



## **SIMULATION AND EXPERIMENTAL STUDY OF CONSENSUS ALGORITHMS FOR MULTIPLE MOBILE ROBOTS WITH INFORMATION FEEDBACK**

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**ABSTRACT**—In this paper, we study the problem of consensus building in multi-robot systems with information feedback. We show how information feedback can be incorporated into the consensus building process so as to improve the robustness and situational awareness of the whole team. We detail the strategies of introducing feedback to the consensus building process through information flow, time-varying weights, external feedback terms, and reference states, and perform simulations and experimental studies of these strategies.

**Key Words:** Consensus building; Distributed robotics; Multi-robot systems; Cooperative control

### **1. INTRODUCTION**

Autonomous robotic vehicles are expected to find potential applications in military operations, search and rescue, environment monitoring, commercial cleaning, material handling, and homeland security. While single robots performing solo missions can yield some benefits, greater benefits will come from the cooperation of teams of robots.

As an inherently distributed strategy for multi-robot coordination, consensus algorithms have recently been studied extensively in the context of cooperative control of multi-robot systems [1-9]. In fact, these algorithms have a historic perspective in [10-12]. The consensus algorithms require only local neighbor-to-neighbor information exchange among the robots. The basic idea for consensus is that each robot updates its information state based on the information states of its local, possibly time-varying neighbors in such a way that the final information state of each robot converges to a common value. This basic idea can be extended to deal with the case that each robot's information states converge to desired relative deviations or to incorporate different group behaviors into the consensus building process. Consensus algorithms have applications in multi-robot rendezvous [13, 14], formation control [1, 15], flocking [16, 17], attitude alignment [18, 19], decentralized task assignment [20], sensor networks [21-23], etc.

Most consensus algorithms studied in the literature do not take into account robot performance and situational states in the consensus building process. For example, in some formation control problems, where the formation is moving through space, the information states of each robot might be dynamically evolving over time according to some inherent dynamics. Also in most cooperative control problems, the information states of each robot might be affected by robot performance, environmental information, or sensor measurement. As a result, it is essential to incorporate robot performance and situational states into the consensus building process as a form of feedback.

The main contribution of this paper is to study how information feedback can be incorporated into the consensus building process so as to improve the robustness and situational awareness of the whole team. We overview strategies of introducing information feedback to the consensus building process through information flow, time-varying weights, external feedback terms, and reference states. All of the strategies will be demonstrated in simulations or on a wheeled mobile robot experimental platform.

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## 2. BACKGROUND AND PRELIMINARIES

It is natural to model information exchange among robots by directed or undirected graphs. A *directed graph* consists of a pair  $(N, E)$ , where  $N$  is a finite nonempty set of nodes, and  $E \subseteq N \times N$  is a set of ordered pairs of nodes, called *edges*. An edge  $(i, j)$  in a directed graph denotes that robot  $j$  can obtain information from robot  $i$ , but not necessarily vice versa. In contrast, the pairs of nodes in an *undirected graph* are unordered, where an edge  $(i, j)$  denotes that robots  $i$  and  $j$  can obtain information from one another. An undirected graph can be considered a special case of a directed graph, where an edge  $(i, j)$  in the undirected graph corresponds to edges  $(i, j)$  and  $(j, i)$  in the directed graph. In a directed graph, if there is an edge from node  $i$  to node  $j$ , then  $i$  is defined as the *parent node*, and  $j$  is defined as the *child node*.

A *directed path* is a sequence of edges in a directed graph of the form  $(i_1, i_2), (i_2, i_3), \dots$ , where  $i_j \in N$ . An *undirected path* in an undirected graph is defined analogously. In a directed graph, a *cycle* is a path that starts and ends at the same node. A directed graph is *strongly connected* if there is a directed path from every node to every other node. An undirected graph is *connected* if there is a path between every distinct pair of nodes. A *directed tree* is a directed graph, where every node has exactly one parent except for one node, called the *root*, which has no parent, and the root has a directed path to every other node. In a directed tree, each edge has a natural orientation away from the root, and no cycle exists. In the case of undirected graphs, a *tree* is a graph in which every pair of nodes is connected by exactly one path.

A *directed spanning tree* of a directed graph is a directed tree formed by graph edges that connect all of the nodes of the graph. An *undirected spanning tree* in an undirected graph is defined analogously. A graph *has a directed spanning tree* if a directed spanning tree is a subset of the graph. A directed graph has a directed spanning tree if and only if there exists at least one node having a directed path to all of the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected. The *union* of a group of graphs is a graph with nodes given by the union of the node sets and edges given by the union of the edge sets of the group of graphs.

Suppose that there are  $n$  robots in the team. The *adjacency matrix*  $A = [a_{ij}] \in R^{n \times n}$  of a directed graph is defined as  $a_{ii} = 0$  and  $a_{ij} > 0$  if  $(j, i) \in E$ , where  $i \neq j$ . The adjacency matrix of an undirected graph is defined analogously except that  $a_{ij} = a_{ji}$ , for all  $i \neq j$ , since  $(j, i) \in E$  implies  $(i, j) \in E$ .

Let the matrix  $L = [\ell_{ij}] \in R^{n \times n}$  be defined as  $\ell_{ii} = \sum_{j \neq i} a_{ij}$  and  $\ell_{ij} = -a_{ij}$ , where  $i \neq j$ . The matrix  $L$  satisfies the conditions

$$\begin{aligned} \ell_{ij} &\leq 0, \quad i \neq j, \\ \sum_{j=1}^n \ell_{ij} &= 0, \quad i = 1, \dots, n. \end{aligned} \tag{1}$$

For an undirected graph the *Laplacian matrix*  $L$  has the property of symmetric positive semidefiniteness [24]. However, the matrix  $L$  for a directed graph does not have this property. For a directed graph, the matrix  $L$  is sometimes called the *directed graph Laplacian* or *nonsymmetric Laplacian* in the literature. In the case of undirected graphs, all of the eigenvalues of  $L$  are nonnegative. In the case of directed graphs, all of the eigenvalues of  $L$  have nonnegative real parts.

In both cases of directed graphs and undirected graphs, 0 is an eigenvalue of  $L$  with an associated eigenvector  $\mathbf{1}$ , where  $\mathbf{1}$  is an  $n \times 1$  column vector of all ones. In the case of undirected graphs, 0 is a simple eigenvalue of  $L$  and all of the other eigenvalues are positive if and only if the undirected graph is

connected [25]. In the case of directed graphs, 0 is a simple eigenvalue of  $L$  and all of the other eigenvalues have positive real parts if and only if the directed graph has a directed spanning tree [26].

Let  $I_n$  denote the  $n \times n$  identity matrix. Given a matrix  $S = [s_{ij}] \in R^{n \times n}$ , the *directed graph of  $S$* , denoted by  $\Gamma(S)$ , is the directed graph with node set  $N = \{1, \dots, n\}$  such that there is an edge in  $\Gamma(S)$  from  $j$  to  $i$  if and only if  $s_{ij} \neq 0$  [27, p. 357].

### 3. FUNDAMENTAL CONSENSUS ALGORITHM

Consider information states with dynamics given by

$$\dot{\xi}_i = u_i, \quad i = 1, \dots, n \quad (2)$$

where  $\xi_i \in R^m$  denotes the information state of the  $i^{\text{th}}$  robot and  $u_i \in R^m$  is the control input. For example, the information state might be robot position, velocity, oscillation phase, or decision variable. A consensus algorithm is proposed in [2-4, 7] as

$$u_i = -\sum_{j=1}^n g_{ij} k_{ij} (\xi_i - \xi_j), \quad (3)$$

where  $k_{ij} > 0$ ,  $g_{ii} = 0$ , and  $g_{ij}$  is 1 if information flows from robot  $j$  to robot  $i$  and 0; otherwise. Note that  $k_{ij}$  denotes the weight for the information-exchange link  $(j, i)$ .

By applying algorithm (3), (2) can be written in matrix form as

$$\dot{\xi} = -(L \otimes I_m) \xi,$$

where  $\xi = [\xi_1^T, \dots, \xi_n^T]^T$ ,  $\otimes$  denotes the Kronecker product, and  $L = [\ell_{ij}] \in R^{n \times n}$  is given as  $\ell_{ii} = \sum_{j \neq i} g_{ij} k_{ij}$  and  $\ell_{ij} = -g_{ij} k_{ij}$ ,  $\forall i \neq j$ . Note that  $L$  satisfies property (1).

Consensus is *achieved* among the  $n$  robots if  $\xi_i(t) \rightarrow \xi_j(t)$ ,  $\forall i \neq j$ , as  $t \rightarrow \infty$ . With the consensus algorithm (3), the final consensus value is a weighted average of the robots' initial information states. Note that the final consensus value is generally *a priori* unknown and depends on the information-exchange topologies as well as weights  $k_{ij}$ .

In this paper, we assume a directed information-exchange topology to take into account the case where sensors might have a limited field of view in the case of information exchange through local sensing. Note that undirected information exchange is a special case of directed information exchange.

Under a fixed information-exchange topology, (3) achieves consensus asymptotically if and only if the information-exchange topology has a directed spanning tree [26]. Under switching information-exchange topologies, (3) *achieves* consensus asymptotically if there exist infinitely many consecutive, uniformly-bounded time intervals such that the union of the information-exchange topologies across each time interval has a directed spanning tree [7].

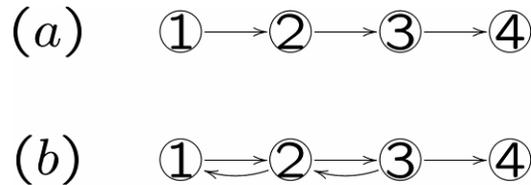
### 4. CONSENSUS BUILDING WITH INFORMATION FEEDBACK

In this section, we show strategies that introduce information feedback to the consensus building process. The strategies include information flow, time-varying weights, external feedback terms, and reference states. All of the strategies will be demonstrated in simulations or on a wheeled mobile robot experimental platform.

#### 4.1 Consensus Building with Information Feedback through Information Flow

The most straightforward strategy to introduce feedback to the consensus building process is through information flow between local neighbors. Information flow among the robots has an effect on the final consensus value. With (3) under a fixed information-exchange topology, the final consensus value is given by  $\xi^* = \sum_{i=1}^n \alpha_i \xi_i(0)$ , where  $\alpha = [\alpha_1, \dots, \alpha_n]^T$  is a left eigenvector of  $-L$  associated with the eigenvalue zero with  $\alpha_i \geq 0$  and  $\sum_{i=1}^n \alpha_i = 1$  [26]. Note that  $\alpha_i > 0$  if robot  $i$  has a directed path to all of the other robots in the information-exchange topology and  $\alpha_i = 0$  if there does not exist such a directed path [26]. As a result, if a robot wants to contribute to the final consensus value, its information needs to flow to all of the other robots in the team directly or indirectly. Information flow among the robots can also be applied to increase the redundancy and robustness of the whole team in the case of failures of certain information-exchange links. For example, if robot  $j$  only receives data due to its station as strictly a child in the directed information-exchange topology or due to unreliable state data transmission, any disturbance to this robot will cause inaccuracy in the team performance. However, if robot  $j$  is also a parent of another robot, then this disturbance feedback is propagated to the other robot and the other robot can adjust its motion relative to the robot being disturbed so as to mitigate the effect of the disturbance.

Consider two information-exchange topologies shown in Figure 1. Case (a) corresponds to a leader-follower topology where robot  $j+1$  follows robot  $j$ ,  $j=1,2,3$ . Case (b) corresponds to a topology where feedback is introduced from followers to leaders through information flow. Note that the final consensus value with Case (a) is  $\xi_1(0)$  while the final consensus value with Case(b) is a weighted average of  $\xi_1(0)$ ,  $\xi_2(0)$ , and  $\xi_3(0)$ . Also note that in Case (a) if robot 3 is perturbed by disturbance, robots 1 and 2 are unaware of this disturbance and their motions remain unaffected. However, in Case (b) if robot 3 is perturbed by disturbance, robots 1 and 2 are able to adjust their motions according to the motion of robot 3 so as to maintain better team performance due to the information flow from robot 3 to robots 1 and 2 directly or indirectly.



**Figure 1. Information-exchange topologies among four robots. Figure 1(a) denotes a leader-follower topology while Figure 1(b) denotes a topology with information flow introduced from followers to leaders.**

To illustrate, we study the effect of information flow with the following simulation. The consensus algorithm (3) is applied to drive four mobile robots to achieve rendezvous under two different information-exchange topologies. In our simulations, we choose  $k_{ij} \equiv 1$ , which implies that the information-exchange links have equal weights. Figure 2 shows the two different time-invariant information-exchange topologies. In particular, Case (a) corresponds to an undirected connected graph while Case (b) corresponds to a directed spanning tree (i.e., a leader-follower topology).

Figure 3 shows the simulation results of the rendezvous application for Cases (a) and (b). The four robots have initial positions at (0,3), (2,3), (2,1), (1,0) m. The trajectories of the four robots are shown in Figure 3. We can see from Figure 3 that the final rendezvous in Case (a) is a weighted average of all of the four robots' initial positions. In contrast, the final rendezvous position in Case (b) is robot 1's initial

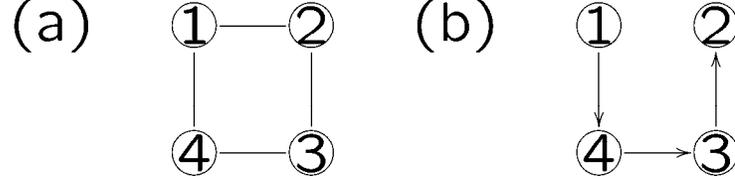


Figure 2. Information-exchange topologies among four robots for the rendezvous application.

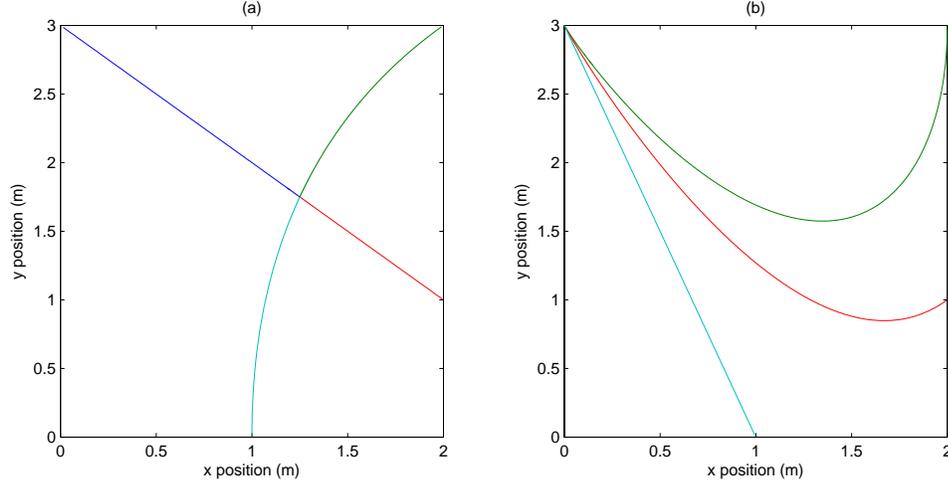


Figure 3. Simulation results of rendezvous for Cases (a) and (b).

position. The simulation results demonstrate how information flow affects the final consensus value since in Case (a) each robot has a directed path to every other robot, but in Case (b) only robot 1 has a directed path to all of the other robots. With information feedback through information flow, the final consensus value can be determined accordingly. Therefore, this strategy can be used in determining final consensus value in related applications.

#### 4.2 Consensus Building with Information Feedback through Time-varying Weights

Information feedback can also be introduced to the consensus building process through time-varying weights. Consider the algorithm

$$u_i = -\sum_{j=1}^n g_{ij} k_{ij}(t, \{\xi_j \mid j \in N_i\}) (\xi_i - \xi_j), \quad (4)$$

where  $N_i$  denotes the neighbor set of robot  $i$ , and  $k_{ij}(\cdot, \cdot)$  denotes the time-varying weights that introduce feedback to the  $i^{\text{th}}$  robot from its local neighbors. Note that  $k_{ij}(\cdot, \cdot)$  represents the coupling strength between robot  $i$  and robot  $j$ . When  $g_{ij} > 0$ , a larger  $k_{ij}$  implies that  $\xi_i$  is driven toward  $\xi_j$  more heavily. The motivation behind (4) is that it may be desirable to weigh the information from different robots differently to represent the time-varying relative confidence of each robot's information or time-varying relative reliability of different communication or sensing links.

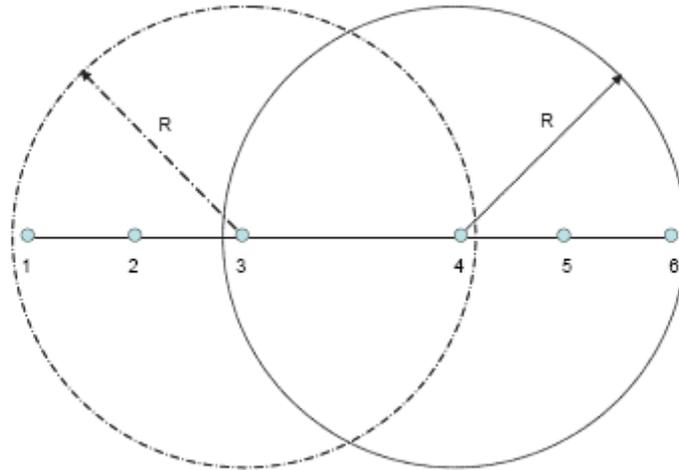
Suppose that  $k_{ij}(\cdot, \cdot)$  is piecewise continuous and uniformly lower and upper bounded. If there exist infinitely many consecutive, uniformly-bounded time intervals such that the union of the information-

exchange graphs across each time interval has a directed spanning tree, then  $\xi_i(t) \rightarrow \xi_j(t)$ ,  $\forall i \neq j$ , asynchronously as  $t \rightarrow \infty$  [7].

In cooperative control systems, robots might move in or out of each other's communication or sensing range. As a result, the information-exchange links among the robots might be established or broken randomly. It is relevant to study how a given connectivity pattern among the robots can be maintained. The problem of preserving connectivity constraints has been discussed recently in [28, 29].

As a preliminary study, we will show how weights  $k_{ij}$  can be adjusted dynamically to guarantee that if the initial information-exchange topology is connected, the possibly switching information-exchange topologies stay connected for all time. For most work in consensus algorithms, weights have been either assumed to be constant or given consideration only so far as to identify that it may be required without actually developing a weighting algorithm for applications.

To understand why the information-exchange links among the robots may be established or broken with the consensus algorithm (3), we analyze (3) by assuming that each robot has equal weights  $k_{ij}$  and a fixed information-exchange range. Figure 4 shows the initial positions of six robots, where  $R$  represents the information-exchange range of each robot.<sup>1</sup> Here we assume that the robots can only move along the horizontal axis for simplicity. Note that the neighbors of robot 3 are robots 1, 2, and 4 while the neighbors of robot 4 are robots 3, 5, and 6. Also note that with the information-exchange range  $R$ , the initial information-exchange topology of the six robots is connected. With the consensus algorithm (3), robot 3 will move to the left while robot 4 will move to the right. As a result, the information-exchange link between robots 3 and 4 will be broken and cause the team to fail to rendezvous. While rendezvous is achievable in isolated groups, overall rendezvous of the team cannot be guaranteed.



**Figure 4. Initial positions of six robots with a limited information-exchange range.**

The problem of link breakage that arises with the constant weights  $k_{ij}$  comes when the number of neighboring robots on one side of a robot is larger than the number of the robots on the other side within the information-exchange range. Understanding the cause of the link breakage, we adjust the weights in (4) as follows: For simplicity, we consider the one-dimensional case. The multi-dimensional case can be dealt with similarly. Let  $p$  and  $q$  be the number of robots on either side of some center robot  $i$  within the information-exchange range  $R$  with  $p \geq q$ . The weights  $k_{ij}$  are defined as  $k_{ij} = q/p$  if robot  $j$  is on

<sup>1</sup> In other words, each robot can receive information from all of the robots that are within the disc of radius  $R$  centered at the robot.

the side with  $p$  neighbors and  $k_{ij} = 1$  if robot  $j$  is on the side with  $q$  neighbors. The scaling of the weights prevents “weak link” breakage and ensures rendezvous with (4) given any number of robots in any initial configuration where the communication topology is initially connected.

To illustrate, we apply (4) to drive multiple mobile robots to reach rendezvous with time-varying weights. It is assumed that each robot has the same limited information-exchange range of 7 meters, and the information-exchange topology is initially connected.

Figure 5 shows the case that the weights  $k_{ij} \equiv 1$ . Note that the connectivity of the information-exchange topology cannot be maintained, and the robots form two separated subgroups. In contrast, Figure 6 shows the case that the weights  $k_{ij}$  are adjusted dynamically according to  $k_{ij} = q/p$  as described above. Note that under the same initial conditions as in Figure 5 the connectivity among those robots is maintained, and the team reaches rendezvous. Therefore, consensus can be reached if the time-varying weights are chosen properly.

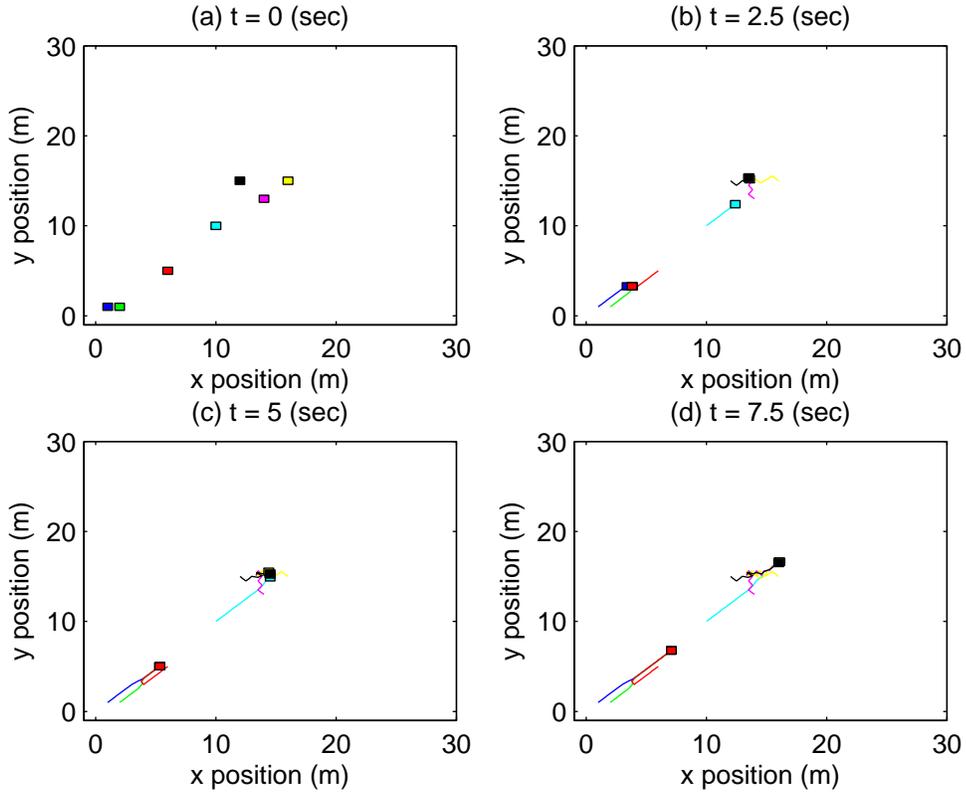
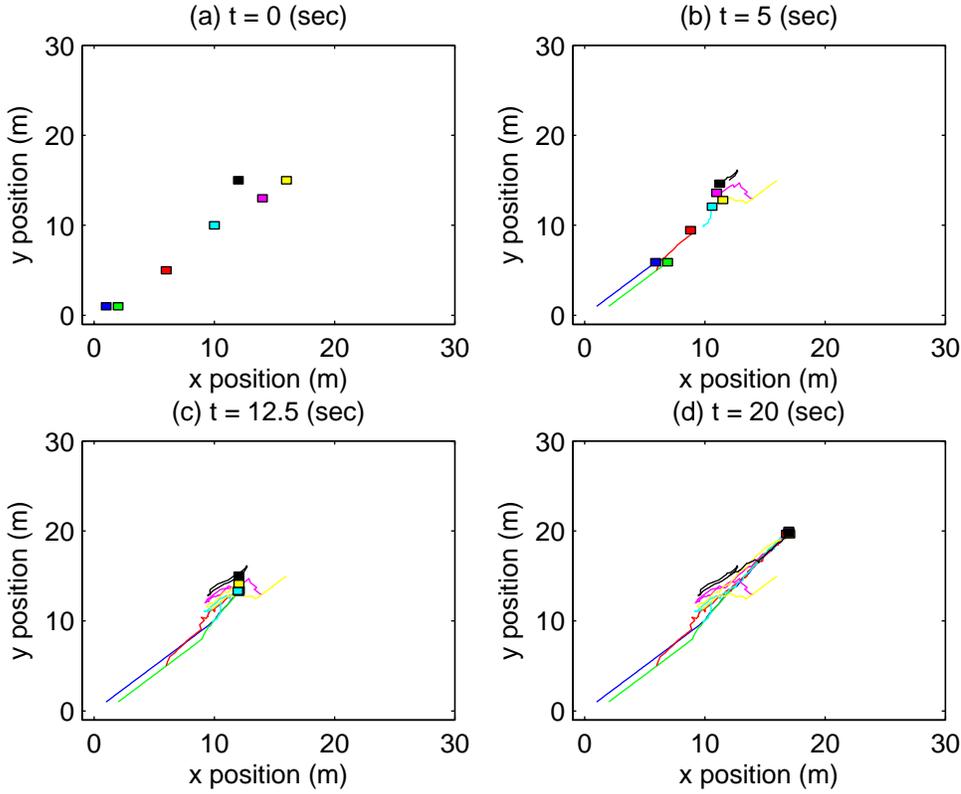


Figure 5. Rendezvous of seven robots with fixed weights  $k_{ij} \equiv 1$ .

#### 4.3 Consensus Building with Information Feedback through External Feedback Terms

Another strategy to introduce feedback to the consensus building process is through external feedback terms. Consider the algorithm

$$u_i = -\sum_{j=1}^n g_{ij} k_{ij} (\xi_i - \xi_j) + \rho_i(t, \{x_j \mid j \in N_i\}, \{x_\ell \mid \ell \in J_i\}) \quad (5)$$



**Figure 6. Rendezvous of seven robots with time-varying weights.**

where  $x_\ell$  denotes the situational state of the  $\ell^{\text{th}}$  robot,  $J_i$  denotes the set of robots whose situational states are available to robot  $i$ , and  $\rho_i(\cdot, \cdot, \cdot)$  denotes a feedback term introduced to the  $i^{\text{th}}$  robot from its local neighbors. As a result, the consensus building process of each robot will be affected by the performance of its local neighbors, which serves as a form of information feedback and therefore improves the robustness of the whole team.

Suppose that  $k_{ij}(t) > 0$  is piecewise continuous and uniformly lower and upper bounded. Under the assumption that there exist infinitely many consecutive, uniformly-bounded time intervals such that the union of the information-exchange graph across each time interval has a directed spanning tree, if  $\|\rho_i - \rho_j\|$  is bounded, so is  $\|\xi_i - \xi_j\|$ ,  $\forall i \neq j$ . Furthermore, if  $\|\rho_i - \rho_j\| \rightarrow 0$ , so is  $\|\xi_i - \xi_j\|$ ,  $\forall i \neq j$ . The proof of the above argument can be found in [30] and [31].

To illustrate, we consider an orientation alignment problem for wheeled multiple mobile robots. We experimentally validate the algorithm (5) on a multi-robot platform which is shown in Figure 7. The mobile robots can communicate with each other through ethernet with TCP/IP protocols. The robots rely on encoder data for their position and orientation information.

In our experiments, we emulate limited inter-robot information exchange by simply disallowing the use of information obtained from certain members of the group although every robot can share information with every other robot. By doing so, we can test distributed cooperative control algorithms that involve only local neighbor-to-neighbor information exchange due to limited communication or sensing. Figure 8 shows the local neighbor-to-neighbor information-exchange topology among four robots, where a link from  $j$  to  $i$  denotes that robot  $i$  can receive information from robot  $j$ .



Figure 7. Multi-robot experimental platform at Utah State University.

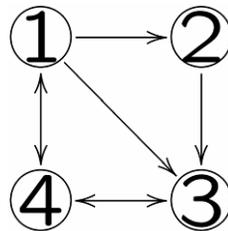


Figure 8. Information-exchange topology among four robots.

Suppose that the orientation kinematics of the  $i^{\text{th}}$  robot is modeled by

$$\dot{\theta}_i = \omega_i, \quad (6)$$

where  $\theta_i$  is the orientation and  $\omega_i$  is the angular speed of the robot. We consider an orientation alignment algorithm as

$$\omega_i = -\sum_{j=1}^n g_{ij} k_{ij} (\theta_i - \theta_j) + \omega^d, \quad i = 1, \dots, 4 \quad (7)$$

where  $\alpha > 0$ , and  $\omega^d = 7.2$  deg/sec is the desired angular speed for the team of robots. In (7) the second term is the external feedback term. The objective of (7) is to guarantee that  $\theta_i(t) \rightarrow \theta_j(t)$  and  $\omega_i \rightarrow \omega^d$  as  $t \rightarrow \infty$ .

Figures 9 and 10 show, respectively, the orientations and angular speeds of the four robots. Although the four robots have different initial orientations, they align their orientations with their adjacent neighbors asymptotically while spinning at the desired angular speed. As shown in Figure 10, the angular speeds of the robots converge to a value near 7.2 deg/sec.

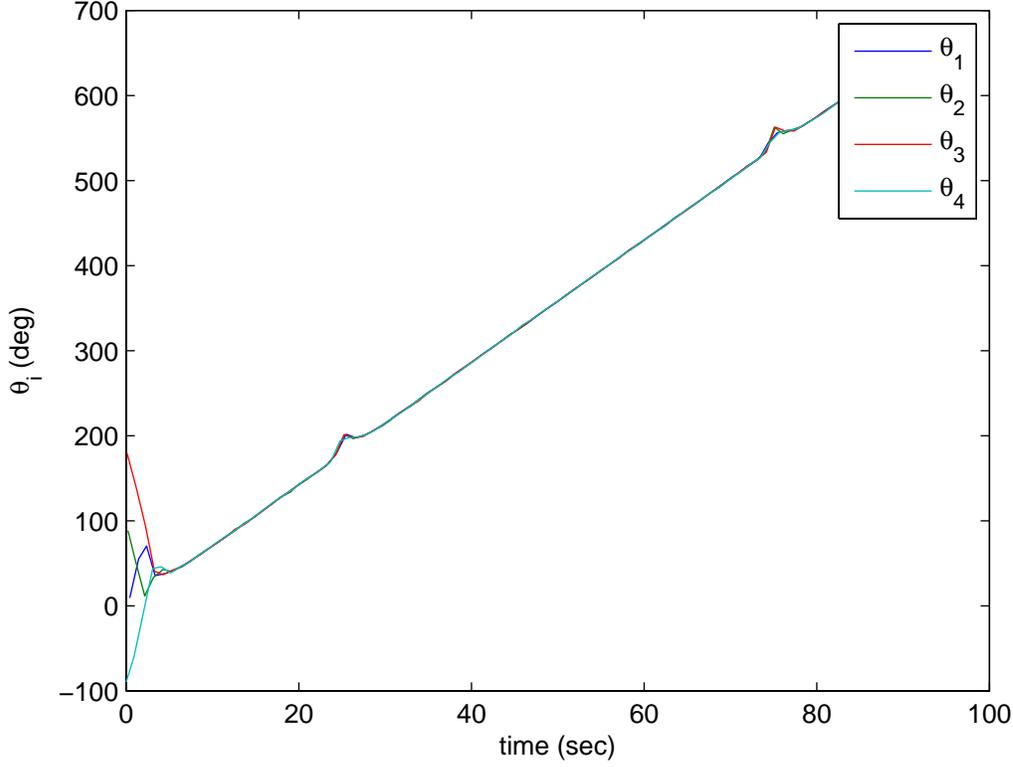


Figure 9. Orientation of the four robots with an external feed term.

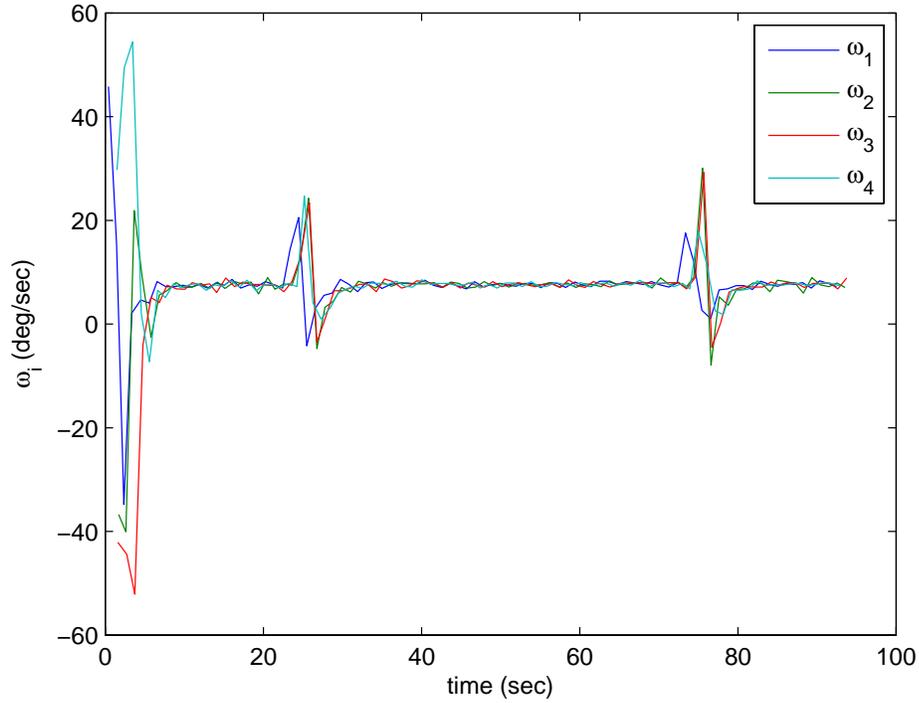
#### 4.4 Consensus Building with Information Feedback through Reference States

Information feedback to the consensus building process can also be introduced through a reference state, which might be a function of robot/environmental dynamics. Let  $\xi^r \in R^m$  be the reference state for the team. Suppose that  $\xi^r$  satisfies the dynamics

$$\dot{\xi}^r = f(t, \xi^r), \quad (8)$$

where  $f(.,.)$  is piecewise continuous in  $t$  and locally Lipschitz in  $\xi^r$ . When only a portion of the robots have access to  $\xi^r$ , we introduce the reference state to the consensus building process by the algorithm

$$u_i = \frac{1}{\eta_i} \sum_{j=1}^n g_{ij} k_{ij} [\dot{\xi}_j - \gamma_i (\xi_i - \xi_j)] + \frac{1}{\eta_i} g_{i(n+1)} \alpha_i [\dot{\xi}^r - \gamma_i (\xi_i - \xi^r)] \quad (9)$$



**Figure 10. Angular speeds of the four robots with an external feed term.**

where  $k_{ij} > 0$ ,  $\gamma_i > 0$ ,  $\alpha_i > 0$ ,  $g_{ij}$  is 1 if information flows from robot  $j$  to robot  $i$  and 0 otherwise,  $g_{i(n+1)}$  is 1 if robot  $i$  has access to  $\xi^r$  and 0 otherwise, and  $\eta_i = g_{i(n+1)}\alpha_i + \sum_{j=1}^n g_{ij}k_{ij}$ . The objective of (9) is to drive  $\xi_i(t) \rightarrow \xi^r(t)$ ,  $i = 1, \dots, n$ , as  $t \rightarrow \infty$ .

Let  $G = [g_{ij}] \in \mathbb{R}^{(n+1) \times (n+1)}$  be the adjacency matrix, where  $g_{ij}$ ,  $\forall i, j = 1, \dots, n$ , is 1 if information flows from robot  $j$  to robot  $i$  and 0 otherwise,  $g_{i(n+1)}$  is 1 if robot  $i$  has access to  $\xi^r$  and 0 otherwise, and  $g_{(n+1)k}$ ,  $\forall k = 1, \dots, n+1$ . Note that (9) guarantees that  $\xi_i(t) \rightarrow \xi^r(t)$  as  $t \rightarrow \infty$  if and only if the directed graph of  $G$  has a directed spanning tree.<sup>2</sup> The proof of the above argument can be found in [32]. Note that with the consensus algorithm (3) consensus is reached on a constant value equal to the weighted average of the initial information states. In contrast, the algorithm (9) reaches consensus a time-varying reference state.

To illustrate, we apply the following algorithm for orientation alignment

$$\omega_i = \frac{1}{\eta_i} \sum_{j=1}^n g_{ij}k_{ij}[\omega_j - \gamma_i(\theta_i - \theta_j)] + \frac{1}{\eta_i} g_{i(n+1)}\alpha_i[\omega^d - \gamma_i(\theta_i - \theta^d)] \quad (10)$$

where  $\theta^d$  and  $\omega^d$  denote the reference orientation and angular velocity for each robot. Here  $\theta_i$ ,  $\theta^d$  and  $\omega^d$  play the roles of  $\xi_i$ ,  $\xi^r$  and  $\xi^r$  respectively in (9).

<sup>2</sup> Treat  $\xi^r$  as a virtual robot with index  $n+1$ . This condition is equivalent to the condition that  $\xi^r$  is the only node that has a directed path to all of the robots in the team.

In our experiment, the reference angular velocity  $\omega^d$  is chosen as 7.2 deg/sec. Figure 11 shows the information-exchange topology among the four robots, where  $i_L$  denotes that robot  $i$  has access to  $\theta^d$  and  $\omega^d$ . Figures 12 and 13 show, respectively, the orientations and angular speeds of the four robots. As shown in Figure 12, the four robots with different initial orientations can synchronize their orientations while following the reference orientation. As shown in Figure 13, the angular speeds of the robots converge to a value near 7.2 deg/sec.

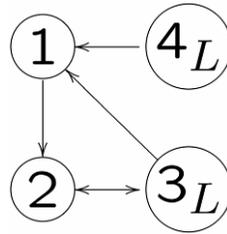


Figure 11. Inter-robot information exchange topology with a reference state.

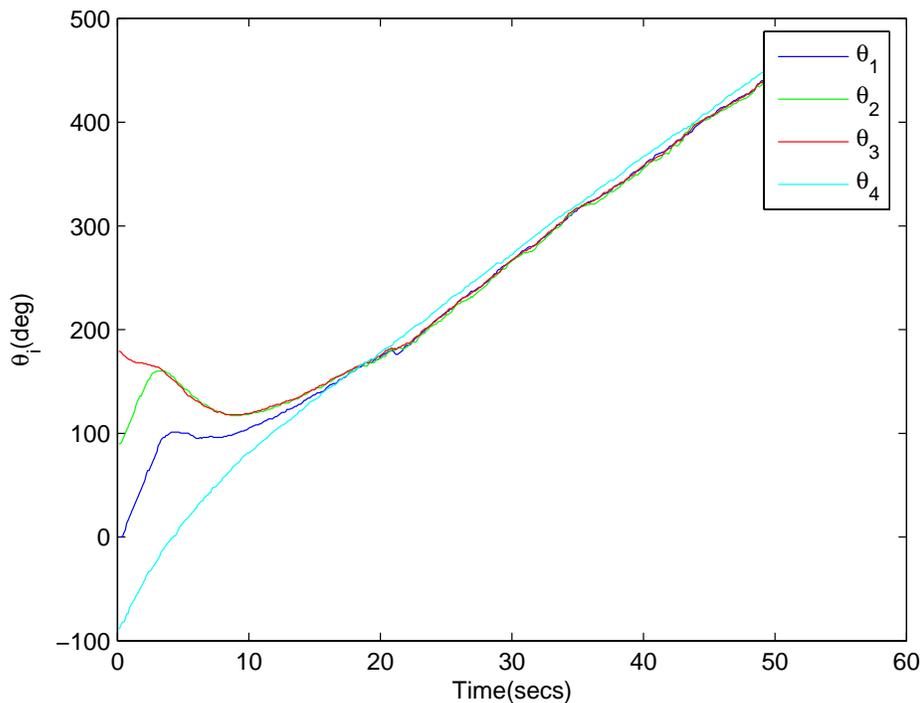
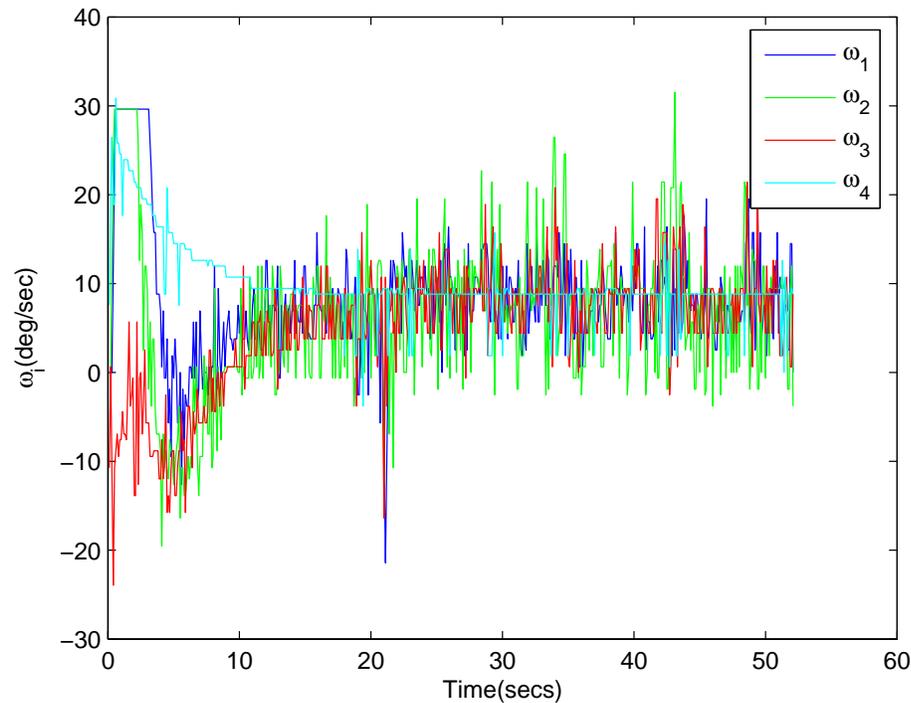


Figure 12. Orientations of the four robots with a reference state.

#### 4.5 Discussion

We have presented four strategies of introducing information feedback to the consensus building process. Each strategy might be appropriate and useful in a different context. The first two consensus feedback strategies through information flow and time-varying weights are internal feedback strategies. In particular, the consensus feedback strategy through information flow can help to determine an appropriate information-exchange topology for a multi-robot team. The consensus strategy through time-varying weights can be used to preserve certain connectivity pattern for the information-exchange topology. The



**Figure 13. Angular speeds of the four robots with a reference state.**

combination of these two strategies affects the final consensus equilibrium. The last two consensus feedback strategies through external feedback terms and reference states are external feedback strategies. In particular, the consensus feedback strategy through external feedback terms can be used for multi-robot formation control with collision avoidance capabilities. The external feedback terms can incorporate environment data to each robot. As a result, each robot can achieve collision avoidance with the obstacles or its neighbors. The consensus feedback strategy with reference states allows a group reference command or state to be injected to the team through only one or a portion of the robots. As a result, there is no need for all of the robots in the team to have access to the group reference command or state, which reduces information-exchange requirement. While the consensus feedback strategy with external feedback terms can incorporate different situational states to the team, the consensus strategy with reference states requires the existence of a common group reference for the team. In real applications, one strategy or a combination of multiple strategies can be applied.

## 5. CONCLUSION

We have studied the problem of consensus building in multi-robot systems with information feedback. Four strategies of introducing feedback to the consensus building process have been presented including information flow, time-varying weights, external feedback terms, and reference states. Illustrative examples have also been demonstrated in simulation and an experimental platform as a proof of concept. Future work will apply the strategies to a multiple UAV formation flying problem.

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