Engineering Notes

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Formation Keeping and Attitude Alignment for Multiple Spacecraft Through Local Interactions

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I. Introduction

ADVANCES in networking and distributed computing enable numerous applications for multivehicle systems including space-based observations, future combat systems, smart homes, enhanced surveillance systems, hazardous material handling systems, and reconfigurable sensing systems. In some applications, it is desirable that multiple vehicles maintain a geometric configuration and achieve relative attitude alignment. One example is deep-space interferometry (see [1,2] and references therein), where a fleet of networked spacecraft are required to perform a sequence of formation maneuvers while maintaining relative attitudes accurately.

In multivehicle coordination, the interplay between information-exchange topologies and control plays an important role. In [3] information-exchange techniques are studied to improve stability margins and accuracy for vehicle formations, where an information flow filter provides each vehicle with the formation center so that this information can be used by each vehicle as a reference. In [4–6] consensus algorithms for single-integrator dynamics are studied in the context of undirected or directed switching information-exchange topologies. Extensions to double-integrator dynamics are discussed in [7,8], in which flocking algorithms are addressed to guarantee separation, alignment, and cohesion behaviors in a group of vehicles under undirected information exchange.

In the area of spacecraft formation flying, [9–11] study the problem of formation keeping and attitude alignment for multiple spacecraft via information exchange with one or two adjacent neighbors. In [9], a leader–follower approach is applied, where each spacecraft tracks its leader's position and attitude, and information only flows from leaders to followers. Although the leader–follower approach is easy to understand and implement, there are limitations. For example, the unique team leader, to which the reference state for the formation is only available, is a single point of failure for the whole group. In addition, there is no explicit feedback from the followers to the leaders: if the follower is perturbed by disturbance, formation keeping and attitude alignment cannot be maintained. In [10,11], the control law for each spacecraft is a function of the states of its two adjacent neighbors. As a result, group feedback is introduced in the team through coupled dynamics between the spacecraft. However, [10,11] require a bidirectional ring communication topology, which is rather restrictive in the sense that each spacecraft needs to explicitly identify its two adjacent neighbors in the group to form the ring. In addition, [10,11] require that the reference state for the formation be available to every group member, which may not be realistic in the presence of communication bandwidth and range limitations.

The main purpose of this note is to address the problem of formation keeping and attitude alignment under a general directed information-exchange topology when the reference state for the formation may only be available to a part of the group members and these group member may not have a directed path to all of the other spacecraft. The contributions of the current note are twofold. First, we propose a formation keeping control law that guarantees that multiple spacecraft can maintain a given formation configuration during formation maneuvers with local neighbor-to-neighbor information exchange when the time-varying reference position and velocity for the virtual center of the formation are only available to a part of the group members. Second, we propose a attitude alignment control law that guarantees that multiple spacecraft can follow a given time-varying reference attitude with local neighbor-to-neighbor information exchange when the reference attitude is only available to a part of the group members. It is worthwhile to mention that although we study the problem of formation keeping and attitude alignment in the context of deep-space spacecraft formation flying, the results hereafter are valid for other rigid bodies that satisfy the same dynamics.

Compared to other work in spacecraft attitude control (e.g., [12]), the emphasis of this note lies in the analysis of how interspacecraft information exchange plays a key role in formation keeping and attitude alignment. Compared to [7,8], the proposed formation keeping control law takes into account the general case of directed information exchange in the presence of a time-varying reference state. In addition, the proposed attitude alignment control law extends the consensus algorithms from single- or double-integrator dynamics as addressed in [3–8] to rigid-body rotational dynamics while taking into account the general case of directed information exchange in the presence of a time-varying reference state. The proposed control laws allow information to flow from any spacecraft to any other spacecraft to introduce information feedback and coupling between neighboring spacecraft so as to increase redundancy and robustness to the whole group in the case of failures of certain information-exchange links, which generalizes the leader–follower approach (e.g., [9]) and the behavioral approach requiring a bidirectional ring information-exchange topology (e.g., [10,11]).

II. Background and Preliminaries

A. Notations

Let $\mathbf{1}_p$ denote the $p \times 1$ column vector of all ones. Let I_p denote the $p \times p$ identity matrix. Given a real scalar γ, we use $\gamma > 0$ to denote that γ is positive. Given a $p \times p$ real matrix P, we use $P > 0$ to denote that P is symmetric positive definite. In the following, a lower case symbol denotes a scalar or vector whereas an upper case symbol denotes a matrix. Also let $\mathbb{R}$ denote the set of real numbers, and $\mathbb{R}^+$ denote the set of positive real numbers.

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Unit quaternions are used to represent spacecraft attitudes in this note. A unit quaternion is defined as $q = [\hat{q}^T, \bar{q}]^T \in \mathbb{R}^4$, where $\hat{q} = a \cdot \sin(\frac{\phi}{2}) \in \mathbb{R}^3$ denotes the vector part and $\bar{q} = \cos(\frac{\phi}{2}) \in \mathbb{R}$ denotes the scalar part of the unit quaternion. In this notation, $a \in \mathbb{R}^3$ is a unit vector, known as the Euler axis, and $\phi \in \mathbb{R}$ is the rotation angle about a, called the Euler angle. Note that $q^T q = 1$ by definition. A unit quaternion is not unique because q and $-q$ represent the same attitude. However, uniqueness can be achieved by restricting ϕ to the range $0 \leq \phi \leq \pi$ so that $\bar{q} \geq 0$ [13].

The product of two unit quaternions p and q is defined by

$$qp = \begin{bmatrix} \bar{q}\,\hat{p} + \bar{p}\,\hat{q} + \hat{q} \times \hat{p} \\ \bar{q}\,\bar{p} - \hat{q}^T \hat{p} \end{bmatrix}$$

which is also a unit quaternion. The conjugate of the unit quaternion q is defined by $q^* = [-\hat{q}^T, \bar{q}]^T$. The conjugate of qp is given by $(qp)^* = p^* q^*$. The multiplicative identity quaternion is denoted by $\boldsymbol{q}_I = [0, 0, 0, 1]^T$, where $qq^* = q^* q = \boldsymbol{q}_I$ and $q\boldsymbol{q}_I = \boldsymbol{q}_I q = q$ [13].

B. Spacecraft Dynamics

In this note, we assume that the spacecraft translational dynamics and rotational dynamics are decoupled and address the translational problem and the attitude control problem separately.

Spacecraft translational dynamics are given by

$$\dot{r}_i = v_i, \qquad m_i \dot{v}_i = f_i, \qquad i = 1, \ldots, n \tag{1}$$

where n is the total number of spacecraft in the group, $r_i \in \mathbb{R}^3$ and $v_i \in \mathbb{R}^3$ denote the position and velocity of the ith spacecraft, respectively, and $m_i \in \mathbb{R}^+$ and $f_i \in \mathbb{R}^3$ are, respectively, the mass and control force associated with the ith spacecraft.

Spacecraft attitude dynamics are given by

$$\begin{aligned} \dot{\widehat{q_i}} &= -\frac{1}{2}\omega_i \times \widehat{q_i} + \frac{1}{2}\overline{q_i}\omega_i, \qquad \dot{\overline{q_i}} = -\frac{1}{2}\omega_i \cdot \widehat{q_i} \\ J_i \dot{\omega}_i &= -\omega_i \times (J_i \omega_i) + \tau_i, \qquad i = 1, \ldots, n \end{aligned} \tag{2}$$

where $\widehat{q_i} \in \mathbb{R}^3$ and $\overline{q_i} \in \mathbb{R}$ are, respectively, the vector and scalar parts of the unit quaternion of the ith spacecraft, $\omega_i \in \mathbb{R}^3$ is the angular velocity, and $J_i \in \mathbb{R}^{3\times 3}$ and $\tau_i \in \mathbb{R}^3$ are, respectively, the inertia tensor and control torque [13].

C. Graph Theory

It is natural to model information exchange between spacecraft by directed or undirected graphs. The readers are referred to [14] for an introduction to graph theory. A directed graph consists of a pair $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a finite nonempty set of nodes, and $\mathcal{E} \in \mathcal{N} \times \mathcal{N}$ is a set of ordered pairs of nodes, called *edges*. An edge (i, j) in a directed graph denotes that spacecraft j can obtain information from spacecraft i, but not necessarily vice versa. In contrast, the pairs of nodes in an undirected graph are unordered, where an edge (i, j) denotes that spacecraft i and j can obtain information from one another. Note that an undirected graph can be considered a special case of a directed graph, where an edge (i, j) in the undirected graph corresponds to edges (i, j) and (j, i) in the directed graph. In a directed graph, if there is an edge from node i to node j, then i is defined as the parent node, and j is defined as the child node.

A directed path is a sequence of edges in a directed graph of the form $(i_1, i_2), (i_2, i_3), \ldots$, where $i_j \in \mathcal{N}$. An undirected path in an undirected graph is defined accordingly. In a directed graph, a cycle is a path that starts and ends at the same node. A directed graph is *strongly connected* if there is a directed path from every node to every other node. An undirected graph is *connected* if there is a path between every distinct pair of nodes. A rooted directed tree is a directed graph, where every node has exactly one parent except for one node, called *root*, which has no parent, and the root has a directed path to every other node. Note that in a rooted directed tree, each edge has a natural orientation away from the root, and no cycle exists. In the case of undirected graphs, a tree is a undirected graph in which every pair of nodes is connected by exactly one path.

A rooted directed spanning tree of a directed graph is a rooted directed tree formed by graph edges that connect all of the nodes of the graph. A directed graph *has* or *contains* a rooted directed spanning tree if a rooted directed spanning tree is a subset of the directed graph. Note that a directed graph has a rooted directed spanning tree if and only if there exists at least one node having a directed path to all of the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a rooted directed spanning tree is a weaker condition than being strongly connected.

Suppose that there are p nodes in the graph. The adjacency matrix $G = [g_{ij}] \in \mathbb{R}^{p\times p}$ of a graph is defined as $g_{ii} = 0$ and $g_{ij} = 1$ if $(j, i) \in \mathcal{E}$ where $i \neq j$. For a weighted graph, G is defined as $g_{ii} = 0$ and $g_{ij} > 0$ if $(j, i) \in \mathcal{E}$ where $i \neq j$. Note that the adjacency matrix of an undirected graph is symmetric because $(j, i) \in \mathcal{E}$ implies $(i, j) \in \mathcal{E}$. However, the adjacency matrix of a directed graph does not have this property. Let $L = [\ell_{ij}] \in \mathbb{R}^{p\times p}$ be defined as $\ell_{ii} = \sum_{j\neq i} g_{ij}$ and $\ell_{ij} = -g_{ij}$ where $i \neq j$. The matrix L satisfies the conditions

$$\ell_{ij} \leq 0, \qquad i \neq j, \qquad \sum_{j=1}^{p} \ell_{ij} = 0, \qquad i = 1, \ldots, p \tag{3}$$

For an undirected graph, L is called the *graph Laplacian* [14], which is symmetric positive semidefinite. However, L for a directed graph does not have this property.

In the case of an undirected graph, L has a simple zero eigenvalue with an associated eigenvector $\mathbf{1}_p$ and all of the other eigenvalues are positive if and only if the undirected graph is connected [14]. In the case of a directed graph, L has a simple zero eigenvalue with an associated eigenvector $\mathbf{1}_p$ and all of the other eigenvalues have positive real parts if and only if the directed graph has a rooted directed spanning tree [6]. Let $x = [x_1, \ldots, x_p]^T$, where $x_j \in \mathbb{R}$, $j = 1, \ldots, p$, and $y = [y_1^T, \ldots, y_p^T]^T$, where $y_j \in \mathbb{R}^m$, $j = 1, \ldots, p$. Under the conditions of both cases, $Lx = 0$ implies that $x = \alpha \mathbf{1}_p$ (i.e., $x_1 = \cdots = x_p$), where $\alpha \in \mathbb{R}$, and $(L \otimes I_m) y = 0$, where $\otimes$ is the Kronecker product, implies that $y = \mathbf{1}_p \otimes \beta$ (i.e., $y_1 = \cdots = y_p$), where $\beta \in \mathbb{R}^m$.

The directed graph of a $p \times p$ real matrix $S = [s_{ij}] \in \mathbb{R}^{p\times p}$, denoted by $\Gamma(S)$, is the directed graph on p nodes such that there is an edge in $\Gamma(S)$ from j to i if and only if $s_{ij} \neq 0$ (cf. [15]).

III. Formation Keeping with a Time-Varying Group Reference Trajectory

In this section, we consider the case where multiple spacecraft maintain a formation configuration while the whole group, that is, the virtual center of the formation, follows a time-varying reference trajectory. Let $r_0^d(t) \in \mathbb{R}^3$ and $v_0^d(t) \in \mathbb{R}^3$ denote the time-varying reference position and velocity for the virtual center of the formation. Suppose that r_0^d and v_0^d satisfy the translational dynamics given by

$$\dot{r}_0^d = v_0^d, \qquad m_0^d \dot{v}_0^d = f_0^d \tag{4}$$

where $m_0^d \in \mathbb{R}^+$ and $f_0^d \in \mathbb{R}^3$ denote, respectively, the virtual mass and force of the group. The goal is to guarantee that $r_i \to r_0^d(t) + r_{ig}(t)$ and $v_i \to v_0^d(t) + v_{ig}(t)$, $\forall\ i$, where $r_{ig} \in \mathbb{R}^3$ and $v_{ig} \in \mathbb{R}^3$ denote the desired position and velocity deviation vector of the ith spacecraft from the virtual center of the formation with $\dot{r}_{ig} = v_{ig}$. Note that once the virtual center of the formation is specified, $r_{ig}(t)$ and $v_{ig}(t)$, $i = 1, \ldots, n$, are determined according to the desired formation configuration. The formation maneuvers are determined by r_0^d and v_0^d.

Next, we consider two special cases where either all of the spacecraft in the team or the unique team leader has access to the reference model (4) and then consider the general case where only a part of the spacecraft have access to the reference model. In the following we assume that all the vectors in each control law have been appropriately transformed and represented in the same coordinate frame.

When each spacecraft has access to the reference model, the control force for each spacecraft is designed as

$$f_i = m_i\{\dot{v}_0^d + \dot{v}_{ig} - \alpha[(r_i - r_0^d - r_{ig}) + \gamma(v_i - v_0^d - v_{ig})] - \sum_{j=1}^{n} g_{ij}k_{ij}[(r_i - r_j - r_{ig} + r_{jg}) + \gamma(v_i - v_j - v_{ig} + v_{jg})]\} \tag{5}$$

where $\alpha \in \mathbb{R}^+$, $\gamma \in \mathbb{R}^+$, $k_{ij} \in \mathbb{R}^+$, $g_{ii} \triangleq 0$, and g_{ij} is 1 if information flows from spacecraft j to spacecraft i and 0; otherwise, $\forall\ i \neq j$. The control law (5) is motivated by [16].

With the control force (5) and the reference model (4), Eq. (1) can be written in matrix form as

$$\begin{bmatrix} \dot{\hat{r}} \\ \ddot{\hat{r}} \end{bmatrix} = \left(\underbrace{\begin{bmatrix} 0_{n\times n} & I_n \\ -(\alpha I_n + L) & -\gamma(\alpha I_n + L) \end{bmatrix}}_{\Sigma} \otimes I_3 \right) \begin{bmatrix} \hat{r} \\ \dot{\hat{r}} \end{bmatrix}$$

where $\hat{r} = [\hat{r}_1^T, \ldots, \hat{r}_n^T]^T$ with $\hat{r}_i = r_i - r_0^d - r_{ig}$, and $L = [\ell_{ij}] \in \mathbb{R}^{n\times n}$ is given as $\ell_{ii} = \sum_{j\neq i} g_{ij}k_{ij}$ and $\ell_{ij} = -g_{ij}k_{ij}$, $\forall\ i \neq j$. Note that L satisfies the property (3). Also note that all eigenvalues of $-(\alpha I_n + L)$ have negative real parts according to Gershgorin's disc theorem [15]. Theorem 6 in [17] shows that if

$$\gamma > \max_{i=1,\ldots,n} \sqrt{\frac{2}{|\nu_i| \cos(\frac{\pi}{2} - \tan^{-1} \frac{-\mathrm{Re}(\nu_i)}{\mathrm{Im}(\nu_i)})}} \tag{6}$$

where ν_i is the ith eigenvalue of $-(\alpha I_n + L)$, and $\mathrm{Re}(\cdot)$ and $\mathrm{Im}(\cdot)$ represent the real and imaginary parts of a number, respectively, then all eigenvalues of Σ have negative real parts. Therefore, if γ satisfies (8), then $\hat{r}_i \to 0$ and $\dot{\hat{r}}_i \to 0$, which implies that $r_i \to r_0^d + r_{ig}$ and $v_i \to v_0^d + v_{ig}$, $i = 1, \ldots, n$. Note that when each spacecraft has access to the reference model, the information-exchange topology does not affect the convergence result as long as γ satisfies (6). However, the existence of intervehicle information exchange improves the formation maintenance accuracy during the transition as shown in [16].

To illustrate from a graphical point of view, consider the information-exchange topology given by Fig. 1, where each spacecraft has access to the reference model. Here we treat node r_0^d as a virtual spacecraft denoting the reference model. Note that there exists a link from node r_0^d to every spacecraft in the team.

In the leader–follower strategy [9], only the unique team leader has access to the reference model, and each vehicle except the team leader has exactly one parent vehicle. Suppose that spacecraft k is the unique team leader. The control force for each spacecraft is designed as

$$f_i = m_i[\dot{v}_0^d + \dot{v}_{ig} - K_{ri}(r_i - r_0^d - r_{ig}) - K_{vi}(v_i - v_0^d - v_{ig})], \quad i = k \tag{7}$$

$$f_i = m_i[\dot{v}_{i_\ell} + \dot{v}_{ig} - \dot{v}_{i_\ell g} - K_{ri}(r_i - r_{i_\ell} - r_{ig} + r_{i_\ell g}) - K_{vi}(v_i - v_{i_\ell} - v_{ig} + v_{i_\ell g})], \quad i \neq k \tag{8}$$

where $K_{ri} \in \mathbb{R}^{3\times 3} > 0$, $K_{vi} \in \mathbb{R}^{3\times 3} > 0$, and spacecraft i_ℓ is the leader, that is, the parent, of spacecraft i.[†]

With the control laws (7) and (8) and the reference model (4), Eq. (1) can be written as

[†]That is, information flows from spacecraft i_ℓ to spacecraft i in the information-exchange topology. However, spacecraft i_ℓ might not be the team leader.

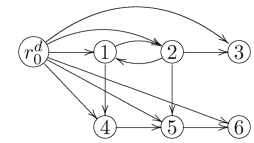

Fig. 1 Information-exchange topology where each spacecraft has access to the reference model.

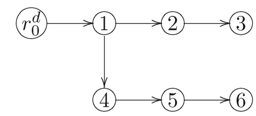

Fig. 2 A leader–follower topology where only the team leader has access to the reference model.

$$\ddot{\tilde{r}}_i = -K_{ri}\tilde{r}_i - K_{vi}\tilde{v}_i \tag{9}$$

where $\tilde{r}_k = r_k - r_0^d - r_{kg}$, $\tilde{v}_k = v_k - v_0^d - v_{kg}$, $\tilde{r}_i = r_i - r_{i_\ell} - r_{ig} + r_{i_\ell g}$, $i \neq k$, and $\tilde{v}_i = v_i - v_{i_\ell} - v_{ig} + v_{i_\ell g}$, $i \neq k$. Note that Eq. (9) implies that $\tilde{r}_i \to 0$ and $\tilde{v}_i \to 0$ because K_{ri} and K_{vi} are symmetric positive definite matrices, which in turn implies that $r_k - r_{kg} \to r_0^d$, $v_k - v_{kg} \to v_0^d$, $r_i - r_{ig} \to r_{i_\ell} - r_{i_\ell g}$, $i \neq k$, and $v_i - v_{ig} \to v_{i_\ell} - v_{i_\ell g}$, $i \neq k$. Therefore, it follows from [9] that $r_i \to r_0^d + r_{ig}$ and $v_i \to v_0^d + v_{ig}$, $i = 1, \ldots, n$, under the leader–follower topology.[‡]

To illustrate, consider the information-exchange topology shown in Fig. 2, where spacecraft 1 is the unique team leader, spacecraft j is the leader of spacecraft $j+1$, $j = 2, 4, 5$, and spacecraft 1 is the leader of spacecraft 2 and 4. Note that node r_0^d has a link only to the team leader.

Note that in the leader–follower topology, information only flows from leaders to followers. When a follower is perturbed by disturbance, the leaders are unaware of this disturbance, and their motions remain unaffected. It might be intuitive to introduce information flow from followers to leaders to introduce feedback so as to improve group robustness. However, it is not clear how information from the followers can be incorporated into the control laws for the leaders without affecting the stability result.

In the general case that the information-exchange topology may or may not have a rooted directed spanning tree and one or more spacecraft may have access to the reference model, we propose the control force

$$f_i = m_i\Big\{\dot{v}_{ig} + \tfrac{1}{\kappa_i}\textstyle\sum_{j=1}^{n} g_{ij}[\dot{v}_j - \dot{v}_{jg} - K_{ri}(r_i - r_j - r_{ig} + r_{jg}) - K_{vi}(v_i - v_j - v_{ig} + v_{jg})] + \frac{1}{\kappa_i} g_{i(n+1)}[\dot{v}_0^d - K_{ri}(r_i - r_0^d - r_{ig}) - K_{vi}(v_i - v_0^d - v_{ig})]\Big\} \tag{10}$$

where $g_{ii} \triangleq 0$, g_{ij}, $\forall\ i, j \in \{1, \ldots, n\}$ is 1 if information flows from spacecraft j to spacecraft i and 0 otherwise, $g_{i(n+1)}$ is 1 if spacecraft i has access to the reference model and 0 otherwise, $\kappa_i = \sum_{j=1}^{n+1} g_{ij}$, $K_{ri} \in \mathbb{R}^{3\times 3} > 0$, and $K_{vi} \in \mathbb{R}^{3\times 3} > 0$.

We have the following theorem for formation keeping with a time-varying group reference trajectory under a general directed information-exchange topology.

Theorem 3.1: Let $G = [g_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$, where g_{ij} and $g_{i(n+1)}$, $1 \leq i,\ j \leq n$, are as defined after Eq. (10) and $g_{(n+1)i} = 0$, $i = 1, \ldots, n+1$. With the control force (10), $r_i \to r_0^d + r_{ig}$ and $v_i \to v_0^d + v_{ig}$ asymptotically if and only if the directed graph of G has a rooted directed spanning tree (with node r_0^d being the root).[§]

[‡]A leader–follower topology corresponds to a directed graph that is itself a rooted directed spanning tree.

[§]Equivalently, r_0^d is the only node that has a directed path to all of the spacecraft in the team.

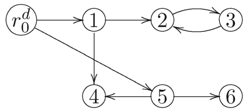

Fig. 3 A general information-exchange topology, where only a part of the spacecraft have access to the reference model and the original topology without node r_0^d does not have a rooted directed spanning tree.

Proof: Let $\tilde{r}_{n+1} \equiv r_0^d$ and $\tilde{v}_{n+1} \equiv v_0^d$. Note that $\ddot{r}_0^d = \dot{v}_0^d$ and $\ddot{r}_j = \dot{v}_j$, $j = 1, \ldots, n$. With (10), Eq. (1) can be rewritten as

$$\kappa_i \ddot{\tilde{r}}_i = \sum_{j=1}^{n+1} g_{ij}[\ddot{\tilde{r}}_j - K_{ri}(\tilde{r}_i - \tilde{r}_j) - K_{vi}(\tilde{v}_i - \tilde{v}_j)], \qquad i = 1, \ldots, n$$

where $\tilde{r}_i = r_i - r_{ig}$ and $\tilde{v}_i = \dot{\tilde{r}}_i$. Then it follows that $\ddot{\sigma}_i = -K_{ri}\sigma_i - K_{vi}\dot{\sigma}_i$, $i = 1, \ldots, n$, where $\sigma_i = \sum_{j=1}^{n+1} g_{ij}(\tilde{r}_i - \tilde{r}_j)$. Then we know that $\sigma_i \to 0$ and $\dot{\sigma}_i \to 0$, $i = 1, \ldots, n$, because K_{ri} and K_{vi} are symmetric positive definite matrices. Let $L_\sigma = [\ell_{ij}]$ be an $(n+1) \times (n+1)$ matrix, where $\ell_{ii} = \sum_{j=1, j\neq i}^{n+1} g_{ij}$ and $\ell_{ij} = -g_{ij}$, $\forall\ i \neq j$. Note that L_σ satisfies the property (3). Also note that all entries of the $n+1$th row of L_σ are zero. In addition, note that $\sigma_i \to 0$ and $\dot{\sigma}_i \to 0$, $i = 1, \ldots, n$, as well as a dummy equation $0 \to 0$, $i = n+1$, can be written in matrix form as $(L_\sigma \otimes I_3)\tilde{r} \to 0$ and $(L_\sigma \otimes I_3)\tilde{v} \to 0$, respectively, where $\tilde{r} = [\tilde{r}_1^T, \ldots, \tilde{r}_{n+1}^T]^T$ and $\tilde{v} = \dot{\tilde{r}}$. Therefore, it follows that $\tilde{r}_i \to \tilde{r}_j$ and $\tilde{v}_i \to \tilde{v}_j$, $\forall i, j \in \{1, \ldots, n+1\}$, if and only if the directed graph of G has a rooted directed spanning tree from Sec. II.C, which in turn implies that $r_i \to r_0^d + r_{ig}$ and $v_i \to v_0^d + v_{ig}$, $i = 1, \ldots, n$, because $\tilde{r}_{n+1} \equiv r_0^d$ and $\tilde{v}_{n+1} \equiv v_0^d$. □

To illustrate, consider the information-exchange topology shown in Fig. 3, where only spacecraft 1 and 5 have access to the reference model. Note that although neither spacecraft 1 nor spacecraft 5 has a directed path to all of the other spacecraft in the team, there exists a directed path from node r_0^d to all of the spacecraft in the team. Therefore, the condition in Theorem 3.1 is satisfied.

Note that with the full access strategy (5), each spacecraft must have access to the reference model. In contrast, the control law (10) does not impose this constraint and allows one or more spacecraft to have access to the reference model. Also note that with the leader–follower strategy (7) and (8), information only flows from leaders to followers, and each spacecraft except the unique team leader has exactly one parent (e.g., no information loops allowed). In contrast, the control law (10) allows information to flow from any spacecraft to any other spacecraft (e.g., followers to leaders) while guaranteeing that the stability result remains unchanged as long as the minimum information-exchange requirement in Theorem 3.1 is satisfied. As a result, information feedback can be introduced through the general information exchange and coupling between neighboring spacecraft, which increases redundancy and robustness to the whole group in the case of failures of certain information-exchange links. The full access strategy (5) can be considered a special case of (10), where each spacecraft has access to the reference model. The leader–follower strategy (7) and (8) can also be considered a special case of (10), where only the unique team leader has access to the reference model, and each spacecraft has at most one neighbor (i.e., its leader).

IV. Attitude Alignment with a Time-Varying Reference Attitude

In this section, we consider the case where multiple spacecraft follow a time-varying reference attitude. Let $q_0^d(t) \in \mathbb{R}^4$ and $\omega_0^d(t) \in \mathbb{R}^3$ denote the time-varying reference attitude and angular velocity, which satisfy the rotational dynamics given by

$$\dot{\hat{q}}_0^d = -\frac{1}{2}\omega_0^d \times \hat{q}_0^d + \frac{1}{2}\bar{q}_0^d\omega_0^d, \qquad \dot{\bar{q}}_0^d = -\frac{1}{2}\omega_0^d \cdot \hat{q}_0^d$$
$$J_0^d\dot{\omega}_0^d = -\omega_0^d \times (J_0^d\omega_0^d) + \tau_0^d \tag{11}$$

where J_0^d and τ_0^d denote, respectively, the virtual inertia tensor and control torque for the group. The goal is to guarantee that $q_i \to q_j \to q_0^d(t)$ and $\omega_i \to \omega_j \to \omega_0^d(t)$, $\forall\ i \neq j$.

In the general case that the reference model is only available to a part of the group members, we propose the control torque to the ith spacecraft as

$$\tau_i = \omega_i \times (J_i\omega_i) + \frac{1}{|\mathcal{N}_i| + 1}J_i\Bigg(\dot{\omega}_0^d + \sum_{j\in\mathcal{N}_i}\dot{\omega}_j\Bigg) - \frac{1}{|\mathcal{N}_i| + 1}\Bigg\{k_{qi}\widehat{p_{\pi_i}} + K_{\omega i}\Big[(\omega_i - \omega_0^d) + \sum_{j\in\mathcal{N}_i}(\omega_i - \omega_j)\Big]\Bigg\} \tag{12}$$

$i \in \mathcal{L}$

$$\tau_i = \omega_i \times (J_i\omega_i) + \frac{1}{|\mathcal{N}_i|}J_i\sum_{j\in\mathcal{N}_i}\dot{\omega}_j - \frac{1}{|\mathcal{N}_i|}\Bigg[k_{qi}\widehat{q_{\pi_i}} + K_{\omega i}\sum_{j\in\mathcal{N}_i}(\omega_i - \omega_j)\Bigg] \tag{13}$$

$i \notin \mathcal{L}$

where $\mathcal{N}_i$ denotes the set of neighboring spacecraft whose information is available to spacecraft i, $|\mathcal{N}_i|$ denotes the cardinality of $\mathcal{N}_i$, $\mathcal{L}$ denotes the set of spacecraft to which the reference model (11) is available, $k_{qi} \in \mathbb{R}^+$, $K_{\omega i} \in \mathbb{R}^{3\times3} > 0$, $p_{\pi_i} = [\prod_{j\in\mathcal{N}_i}(q_j^* q_i)]q_0^{d*}q_i$, and $q_{\pi_i} = \prod_{j\in\mathcal{N}_i}(q_j^* q_i)$. We assume that $i \notin \mathcal{N}_i$. Note that $j \in \mathcal{N}_i$ does not imply that $i \in \mathcal{N}_j$ in the case of directed information exchange.

Theorem 4.1: Let $G = [g_{ij}] \in \mathbb{R}^{(n+1)\times(n+1)}$, where g_{ij}, $i, j \in \{1, \ldots, n\}$ is 1 if information flows from spacecraft j to spacecraft i and 0 otherwise, $g_{i(n+1)}$, $i = 1, \ldots, n$ is 1 if the reference model is available to spacecraft i and 0 otherwise, and $g_{(n+1)j} = 0$, $j = 1, \ldots, n+1$. With the control torques (12) and (13), if the directed graph of G has a rooted directed spanning tree,[¶] then $\widehat{p_{\pi_i}} \to 0$, $i \in \mathcal{L}$, $\widehat{q_{\pi_i}} \to 0$, $i \notin \mathcal{L}$, and $\omega_i \to \omega_0^d$, $i = 1, \ldots, n$, asymptotically.

Proof: Let $q_{n+1} \equiv q_0^d$ and $\omega_{n+1} \equiv \omega_0^d$. Also let $\mathcal{J}_i = \mathcal{N}_i$ if $g_{i(n+1)} = 0$ and $\mathcal{J}_i = \mathcal{N}_i \cup \{n+1\}$ if $g_{i(n+1)} = 1$. Then Eqs. (12) and (13) can be rewritten as

$$\tau_i = \omega_i \times (J_i\omega_i) + \frac{1}{|\mathcal{J}_i|}J_i\sum_{j\in\mathcal{J}_i}\dot{\omega}_j - \frac{1}{|\mathcal{J}_i|}\Bigg[k_{qi}\widehat{s_{\pi_i}} + K_{\omega i}\sum_{j\in\mathcal{J}_i}(\omega_i - \omega_j)\Bigg] \tag{14}$$

$i = 1, \ldots, n$

where $s_{\pi_i} = \prod_{j\in\mathcal{J}_i}(q_j^* q_i)$. Combining Eqs. (2) and (14), gives

$$J_i\dot{\omega}_{\sigma_i} = -k_{qi}\widehat{s_{\pi_i}} - K_{\omega i}\omega_{\sigma_i}, \qquad i = 1, \ldots, n \tag{15}$$

where $\omega_{\sigma_i} = \sum_{j\in\mathcal{J}_i}(\omega_i - \omega_j)$. In [12], it shows that if the unit quaternion and angular velocity pairs (q_k, ω_k) and (q_ℓ, ω_ℓ) both satisfy the quaternion kinematics defined by the first two equations in Eq. (2), then the unit quaternion and angular velocity pair $(q_\ell^* q_k, \omega_k - \omega_\ell)$ also satisfies the quaternion kinematics. It is straightforward to extend this argument by induction to show that the unit quaternion and angular velocity pair $(s_{\pi_i}, \omega_{\sigma_i})$ also satisfies the quaternion kinematics. Thus Eq. (15) implies that $\widehat{s_{\pi_i}} \to 0$ and $\omega_{\sigma_i} \to 0$, $i = 1, \ldots, n$ according to [12]. Let $L_\omega = [\ell_{ij}]$ be an $(n+1) \times (n+1)$ matrix, where $\ell_{ii} = \sum_{j=1, j\neq i}^{n+1} g_{ij}$ and $\ell_{ij} = -g_{ij}$, $i \neq j$.

[¶]Define a virtual node $n+1$ representing the reference model (11). With G defined in Theorem 4.1, where $g_{(n+1)j} = 0$, $j = 1, \ldots, n+1$, the condition that the directed graph of G has a rooted directed spanning tree implies that node $n+1$ is the root of the rooted directed spanning tree. This condition is also equivalent to the condition that node $n+1$ is the only node that has a directed path to all of the spacecraft in the group. That is, the reference states q_0^d, ω_0^d, and $\dot{\omega}_0^d$ can flow to any spacecraft in the team directly or indirectly.

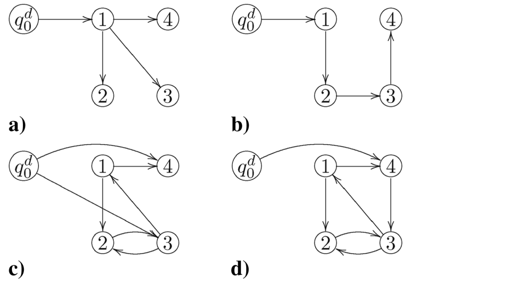

Fig. 4 Information-exchange topologies between four spacecraft where Figs. 4a and 4b correspond to the leader–follower approach and Figs. 4c and 4d correspond to the control laws (12) and (13).

Note that L_ω satisfies the property (3) and all of the entries of the $n+1$th row of L_ω are zero. Also note that $\omega_{\sigma_i} \to 0$, $i = 1, \ldots, n$, together with a dummy equation $0 \to 0$, $i = n+1$, can be written in matrix form as $(L_\omega \otimes I_3)\omega \to 0$, where $\omega = [\omega_1^T, \ldots, \omega_{n+1}^T]^T$. Noting that the directed graph of G has a rooted directed spanning tree, we know that $\omega_i \to \omega_j$, $i, j \in \{1, \ldots, n+1\}$ from Sec. II.C, which in turn implies that $\omega_i \to \omega_0^d$, $\forall\ i$, because $\omega_{n+1} \equiv \omega_0^d$. □

In the information-exchange topology, if a node k has exactly one parent, node ℓ, then $\widehat{s_{\pi_k}} = \widehat{q_\ell^* q_k} \to 0$ implies that $q_k \to q_\ell$. That is, spacecraft k approaches the reference attitude q_0^d if $\ell = n+1$, or spacecraft k and ℓ approach the same attitude if $\ell \neq n+1$. As a result, edge (ℓ, k) can be deleted in the information-exchange topology, and nodes k and ℓ can be combined as one single node whose incoming and outgoing edges are the union of the incoming and outgoing edges of nodes k and ℓ. By repeating this procedure, we can simplify the information-exchange topology. If the information-exchange topology can be simplified to a directed graph with only one node, then $\widehat{p_{\pi_i}} \to 0$, $i \in \mathcal{L}$ and $\widehat{q_{\pi_i}} \to 0$, $i \notin \mathcal{L}$, directly imply that $q_i \to q_j \to q_0^d$. Similar to the formation keeping case, the leader–follower approach for attitude alignment (e.g., [9]) can be considered a special case of the control laws (12) and (13), where each spacecraft has at most one neighbor (i.e., its leader).

As an illustrative example, Fig. 4 shows the information-exchange topologies between four spacecraft, where node q_0^d (i.e., node $n+1$) denotes the reference model and node i, $i = 1, \ldots, 4$ denotes the ith spacecraft. Note that in Fig. 4 a link from node q_0^d to node j denotes that the reference model is available to spacecraft j. In particular, the leader–follower approach (e.g., [9]) corresponds to Figs. 4a and 4b, where each spacecraft except the team leader has only one parent node. In contrast, the control laws (12) and (13) correspond to Figs. 4c and 4d, which are more general than Figs. 4a and 4b in the sense that information can flow between all spacecraft to introduce feedback between neighbors and the reference model may be available to one or more spacecraft in the group. Note that node q_0^d has a directed path to all of the spacecraft in the group in Figs. 4a–4d.

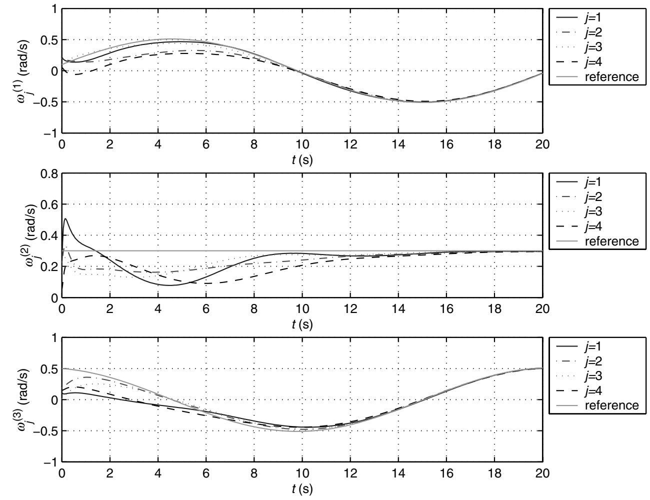

Fig. 6 Actual angular velocities of each spacecraft and the reference angular velocity ω_0^d.

Also note that Figs. 4a–4d can all be simplified to a directed graph with only one node, which directly implies that $q_i \to q_j \to q_0^d$ with the control laws (12) and (13).

V. Simulation

In this section, we only show an attitude alignment example due to space limitations. We apply the control laws (12) and (13) to guarantee that four spacecraft follow a time-varying reference attitude $q_0^d(t)$ and angular velocity $\omega_0^d(t)$.

In the simulation, we let $\tau_0^d = [0, 0, 0]^T$, $J_0^d = \mathrm{diag}\{1, 2, 1\}$, $q_0^d(0) = [0, 0, 0, 1]^T$, and $\omega_0^d(0) = [0.1, 0.3, 0.5]^T$. We also choose $q_i(0)$ and $\omega_i(0)$, $i = 1, \ldots, 4$ randomly. Also let $k_{qi} = 1$ and $K_{\omega i} = 2I_3$, $i = 1, \ldots, 4$ in Eqs. (12) and (13). The information flow between the four spacecraft is shown in Fig. 4c, in which the reference model is only available to spacecraft 3 and 4. In the following, a superscript (j) denotes the jth component of a quaternion or vector.

Figure 5 shows the actual attitudes of each spacecraft and the reference attitude. Figure 6 shows the actual angular velocities of each spacecraft and the reference angular velocity. Note that the actual attitudes and angular velocities of each spacecraft converge to their reference values. Figure 7 shows the control torques of each spacecraft.

VI. Conclusions

We have studied formation keeping and attitude alignment for multiple spacecraft with local neighbor-to-neighbor information

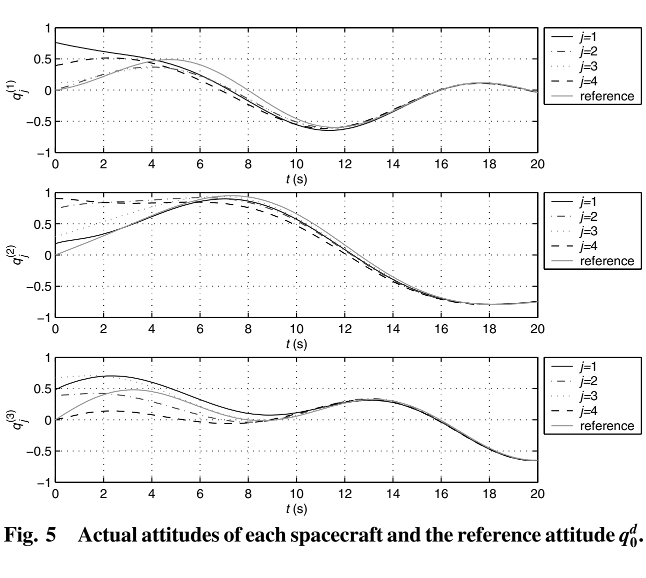

Fig. 5 Actual attitudes of each spacecraft and the reference attitude q_0^d.

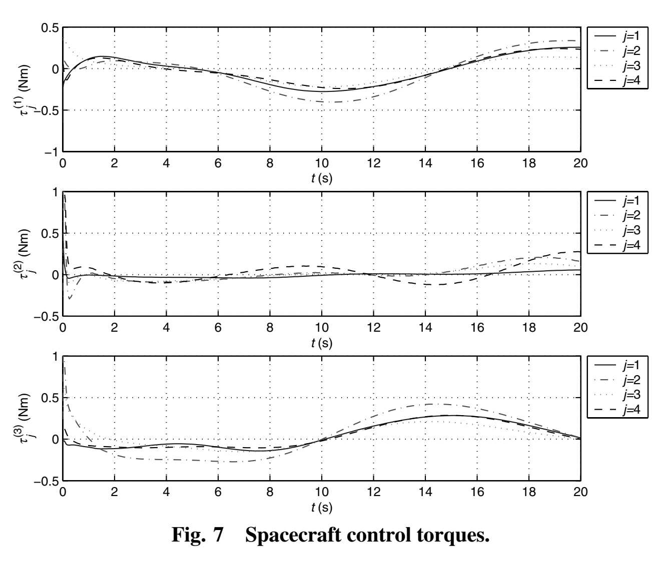

Fig. 7 Spacecraft control torques.

exchange. Control laws have been proposed for formation keeping and attitude alignment during formation maneuvers in the presence of arbitrary information loops or feedback between neighboring spacecraft. Simulation results on reference attitude tracking for multiple spacecraft have demonstrated the effectiveness of our approach. In this note, we have not addressed the robustness of the proposed control laws in the presence of model uncertainties and noise, the issue of collision avoidance, and the problem of coupling between translation dynamics and rotational dynamics. These will be topics of future research.

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