

## Discussion on: “Consensus of Second-Order Delayed Multi-Agent Systems with Leader-Following”

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The paper by H. Yang, X. Zhu, and S. Zhang studied a second-order leader-following consensus algorithm in multi-agent systems with directed network topologies and heterogeneous input delays, which generalized the previous work [1] from the case of symmetric coupling weights to that of asymmetric coupling weights. Some sufficient conditions for reaching second-order consensus in a leader-following context were derived by utilizing the Gershgorin disc theorem and the curvature theory. It was shown that to ensure leader-following consensus, the input delays should be less than some critical values determined by the network topology and the control gains.

Consensus algorithms in multi-agent systems have been extensively studied in the literature. Existing works have different focuses and might be categorized in a variety of ways. In particular, different consensus algorithms can be designed and analyzed for different system models, among which the most studied models are single-integrator dynamics [2], [3], [4], [5] and double-integrator dynamics [6], [7], [8], [9]. In addition, besides leaderless consensus algorithms, it is often of interest to study leader-following consensus algorithms, i.e., consensus in the presence of a leader or a reference [10], [11]. This paper discussed a leader-following consensus algorithm for double-integrator dynamics when the leader has a constant velocity. The limitation of the algorithm (i.e., its incapability of tracking a leader with a time-varying velocity) has been commented by the authors in Remark 1.

There usually exist delays in networked systems due to the finite speed of information transmission and processing. In order to discuss the influence of the delays, one needs to first model the delays, which are usually classified as input delays and communication delays. The input delays can be caused by information processing while the communication delays can be caused by information propagation from one agent to another. Consensus in the presence of input delays and communication delays are considered in [5], [7], [12], [13] and [14], [15], [16], respectively. Consensus in the presence of both input and communication delays is considered in [17]. While it is usually assumed that the delays are uniformly fixed [5], the extension to the case of multiple (non-uniform or diverse) time-varying delays is also discussed [18], [19], which take into account a more general case. In this paper, delays were modeled as multiple fixed input delays.

When studying the influence of the delays on consensus algorithms, it is desirable to find some sufficient conditions to guarantee the stability of the closed-loop system. Two approaches are generally adopted for the analysis, i.e., the frequency-domain approach and the time-domain approach. The frequency-domain approach is the Nyquist stability criterion [1], [17] while the time-domain approach is the Lyapunov-Krasovskii Theorem [13], [18] or the Lyapunov-Razumikhin Theorem [20]. The frequency-domain approach is often only applicable to particular problems while the time-domain approach is more general. For example, the frequency-domain approach might not be feasible when the network

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topologies are switching or the delays are time-varying while the time-domain approach is still applicable in this case [18]. On the other hand, the stability conditions derived by the time-domain approach are often related to some linear matrix inequality (LMI) conditions, which might not be straightforward to directly reveal the relationship between the network structure and the bound of the delays in the stability conditions. In contrast, the stability conditions derived by the frequency-domain approach can usually explicitly show the feasible bound of the delays in relation to the network structure and are hence easy to understand and implement in real applications. This paper adopted the frequency-domain approach as its analysis tool.

Although much work has been done on consensus algorithms in the presence of delays, there are still several interesting directions worthy of further investigations. For example, in a leader-following context, assume that the leader evolves according to the following dynamics

$$\begin{aligned}\dot{x}_0(t) &= v_0(t) \\ \dot{v}_0(t) &= f(x_0(t), v_0(t), t),\end{aligned}\quad (1)$$

where  $x_0 \in \mathbb{R}^m$  and  $v_0 \in \mathbb{R}^m$  are, respectively, the position and velocity of the leader, and  $f$  is a nonlinear function. Then, consider the follower dynamics modeled by

$$\begin{aligned}\dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= f(x_i(t), v_i(t), t) + u_i(t - T_i(t)),\end{aligned}\quad (2)$$

where  $x_i \in \mathbb{R}^m$  and  $v_i \in \mathbb{R}^m$  are, respectively, the position and velocity of agent  $i$ ,  $u_i$  is the control input, and  $T_i(t)$  is the time-varying input delay,  $i = 1, 2, \dots, n$ .

In this discussion, we consider the following general consensus algorithm

$$\begin{aligned}u_i(t) &= - \sum_{j=1}^n a_{ij}(t) \{ [x_i(t) - x_j(t - T_{ij}(t))] \\ &\quad + \gamma [v_i(t) - v_j(t - T_{ij}(t))] \} \\ &\quad - b_i(t) \{ [x_i(t) - x_0(t - T_{ij}(t))] \\ &\quad + \gamma [v_i(t) - v_0(t - T_{ij}(t))] \},\end{aligned}\quad (3)$$

where  $a_{ij}(t)$  is the  $(i, j)$ th entry of the adjacency matrix associated with the  $n$  followers at time  $t$ ,  $b_i(t)$  is the linking weight from the leader to agent  $i$  at time  $t$ ,  $T_{ij}(t)$  is the time-varying communication delay, and  $\gamma$  is the coupling strength.

For the leaderless consensus problem, if there are no communication delays ( $T_{ij} = 0$ ), the input delays are uniformly fixed ( $T_i(t) = T$ ), the network topology is static ( $a_{ij}(t) = a_{ij}$  and  $b_i(t) = b_i$ ,  $\forall i \neq j, = 1, 2, \dots, n$ ), and the system is linear ( $f = 0$ ), a necessary and sufficient condition for reaching second-order consensus has been established in [7]. This work was then extended in [YuSMCB] to the nonlinear case where  $f(x_i, v_i, t)$  is a general nonlinear function that determines the final asymptotic velocity, and some sufficient conditions for reaching second-order consensus were derived. The approaches used in the linear [7] and nonlinear [8] cases are, respectively, eigenvalue analysis and Lyapunov functions. Recently, [9] has extended the results in [7] to the case of switching network topologies and derived some sufficient conditions for reaching second-order consensus based on the Lyapunov-Razumikhin theorem. It is worthwhile to mention that most approaches used to study the leaderless consensus problem can be directly applied to study the leader-following consensus problem. Here, the problem described by (1)-(3) is very general in the sense that it takes into account the time-varying communication and input delays, switching network topologies, and nonlinear dynamics. Therefore, the delay model used in this paper can be viewed as a special case of the model in this discussion.

In particular, we will extend the delay model in this paper to a more general one. As in this paper, we use double-integrator dynamics to describe the system model and study a fixed network topology. However, we consider the case where there exist both multiple fixed input delays and multiple fixed communication delays. Then, equation (6) in this paper is extended to

$$\begin{aligned}\dot{x}_i(t) &= v_i(t) \\ \dot{v}_i(t) &= - \sum_{j=1}^n a_{ij} \{ [x_i(t - T_i) - x_j(t - T_i - T_{ij})] \\ &\quad + \gamma [v_i(t - T_i) - v_j(t - T_i - T_{ij})] \\ &\quad - b_i \{ [x_i(t - T_i) - x_0(t - T_i - T_{ij})] \\ &\quad + \gamma [v_i(t - T_i) - v_0(t - T_i - T_{ij})] \},\end{aligned}\quad (4)$$

where  $T_i$  and  $T_{ij}$  are, respectively, the fixed input and communication delays. Here we have let  $k_i = \alpha_i = 1$  and  $\gamma_i = \gamma, \forall i$ . By doing so, we will not lose the generality because we assume that  $a_{ij}$  might not be equal to  $a_{ji}$ . Define  $\delta x_i(t) = x_i(t) - x_0(t)$  and  $\delta v_i(t) = v_i(t) - v_0(t)$ . Equation (4) can be written as

$$\begin{aligned}
\delta\dot{x}_i(t) &= \delta v_i(t) \\
\delta\dot{v}_i(t) &= -\sum_{j=1}^n a_{ij}\{\delta x_i(t-T_i) - \delta x_j(t-T_i-T_{ij})\} \\
&\quad + \gamma\{\delta v_i(t-T_i) - \delta v_j(t-T_i-T_{ij})\} \\
&\quad - b_i[\delta x_i(t-T_i) + \gamma\delta v_i(t-T_i)].
\end{aligned} \tag{5}$$

Here we have assumed that  $v_0 = 0$  and hence  $x_0$  is a constant. This implies that  $x_0(t-T_i) = x_0(t-T_i-T_{ij})$ . The analysis on the case when  $v_0$  is a non-zero constant is almost the same except that we can only get  $x_i - x_0 \rightarrow e$  as  $t \rightarrow \infty$  due to the existence of the communication delays, where  $e$  is a non-zero constant determined by the values of the communication delays and  $v_0$ . Equation (5) can be written in matrix form as

$$\begin{aligned}
\begin{bmatrix} \delta\dot{x}(t) \\ \delta\dot{v}(t) \end{bmatrix} &= \begin{bmatrix} \mathbf{0}_{n \times n} & I_n \\ \mathbf{0}_{n \times n} & \mathbf{0}_{n \times n} \end{bmatrix} \begin{bmatrix} \delta x(t) \\ \delta v(t) \end{bmatrix} \\
&+ \sum_{k=1}^n \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{0}_{n \times n} \\ -D_k & -\gamma D_k \end{bmatrix} \begin{bmatrix} \delta x(t-\tau_k) \\ \delta v(t-\tau_k) \end{bmatrix} \\
&+ \sum_{k=n+1}^r \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{0}_{n \times n} \\ A_k & \gamma A_k \end{bmatrix} \begin{bmatrix} \delta x(t-\tau_k) \\ \delta v(t-\tau_k) \end{bmatrix},
\end{aligned} \tag{6}$$

where  $\mathbf{0}_{n \times n}$  is the  $n \times n$  matrix with all zero entries,  $I_n$  is the  $n \times n$  identity matrix,  $\delta x = [\delta x_1, \dots, \delta x_n]^T$ ,  $\delta v = [\delta v_1, \dots, \delta v_n]^T$ ,  $\tau_k = T_i, i = 1, \dots, n, \tau_k = T_i + T_{ij}, k = n+1, \dots, r, i, j = 1, \dots, n, r = \frac{n(n+1)}{2}$ , and  $D_k = [D_{kij}]$  and  $A_k = [A_{kij}]$  are the corresponding coefficient matrices associated with the delay

$\tau_k$ , where  $D_{kij} = \begin{cases} \sum_{j=1}^n a_{ij}, & i=j, \tau_k = T_i \\ 0 & \text{otherwise} \end{cases}$ ,  $A_{kij} = \begin{cases} a_{ij}, & i \neq j, \tau_k = T_i + T_{ij} \\ 0, & \text{otherwise} \end{cases}$ . Define  $\delta X = [\delta x^T, \delta v^T]^T$ ,

$A_0 = \begin{bmatrix} \mathbf{0}_{n \times n} & I_n \\ \mathbf{0}_{n \times n} & \mathbf{0}_{n \times n} \end{bmatrix}$ ,  $A_k = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{0}_{n \times n} \\ -D_k & -\gamma D_k \end{bmatrix}$ ,  $k = 1, \dots, n$ , and  $A_k = \begin{bmatrix} \mathbf{0}_{n \times n} & \mathbf{0}_{n \times n} \\ A_k & -\gamma A_k \end{bmatrix}$ ,  $k = n+1, \dots, r$ .

Note here  $\sum_{r=0}^r A_k = \begin{bmatrix} \mathbf{0}_{n \times n} & I_n \\ -(L+B) & -\gamma(L+B) \end{bmatrix}$ , where  $L$  is the Laplacian matrix corresponding to the adjacency matrix associated with the directed graph for the  $n$  followers, and  $B = \text{diag}\{b_i\}$ . Also note that it is easy to show that all eigenvalues of  $\begin{bmatrix} \mathbf{0}_{n \times n} & I_n \\ -(L+B) & -\gamma(L+B) \end{bmatrix}$  are on the open left half

plane if the leader is a globally reachable node and  $\gamma > \max\{\sqrt{\frac{\text{Im}(\mu_i)^2}{\text{Re}(\mu_i)|\mu_i|^2}}\}$  [21], where  $\mu_i$  is the  $i$ th eigenvalue of

$L+B$ . Then, consider the following Lyapunov function candidate

$$\begin{aligned}
V &= [\delta X + \sum_{k=1}^r \mathcal{A}_{\parallel} \int_{-\tau_k}^0 \delta X(t+\theta) [\theta]^T \mathcal{P} [\delta X \\
&\quad + \sum_{k=1}^r \mathcal{A}_{\parallel} \int_{-\tau_k}^0 \delta X(t+\theta) d\theta] \\
&\quad + \sum_{k=1}^r \int_{-\tau_k}^0 \int_{t+\theta}^t \delta X(\xi)^T S_k \delta X(\xi) d\xi d\theta],
\end{aligned}$$

where  $P$  and  $S_k$  are symmetric positive-definite matrices. By following a similar analysis to that in [22], the stability of the closed-loop system (6) can be guaranteed under proper LMI conditions. Note that the condition that all eigenvalues of  $\sum_{r=0}^r \mathcal{A}_k$  are on the open left half plane implies that there exists a  $P$  to ensure that  $(\sum_{r=0}^r \mathcal{A}_k)^T P + P(\sum_{r=0}^r \mathcal{A}_k)$  is negative-definite and further implies the existence of the LMI conditions.

Until now, it is still quite difficult to solve the consensus problem described by (1)–(3). While we have only focused on one simple direction in this discussion, there are several interesting possibilities deserving further investigations. For example, it might be interesting to consider the case of leader-following consensus in the presence of a leader with a time-varying velocity available to only a portion of the followers when there exist both communication and input delays. It might also be appealing to analyze the stability conditions on nonlinear consensus algorithms in the presence of multiple time-varying delays. The research on the comparison between the influence of nonuniform time-varying delays and uniform fixed delays on consensus algorithms might also lead to some interesting results. In addition, instead of considering delays as a disadvantage, it might be possible to consider delays as an advantage. That is, some performance of the closed-loop system (such as the convergence speed) might be improved if delayed information is introduced to the consensus algorithms intentionally.

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## Final Comments by the Authors

### H.-Y. Yang, X.-L. Zho, S.-Y. Zhang

In the paper [1], we studied a second-order leader-following consensus algorithm in multi-agent systems with directed network topologies and heterogeneous input delays, which generalized the previous work [2] from the case of symmetric coupling weights to that of asymmetric coupling weights. By applying the generalized Nyquist criterion of the frequency domain, the consensus algorithm with heterogeneous input delays was analyzed. By utilizing the Greshgorin's disc theorem and curvature theory, the consensus motion of delayed multiple-agent algorithm with leader-following was studied, and decentralized consensus conditions for the multi-agent systems with asymmetric coupling weights were obtained, where sufficient conditions for reaching consensus use only local information of each agent.

In the paper "Discussion on: "Consensus of Second-Order Delayed Multi-Agent Systems with Leader-Following"" written by Meng, Yu and Ren, a

second-order leader-following consensus algorithm with diverse input delays and diverse communication delays was considered by applying the Lyapunov-Krasovskii Theorem of the time-domain approach that is different from the Nyquist stability criterion of the frequency-domain approach in [1].

Consensus means that a team of agents reaches an agreement on a common value by negotiating with their neighbors. Recently, consensus algorithms in multi-agent systems have been extensively studied in the literature ([1] and its references). In the current studies of the agent related problems, leader-following consensus algorithm under a leader or a reference has become one of the main research topic. In the paper [1], we studied a leader-following consensus algorithm for double-integrator dynamics when the leader has a constant velocity. In the discussion paper written by Meng, Yu and Ren, a leader-following consensus algorithm is considered when the

leader's velocity is zero i.e., the position of the leader is fixed.

In leader-following multi-agent systems, suppose the dynamics of the leader is determined by

$$\dot{x}_0(t) = v_0(t), \quad (1)$$

where  $x_0 \in R^m$  and  $v_0 \in R^m$  are the position and the velocity of the leader, respectively. Then, a consensus protocol of multi-agent systems for the follower can be modeled by

$$\begin{aligned} \dot{x}_i(t) &= v_i(t), \\ \dot{v}_i(t) &= u_i(t), \quad i = 1, \dots, n, \end{aligned} \quad (2)$$

where  $x_i \in R^m$ ,  $v_i \in R^m$ , and  $u_i \in R^m$ , are the position, velocity and acceleration, respectively, of agent  $i$ . The control input  $u_i(t)$  is designed in [1] as, for all  $i \in I$ ,

$$\begin{aligned} u_i(t) &= -k_i \left( \sum_{j=1}^n a_{ij} [\alpha_i (x_i(t) - x_j(t)) \right. \\ &\quad \left. + \gamma_i (v_i(t) - v_j(t))] + b_i [\alpha_i (x_i(t) - x_0(t)) + \gamma_i (v_i(t) - v_0(t))] \right), \end{aligned} \quad (3)$$

where control parameters  $k_i > 0$ ,  $\alpha_i > 0$  and  $\gamma_i > 0$ ,  $b_i$  is the linking weight from agent  $i$  to the leader. Note that  $b_i > 0$  if there is a directed edge from agent  $i$  to the leader; otherwise,  $b_i = 0$ .

Due to the finite speeds of transmission and spreading as well as traffic congestions, there are usually time delays in spreading and communication in reality. Therefore, it is very important to study the delay effect on convergence of consensus protocols. In order to discuss the influence of the delays, one needs to first model the delays, which are usually classified as input delays and communication delays. The input delays can be caused by information processing while the communication delays can be caused by information propagation from one agent to another. In the paper [1], consensus algorithm for double-integrator dynamics (3) with heterogenous input delays is studied.

In the discussion paper, suppose the system is linear, the control gain  $k_i = 1$ ,  $\alpha_i = 1$ ,  $\gamma_i = \gamma$ , and there are diverse input delays  $T_i$  and diverse communication delays  $T_{ij}$ , the consensus algorithm of the system (3) with asymmetric coupling weighted graph is described as

$$\begin{aligned} u_i(t) &= - \sum_{j=1}^n a_{ij} [(x_i(t - T_i) - x_j(t - T_i - T_{ij})) \\ &\quad + \gamma (v_i(t - T_i) - v_j(t - T_i - T_{ij}))] \\ &\quad - b_i [(x_i(t - T_i) - x_0(t - T_i - T_{i0})) \\ &\quad + \gamma (v_i(t - T_i) - v_0(t - T_i - T_{i0}))], \quad i \in I. \end{aligned} \quad (4)$$

By applying the Lyapunov-Krasovskii Theorem of the time-domain approach, the consensus of the algorithm (4) with the assumption that  $v_0 = 0$  (hence  $x_0$  is a constant) is analyzed.

Although the time-domain approach is more general than the frequency-domain approach, the consensus condition derived by the time-domain approach is often related to some linear matrix inequality (LMI) condition, which might not be straightforward to directly reveal the relationship between the network structure and the bound of the delays. Moreover, the parameters of the LMI consensus condition are coupled with that of other agents. However, the consensus conditions in [1] obtained by the frequency-domain approach are decentralized consensus conditions for the multi-agent systems, where the sufficient conditions for reaching consensus use only local information of each agent.

We apply the frequency-domain analysis method in [1] to study the consensus of the algorithm (4) with the same assumption that  $v_0 = 0$ . Then, we can obtain the consensus condition for the leader-following multi-agent systems with diverse input delays and diverse communication delays.

**Theorem 1:** *Suppose the multi-agent systems (1, 2, 4) are composed of  $n$  agents and a leader with a static directed interconnection graph that has the leader as a globally reachable node, and the interconnection graph has asymmetric coupling weights. For each agent the following preconditions are assumed, for all  $i \in I$*

$$\begin{aligned} K_i &< 0.4495, \\ \omega_{i0} &\leq T_i^{-1} \arctan(\gamma \omega_{i0}), \end{aligned} \quad (5)$$

where  $K_i = T_i/\gamma$ ,  $\hat{i} = \arg \max_{i \in I} T_i$  and  $\omega_{i0}$  satisfies

$$\omega_{i0} = \frac{\sqrt{\frac{3K_i - K_i^2 + \sqrt{(3K_i - K_i^2)^2 + 8K_i(1 - K_i)}}{2}}}{T_i} \quad (6)$$

Thus, all the agents in the systems asymptotically converge to the leader's state, if

$$\gamma T_i (2g_i + b_i) < F_i \sin(F_i), \quad (7)$$

where  $g_i = \sum_{l=1}^n a_{il}$ ,  $F_i = \omega_{ic} T_i$ ,  $\omega_{ic}$  is the critical frequency of the Nyquist plot of function  $\bar{G}_i = \frac{1+j\omega\gamma}{j\omega} \frac{e^{-j\omega T_i}}{j\omega}$ , satisfying

$$\tan(\omega_{ic} T_i) = \gamma \omega_{ic}. \quad (8)$$

**Remark 1:** *Theorem 1 gives a decentralized consensus condition for multi-agent systems with diverse com-*

munication delays and diverse input delays, which uses only local information of each agent. This condition is consistent with that given in [1].

**Remark 2:** The consensus condition in Theorem 1 is only dependent on input delays  $T_i$ , but independent of communication delays  $T_{ij}$ . Therefore, suppose the input delays  $T_i = 0$  and  $T_{ij} \neq 0$ , the consensus of the algorithm (4) is reached.

**Remark 3:** The consensus condition obtained by LMI in discussion paper can not have the features described in Remark 1 and Remark 2.

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