

# Distributed Attitude Consensus among Multiple Networked Spacecraft

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**Abstract**—In this paper we consider the distributed attitude consensus problem among multiple networked spacecraft in deep space, which extends the standard information consensus results in the literature to rigid body rotational dynamics. We propose control laws for three different cases. In the first case, multiple spacecraft align their attitudes during formation maneuvers with zero final angular velocities under an undirected communication topology. In the second case, attitudes are aligned in the presence of nonzero final angular velocities under an undirected communication topology. In the third case, attitude consensus under a directed information exchange topology is addressed. Simulation results for attitude consensus among six spacecraft show the effectiveness of our approach.

## I. INTRODUCTION

In recent years advances in networking and distributed computing makes possible numerous applications for multi-agent systems. One example is space-based interferometry, where a formation of networked spacecraft could be used to synthesize a space-based interferometer with base-lines reaching tens to hundreds of kilometers as an alternative to traditional monolithic spacecraft.

The research of spacecraft formation flying has received significant attention in the literature. Recent results have been reported in [1], [2], [3], [4], [5], [6], to name a few.

In interferometry applications, it is essential that spacecraft maintain relative or the same attitudes during formation maneuvers. In the following we use the term *attitude consensus* or *alignment* to refer to the case that multiple spacecraft maintain the same relative attitudes to their reference frames respectively. Leader-follower and behavioral approaches are two techniques to tackle the attitude consensus problem. In the leader-follower approach [1], each follower in the team simply tracks the attitude of a designated leader. This approach has the advantage that the formation flying problem reduces to well-studied tracking problems. However, no information feedback is introduced from the followers to the leader and the leader becomes a single point of failure. As a comparison, in the behavioral approach [5], the control torque for each spacecraft is a function of the attitudes and angular velocities of two adjacent neighbors. As a result, group feedback is introduced in the team through coupled dynamics between spacecraft.

The main contribution of this paper is to extend the previous synchronized spacecraft rotation results reported in [5], [6] to a more general scenario. Rather than requiring a restricted bidirectional ring topology, we show

that attitude consensus among multiple networked spacecraft can be achieved as long as the undirected communication topology is connected (Theorem 3.1). As a result, there is no need for each spacecraft to explicitly identify its two adjacent neighbors in the team to form a bidirectional ring. In addition, we generalize the synchronized rotation results in [5], [6] to the case that attitude is aligned in the presence of nonzero final angular velocities (Theorem 3.2). This may be appropriate for applications where multiple spacecraft are required to rotate at the same rate and maintain the same attitudes. Furthermore, attitude consensus with a unidirectional information exchange topology is also discussed (Theorem 3.3). It is worthwhile to mention that although we use PD-like control laws for attitude consensus, existing formation control results developed for double integrator dynamics are not directly applicable to spacecraft attitude dynamics due to the inherent nonlinearity in quaternion kinematics. The extension from double integrator dynamics to spacecraft attitude dynamics is nontrivial. This paper extends the consensus type problem with single or double integrator dynamics (e.g. [7], [8], [9]) to rigid body rotational dynamics.

## II. BACKGROUND AND PRELIMINARIES

### A. Spacecraft Attitude Dynamics

We use unit quaternions to represent spacecraft attitudes in this paper. A unit quaternion is defined as  $q = [\hat{q}^T, \bar{q}]^T$ , where  $\hat{q} = a \cdot \sin(\frac{\phi}{2}) \in \mathbb{R}^3$  and  $\bar{q} = \cos(\frac{\phi}{2}) \in \mathbb{R}$ . In this notation,  $a$  is a unit vector, known as the Euler axis, and  $\phi$  is the rotation angle about  $a$ , called the Euler angle. Note that  $q^T q = 1$  by definition. A unit quaternion is not unique since  $q$  and  $-q$  represent the same attitude. However, uniqueness can be achieved by restricting  $\phi$  to the range  $0 \leq \phi \leq \pi$  so that  $\bar{q} \geq 0$  [10]. In the remainder of the paper, we restrict  $\bar{q}$  to be nonnegative (see [5] for conditions under which  $\bar{q}(t) \geq 0$ ,  $\forall t \geq 0$ , if  $\bar{q}(0) \geq 0$ ).

The product of two unit quaternions  $p$  and  $q$  is defined by

$$qp = \begin{bmatrix} \bar{q}\hat{p} + \bar{p}\hat{q} + \hat{q} \times \hat{p} \\ \bar{q}\bar{p} - \hat{q}^T \hat{p} \end{bmatrix},$$

which is also a unit quaternion. The conjugate of the unit quaternion  $q$  is defined by  $q^* = [-\hat{q}^T, \bar{q}]^T$ . The conjugate of  $qp$  is given by  $(qp)^* = p^* q^*$ . The multiplicative identity quaternion is denoted by  $\mathbf{q}_I = [0, 0, 0, 1]^T$ , where  $qq^* = q^* q = \mathbf{q}_I$  and  $q\mathbf{q}_I = \mathbf{q}_I q = q$ .

Spacecraft attitude dynamics are given by

$$\begin{aligned} \dot{\hat{q}}_i &= -\frac{1}{2}\omega_i \times \hat{q}_i + \frac{1}{2}\bar{q}_i \omega_i, & \dot{\bar{q}}_i &= -\frac{1}{2}\omega_i \cdot \hat{q}_i \\ J_i \dot{\omega}_i &= -\omega_i \times (J_i \omega_i) + \tau_i, \end{aligned} \quad (1)$$

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where  $\hat{q}_i$  and  $\bar{q}_i$  are vector and scalar parts of the unit quaternion of the  $i^{\text{th}}$  spacecraft,  $\omega_i$  is the angular velocity, and  $J_i$  and  $\tau_i$  are inertia tensor and control torque.

### B. Graph Theory

It is natural to model information exchange between spacecraft by directed/undirected graphs. A digraph (directed graph) consists of a pair  $(\mathcal{N}, \mathcal{E})$ , where  $\mathcal{N}$  is a finite nonempty set of nodes and  $\mathcal{E} \in \mathcal{N}^2$  is a set of ordered pairs of nodes, called edges. As a comparison, the pairs of nodes in an undirected graph are unordered. If there is a directed edge from node  $v_i$  to node  $v_j$ , then  $v_i$  is defined as the parent node and  $v_j$  is defined as the child node. A directed path is a sequence of ordered edges of the form  $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \dots$ , where  $v_{i_j} \in \mathcal{N}$ , in a digraph. An undirected path in an undirected graph is defined accordingly. A digraph is called strongly connected if there is a directed path from every node to every other node. An undirected graph is called connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node, except the root, has exactly one parent. A directed spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a graph has (or contains) a directed spanning tree if there exists a directed spanning tree being a subset of the graph. Note that the condition that a digraph has a directed spanning tree is equivalent to the case that there exists at least one node having a directed path to all the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected.

The adjacency matrix  $G = [g_{ij}]$  of a weighted graph is defined as  $g_{ii} = 0$  and  $g_{ij} > 0$  if  $(j, i) \in \mathcal{E}$  where  $i \neq j$ . The Laplacian matrix of the weighted graph is defined as  $L = [l_{ij}]$ , where  $l_{ii} = \sum_{j \neq i} g_{ij}$  and  $l_{ij} = -g_{ij}$ ,  $\forall i \neq j$ . For an undirected graph, the Laplacian matrix is symmetric positive semi-definite. This property does not hold for a digraph (nonsymmetric) Laplacian matrix.

In the case of an undirected interaction graph, the graph Laplacian has a simple zero eigenvalue if and only if the graph is connected [11]. In the case of a directed interaction graph, the digraph Laplacian has a simple zero eigenvalue if and only if the digraph has a directed spanning tree [8]. In both cases,  $\mathbf{1}$  is the eigenvector of the graph (digraph) Laplacian associated with eigenvalue zero, where  $\mathbf{1}$  denotes the  $n \times 1$  column vector of all ones.

### C. Matrix Theory

Let  $I_n$  denote the  $n \times n$  identity matrix. Given a real scalar  $q$ , we use  $q > 0$  to denote that  $q$  is positive. Given an  $n \times n$  real matrix  $P$ , we use  $P \succ 0$  to denote that matrix  $P$  is symmetric positive definite.

The digraph of an  $n \times n$  real matrix  $A$ , denoted by  $\Gamma(A)$ , is the digraph on  $n$  nodes such that there is a directed edge in  $\Gamma(A)$  from  $v_j$  to  $v_i$  if and only if  $a_{ij} \neq 0$  (c.f. [12]).

## III. ATTITUDE CONSENSUS AMONG MULTIPLE NETWORKED SPACECRAFT

In this section, we propose control laws for attitude consensus among multiple networked spacecraft. In the following we assume that all the vectors in each control law have been appropriately transformed and represented in the same coordinate frame.

Before moving on, we need the following lemma for our main results.

*Lemma 3.1:* If the unit quaternion and angular velocity pairs  $(q_k, \omega_k)$  and  $(q_\ell, \omega_\ell)$  satisfy the quaternion kinematics defined by the first two equations in Eq. (1), then the unit quaternion and angular velocity pair  $(q_\ell^* q_k, \omega_k - \omega_\ell)$  also satisfies the quaternion kinematics. In addition, if  $V_q = \|q_\ell^* q_k - \mathbf{q}_I\|^2$ , then  $\dot{V}_q = (\omega_k - \omega_\ell)^T \widehat{q_\ell^* q_k}$ , where  $\widehat{p}$  denotes the vector part of quaternion  $p$ .

*Proof:* see [13]. ■

We first consider the case that multiple spacecraft align their attitudes during formation maneuvers and their angular velocities approach zero under an undirected communication topology. The control torque to the  $i^{\text{th}}$  spacecraft is proposed as

$$\tau_i = -k_G \widehat{q^{d*} q_i} - D_{G_i} \omega_i - \sum_{j=1}^n g_{ij} [a_{ij} \widehat{q_j^* q_i} + b_{ij} (\omega_i - \omega_j)], \quad (2)$$

where  $k_G \geq 0$ ,  $D_{G_i} \in \mathbb{R}^{3 \times 3} \succ 0$ ,  $a_{ij} = a_{ji} > 0$ ,  $b_{ij} = b_{ji} > 0$ ,  $g_{ii} \triangleq 0$ , and  $g_{ij}$  is 1 if spacecraft  $i$  receives information from spacecraft  $j$  and 0 otherwise. Note that control law (2) is model independent (i.e., no  $J_i$ ). Also note that although certain torque feedback can be chosen to linearize the last equation in Eq. (1), the quaternion kinematics represented by the first two equations in Eq. (1) are inherently nonlinear. This feature makes the spacecraft attitude consensus problem more complicated than formation control problems for systems modeled by single or double integrator dynamics.

Compared to the control law in [6], where a bidirectional ring topology is assumed, there is no need to identify two adjacent spacecraft in the group in Eq. (2). Each spacecraft simply communicates with all the other spacecraft that are in its communication range. As long as the communication graph is connected, attitude consensus can be achieved as shown in the following.

We have the following theorem for attitude consensus among multiple networked spacecraft in the first case.

*Theorem 3.1:* Assume that the control torque is given by Eq. (2) and the communication graph is undirected and connected. If  $k_G > 0$ , then  $q_i \rightarrow q_j \rightarrow q^d$  and  $\omega_i \rightarrow 0$  asymptotically,  $\forall i \neq j$ . If  $k_G = 0$ , then  $q_i \rightarrow q_j$  and  $\omega_i \rightarrow 0$  asymptotically,  $\forall i \neq j$ .

*Proof:* Consider a Lyapunov function candidate:

$$V = k_G \sum_{i=1}^n \|q^{d^*} q_i - \mathbf{q}_I\|^2 + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} a_{ij} \|q_j^* q_i - \mathbf{q}_I\|^2 + \frac{1}{2} \sum_{i=1}^n (\omega_i^T J_i \omega_i).$$

Applying Lemma 3.1, the derivative of  $V$  is

$$\dot{V} = k_G \sum_{i=1}^n \omega_i^T \widehat{q^{d^*} q_i} + \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} a_{ij} (\omega_i - \omega_j)^T \widehat{q_j^* q_i} + \sum_{i=1}^n \omega_i^T (\tau_i - \omega_i \times J_i \omega_i). \quad (3)$$

Note that  $\omega_i^T (\omega_i \times J_i \omega_i) = 0$  and

$$\begin{aligned} & \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} a_{ij} (\omega_i - \omega_j)^T \widehat{q_j^* q_i} \\ &= \frac{1}{2} \sum_{i=1}^n \omega_i^T \left( \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} \right) - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} a_{ij} \omega_j^T \widehat{q_j^* q_i} \\ &= \frac{1}{2} \sum_{i=1}^n \omega_i^T \left( \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} \right) - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ji} a_{ji} \omega_j^T \widehat{q_j^* q_i} \\ &= \frac{1}{2} \sum_{i=1}^n \omega_i^T \left( \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} \right) + \frac{1}{2} \sum_{j=1}^n \sum_{i=1}^n g_{ji} a_{ji} \omega_j^T \widehat{q_i^* q_j} \\ &= \frac{1}{2} \sum_{i=1}^n \omega_i^T \left( \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} \right) + \frac{1}{2} \sum_{j=1}^n \omega_j^T \left( \sum_{i=1}^n g_{ji} a_{ji} \widehat{q_i^* q_j} \right) \\ &= \sum_{i=1}^n \omega_i^T \left( \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} \right), \end{aligned}$$

where we have used the fact that  $g_{ij} = g_{ji}$  and  $a_{ij} = a_{ji}$  to obtain the second equality, and we have switched the order of the summation signs and have used the fact that  $\widehat{q_j^* q_i} = -\widehat{q_i^* q_j}$  to obtain the third equality.

As a result, Eq. (3) becomes

$$\dot{V} = \sum_{i=1}^n \omega_i^T (k_G \widehat{q^{d^*} q_i} + \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} + \tau_i).$$

With control law (2), the derivative of  $V$  becomes

$$\dot{V} = - \sum_{i=1}^n (\omega_i^T D_{G_i} \omega_i) - \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} b_{ij} \|\omega_i - \omega_j\|^2 \leq 0, \quad (4)$$

where we have used the fact that

$$\sum_{i=1}^n \omega_i^T \sum_{j=1}^n g_{ij} b_{ij} (\omega_i - \omega_j) = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} b_{ij} \|\omega_i - \omega_j\|^2.$$

We consider the following two subcases:

**Subcase A:**  $k_G > 0$  and  $D_{G_i} \succ 0$ .

Let  $\Omega = \{q^{d^*} q_i - \mathbf{q}_I, \omega_i | \dot{V} = 0\}$ . Also let  $\bar{\Omega}$  be the largest invariant set in  $\Omega$ . On  $\bar{\Omega}$ ,  $\dot{V} \equiv 0$ , which implies that  $\omega_i \equiv 0$ ,  $i = 1, \dots, n$ . Because  $\omega_i \equiv 0$ , we know that

$$k_G \widehat{q^{d^*} q_i} + \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} = 0, \quad i = 1, \dots, n. \quad (5)$$

from Eqs. (1) and (2).

Noting that  $q_j^* q_i = q_j^* (q^d q^{d^*}) q_i = (q_j^* q^d) (q^{d^*} q_i)$ , we rewrite Eq. (5) as

$$\widehat{p_i^* q_i} = 0, \quad (6)$$

where

$$p_i = k_G \mathbf{q}_I + \sum_{j=1}^n g_{ij} a_{ij} q^{d^*} q_j. \quad (7)$$

Also note that Eq. (6) is equivalent to

$$-\overline{q^{d^*} q_i} \widehat{p_i} + \widehat{p_i} \widehat{q^{d^*} q_i} + \widehat{q^{d^*} q_i} \times \widehat{p_i} = 0. \quad (8)$$

Motivated by [5], we multiply Eq. (8) by  $(\widehat{q^{d^*} q_i} \times \widehat{p_i})^T$  and get

$$\left\| \widehat{q^{d^*} q_i} \times \widehat{p_i} \right\|^2 = 0. \quad (9)$$

Combining Eqs. (8) and (9), gives

$$-\overline{q^{d^*} q_i} \widehat{p_i} + \widehat{p_i} \widehat{q^{d^*} q_i} = 0. \quad (10)$$

Using Eq. (7), we rewrite Eq. (10) as

$$\begin{aligned} & -\overline{q^{d^*} q_i} \sum_{j=1}^n g_{ij} a_{ij} \widehat{q^{d^*} q_j} \\ & + (k_G + \sum_{j=1}^n g_{ij} a_{ij} \overline{q^{d^*} q_j}) \widehat{q^{d^*} q_i} = 0, \quad i = 1, \dots, n. \quad (11) \end{aligned}$$

Note that Eq. (11) can be written in matrix form as

$$(k_G I_n \otimes I_3 + L \otimes I_3) \hat{q}_s = 0,$$

where  $\otimes$  is the Kronecker product,  $I_3$  is the  $3 \times 3$  identity matrix,  $\hat{q}_s$  is a column vector stack composed of  $\widehat{q^{d^*} q_\ell}$ ,  $\ell = 1, \dots, n$ , and  $L = [\ell_{ij}]$  is a Laplacian matrix with  $\ell_{ii} = \sum_{j=1}^n g_{ij} a_{ij} \overline{q^{d^*} q_i}$  and  $\ell_{ij} = -g_{ij} a_{ij} \overline{q^{d^*} q_i}$ .

With the assumption that  $\overline{q^{d^*} q_i} \geq 0$ , we see that matrix  $k_G I_n + L$  is strictly diagonally dominant and therefore has full rank, which in turn implies that  $\hat{q}_s = 0$ . Therefore, we see that  $\widehat{q^{d^*} q_i} = 0$ ,  $i = 1, \dots, n$ , which implies that  $q_i = q^d$ .

Therefore, by LaSalle's invariance principle  $q^{d^*} q_i - \mathbf{q}_I \rightarrow 0$  and  $\omega_i \rightarrow 0$  asymptotically. Equivalently, we know that  $q_i \rightarrow q_j \rightarrow q^d$ ,  $\forall i \neq j$ , and  $\omega_i \rightarrow 0$ ,  $i = 1, \dots, n$ .

**Subcase B:**  $k_G = 0$  and  $D_{G_i} \succ 0$ .

In this case, let  $\Omega = \{(q_j^* q_i - \mathbf{q}_I, \omega_i) | \dot{V} = 0\}$ . Also let  $\bar{\Omega}$  be the largest invariant set in  $\Omega$ . On  $\bar{\Omega}$ ,  $\dot{V} \equiv 0$ , which implies that  $\omega_i \equiv 0$ ,  $i = 1, \dots, n$ . Then we can follow the same arguments as in Subcase A by letting  $p_i = \sum_{j=1}^n g_{ij} a_{ij} q_j$ . After some manipulation, we can show that

$$(L \otimes I_3) \hat{q}_t = 0,$$

where  $\hat{q}_t$  is a column vector stack composed of  $\hat{q}_\ell$ ,  $\ell = 1, \dots, n$ , and  $L = [\ell_{ij}]$  is a Laplacian matrix with  $\ell_{ii} = \sum_{j=1}^n g_{ij} a_{ij} q_i$  and  $\ell_{ij} = -g_{ij} a_{ij} q_i$ .

Noting that the undirected communication graph is connected, we know that  $\hat{q}_t = \mathbf{1} \otimes \hat{q}_0$ , where  $\hat{q}_0 \in \mathbb{R}^3$ . Therefore, we see that  $q_i = q_j$ ,  $\forall i \neq j$ , which implies that  $q_j^* q_i = \mathbf{q}_I$ .

Therefore, by LaSalle's invariance principle  $q_j^* q_i - \mathbf{q}_I \rightarrow 0$  and  $\omega_i \rightarrow 0$  asymptotically. Equivalently, we know that  $q_i \rightarrow q_j$ ,  $\forall i \neq j$ , and  $\omega_i \rightarrow 0$ ,  $i = 1, \dots, n$ . ■

We also consider the case that multiple spacecraft align their attitudes but with nonzero angular velocities under an undirected communication topology. We propose the following control torque:

$$\tau_i = \omega_i \times J_i \omega_i - J_i \sum_{j=1}^n g_{ij} [a_{ij} \widehat{q_j^* q_i} + b_{ij} (\omega_i - \omega_j)]. \quad (12)$$

Note that control law (12) is model dependent in the sense that  $J_i$  is required to be known.

*Theorem 3.2:* With control torque given by Eq. (12), if the communication graph is undirected and connected, then  $q_i \rightarrow q_j$ , and  $\omega_i \rightarrow \omega_j$  asymptotically,  $\forall i \neq j$ .

*Proof:* Consider a Lyapunov function candidate:

$$V = \frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} a_{ij} \|q_j^* q_i - \mathbf{q}_I\|^2 + \frac{1}{2} \sum_{i=1}^n \omega_i^T \omega_i.$$

Following a similar procedure to that of Theorem 3.1, we obtain

$$\dot{V} = -\frac{1}{2} \sum_{i=1}^n \sum_{j=1}^n g_{ij} b_{ij} \|\omega_i - \omega_j\|^2 \leq 0.$$

Let  $\Omega = \{(q_j^* q_i - \mathbf{q}_I, \omega_i | \dot{V} = 0)\}$ . Also let  $\bar{\Omega}$  be the largest invariant set in  $\Omega$ . On  $\bar{\Omega}$ ,  $\dot{V} \equiv 0$ , which implies that  $\omega_i = \omega_j$ ,  $\forall i \neq j$ , since the undirected communication graph is connected. Therefore, we see that  $\dot{\omega}_i = \dot{\omega}_j$ . As a result, we know that  $\dot{\omega} \in \text{span}\{\mathbf{1} \otimes \eta\}$ , where  $\dot{\omega} = [\dot{\omega}_1, \dots, \dot{\omega}_n]^T$  and  $\eta$  is a  $3 \times 1$  vector.

With control torque (12), we know that

$$\dot{\omega}_i = -\sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i}, \quad i = 1, \dots, n.$$

We also know that

$$\sum_{i=1}^n \eta^T \dot{\omega}_i = -\sum_{i=1}^n \eta^T \left( \sum_{j=1}^n g_{ij} a_{ij} \widehat{q_j^* q_i} \right) = 0,$$

where we have used the fact that  $\widehat{q_j^* q_i} = -\widehat{q_i^* q_j}$ . As a result, we see that  $\dot{\omega}$  is orthogonal to  $\text{span}\{\mathbf{1} \otimes \eta\}$ .

Therefore, we conclude that  $\dot{\omega} = 0$ . Following the proof for Subcase B in Theorem 3.1, we see that  $q_i = q_j$  since the undirected graph is connected. By LaSalle's invariance principle  $q_i \rightarrow q_j$  and  $\omega_i \rightarrow \omega_j$  asymptotically. ■

Note that if the undirected graph is not connected, then the above conclusion is still true for each connected subgroup. If it is desirable that spacecraft maintain different relative attitudes during formation maneuvers, we can replace each  $q_i$  by  $q_i q_{\delta_i}$ , where  $q_{\delta_i}$  is a constant quaternion defining relative attitudes between spacecraft. As a result,  $q_i q_{\delta_i} \rightarrow q_j q_{\delta_j}$ , that is,  $q_j^* q_i \rightarrow q_{\delta_j} q_{\delta_i}^*$ . In addition, we can extend control torque (2) such that  $q_i \rightarrow q_j$  and  $\omega_i \rightarrow \omega^d$  by deleting the term  $-k_G q^d q_i$  and replacing the term  $-D_{G_i} \omega_i$  by the term  $-D_{G_i} (\omega_i - \omega^d)$ .

We finally consider the case that multiple spacecraft align their attitudes under a directed information flow topology.

Let  $\mathcal{J}_i$  denote the set of spacecraft whose information is available to the  $i^{\text{th}}$  spacecraft. Let  $|\mathcal{J}_i|$  denote the cardinality of  $\mathcal{J}_i$ . We assume that  $i$  is not in set  $\mathcal{J}_i$ . Note that  $j \in \mathcal{J}_i$  does not imply  $i \in \mathcal{J}_j$  under the directed information flow topology. Also let  $q_i^r$  and  $\omega_i^r$  denote the desired attitude and angular velocity of the  $i^{\text{th}}$  spacecraft.

Suppose that  $|\mathcal{J}_i| \geq 1$ . Define  $q_i^r = q_i (\prod_{j \in \mathcal{J}_i} (q_j^* q_i))^*$  and  $\omega_i^r = \omega_i - \sum_{j \in \mathcal{J}_i} (\omega_i - \omega_j)$ . Note that  $q_i^r$  and  $\omega_i^r$  also satisfy the quaternion kinematics in this case. The control torque is defined as

$$\tau_i = \omega_i \times J_i \omega_i + \frac{1}{|\mathcal{J}_i|} [J_i \sum_{j \in \mathcal{J}_i} \dot{\omega}_j - k_{qi} \widehat{q_i^r q_i} - K_{\omega_i} (\omega_i - \omega_i^r)], \quad (13)$$

where  $k_{qi} > 0$  and  $K_{\omega_i} \in \mathbb{R}^{3 \times 3} \succ 0$ .

*Theorem 3.3:* With control torque (13), attitude consensus is achieved among a team of spacecraft only if the directed information flow graph contains a directed spanning tree. If the information flow graph contains a directed spanning tree, then  $\prod_{j \in \mathcal{J}_i} (q_j^* q_i) \rightarrow 0$  and  $\omega_i \rightarrow \omega_j$  asymptotically,  $\forall i \neq j$ . In particular, if the information flow graph is a unidirectional ring topology, then  $q_i \rightarrow q_j$  and  $\omega_i \rightarrow \omega_j$  asymptotically,  $\forall i \neq j$ .

*Proof:* For the first statement, if the directed information flow graph does not contain a directed spanning tree, there exist either multiple separated groups or multiple leaders. In the former case, there is no interaction between the separated subgroups, which implies that attitude consensus cannot be achieved between these subgroups. In the latter case, the attitudes of the multiple leaders are not affected by any other spacecraft in the team, which imply that attitude consensus cannot be achieved between these leaders.

For the second statement, note that with control law (13) Eq. (1) can be written as  $J_i \dot{\omega}_i = J_i \dot{\omega}_i^r - k_{qi} \widehat{q_i^r q_i} - K_{\omega_i} (\omega_i - \omega_i^r)$ , which implies that  $\widehat{q_i^r q_i} \rightarrow 0$  and  $\omega_i \rightarrow \omega_i^r$ ,  $i = 1, \dots, n$ , according to [1]. Equivalently, we know that  $\prod_{j \in \mathcal{J}_i} (q_j^* q_i) \rightarrow 0$  and  $\sum_{j \in \mathcal{J}_i} (\omega_i - \omega_j) \rightarrow 0$ ,  $i = 1, \dots, n$ , asymptotically. Noting that the information flow graph contains a directed spanning tree, we know that  $\omega_i \rightarrow \omega_j$ ,  $\forall i \neq j$ .

For the third statement, if the information flow graph is a unidirectional ring topology, we number an arbitrary spacecraft in the team as spacecraft  $S_1$  and number the other spacecraft consecutively as spacecraft  $S_2$  to  $S_n$  following the order of information flow. As a result, we see that  $q_1^r = q_n$  and  $q_i^r = q_{i-1}$ , where  $i = 2, \dots, n$ . With control torque (13), we see that  $q_1 \rightarrow q_n$  and  $q_j \rightarrow q_{j-1}$ ,  $j = 2, \dots, n$ , which implies that  $q_i \rightarrow q_j$ ,  $\forall i \neq j$ . ■

In the case that there exists a unique team leader, denoted as spacecraft  $\ell$ , we assume that  $q_\ell^r$  and  $\omega_\ell^r$  are defined by a mission planner (i.e.,  $|\mathcal{J}_\ell| = 0$ ) and they satisfy the quaternion kinematics. The control torque for spacecraft  $\ell$  is defined as

$$\tau_\ell = \omega_\ell \times J_\ell \omega_\ell + J_\ell \dot{\omega}_\ell^r - k_{q\ell} \widehat{q_\ell^r q_\ell} - K_{\omega_\ell} (\omega_\ell - \omega_\ell^r).$$

The conclusion of Theorem 3.3 also holds if the information flow graph contains a directed spanning tree with spacecraft  $\ell$  being the root. In particular, if the information flow graph is itself a spanning tree with spacecraft  $\ell$  being the root, then  $q_i \rightarrow q_\ell^*$ ,  $\forall i$ , which corresponds to the leader-follower topology in [1].

#### IV. SIMULATION RESULTS

In this section, we simulate a scenario where six spacecraft align their attitudes through local information exchange. We will consider the first two cases in Section III due to space limitation. The undirected communication topology is shown by Fig. 1.

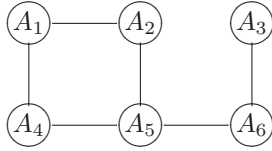


Fig. 1. Communication graph for Cases 1 and 2.

The spacecraft specifications are shown in Table I. The control parameters and control laws used for each case are shown in Table II. In the following, let  $q^d = [0, 0, 0, 1]^T$  and choose  $q_i(0)$  and  $\omega_i(0)$  randomly. Also assume that the control torque of each spacecraft satisfies  $|\tau_i^{(j)}| \leq 1$  Nm, where  $j = 1, 2, 3$  denotes each component of the control torque.

TABLE I  
SPACECRAFT SPECIFICATIONS.

|       |   |
|-------|---|
| $J_1$ | [1 0.1 0.1; 0.1 0.1 0.1; 0.1 0.1 0.9] kg m <sup>2</sup>       |
| $J_2$ | [1.5 0.2 0.3; 0.2 0.9 0.4; 0.3 0.4 2.0] kg m <sup>2</sup>     |
| $J_3$ | [0.8 0.1 0.2; 0.1 0.7 0.3; 0.2 0.3 1.1] kg m <sup>2</sup>     |
| $J_4$ | [1.2 0.3 0.7; 0.3 0.9 0.2; 0.7 0.2 1.4] kg m <sup>2</sup>     |
| $J_5$ | [0.9 0.15 0.3; 0.15 1.2 0.4; 0.3 0.4 1.2] kg m <sup>2</sup>   |
| $J_6$ | [1.1 0.35 0.45; 0.35 1.0 0.5; 0.45 0.5 1.3] kg m <sup>2</sup> |

TABLE II  
CONTROL PARAMETERS FOR DIFFERENT CASES.

|          |          |   |
|----------|----------|---|
| Case 1-A | Eq. (2)  | $k_G = 1, D_{Gi} = 2I_3, a_{ij} = 5, b_{ij} = 10$ |
| Case 1-B | Eq. (2)  | $k_G = 0, D_{Gi} = 2I_3, a_{ij} = 5, b_{ij} = 10$ |
| Case 2   | Eq. (12) | $a_{ij} = 5, b_{ij} = 10$                         |

In each plot hereafter, we use a superscript ( $j$ ) to denote the  $j^{\text{th}}$  component of a quaternion or vector. Figs. 2 and 3 show respectively the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 1-A. Note that each spacecraft converges to its desired attitude while aligns their attitudes during the transition. Also note that the angular velocities of each spacecraft converge to zero. The plot for the control torques is omitted due to space limitation.

Figs. 4 and 5 show respectively the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 1-B. Note that each spacecraft converges to the same attitude and the angular velocities of each spacecraft converge to zero.

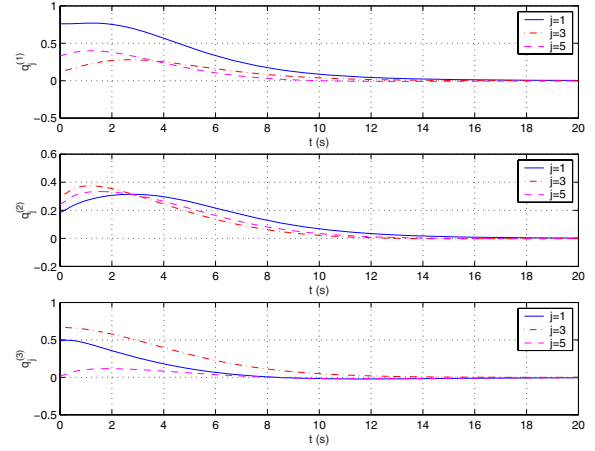


Fig. 2. Spacecraft attitudes in Case 1-A.

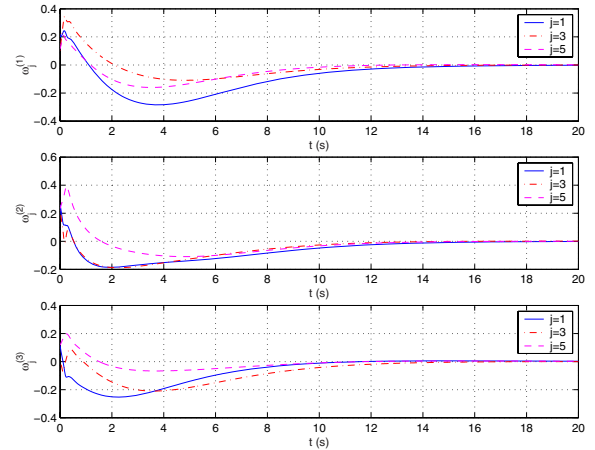


Fig. 3. Spacecraft angular velocities in Case 1-A.

Figs. 6 and 7 show respectively the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 2. Note that each spacecraft converges to the same attitude and the same (possibly nonzero) angular velocity.

#### V. CONCLUSION AND FUTURE WORK

We have considered the distributed attitude consensus problem among a team of spacecraft. We have proposed control laws and shown conditions under which attitudes are aligned with zero or nonzero final angular velocities under an undirected communication topology. The case of a directed information flow topology is also discussed. Simulation results have shown a scenario where six spacecraft align their attitudes through local information exchange.

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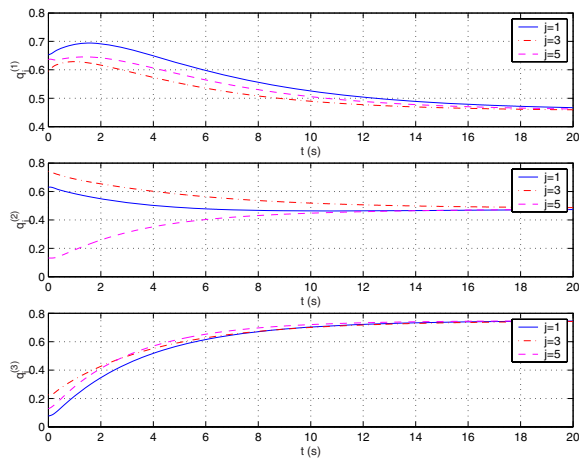


Fig. 4. Spacecraft attitudes in Case 1-B.

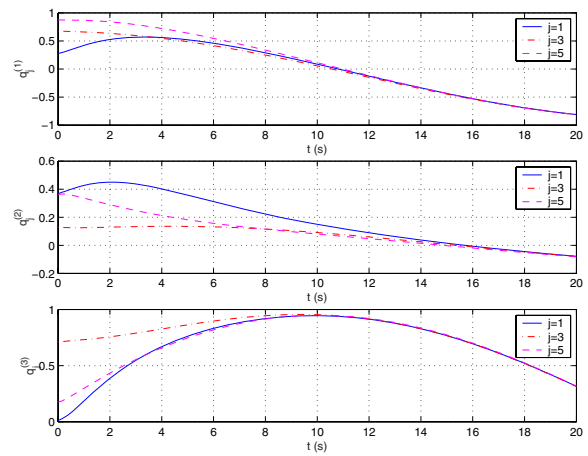


Fig. 6. Spacecraft attitudes in Case 2.

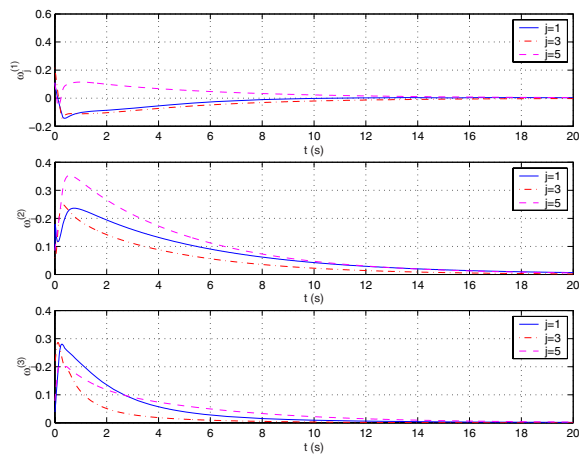


Fig. 5. Spacecraft angular velocities in Case 1-B.

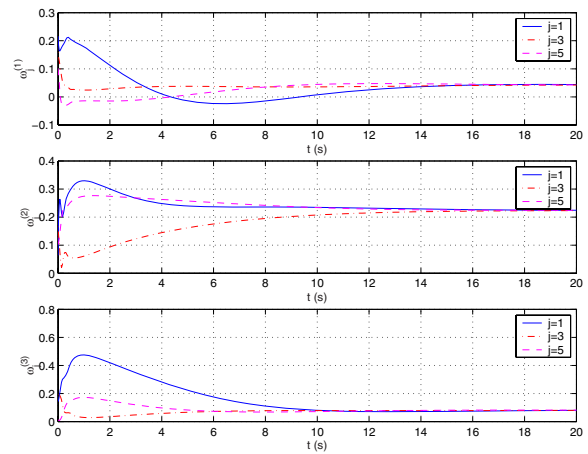


Fig. 7. Spacecraft angular velocities in Case 2.

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