

# Decentralization of Coordination Variables in Multi-vehicle Systems

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**Abstract**—Equipping each vehicle in a cooperative team with a global picture of the group helps vehicles direct toward specified desired group goals and achieve desirable group behaviors. In this paper, we instantiate the group level information formalized as “coordination variables” on each vehicle. We then develop consensus strategies to guarantee that each coordination variable instantiation converges to a sufficiently common value in the case that the coordination variable is driven by a common input or inputs with bounded inconsistency. We show conditions under which consensus can be achieved for each coordination variable instantiation and provide boundedness analyses for the inconsistency of different instantiations when inconsistent inputs exist. The effectiveness of the proposed strategies is demonstrated through a formation control example.

## I. INTRODUCTION

Coordinated control of multiple vehicles has received significant attention in the control and robotics communities in recent years regarding the benefits of using many inexpensive, simple systems to replace a single monolithic, expensive, and complicated system.

Two levels of information will be distinguished in a cooperative team in this paper. One is the group level information, which represents the group coordination objectives (e.g. group behaviors or goals). The other is the vehicle level information (e.g. vehicle states). Following the terminology in [1], we refer the group level information as “coordination variables”. In a centralized scheme for multi-vehicle coordination, the coordination variable is often implemented at a central location and broadcast to all the vehicles, which has the weakness of a single point of failure and lack of scalability. As a comparison, a decentralized scheme usually achieves more reliability and robustness but vehicles in the team often lack a global picture about the whole group, that is, the coordination variable. It is feasible and worthwhile to give each vehicle in the team the global picture of the group (see e.g. [2]). A possible approach to realize this is to instantiate the coordination variable on each vehicle (see e.g. [3], [1]). However, due to dynamically changing local situational awareness, discrepancies may appear for different coordination variable instantiations. As a result, we need to develop strategies to ensure that all coordination variable instantiations are sufficiently common (see [4], [5], [6], [7], [8] for recent study in information consensus).

The main contribution of this paper is a framework that decentralizes the coordination variables. The scheme avoids a single point of failure, requires only local information exchange, and is scalable to a large number of vehicles.

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## II. DECENTRALIZED COORDINATION FRAMEWORK

One way to coordinate multi-vehicle systems is through a centralized coordination scheme, where the coordination variable is implemented at a central location and then broadcast to each vehicle in the team [9]. However, as the number of vehicles in the team increases, this scheme may result in degraded overall system performance due to heavy communication overhead at the central location. In addition, the central location is a single point of failure for the whole system.

As an alternative, each vehicle in the team can instantiate a local copy of the centralized coordination scheme. Fig. 1 shows a decentralized coordination framework. In Fig. 1 each vehicle has a local copy of a discrete event supervisor, denoted by  $\mathbf{G}_i$ , which outputs a sequence of desired coordination variable values, denoted by  $y_{G_i} = \xi^{d(k)}$ ,  $k = 1, \dots, K$ . In addition, each vehicle instantiates a local copy of a consensus module, denoted by  $\mathbf{C}_i$ . Each consensus module obtains coordination variable instantiations, denoted by  $\xi_i$  for the  $i^{\text{th}}$  instantiation, from its local (time-varying) neighbors and implements algorithms to guarantee that each coordination variable instantiation comes into consensus as well as achieves its current desired state. Feedback from  $\mathbf{C}_i$  to  $\mathbf{G}_i$  is achieved through performance measure  $z_{C_i}$ , which describes how far the  $i^{\text{th}}$  coordination variable instantiation is from its desired state. The module  $\mathbf{K}_i$  is the local vehicle controller, which receives  $\xi_i$  from the consensus module, converts  $\xi_i$  to the desired states for the  $i^{\text{th}}$  vehicle, and then controls the actual state for the  $i^{\text{th}}$  vehicle to track its desired state. Formation feedback from the  $i^{\text{th}}$  local vehicle controller to the  $i^{\text{th}}$  consensus module  $\mathbf{C}_i$  is achieved through performance measure  $z_i$ . As a result, the evolution of the coordination variable instantiations can be modified according to vehicle performance to improve group robustness. The module  $\mathbf{V}_i$  is the  $i^{\text{th}}$  vehicle, with control input  $u_i$  and output  $y_i$ .

With the decentralized coordination framework, coordinated control problems can be decoupled into two subtasks. One is to design consensus algorithms such that each coordination variable instantiation converges to a sufficiently common value. The other is to develop local coordination algorithms based on each coordination variable instantiation to achieve a given coordination objective. At the group level, we assume that the coordination variable instantiations are driven into consensus through inter-vehicle communications, where a group level communication topology is used to represent the communication links among vehicles for the coordination variable instantiations. At the vehicle level, control laws for each vehicle are based on its own

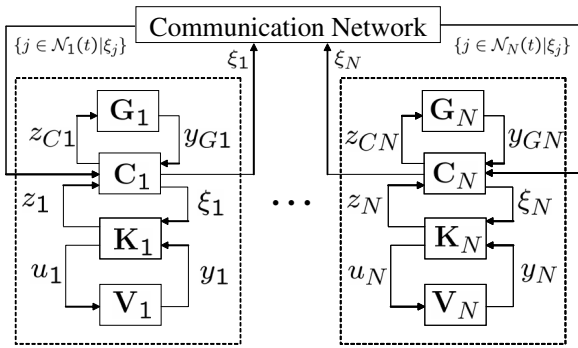


Fig. 1. A decentralized coordination framework.

states, its coordination variable instantiation, and available neighboring vehicle information through local sensing or communication. Associated with each vehicle is a sensing topology or a transmission topology (see e.g. [2]) for vehicle level information. Note that the vehicle level sensing or transmission topology may be different from the group level communication topology. Both levels of topologies may change dynamically. In this paper, we focus on the group level topology and assume that there is no vehicle level topology for simplicity. We also assume that the local control laws for each vehicle are given and only focus on consensus and evolution of the coordination variable instantiations.

In [3], a decentralized scheme is proposed with the requirement that the communication topology forms a fixed bidirectional ring. As a comparison, the current framework generalizes the one in [3] by allowing random packet loss for each communication link as well as dynamically changing, sparse, and intermittent intervehicle communication topologies. In addition, in the current framework, there is no need to identify two adjacent neighbors in order to form a ring topology as in [3] since at each time each vehicle simply communicates with any available local neighbors.

### III. CONSENSUS AND EVOLUTION OF COORDINATION VARIABLE INSTANTIATIONS

#### A. Definitions

A directed graph  $\mathcal{G}$  will be used to model the group level communication topology. In  $\mathcal{G}$ , the  $i^{\text{th}}$  node represents the  $i^{\text{th}}$  vehicle  $\mathcal{A}_i$  and a directed edge from  $\mathcal{A}_i$  to  $\mathcal{A}_j$  denoted as  $(\mathcal{A}_i, \mathcal{A}_j)$  represents a unidirectional information exchange link from  $\mathcal{A}_i$  to  $\mathcal{A}_j$ , that is, vehicle  $j$  can receive or obtain information from vehicle  $i$ ,  $(i, j) \in \mathcal{I}$ , where  $\mathcal{I} = \{1, 2, \dots, n\}$ . Noting that the communication topology may be dynamically changing, we let  $\bar{\mathcal{G}} = \{\mathcal{G}_1, \mathcal{G}_2, \dots, \mathcal{G}_M\}$  denote the set of all possible directed interaction graphs defined for  $\mathcal{A}$ . The union of a group of directed graphs  $\{\mathcal{G}_{\ell_1}, \mathcal{G}_{\ell_2}, \dots, \mathcal{G}_{\ell_m}\} \subset \bar{\mathcal{G}}$  is a directed graph with nodes given by  $\mathcal{A}_i$ ,  $i \in \mathcal{I}$ , and edge set given by the union of the edge sets of  $\mathcal{G}_{\ell_j}$ , where  $\ell_j \in \{1, 2, \dots, M\}$ .

A directed path in graph  $\mathcal{G}$  is a sequence of edges  $(\mathcal{A}_{k_1}, \mathcal{A}_{k_2}), (\mathcal{A}_{k_2}, \mathcal{A}_{k_3}), (\mathcal{A}_{k_3}, \mathcal{A}_{k_4}), \dots$  in that graph, where  $k_j \in \mathcal{I}$ . Graph  $\mathcal{G}$  is called strongly connected if there is a

directed path from  $\mathcal{A}_i$  to  $\mathcal{A}_j$  and  $\mathcal{A}_j$  to  $\mathcal{A}_i$  between any pair of distinct nodes  $\mathcal{A}_i$  and  $\mathcal{A}_j$ ,  $\forall (i, j) \in \mathcal{I}$ . A directed tree is a directed graph, where every node, except the root, has exactly one parent. A (directed) spanning tree of a directed graph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph has or contains a (directed) spanning tree if a subset of the edges forms a (directed) spanning tree.

Let  $\mathbf{1}$  denote an  $n \times 1$  column vector with all entries equal to 1. A matrix  $A = [a_{ij}] \in M_n(\mathbb{R})$  is nonnegative, denoted as  $A \geq 0$ , if all its entries are nonnegative. Furthermore, if all its row sums are +1,  $A$  is said to be a (row) stochastic matrix.

#### B. Consensus Algorithm

Let  $\mathbb{N}$  and  $\mathbb{N}^+$  denote the set of nonnegative integers and positive integers respectively. Given  $T_s$  as the sampling period, a discrete-time consensus scheme is given by

$$\xi_i[k+1] = \frac{1}{\sum_{j=1}^n \alpha_{ij}[k]g_{ij}[k]} \sum_{j=1}^n \alpha_{ij}[k]g_{ij}[k]\xi_j[k] + v_i[k], \quad (1)$$

where  $k \in \mathbb{N}$  is the discrete-time index,  $(i, j) \in \mathcal{I}$ ,  $v_i[k]$  denotes the input at time  $t = kT_s$ ,  $\alpha_{ij}[k] > 0$  are uniformly lower and upper bounded,  $g_{ii}[k] \triangleq 1$ , and  $g_{ij}[k]$ ,  $\forall j \neq i$ , is 1 if information flows from  $\mathcal{A}_j$  to  $\mathcal{A}_i$  and 0 otherwise. Note that both  $\alpha_{ij}$  and  $g_{ij}$  may be time-varying.

Assume that  $\xi_i \in \mathbb{R}^m$ . Eq. (1) can be written in matrix form as

$$\xi[k+1] = (D[k] \otimes I_m)\xi[k] + v[k], \quad (2)$$

where  $\xi = [\xi_1^T, \dots, \xi_n^T]^T$ ,  $v = [v_1^T, \dots, v_n^T]^T$ ,  $\otimes$  denotes the Kronecker product,  $I_m$  denotes the  $m \times m$  identity matrix, and  $D[k] = [d_{ij}[k]]$ ,  $(i, j) \in \mathcal{I}$ , with  $d_{ij}[k] = \frac{\alpha_{ij}[k]g_{ij}[k]}{\sum_{j=1}^n \alpha_{ij}[k]g_{ij}[k]}$ .

#### C. Analysis

In the following, we only provide an analysis for the case  $\xi_i \in \mathbb{R}$  for simplicity. All results remain valid in the case of  $\xi_i \in \mathbb{R}^m$ ,  $i \in \mathcal{I}$ . Before moving on, we require the following lemma.

**Lemma 3.1:** If graph  $\mathcal{G}$  has a (directed) spanning tree, then 1 is the unique eigenvalue of  $D$  with maximum modulus and  $\lim_{m \rightarrow \infty} D^m \rightarrow \mathbf{1}\mu^T$ , where  $m \in \mathbb{N}^+$  and  $\mu = [\mu_1, \dots, \mu_n]^T \geq 0$  satisfies  $D^T\mu = \mu$  and  $\mathbf{1}^T\mu = 1$ .

*Proof:* Follows from Corollary 3.5 and Lemma 3.7 in [8]. ■

Note that the solution to Eq. (2) is given by  $\xi[k] = D^k\xi[0] + \sum_{i=1}^k D^{k-i}v[i]$ ,  $\forall k \in \mathbb{N}^+$ . We have the following theorem regarding  $\xi_i[k]$  and  $\|\xi_i[k] - \xi_j[k]\|$  as  $k \rightarrow \infty$ .

**Theorem 3.1:** Given discrete-time scheme (1), assume that graph  $\mathcal{G}$  has a (directed) spanning tree. If  $v_1[k] = \dots = v_n[k] = v_*[k]$ ,  $\forall k \in \mathbb{N}$ , then  $\xi_i[k] \rightarrow \mu^T\xi[0] + \sum_{j=1}^k v_*[k-j]$ ,  $\forall i \in \mathcal{I}$ , as  $k \rightarrow \infty$  asymptotically. If  $\|v_i[k] - v_j[k]\|$  is uniformly bounded,  $\forall i \neq j$ , so is  $\|\xi_i[k] - \xi_j[k]\|$ .

*Proof:* For the first statement of the lemma, we know that  $D^k \rightarrow \mathbf{1}\mu^T$  as  $k \rightarrow \infty$  and  $\mu^T\mathbf{1} = 1$  from Lemma 3.1. Note

that  $v[k] = \mathbf{1}v_*[k]$ . Also note that  $D^i \mathbf{1} = \mathbf{1}, \forall i \in \mathbb{N}$ , since  $\mathbf{1}$  is an eigenvector of  $D$  and therefore an eigenvector of  $D^i$ ,  $\forall i \in \mathbb{N}$ , associated with eigenvalue 1. Therefore, following the solution to Eq. (1), we know that using discrete-time scheme (1) consensus can be achieved asymptotically if each coordination variable instantiation is driven by a common input.

For the second statement of the lemma, let  $\xi_{ij} = \xi_i - \xi_j$  be the consensus error variables. Note that  $\xi_{ij} = \xi_{1j} - \xi_{1i}$ . Define the consensus error vector as  $\tilde{\xi} = [\xi_{12}, \xi_{13}, \dots, \xi_{1n}]^T$ . Also let  $v_{ij} = v_i - v_j$  and  $\tilde{v} = [v_{12}, v_{13}, \dots, v_{1n}]^T$ . We get  $\dot{\tilde{\xi}} = \Delta \tilde{\xi} + \tilde{v}$ , where  $\Delta$  is a  $(n-1) \times (n-1)$  matrix that can be derived from Eq. (1).

From the first statement of the lemma, we know that  $\tilde{\xi} \rightarrow 0$  asymptotically if  $\tilde{v} = 0$ , which implies that  $\Delta$  is a stable matrix. Therefore,  $\|\tilde{\xi}\|$  is bounded for bounded  $\|\tilde{v}\|$ , which in turn implies that  $\|\xi_i - \xi_j\|$  is bounded for bounded  $\|v_i - v_j\|$ . ■

*Corollary 3.2:* Let  $v_i, \forall i \in \mathcal{I}$ , be arbitrary constants. Given discrete-time scheme (1), if graph  $\mathcal{G}$  has a (directed) spanning tree then  $\|\xi_i[k] - \xi_j[k]\|$  is uniformly bounded and  $\xi_i[k] - \xi_j[k]$  approaches a constant value as  $k \rightarrow \infty$ .

By using the results in [8], we can show that the second statement of Theorem 3.1 is still valid even in the case of switching communication topologies as long as there exist infinitely many consecutive uniformly bounded time intervals such that the union of the interaction graph across each such interval has a (directed) spanning tree.

Note that although each  $\xi_i[k], i \in \mathcal{I}$ , may become unbounded as  $k \rightarrow \infty$  when driven by an input, their inconsistency is guaranteed to be bounded by the above analyses. Also note that if  $\|v_i[k]\|$  is uniformly bounded for each  $i \in \mathcal{I}$ , the condition that  $\|v_i[k] - v_j[k]\|$  is uniformly bounded is trivially satisfied. If each vehicle evolves according to some nonlinear dynamics  $f(k, \xi_i, u[k])$ , where  $u$  is the common exogenous input to each vehicle, then we can let  $v_i[k] = f(k, \xi_i, u[k])$  in Eq. (1). In this case, consensus is not guaranteed to be achieved asymptotically in general although a similar analysis to Theorem 3.1 guarantees that  $\|\xi_i[k] - \xi_j[k]\|$  is uniformly bounded if  $\|f(k, \xi_i, u[k]) - f(k, \xi_j, u[k])\|$  is uniformly bounded,  $\forall i \neq j$ .

Consider a consensus example for five vehicles with a communication topology given by Fig. 2. We assume that  $v_i[k] = \sin(2\xi_i[k])$  and  $\alpha_{ij} = 1$ . Fig. 3 shows the difference between  $\xi_i$  and  $\xi_{i+1}, i = 1, \dots, 4$ . We can see that consensus is not achieved but the difference of the information state between vehicles is uniformly bounded since  $|\sin(2\xi_i[k])|$  is uniformly bounded.

#### IV. APPLICATION TO FORMATION CONTROL

In this section, we apply the decentralized coordination framework to a multi-vehicle formation control scenario where 25 mobile robots need to preserve a formation shape when performing formation maneuvers.

The kinematic equations of a fully actuated mobile robot

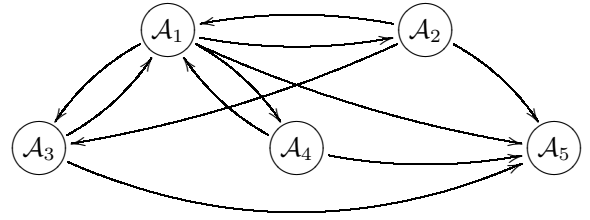


Fig. 2. Communication topology with five vehicles.

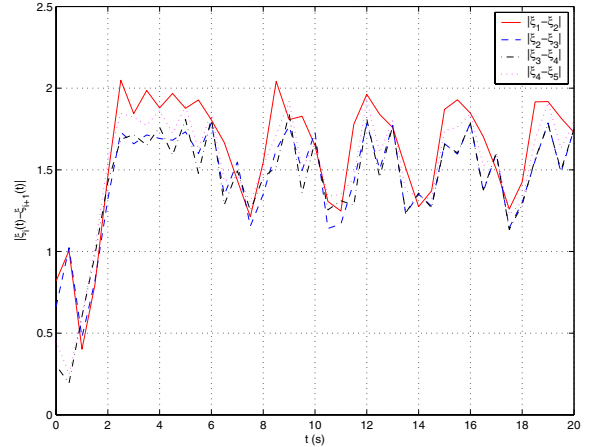


Fig. 3. Consensus with  $v_i[k] = \sin(2\xi_i[k]), i = 1, \dots, 5$ .

are

$$\dot{z}_i = u_i, \quad i = 1, \dots, 25 \quad (3)$$

where  $z_i = [x_i, y_i]^T$  represents the position of the  $i^{\text{th}}$  robot, and  $u_i = [u_{xi}, u_{yi}]^T$  represents the control input. Note that Eq. (3) also denotes the kinematics for a nonholonomic mobile robot after feedback linearization for a fixed point off the center of the wheel axis. While feedback linearization will result in loss of robot orientation information in the case of nonholonomic mobile robots, the decentralized coordination framework is sufficiently general to tackle the case without feedback linearization by employing tracking control laws that account for nonholonomic constraints. Note that although we use very simple robot kinematics to illustrate the idea of decentralization of coordination variables here, the decentralized coordination framework is applicable to coordinated control of vehicles with complicated dynamics.

For our tests, the 25 robot team is required to preserve a desired formation shape as shown in Fig. 4, where squares represent the desired positions of each robot and the two perpendicular arrows located at the virtual center of the formation denoted formation frame  $\mathcal{C}_F$ . In Fig. 4 one robot is located at the origin of the formation frame while the others are uniformly distributed along circles centered at the origin of the formation frame with a radius of 20 meters and 40 meters respectively.

Define the coordination variable as  $\xi(t) = [x_0(t), y_0(t), \theta_0(t)]^T$ , where  $(x_0(t), y_0(t))$  and  $\theta_0(t)$  denote the position and orientation of formation frame  $\mathcal{C}_F$

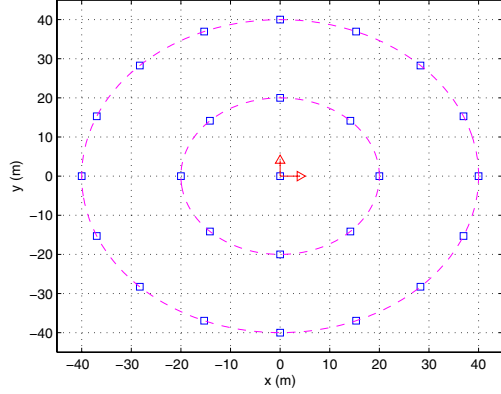


Fig. 4. Formation of 25 robots with central formation frame  $\mathcal{C}_F$ .

respectively. Given  $\xi(t)$ , the desired trajectory for each robot can in turn be defined as

$$z_i^d(t) = \begin{bmatrix} x_0(t) \\ y_0(t) \end{bmatrix} + \begin{bmatrix} \cos(\theta_0(t)) & -\sin(\theta_0(t)) \\ \sin(\theta_0(t)) & \cos(\theta_0(t)) \end{bmatrix} \begin{bmatrix} \tilde{x}_{i0}^d(t) \\ \tilde{y}_{i0}^d(t) \end{bmatrix},$$

where  $(\tilde{x}_{i0}^d(t), \tilde{y}_{i0}^d(t))$  is the specified desired deviation of each robot from the formation center. To further simplify the problem, we parameterize the coordination variable by  $s \in \mathbb{R}$ , which is a function of time (see e.g., [10]). As a result, the coordination variable can be defined as  $\xi(s(t)) = [x_0(s(t)), y_0(s(t)), \theta_0(s(t))]^T$ . In this case, we can certainly instantiate  $\xi$  on each vehicle, denoted by  $\xi_i$ , and apply consensus algorithms to guarantee that each instantiation converges to a sufficiently common value. However, noting that parameter  $s$  represents the minimum amount of information needed by each robot to coordinate its motion with the group, we can also instantiate parameter  $s$  on each robot as  $s_i$  and drive each instantiation into consensus via intervehicle communications. By doing so, the amount of information that needs to be communicated between vehicles is reduced in the sense that  $\xi_i \in \mathbb{R}^3$  while  $s_i \in \mathbb{R}$ . Compared to a centralized scheme where parameter  $s$  is implemented at a central location and broadcast to all the robots, this decentralized implementation overcomes the drawback of a single point of failure.

The discrete-time consensus scheme is given by

$$s_i[k+1] = \frac{1}{\sum_{j=1}^n \alpha_{ij}[k]g_{ij}[k]} \sum_{j=1}^n \alpha_{ij}[k]g_{ij}[k](s_j[k] + w_{ij}[k]) + \lambda_i,$$

where  $\alpha_{ij}[k]$  are chosen as arbitrary positive constants,  $w_{ij}[k]$  denotes the communication noise associated with the communication channel from robot  $j$  to robot  $i$ ,  $\lambda_i$  is the input, and  $g_{ij}[k]$  are 1 if information flows from robot  $j$  to robot  $i$  and 0 otherwise. In the simulation, we choose the sample period as  $T_s = 0.5$  (sec). Then each robot  $i$  can track its desired states specified by its parameter instantiation  $s_i$

based on a simple tracking law

$$u_i = \dot{z}_i^d(s_i) - \gamma_i(z_i - z_i^d(s_i)),$$

where  $\gamma_i > 0$ .

We simulate the case where formation frame  $\mathcal{C}_F$  follows a trajectory of a circle with radius 200 meters. In the simulation, we let  $x_0(s) = 200 \cos(\frac{2\pi}{S}s)$ ,  $y_0(s) = 200 \sin(\frac{2\pi}{S}s)$ , and  $\theta_0(s) = \frac{2\pi}{S}s$ , where  $S$  specifies the period of the desired trajectory for the formation frame in terms of parameter  $s$ . That is, when  $s$  evolves from 0 to  $S$ , the trajectory of the formation frame completes one cycle. We assume that each robot has a communication range of 30 meters. Taking into account random communication packet losses, we assume that it is possible that the  $i^{\text{th}}$  robot can obtain information from the  $j^{\text{th}}$  robot but not vice versa at a certain time. That is, the communication graph is generally bidirectional but may be sporadically unidirectional over one or more time steps. Specifically, we assume that there are 20% communication packet losses for any existing communication link, which implies that the union of the communication topologies across each sufficient large time interval will generically have a (directed) spanning tree. We also assume that each robot has limited control authority such that  $|u_{xi}| \leq 1$  and  $|u_{yi}| \leq 1$ . In the following, we assume that there is no collision avoidance between robots.

Table I shows parameter values used in each of three test cases. In Cases 1 and 2, each  $s_i$  is driven by a common exogenous input in the presence of communication noise. In Case 3, each  $s_i$  is driven by inputs with bounded inconsistency, particularly a nonlinear signal representing group feedback information, in the presence of communication noise. The simple case where there is no communication noise is straightforward and will not be considered.

TABLE I  
PARAMETER VALUES USED IN CASE  $i$ ,  $i = 1, 2, 3$ .

Case Number	Parameter Values
Case 1	$S = 500$ (sec), $\lambda_i = \frac{T_s}{4}$
Case 2	$S = 300$ (sec), $\lambda_i = \frac{T_s}{4}$
Case 3	$S = 300$ (sec), $\lambda_i = \frac{T_s}{4} \eta(\ z_i - z_i^d\ )$

In Case 1, each instantiation of parameter  $s$  is driven by the same input  $\lambda_i = \frac{T_s}{4}$ . Fig. 5 shows formation maneuvers of the 25 robots at  $t = 0, 400, 800,$  and  $1200$  (sec) respectively in Case 1. The green circle represents the desired trajectory of the formation center, square vertices denote the actual locations of each robot, and star vertices denote the desired locations of each robot. Fig. 6 shows consensus of  $s_i$  with random communication noise in Case 1. We can see that the difference between each instantiation is bounded. Fig. 7 shows the tracking errors and formation keeping errors with random communication noise in Case 1. Here  $\text{dist}(a, b)$  is defined as  $\|a - b\|$ . Note that the desired distances between vehicles (1, 7) and (2, 10) are 20 meters. Due to the fact

<sup>1</sup>Not necessarily  $t \in [0, S]$  since  $s$  is a function of  $t$

that the formation center evolves at a relatively low speed ( $S = 500$  (sec)), the formation is preserved well even if there exists communication noise and each robot has limited control authority.

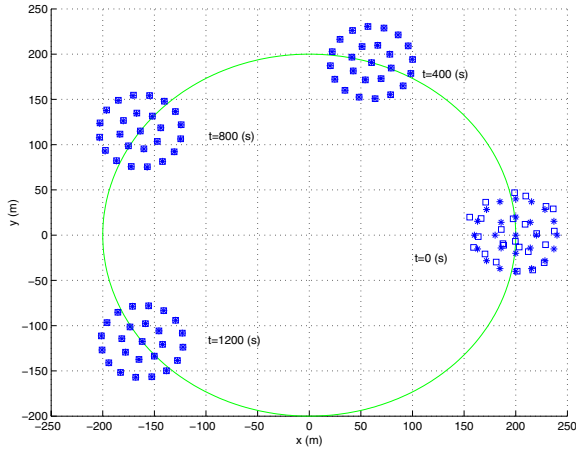


Fig. 5. Virtual structure maintenance for Case 1.

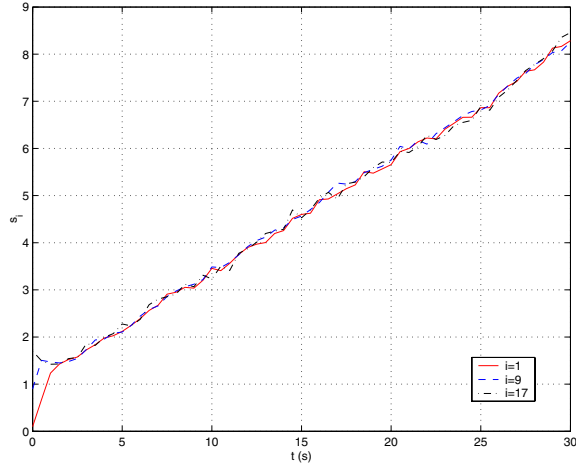


Fig. 6. Consensus of  $s_i$  with random communication noise in Case 1.

In Case 2, each instantiation of parameter  $s$  is also driven by the same input  $\lambda_i = \frac{T_s}{4}$ . However, in this case the formation center evolves at a higher speed ( $S = 300$  (sec)). Fig. 8 shows consensus of  $s_i$  with random communication noise in Case 2. Fig. 9 shows the tracking errors and formation keeping errors with random communication noise in Case 2. Compared to Case 1, formation is not preserved well due to the limited control authority of each robot to track trajectories evolving relatively fast.

In Case 3, we replace the common exogenous input with group feedback from each vehicle to its instantiation of parameter  $s$  by defining  $\lambda_i = \frac{T_s}{4} \eta(\|z_i - z_i^d\|)$ , where  $\eta(\cdot)$  is defined in such a way that  $\eta(\|z_i - z_i^d\|) = 1$  if  $\|z_i - z_i^d\| \leq \epsilon$  and  $0 < \eta(\|z_i - z_i^d\|) < 1$  decreases as  $\|z_i - z_i^d\|$  increases. As a result, if the tracking error for the  $i^{\text{th}}$  robot is below a certain bound  $\epsilon$ , the  $i^{\text{th}}$  instantiation of

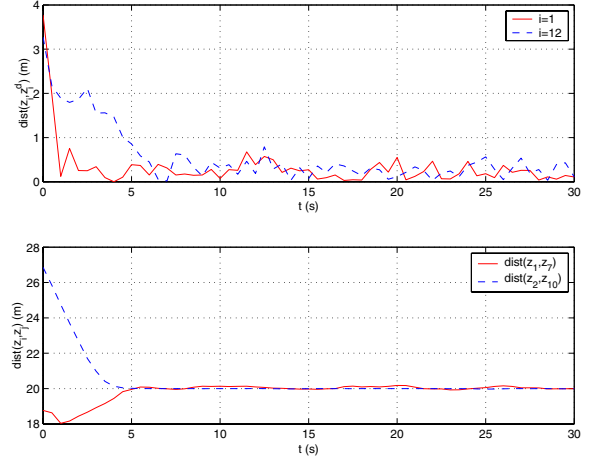


Fig. 7. Tracking errors and formation keeping errors with random communication noise in Case 1.

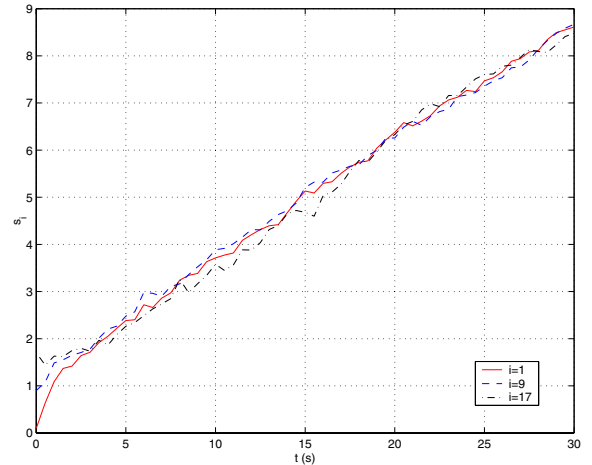


Fig. 8. Consensus of  $s_i$  with random communication noise in Case 2.

parameter  $s$  evolves at a nominal velocity. If the tracking error for the  $i^{\text{th}}$  robot exceeds the bound, the  $i^{\text{th}}$  instantiation of parameter  $s$  evolves more slowly as the tracking error increases. In this paper, we simply define function  $\eta(\cdot)$  as

$$\eta(x) = \begin{cases} 1, & x \leq \epsilon \\ \frac{1}{1+k(x-\epsilon)}, & x > \epsilon \end{cases}$$

where  $\epsilon = 0.2$  and  $k = 100$ . Fig. 10 shows consensus of  $s_i$  with random communication noise in Case 3. Fig. 11 shows the tracking errors and formation keeping errors with random communication noise in Case 3. In this case, formation is preserved well even if  $S$  is also chosen to be 300 (sec) as in Case 2. By comparing Figs. 8 and 10, we can see that each  $s_i$  in Case 3 evolves more slowly than those in Case 2 due to the effect of group feedback. Note that there exists inconsistency between  $\lambda_i = \frac{T_s}{4} \eta(\|z_i - z_i^d\|)$  due to the inconsistency between  $z_i - z_i^d$ . However, noting that  $|\lambda_i|$  is bounded, we see that the inconsistency between  $\lambda_i$  is bounded since  $|\lambda_i - \lambda_j| \leq |\lambda_i| + |\lambda_j|$ . As expected, the

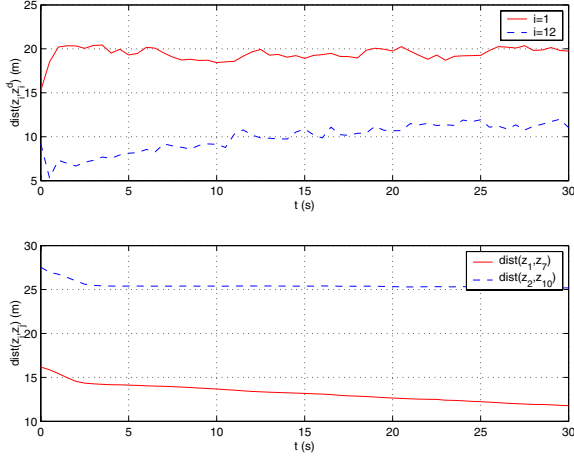


Fig. 9. Tracking errors and formation keeping errors with random communication noise in Case 2.

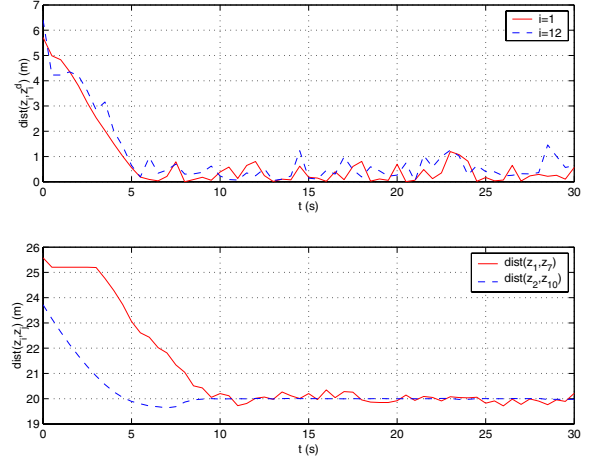


Fig. 11. Tracking errors and formation keeping errors with random communication noise in Case 3.

inconsistency between  $s_i$  is bounded as shown in Fig. 10.

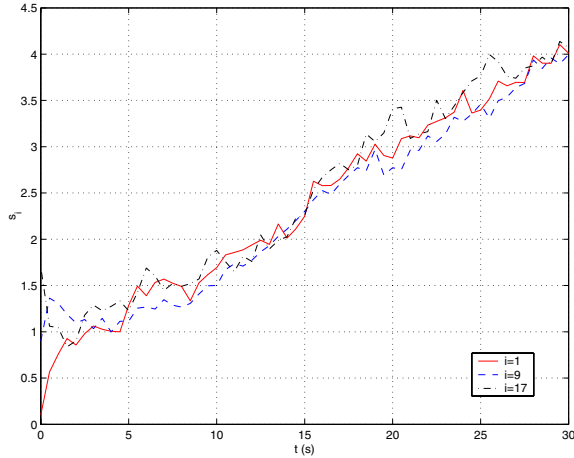


Fig. 10. Consensus of  $s_i$  with random communication noise in Case 3.

## V. CONCLUSION AND FUTURE WORK

This paper has addressed the problem of decentralization of coordination variables in multi-vehicle systems via consensus strategies. We have shown conditions under which consensus can be achieved for each coordination variable instantiation driven by a common input and performed boundedness analyses for the inconsistency between vehicles when there are inconsistent inputs (e.g., communication noise). An application to multi-robot formation maneuvering has been presented to show the effectiveness of our results.

## ACKNOWLEDGMENT

The author would like to gratefully acknowledge Profs. Randy Beard, Tim McLain, and Ella Atkins for their technical guidance on the subject.

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