecraft will preserve the formation shape during the maneuver is not guaranteed by this control law. For example, the errors $\left\|r_i(t) - r_i^d(t)\right\|$ and $\left\|q_i(t) - q_i^d(t)\right\|$ for the ith spacecraft may be large during the maneuver, that is, the spacecraft may get out of the desired formation. If the virtual structure moves too fast, the spacecraft could fall far behind their desired positions. If the virtual structure moves too slowly, the maneuver cannot be achieved within a short time. Therefore, we introduce formation feedback from the spacecraft to the virtual structure to overcome these drawbacks. We will introduce nonlinear gains η in the control law for the virtual structure.

Let $X = [X_1^T, X_2^T, \cdots, X_N^T]^T$ and $X^d = [X_1^{d^T}, X_2^{d^T}, \cdots, X_N^{d^T}]^T$, where N is the number of spacecraft in the formation. The performance measure is defined as $\left\|X - X^d\right\|$. We would like to design the nonlinear gains to meet the following requirements. When the spacecraft are out of the desired formation, that is, $\left\|X - X^d\right\|$ is large, the virtual structure will slow down or stop, allowing the spacecraft to regain formation. When the spacecraft are maintaining formation, that is, $\left\|X - X^d\right\|$ is small, the virtual structure will keep moving toward its final goal. A candidate for such gains can be defined as $\eta = K + K_F \left\|X - X^d\right\|^2$, where $K = K^T > 0$ is the gain when there is no formation feedback, and $K_F = K_F^T > 0$ is the formation gain which weights the performance measure $\left\|X - X^d\right\|$. We can see that

$$\begin{aligned} \left\|X - X^d\right\| \to 0 &\Rightarrow \eta \to K \\ \left\|X - X^d\right\| \to \infty &\Rightarrow \eta \to \infty. \end{aligned}$$

We can use nonlinear gains η_v, η_ω, and $\eta_{\dot{\xi}}$ to replace K_v, K_ω, and $K_{\dot{\xi}}$ in (7), where nonlinear gains are defined as follows.

$$\begin{aligned} \eta_v =& K_v + K_F \left\|X - X^d\right\|^2 \\ \eta_\omega =& K_\omega + K_F \left\|X - X^d\right\|^2 \\ \eta_{\dot{\xi}} =& K_{\dot{\xi}} + K_F \left\|X - X^d\right\|^2 . \end{aligned} \tag{11}$$

Of course, we can use different K_F and rise times for pairs (K_r, η_v), (k_q, η_q), and $(K_\xi, \eta_{\dot{\xi}})$ to change the weights of translation, rotation, and expansion/contraction effects. As a result, nonlinear gains slow down or speed up the virtual structure based on how far out of the desired formation the spacecraft are.

The control law for the virtual structure with formation feedback is given as follows.

Lemma 4.3 *Let*

$$\begin{pmatrix} \dot{r}_F \\ \dot{v}_F \\ \dot{q}_F \\ \dot{\omega}_F \\ \dot{\xi}_F \\ \ddot{\xi}_F \end{pmatrix} = \begin{pmatrix} \dot{v}_F \\ \dot{v}_F^d - K_r(r_F - r_F^d) - \eta_v(v_F - v_F^d) \\ \frac{1}{2}\Omega(\omega_F)q_F \\ \dot{\omega}_F^d + k_q\vec{q}_e - \eta_\omega(\omega_F - \omega_F^d) \\ \dot{\xi}_F \\ \ddot{\xi}_F^d - K_\xi(\xi_F - \xi_F^d) - \eta_{\dot{\xi}}(\dot{\xi}_F - \dot{\xi}_F^d) \end{pmatrix}, \tag{12}$$

where $q_e = q_F^* q_F^d$.

If η_v, η_ω, *and* $\eta_{\dot{\xi}}$ *are given by* (11), *then* $\|X_F - X_F^d\| \to 0$ *as* $t \to \infty$.

Proof: We can follow the same procedure as Lemma 4.2 except that we use nonlinear gains η_v, η_ω, and $\eta_{\dot{\xi}}$ to replace the linear gains K_v, K_ω, and $K_{\dot{\xi}}$ respectively everywhere in the proof. Since η_v, η_ω, and $\eta_{\dot{\xi}}$ are positive definite, we can show that $\|X_F - X_F^d\| \to 0$ as $t \to \infty$.

Combined with the control law for each spacecraft, we know that $X_F(t) \to X_F^d(t)$ and $X_i(t) \to X_i^d(t)$ as $t \to \infty$. Formation maneuvers with formation feedback are asymptotically achieved.

When $X_F^d(t)$ is specified for the virtual structure, $X_F(t)$ will track $X_F^d(t)$ according to the control law for the virtual structure with formation feedback. If the formation moves too fast, $\|X - X^d\|$ will increase. As a result of the formation feedback, the virtual structure will slow down for the spacecraft to track their desired states, that is, to keep the formation. Thus $\|X - X^d\|$ will decrease correspondingly, and the formation can keep moving toward its goal with a reasonable speed. As this coupled procedure proceeds with time, the formation maneuver will be asymptotically achieved.

5 Simulation Results

In this section we will consider a group of three spacecraft each with mass given 150 Kg. The desired original positions of the three spacecraft are given by $r_{1F}^d = [8,0,0]^T$, $r_{2F}^d = [0,8,0]^T$, $r_{3F}^d = [0,0,8]^T$ meters and the desired original attitudes are given by $q_{1F}^d = q_{2F}^d = q_{3F}^d = [0,0,0,1]^T$ with respect to the formation frame C_F. We suppose that the three spacecraft start from rest with some initial errors. The three spacecraft will perform a formation maneuver of a combination of translation, rotation, and expansion. The formation will start from rest with inertial position $r_F(0) = [0,0,0]^T$ and inertial attitude $q_F(0) = [0,0,0,1]^T$ to the desired position $r_F^d = [20,20,20]^T$ and desired attitude $q_F^d = [u^T sin(\pi/4), cos(\pi/4)]^T$, where $u = [1/\sqrt{14}, 2/\sqrt{14}, 3/\sqrt{14}]^T$, and expand 1.5 times the original size.

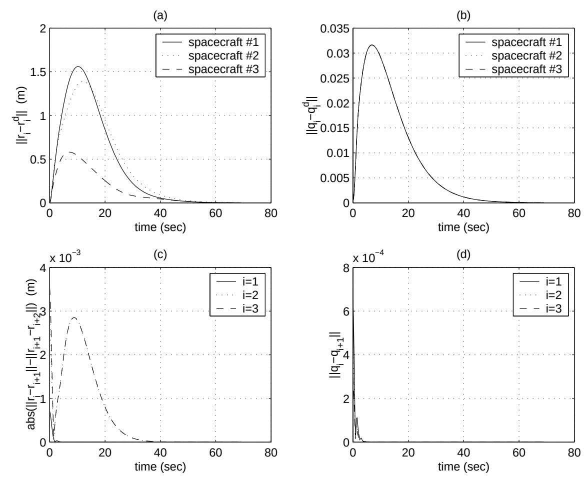

Figure 2: Position and attitude errors without formation feedback (convergence time: 69 sec).

In simulation, we will plot absolute position and attitude errors as well as relative position and attitude errors for each spacecraft to the time when the system converges. When $\|X_F - X_F^d\| + \|X - X^d\| < 0.001$, we say that the system has converged. Absolute position error is represented by absolute difference between actual position and desired position for each spacecraft. Absolute attitude error is represented by absolute difference between actual attitude and desired attitude for each spacecraft. Since the formation shape is an equilateral triangle and the three spacecraft keep the same attitude in the formation, we use absolute difference between lengths of the sides in the equilateral triangle to represent the relative position error and absolute difference between the attitude of each spacecraft to represent the relative attitude error. If the formation is preserved exactly, the relative position and attitude errors should be zero.

In this section, we use a subscript i ($1 \le i \le 3$) which is defined modulo 3 to represent the states for the ith spacecraft. For each figure in this section, in part (a), we plot absolute position errors represented by $\|r_i - r_i^d\|$. In part (b), we plot absolute attitude errors represented by $\|q_i - q_i^d\|$. In part (c), we plot relative position errors represented by$|\,\|r_i - r_{i+1}\| - \|r_{i+1} - r_{i+2}\|\,|$. In part (d), we plot relative attitude errors represented by $\|q_i - q_{i+1}\|$. Note that sometimes some curves may coincide with each other.

Figure 2 shows the formation maneuver without formation feedback. Figure 3 shows the formation maneu-

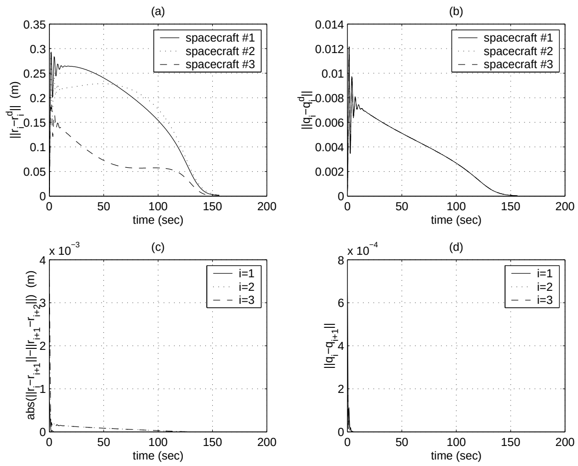

Figure 3: Position and attitude errors with formation feedback (convergence time: 156 sec).

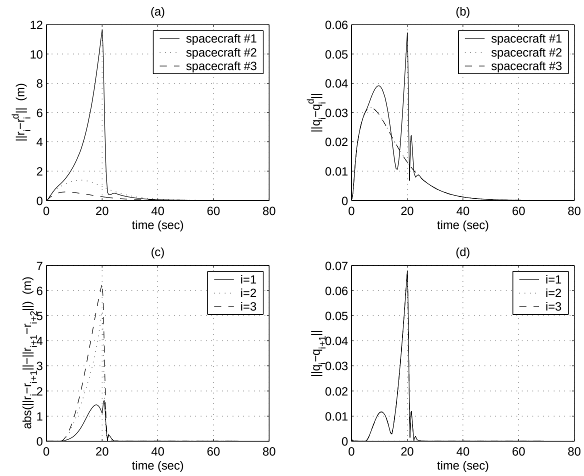

Figure 4: Position and attitude errors without formation feedback when spacecraft #1 fails for 15 seconds (convergence time: 69 sec).

ver with formation feedback. By comparing each part of Figure 2 and Figure 3, we can see that the maximum absolute and relative errors of the system without formation feedback is larger than that of the one with formation feedback. Also the system without formation feedback converges faster than the one with formation feedback when we choose the same rise time for both of them. When we decrease the rise time in Figure 2 to let the system converge faster, the corresponding errors will increase significantly. Similarly, we can also increase the rise time to decrease the errors, but the system will converge more slowly. In Figure 3, since the system has formation feedback, we can choose smaller rise time than that in Figure 2 to let the system converge faster while the errors are still maintained within a reasonable range.

In Figure 4 and 5, we simulate the formation maneuver results when spacecraft #1 fails from 5th to 20th second with and without formation feedback respectively. Since there is no formation feedback in Figure 4, the virtual structure keeps moving toward its final goal even if one of the spacecraft fails for some time. As a result, spacecraft #1 cannot track its desired states satisfactorily, and the system has very large absolute and relative errors during the period when spacecraft #1 fails. In fact, in this case the spacecraft are out of formation for some time. However, in Figure 5, since there is formation feedback, the virtual structure slows down to preserve the formation when one of the spacecraft fails for a period of time. As a result, the system in Figure 5 has much smaller absolute and relative errors than the one in Figure 4. The formation is preserved much better than that in Figure 4 even if spacecraft #1 fails for 15 seconds.

Within the same range of error, the system with formation feedback can choose smaller rise time, and thus converge faster than the one without formation feedback. Within the same range of convergence speed, the system with formation feedback will have smaller errors than the one without formation feedback. We know that absolute and relative errors will decrease when the formation gain K_F increases, but the convergence speed will decrease correspondingly. At the same time, when the formation gain K_F decreases, the system will converge faster, but the absolute and relative errors will increase correspondingly. We also know by simulation that it is hard to choose good rise time beforehand to achieve a good performance in the system without formation feedback. However, a wide range of rise times work well in the system with formation feedback.

6 Conclusion

In this paper we have investigated a novel idea of introducing formation feedback under the scheme of virtual structures through a detailed application of this idea to the problem of synthesizing multiple spacecraft in deep space. Introducing formation feedback from spacecraft to the formation has several advantages. First, the system can achieve a good performance in improving convergence speed and decreasing maneuver errors. Second, formation feedback adds a sense of group stability and robustness to the whole system. Third, formation feedback improves the robustness with respect to choosing gains for different spacecraft. Finally, formation feedback makes formation keeping more robust to synchronization issues and the variability on each spacecraft.

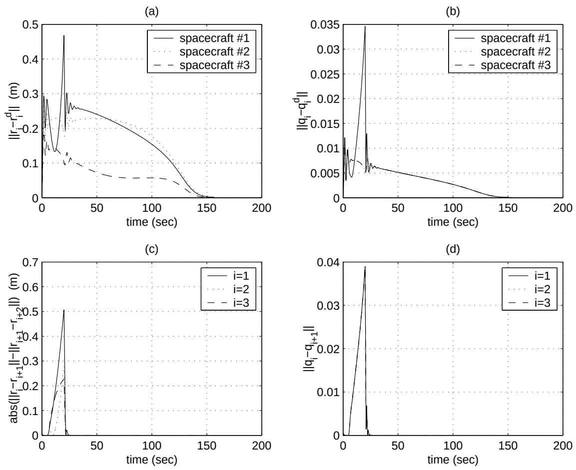

Figure 5: Position and attitude errors with formation feedback when spacecraft #1 fails for 15 seconds (convergence time: 156 sec).

Acknowledgements

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