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Consensus Building in Multi-vehicle Systems with Information Feedback

Wei Ren
Department of Electrical and Computer Engineering
Utah State University
Logan, UT, 84322-4120, USA
wren@engineering.usu.edu

***Abstract*— In this paper, we study the problem of consensus building in multi-vehicle systems with information feedback. We will show how information feedback can be incorporated into the consensus building process so as to improve the robustness and situational awareness of the whole team. We will detail strategies of introducing feedback to the consensus building process through information flow, external feedback terms, state-dependent weighting factors, and reference states. Application examples are also given to illustrate the information feedback strategies.**

***Index Terms*— Consensus building, Multi-vehicle systems, Cooperative control**

I. Introduction

Autonomous vehicle systems are expected to find potential applications in military operations, search and rescue, environment monitoring, commercial cleaning, material handling, and homeland security. While single vehicles performing solo missions will yield some benefits, greater benefits will come from the cooperation of teams of vehicles. Cooperative control of multi-vehicle systems has received significant attention in the control and robotics communities in recent years regarding the benefits of using many inexpensive, simple systems to replace a single monolithic, expensive, and complicated system.

As an inherently distributed strategy for multi-vehicle coordination, consensus algorithms have recently been studied extensively in the context of cooperative control of multi-vehicle systems [1], [2], [3], [4], [5], [6], [7], to name a few. Those algorithms only require local neighbor-to-neighbor information exchange between the vehicles. The basic idea for information consensus is that each vehicle updates its information state based on the information states of its local (possibly time-varying) neighbors in such a way that the final information state of each vehicle converges to a common value. This basic idea can be extended to deal with the case that each vehicle's information states converge to desired relative deviations or to incorporate different group behaviors into the consensus building process. Consensus algorithms have numerous applications in rendezvous problem, cooperative timing, formation control, attitude synchronization, and distributed decision making.

Most consensus algorithms studied in the literature do not take into account vehicle performance, environmental information, sensor measurement, etc. in the consensus building process. For example, in some formation control problems where the formation is moving through space, the information states of each vehicle may be dynamically evolving in time according to some inherent dynamics. Also in most cooperative control problems, the information states of each vehicle may be affected by environmental factors or sensor measurement. As a result, it is essential to incorporate vehicle performance, environmental information, sensor measurement into the consensus building process as a form of feedback.

The main contribution of this paper is to study how information feedback can be incorporated into the consensus building process so as to improve the robustness and situational awareness of the whole team. In particular, we study strategies of introducing information feedback to the consensus building process through information flow, external feedback terms, state-dependent weighting factors, and reference states. The first three strategies will be illustrated by application examples.

II. Background and Preliminaries

It is natural to model information exchange between vehicles by directed/undirected graphs. A digraph (directed graph) consists of a pair $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a finite nonempty set of nodes and $\mathcal{E} \in \mathcal{N}^2$ is a set of ordered pairs of nodes, called edges. As a comparison, the pairs of nodes in an undirected graph are unordered. If there is a directed edge from node v_i to node v_j, then v_i is defined as the parent node and v_j is defined as the child node. A directed path is a sequence of ordered edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \cdots$, where $v_{i_j} \in \mathcal{N}$, in a digraph. An undirected path in an undirected graph is defined accordingly. A digraph is called strongly connected if there is a directed path from every node to every other nodes. An undirected graph is called connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node, except the root, has exactly one parent. A directed spanning tree of a digraph is a directed tree formed by graph

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edges that connect all the nodes of the graph. We say that a graph has (or contains) a directed spanning tree if there exists a directed spanning tree being a subset of the graph. Note that the condition that a digraph has a directed spanning tree is equivalent to the case that there exists at lease one node having a directed path to all the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected. The union of a group of digraphs is a digraph with nodes given by the union of the node sets and edges given by the union of the edge sets of those digraphs.

Fig. 1 shows a directed graph with more than one possible spanning trees, but is not strongly connected. The double arrows denote one possible spanning tree with A_5 as the parent. Spanning trees with A_1 and A_4 as the parent, are also possible.

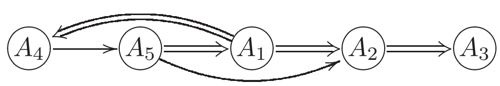

Fig. 1. A directed graph that has more than one possible spanning trees, but is not strongly connected. One possible spanning tree is denoted with double arrows.

III. Fundamental Consensus Algorithm

For information states with dynamics given by

$$\dot{\xi}_i = u_i, \quad i = 1, \cdots, n, \tag{1}$$

where $\xi_i \in I\!R^m$ denotes the information state of the i^{th} vehicle and $u_i \in I\!R^m$ is the control input, consensus algorithm is proposed in [1], [3], [5], [8] as

$$u_i = -\sum_{j=1}^{n} g_{ij} k_{ij} (\xi_i - \xi_j), \tag{2}$$

where $k_{ij} > 0$, $g_{ii} \stackrel{\triangle}{=} 0$, and g_{ij} is 1 if information flows from vehicle j to vehicle i and 0 otherwise, $\forall i \neq j$.

By applying algorithm (2), Eq. (1) can be written in matrix form as

$$\dot{\xi} = -(L \otimes I_m)\xi,$$

where $\xi = [\xi_1^T, \cdots, \xi_n^T]^T$, $\otimes$ denotes the Kronecker product, and $L = [\ell_{ij}] \in I\!R^{n\times n}$ is given as $\ell_{ii} = \sum_{j\neq i} g_{ij} k_{ij}$ and $\ell_{ij} = -g_{ij} k_{ij}$, $\forall i \neq j$.

Consensus is said to be reached among the n vehicles if $\xi_i(t) \rightarrow \xi_j(t)$, $\forall i \neq j$, as $t \rightarrow \infty$. With consensus algorithm (2), the final consensus value is a weighted average of the vehicles' initial information states. Note that the final consensus value is generally *a priori* unknown and will depend on the information exchange topologies as well as weighting factors k_{ij}.

Under a fixed information exchange topology, algorithm (2) achieves consensus asymptotically if and only if the information exchange topology has a (directed) spanning tree [8].

In the following, we assume a directed information exchange topology to take into account the case that sensors may have a limited field of view in the case of information exchange through local sensing. Note that undirected information exchange is a special case of directed information exchange.

IV. Consensus Building with Information Feedback through Information Flow

A. Basic Result

The most straightforward strategy to introduce feedback to the consensus building process is through information flow between local neighbors.

With consensus algorithm (2), the final consensus value is given by $\xi^* = \sum_{i=1}^{n} \alpha_i \xi_i(0)$, where $\alpha = [\alpha_1, \cdots, \alpha_n]^T$ is a nonnegative left eigenvector of $-L$ associated with eigenvalue 0 with $\alpha_i \geq 0$ and $\sum_{i=1}^{n} \alpha_i = 1$ [8]. Note that $\alpha_i \neq 0$ if vehicle i has a directed path to all the other vehicles in the information exchange topology and $\alpha_i = 0$ if there does not exist such a directed path [8]. As a result, if a vehicle wants to contribute to the final consensus value, its information needs to flow to all the vehicles in the team directly or indirectly.

Information flow between vehicles can also be applied to increase the redundancy and robustness of the whole team in the case of failures of certain information exchange links. For example, if vehicle j only receives data due to its station as strictly a "child" in the directed information exchange topology or due to unreliable state data transmission, any disturbance to this vehicle will cause inaccuracy in the vehicle team. However, if vehicle j is also a parent of another vehicle, then this disturbance feedback is propagated to the other vehicle.

B. Illustrative Example

To illustrate, we consider two information exchange topologies shown in Fig. 2. In Fig. 2, Subplot (a) corresponds to a leader-follower topology where vehicle $j+1$ follows vehicle j, $j = 1, 2, 3$. Subplots (b) corresponds to a topology where feedback is introduced from followers to leaders through information flow. Note that the final consensus value with Subplot (a) is $\xi_1(0)$ while the final consensus value with Subplot (b) is a weighted average of $\xi_1(0)$, $\xi_2(0)$, and $\xi_3(0)$. Also note that in Subplot (a) if vehicle 3 is perturbed by disturbance, vehicles 1 and 2 are unaware of this disturbance and their motions remain unaffected. However, in Subplot (b) if vehicle 3 is perturbed by disturbance, vehicles 1 and 2 are able to adjust their motions according to the motion of vehicle 3 so as to maintain better team performance due to the information flow from vehicle 3 to vehicles 1 and 2 directly or indirectly.

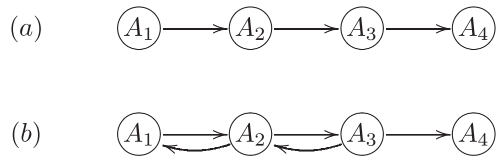

Fig. 2. Information exchange topologies between four vehicles. Subplot (a) denotes a leader-follower topology. Subplots (b) denotes a topology with information flow introduced from followers to leaders.

V. Consensus Building with Information Feedback through External Feedback Terms

A. Basic Result

Another strategy to introduce feedback to the consensus building process is through an external feedback term. Consider the following algorithm:

$$u_i = -\sum_{j=1}^{n} g_{ij} k_{ij} (\xi_i - \xi_j) + \rho_i(t, x_i, \{j \in \mathcal{N}_i | x_j\}), \quad (3)$$

where $\mathcal{N}_i$ denotes the set of vehicles whose information is available to vehicle i, and $\rho_i(\cdot,\cdot,\cdot)$ denotes a feedback term introduced to the i^{th} vehicle from its local neighbors. As a result, the consensus building process of each vehicle will be affected by the performance of its local neighbors, which serves as a form of information feedback and therefore improves the robustness of the whole team.

We need the following lemma to analyze Eq. (3).

Lemma 5.1: Let $\dot{\xi}_i = -\sum_{j=1}^{n} g_{ij}(t) k_{ij}(t)(\xi_i - \xi_j)$, where $k_{ij}(t) > 0$ is piecewise continuous and uniformly lower and upper bounded. If there exist infinitely many consecutive uniformly bounded time intervals such that the union of the information exchange graph across each such interval has a (directed) spanning tree, then $\xi_i \to \xi_j$, $\forall i \neq j$, uniformly exponentially. Furthermore, if $\|\rho_i - \rho_j\|$ is bounded, so is $\|\xi_i - \xi_j\|$, $\forall i \neq j$.

Proof: See [9] and [10]. ■

B. Illustrative Example

We will apply the virtual leader/virtual structure approach (e.g. [11], [12], [13], [14]) to deal with a formation control problem. Let $x_0(s(t))$ denote the parameterized state of the virtual leader of the team, where s is a parameter that incorporates error feedback into the whole system through its evolution [14]. Let $x_i^d(s(t))$ represent the desired state of the i^{th} vehicle, which can be defined from $x_0(s(t))$. Suppose that a pointwise control Lyapunov function (CLF) can be found for a smooth parameterized desired trajectory $x_i^d(s)$, that is, $V_i(x_i, x_i^d(s)) = 0$ at $x_i = x_i^d(s)$ for each $s \in [s_s, s_f]$. We also assume that $V_i(x_i, x_i^d(s)) \to \infty$ if $\left\|x_i - x_i^d(s)\right\| \to \infty$ for any $s \in [s_s, s_f]$.

In [15] CLFs are used to define a formation measure function so that a constrained motion control problem of multiple systems is converted into a stabilization problem for one single system. To represent the formation maintenance accuracy, a formation measure function is defined in [15] as

$$F(x,s) = \sum_{i=1}^{n} \beta_i V_i(x_i(t), x_i^d(s))),$$

where V_i is the pointwise CLF for each vehicle and $\beta_i > 0$. The formation is defined to be preserved if $F(x,s) \le F_U$, where F_U is an upper bound on the formation measure function $F(x,s)$. Also the evolution speed of s is defined as

$$\dot{s} = \begin{cases} \min\left\{ \frac{v_0}{\delta + \|\frac{\partial x_0(s)}{\partial s}\|}, \frac{-\frac{\partial F}{\partial x}\dot{x}}{\delta + |\frac{\partial F}{\partial s}|} \left(\frac{\sigma(F_U)}{\sigma(F(x,s))} \right) \right\}, & s_s \le s < s_f \\ 0, \quad s = s_f \end{cases}, \quad (4)$$

where $\delta > 0$ is a small positive constant, v_0 is the nominal velocity for the formation, and $\sigma(\cdot)$ is a class $\mathcal{K}$ function. Therefore, formation maneuvers are performed in two steps. First, when $s_s \le s < s_f$, the formation is preserved within some boundary given by F_U. Second, when $s = s_f$, each vehicle is regulated to a constant desired state given by $x_i^d(s_f)$ and reaches (eventually) its final goal.

In [15], parameter s is implemented at a central location and broadcast to all the vehicles in the team. The states of each vehicle are also sent to the central location to incorporate error feedback into the evolution of s. Each vehicle then derives its local control law according to the evolution law of s. While this implementation is feasible in the case that a robust central location exists and high bandwidth communication is available, issues such as a single point of failure or stringent intervehicle communication constraints will significantly degrade the overall system performance.

In the following, we study a decentralized scheme, where each vehicle instantiates a local copy of s, denoted as s_i. All the vehicles then exchange their instantiations between local neighbors through intervehicle communication.

The formation measure function for vehicle i is defined as

$$F_i(x_i, s_i) = \beta_i V_i(x_i(t), x_i^d(s_i))),$$

where V_i is the pointwise CLF for each vehicle and $\beta_i > 0$.

The evolution law of s_i is defined as

$$\dot{s}_i = \begin{cases} -\sum_{j=1}^{n} g_{ij}(t) k_{ij}(t)(s_i - s_j) \\ +\min\left\{ \frac{v_0}{\delta + \|\frac{\partial x_0(s_i)}{\partial s_i}\|}, \frac{-\frac{\partial F_i}{\partial x_i}\dot{x}_i}{\delta + |\frac{\partial F_i}{\partial s_i}|} \left(\frac{\sigma(F_{Ui})}{\sigma(F_i(x_i,s_i))} \right) \right\}, \\ \qquad s_s \le s < s_f \\ -\sum_{j=1}^{n} k_{ij}(t) g_{ij}(t)(s_i - s_j), \quad s = s_f \end{cases}, \quad (5)$$

where $k_{ij}(t) > 0$ is piecewise continuous and uniformly lower and upper bounded, $g_{ii}(t) \stackrel{\triangle}{=} 0$, and $g_{ij}(t)$ is 1 if vehicle i receives s_j from vehicle j at time t and 0 otherwise. In Eq. (5), at $s_s \le s < s_f$ the first term is used to drive $s_i \to s_j$, $\forall i \neq j$, and the second term is used to incorporate feedback from the i^{th} vehicle's tracking performance to the evolution speed of s_i.

Note that the evolution speed of s_i depends on $F_i(x_i, s_i)$ and s_j, $j \in \mathcal{N}_i$, where $\mathcal{N}_i$ represents the i^{th} vehicle's (possibly time-varying) local neighbors. Each vehicle then derives its local control law according to the evolution law of s_i.

The formation is defined to be preserved if $|s_i - s_j| \leq s_U$, $\forall i \neq j$, where s_U is an upper bound on the inconsistency of s_i, and $F_i(x_i, s_i) \leq F_{Ui}$, where F_{Ui} is an upper bound on the formation measure function $F_i(x_i, s_i)$.

Note that

$$\min\left\{\frac{v_0}{\delta + \|\frac{\partial x_0(s_i)}{\partial s_i}\|}, \frac{-\frac{\partial F_i}{\partial x_i}\dot{x}_i}{\delta + |\frac{\partial F_i}{\partial s_i}|}\left(\frac{\sigma(F_{Ui})}{\sigma(F_i(x_i, s_i))}\right)\right\}$$

is bounded at $s_s \leq s < s_f$. From Lemma 5.1 we know that $|s_i - s_j|$, $\forall i \neq j$, is bounded at $s_s \leq s < s_f$ if there exist infinitely many consecutive uniformly bounded time intervals such that the union of the information exchange graph across each such interval has a (directed) spanning tree. Furthermore, we know that $s_i \to s_j$, $\forall i \neq j$, at $s = s_f$ under the same condition.

To illustrate, consider fully actuated mobile robot kinematic equations given by

$$\dot{z}_i = u_i, \tag{6}$$

where $z_i = [x_i, y_i]^T$ represents the position of the i^{th} robot, and $u_i = [u_{xi}, u_{yi}]^T$ represents the control input.

We will simulate two robots moving in a spiral formation with $s \in [0, 5]$. The desired distance between these two robots is 10 meters. The center of the line connecting the desired positions of the two robots, i.e., the virtual center of the formation, tracks a trajectory $(x_0(s), y_0(s))$ given by $(0, s)$. Also the line connecting the desired positions of the two robots rotates about its center counterclockwise with an angle given by $\omega_0 s$. The desired states for the two robots are (x_i^d, y_i^d) which are given by $(5\cos(\omega_0 s), s + 5\sin(\omega_0 s))$ and $(-5\cos(\omega_0 s), s - 5\sin(\omega_0 s))$ respectively. The two robots start from rest with some initial errors.

Let $V_i = \frac{1}{2}(x_i - x_i^d)^2 + \frac{1}{2}(y_i - y_i^d)^2$, which is a valid (pointwise in s) CLF for the i^{th} robot. Define $F_i(x_i, s_i) = 2V_i(x_i, x_i^d(s))$ as the formation measure function for vehicle i. The local control laws for each robot are derived using the pointwise CLF so that $x_i \to x_i^d(s)$ and $y_i \to y_i^d(s)$ pointwise in s.

In the decentralized scheme, set $F_{U1} = F_{U2} = 0.1$. In Fig. 3, we plot the desired trajectories for robot #1 and #2. The actual trajectories almost coincide with the desired ones. To see the pattern clearly, we let $s \in [0, 15]$. Assume that robot #1 and #2 obtain one another's instantiation of s through communication. Also assume that there exists inconsistency between s_1 and s_2 at $t = 0$ sec. Fig. 4 shows the inconsistency between s_1 and s_2, denoted by $s_1 - s_2$. Note that $s_1 - s_2$ is bounded and approaches zero as $s_i \to s_f$, $i = 1, 2$. Fig. 5 shows formation measure function $F_i(x_i, s_i)$, $i = 1, 2$. We can see that $F_i(x_i, s_i)$, $i = 1, 2$, are large at $t = 0$ sec due to the inconsistency between s_1 and s_2 at $t = 0$ sec. Then both $F_i(x_i(t), s_i(t))$, $i = 1, 2$, decrease and stay below F_{Ui} as s_i increases. The tracking errors for robot #1 and #2 are shown in Fig. 6.

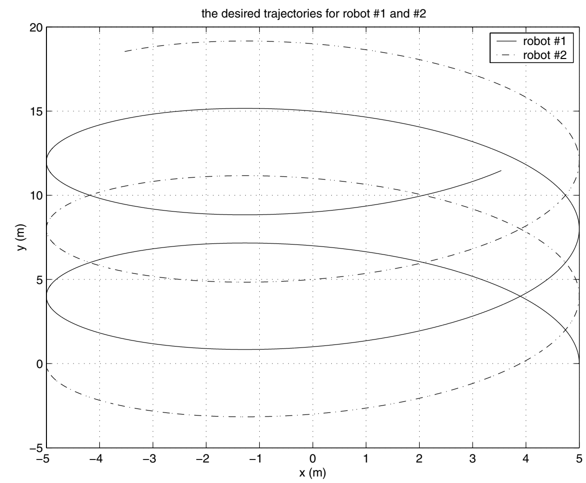

Fig. 3. The desired trajectories for robot #1 and #2.

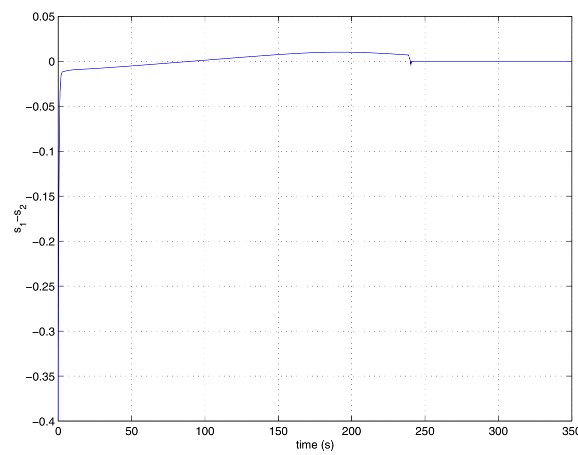

Fig. 4. The inconsistency between s_i using the decentralized scheme.

VI. Consensus Building with Information Feedback through State-dependent Weighting Factors

A. Basic Result

Information feedback can also be introduced to the consensus building process through state-dependent weighting factors. Consider the following algorithm:

$$u_i = -\sum_{j=1}^{n} g_{ij} k_{ij}(t, x_i, \{j \in \mathcal{N}_i | x_j\})(\xi_i - \xi_j), \tag{7}$$

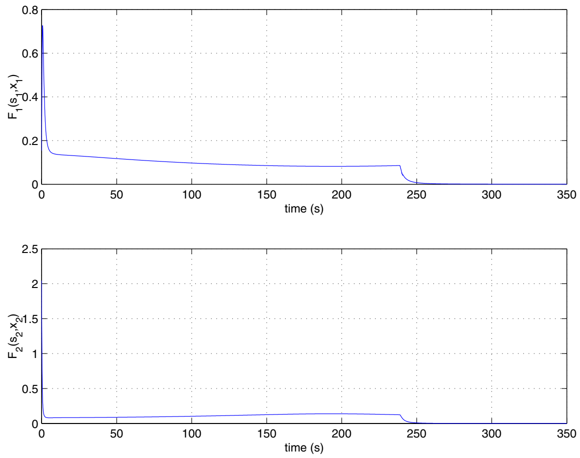

Fig. 5. Formation measure function $F_i(x_i, s_i)$ using the decentralized scheme.

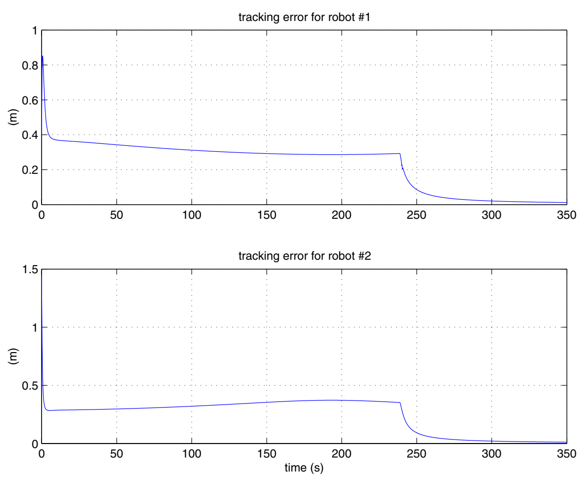

Fig. 6. The tracking errors for robot #1 and #2 using the decentralized scheme.

where $k_{ij}(\cdot, \cdot, \cdot)$ denotes the state-dependent weighting factors that introduce feedback to the i^{th} vehicle from its local neighbors.

B. Illustrative Example

In cooperative control systems, vehicles may move in or out of each other's communication range. As a result, the communication links between the vehicles may be established or broken randomly. It is relevant to study how a given connectivity pattern between the vehicles can be maintained. The problem of preserving connectivity constraints has been discussed in [16], [17] recently. As a preliminary study, we apply consensus algorithm (7) to drive multiple mobile agents to reach a rendezvous position. It is assumed that each agent has a limited communication range and the communication topology is connected initially. Fig. 7 shows the case where the weighting factors k_{ij} are constant. Note that the connectivity of the communication topology cannot be maintained and the agents form two separated subgroups. As a comparison, Fig. 8 shows the case that the weighting factors k_{ij} are adjusted dynamically such that neighboring vehicles do not move out of their communication range. Note that under the same initial conditions the connectivity between those agents is maintained and the team reaches a rendezvous point.

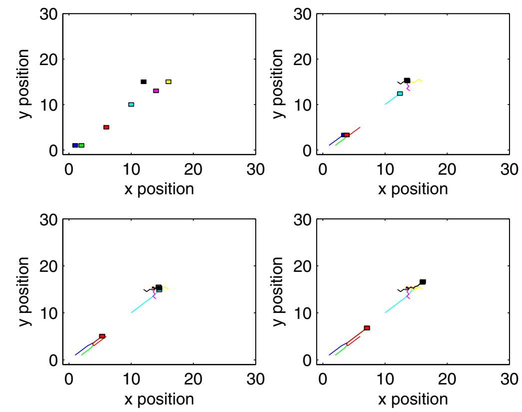

Fig. 7. Rendezvous of seven agents with fixed weighting factors.

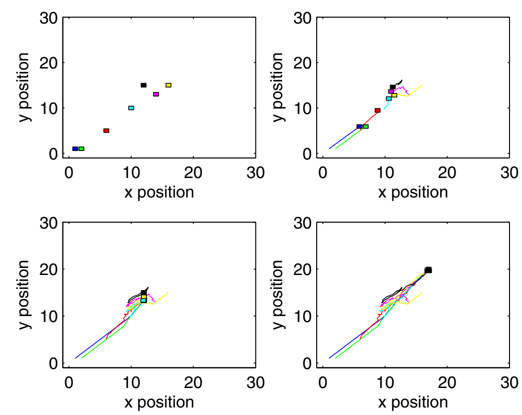

Fig. 8. Rendezvous of seven agents with state-dependent weighting factors.

VII. Consensus Building with Information Feedback through Reference States

Information feedback to the consensus building process can also be introduced through a reference state, which may be a function of vehicle/environmental dynamics or sensor measurement.

Let $\xi^r \in I\!R^m$ be the reference state and assume that ξ^r satisfies the following dynamics:

$$\dot{\xi}^r = f(t, \xi^r). \tag{8}$$

In the case that only a portion of the vehicles have access to ξ^r, we apply the following consensus algorithm:

$$u_i = \frac{1}{\eta_i} \sum_{j=1}^{n} g_{ij} k_{ij} [u_j - \gamma_i(\xi_i - \xi_j)] + \frac{1}{\eta_i} g_{i(n+1)} \alpha_i [f(t, \xi^r) - \gamma_i(\xi_i - \xi^r)], \tag{9}$$

where $k_{ij} > 0$, $\gamma_i > 0$, $\alpha_i > 0$, $g_{ii} \triangleq 0$, g_{ij} is 1 if information flows from vehicle j to vehicle i and 0 otherwise, $g_{i(n+1)}$ is 1 if vehicle i has access to ξ^r and 0 otherwise, and $\eta_i = g_{i(n+1)}\alpha_i + \sum_{j=1}^{n} g_{ij} k_{ij}$.

Note that k_{ij} and α_{ij} in Eq. (9) can be state-dependent weighting factors. For example, we may choose α_{ij} as $\alpha_i(t, x_i, \{j \in \mathcal{N}_i | x_j\})$ so that the performance of a vehicle and its neighbors can affect how accurate the vehicle wants to track the reference state. Also note that under certain circumstances information feedback can also be introduced directly to the reference model as follows:

$$\dot{\xi}^r = f(t, \xi^r, \{\ell \in \mathcal{L} | \xi_\ell\}), \tag{10}$$

where $\mathcal{L}$ denotes the set of vehicles that have access to ξ^r.

Due to space limitation, we will omit the illustrative example for this strategy.

VIII. Conclusion

We have studied the problem of consensus building in multi-vehicle systems with information feedback. Four strategies of introducing feedback to the consensus building process have been presented including information flow, external feedback terms, state-dependent weighting factors, and reference states. Illustrative examples have also been demonstrated as a proof of concept.

IX. ACKNOWLEDGMENTS

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