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Brief paper

Distributed finite-time attitude containment control for multiple rigid bodies[☆]

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ABSTRACT

Distributed finite-time attitude containment control for multiple rigid bodies is addressed in this paper. When there exist multiple stationary leaders, we propose a model-independent control law to guarantee that the attitudes of the followers converge to the stationary convex hull formed by those of the leaders in finite time by using both the one-hop and two-hop neighbors' information. We also discuss the special case of a single stationary leader and propose a control law using only the one-hop neighbors' information to guarantee cooperative attitude regulation in finite time. When there exist multiple dynamic leaders, a distributed sliding-mode estimator and a non-singular sliding surface were given to guarantee that the attitudes and angular velocities of the followers converge, respectively, to the dynamic convex hull formed by those of the leaders in finite time. We also explicitly show the finite settling time.

1. Introduction

As an important branch of cooperative control, distributed cooperative attitude control for multiple rigid bodies has received much research attention in recent years. Compared with traditional centralized coordination approaches, distributed cooperative attitude control achieves more benefits, such as greater efficiency, higher robustness, and less communication requirement.

A ring communication topology was considered in Nair and Leonard (2007), where an energy method was used to guarantee attitude synchronization of multiple rigid bodies. The nonlinear contraction analysis was used in Chung, Ahsun, and Slotine (2009) to analyze the global exponential stability of cooperative tracking control laws for both translational and attitude dynamics in the Lagrange form. Under an undirected or directed communication topology, Ren (2007) presented control laws for an attitude synchronization problem and an attitude regulation problem by using relative attitude and relative angular velocity information. A cooperative attitude tracking problem was solved in VanDyake and Hall (2006), where more accurate definitions of relative attitudes and relative angular velocities were given to guarantee that the proof is more strict. The authors in Bai, Arcak, and Wen (2008) aimed to use only relative attitude and relative angular velocity information to achieve attitude coordination. An adaptive control estimator was used to estimate the angular velocity and the acceleration of a reference under a stringent assumption that the angular velocity of the reference can be parameterized by unknown constants with known time-varying functions. In Sarlettea, Sepulchrea, and Leonard (2009), the authors considered a general undirected connected communication topology while sacrificing to get only a locally asymptotical synchronization result. External disturbances and time delays were considered in cooperative attitude control in Jin, Jiang, and Sun (2008), where a variable structure controller was used to counteract the impact of external disturbances. A leader–follower rigid bodies formation was addressed in Kristiansen, Loria, Chaillet, and Nicklasson (2009). By using an estimator to estimate the angular velocities, the leader tracks the time-varying reference and the follower tracks the leader with only attitude information.

Existing cooperative attitude control algorithms for multiple rigid bodies often focus on leaderless synchronization or cooperative regulation or tracking with only one leader. In contrast, Ji, Ferrari-Trecate, Egerstedt, and Buffa (2008) introduced the multi-leader concept for single-integrator dynamics and proposed a containment control law, where the states of the followers converge to the convex hull formed by the states of the leaders. The authors in Dimarogonas, Tsiotras, and Kyriakopoulos (2009) extended the

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results in Ji et al. (2008) to the case of rigid body attitude containment control, but only discussed the case of multiple stationary leaders with asymptotic rather than finite-time convergence.

It is common that standard attitude control laws can only achieve asymptotic convergence in infinite time and cannot counteract the impact of external disturbances or model uncertainties. In contrast, finite-time control laws offer many benefits including faster convergence, precise performance, and robustness to uncertainties and disturbances. Finite-time attitude stabilization using a non-Lipschitz continuous control law was addressed in Li, Ding, and Li (2009). However, due to the nonlinear property of the quaternion parameters, the proposed control law was rather complex and hard to implement in practical applications. Similar finite-time control laws were given in Hong, Xu, and Huang (2002) and Yu, Yu, Shirinzadeh, and Man (2005) with an emphasis on control of a single robotic manipulator, where Hong et al. (2002) focused on the regulation problem and Yu et al. (2005) focused on the tracking problem. References Li et al. (2009), Hong et al. (2002) and Yu et al. (2005) all focused on control of a single system rather than multiple networked systems.

The analysis on attitude containment control for multiple rigid bodies can serve as an effective tool when there exist multiple leader rigid bodies in remote sensing applications and is applicable to applications involving networked Lagrange systems. In addition, it may be desirable to achieve attitude containment control in finite time. However, finite-time attitude containment control for multiple rigid bodies has not been addressed in the literature. Suppose that there are multiple leaders with known stationary or dynamic attitudes. The goal of this paper is to drive the attitudes of the followers to the limit constrained by the leaders, which is later proved to be the convex hull of the leaders' attitudes, in finite time by expanding on our preliminary work reported in Meng, Ren, and You (2010).

This paper is organized as follows. In Section 2, we introduce basics for rigid body attitude dynamics, graph theory, and containment control. In Section 3, we first propose a model-independent control law for the case of multiple stationary leaders with constant attitudes by using both the one-hop and two-hop neighbors' information. Then we discuss the case of a single stationary leader where the proposed control law uses only the one-hop neighbors' information. In Section 4, we propose a control law for the case of multiple dynamic leaders with time-varying attitudes. Section 5 contains our concluding remarks.

2. Background and preliminaries

2.1. Notations

Given a vector $x = [x_1, \ldots, x_n]^T$ and $\alpha \in \mathbb{R}$, define $x^\alpha = [x_1^\alpha, \ldots, x_n^\alpha]^T$, $\mathrm{sig}(x)^\alpha = [\mathrm{sgn}(x_1)|x_1|^\alpha, \ldots, \mathrm{sgn}(x_n)|x_n|^\alpha]^T$, and $|x|^\alpha = [|x_1|^\alpha, \ldots, |x_n|^\alpha]^T$.

We use $\mathrm{diag}(x)$ to denote the diagonal matrix of a vector x. $\lambda_{\min}(A)$ and $\lambda_{\max}(A)$ are, respectively, the minimum and maximum eigenvalues of the matrix A.

2.2. Rigid body attitude kinematics and dynamics

For a formation of n followers and m leaders, the attitude of each rigid body is represented by Modified Rodriguez Parameters (MRPs) given by $\sigma_i = e_i \tan\frac{\Phi_i}{4}$, $i = 1, \ldots, n+m$, where e_i and Φ_i are the principle axis and the principle angle of the attitude of the ith rigid body (Schaub & Junkins, 2003). Attitude kinematics and dynamics of each rigid body using MRPs are given by Schaub and Junkins (2003)

$$\dot{\sigma}_i = G(\sigma_i)\omega_i, \quad i = 1, \ldots, n+m, \tag{1a}$$

$$J_i\dot{\omega}_i = -\omega_i \times (J_i\omega_i) + \tau_i, \quad i = 1, \ldots, n+m, \tag{1b}$$

where $\sigma_i \in \mathbb{R}^3$ are the MRPs denoting the rotation from the body frame of the ith rigid body to the inertial frame, ω_i is the angular velocity of the ith rigid body with respect to the inertial frame expressed in the body frame of the ith rigid body, $\times$ denotes the cross product between two vectors, and $G(\sigma_i)$ is given by $G(\sigma_i) = \frac{1}{2}(\sigma_i^\times + \sigma_i\sigma_i^T + \frac{1-\|\sigma_i\|^2}{2} I_3)$, where I_3 is the 3×3 identity matrix, $\sigma_i^\times$ denotes a 3×3 skew-symmetric matrix, $\|\sigma_i\| = \sqrt{\sigma_i^T\sigma_i}$ denotes the 2-norm of σ_i, and $J_i \in \mathbb{R}^{3\times 3}$ and $\tau_i \in \mathbb{R}^3$ are, respectively, the inertia tensor and control torque of the ith rigid body. Note that any three-dimensional attitude description may cause a geometric singularity. For MRPs, the singularity problem occurs at $\Phi_i = \pm 360$ deg (Schaub & Junkins, 2003). Consistent with the statement provided in Tsiotras (1996), the stability results obtained in this paper mean the stability of the corresponding kinematics parameters, i.e., the stability is guaranteed for all initial attitudes except for the singular point.

2.3. Attitude dynamics in Lagrange expression

We first transform attitude kinematics (1a) and dynamics (1b) to Lagrange expression (Slotine & Benedetto, 1990; Wong, de Queiroz, & Kapila, 2001)

$$M_i(\sigma_i)\ddot{\sigma}_i + C_i(\sigma_i, \dot{\sigma}_i)\dot{\sigma}_i = G^{-T}(\sigma_i)\tau_i, \tag{2}$$

where $M_i(\sigma_i) = G^{-T}(\sigma_i)J_iG^{-1}(\sigma_i)$, and $C_i(\sigma_i, \dot{\sigma}_i) = -G^{-T}(\sigma_i)J_iG^{-1}(\sigma_i)\dot{G}(\sigma_i)G^{-1}(\sigma_i) - G^{-T}(\sigma_i)(J_iG^{-1}(\sigma_i)\dot{\sigma}_i)^\times G^{-1}(\sigma_i)$. Note that (2) is only valid when the MRPs are nonsingular. Also note that $M_i(\sigma_i)$ is a symmetric positive-definite matrix and $\dot{M}_i(\sigma_i) - 2C_i(\sigma_i, \dot{\sigma}_i)$ is a skew-symmetric matrix. We assume that the measurements for each rigid body are σ_i and $\dot{\sigma}_i$ in this paper. Note that the approaches proposed in this paper are based on Lagrange expression (2). Therefore, the results in this paper can be extended to other attitude representations as long as the attitude kinematics and dynamics based on those representations can be written in a Lagrange form. For example, the extension to Gibbs vector is feasible by using a similar analysis given in Slotine and Benedetto (1990).

2.4. Graph theory notions

Using graph theory, we can model the communication topology among rigid bodies in the formation. A directed graph $\mathcal{G}$ consists of a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{v_1, \ldots, v_p\}$ is a finite, nonempty set of nodes and $\mathcal{E} \in \mathcal{V} \times \mathcal{V}$ is a set of ordered pairs of nodes. An edge (v_i, v_j) denotes that node v_j can obtain information from node v_i, but not necessarily vice versa. All neighbors of node v_i are denoted as $N_i := \{v_j \mid (v_j, v_i) \in \mathcal{E}\}$. An undirected graph $\mathcal{G}$ is defined such that $(v_j, v_i) \in \mathcal{E}$ implies $(v_i, v_j) \in \mathcal{E}$. In this paper, nodes are exemplified as a formation of rigid bodies.

A directed path is a sequence of edges of the form (v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), ... An undirected path in an undirected graph is defined analogously. An undirected graph is connected if there is an undirected path between every pair of distinct nodes.

The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{p\times p}$ associated with the directed graph $\mathcal{G}$ is defined such that a_{ij} is positive if $(v_j, v_i) \in \mathcal{E}$ while $a_{ij} = 0$ otherwise. For the undirected graph $\mathcal{G}$, we assume that $a_{ij} = a_{ji}$. In this paper, we assume that $a_{ii} = 0$, $\forall i$. The (nonsymmetric) Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{p\times p}$ associated with $\mathcal{A}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$. Zero is an eigenvalue of $\mathcal{L}$ with an associated eigenvector $\mathbf{1}_p$, where $\mathbf{1}_p$ is the $p \times 1$ vector of all ones.

2.5. Definitions and assumptions

Definition 1 (*Cao & Ren, 2009*)**.** In this paper, we suppose that there are m leader nodes and n follower nodes in the directed graph $\mathcal{G}_{n+m}$. A node is called a *follower* if the node has a neighbor. A node is called a *leader* if the node has no neighbor. Without losing generality, we let nodes 1 to n represent the followers and nodes $n+1$ to $n+m$ represent the leaders. We also use $L := \{n+1, \ldots, n+m\}$ and $F := \{1, \ldots, n\}$ to denote, respectively, the leader set and the follower set. We separate the directed graph $\mathcal{G}_{n+m}$ as the leader communication topology $\mathcal{G}_m^L$ and the follower communication topology $\mathcal{G}_n^F$, where $\mathcal{G}_m^L := (L, \mathcal{E}^L)$ and $\mathcal{G}_n^F := (F, \mathcal{E}^F)$. In our problems, the leaders do not communicate with each other, which means that $\mathcal{E}^L \in \emptyset$. The communication between different followers are bidirectional, which means that $\mathcal{G}_n^F$ is undirected. In addition, the communication between a leader and a follower is unidirectional with the leader issuing the communication.

Assumption 1. Suppose that for each follower, there exists at least one leader that has a path to the follower.

Definition 2. The adjacency matrix $\mathcal{A}_{n+m} = [a_{ij}] \in \mathbb{R}^{(n+m)\times(n+m)}$ and the Laplacian matrix $\mathcal{L}_{n+m} = [l_{ij}] \in \mathbb{R}^{(n+m)\times(n+m)}$ associated with $\mathcal{G}_{n+m}$ are defined as in Section 2.4. Consistent with Definition 1, each entry of the last m rows of $\mathcal{L}_{n+m}$ is zero because $\mathcal{E}^L \in \emptyset$ and the leaders do not receive information from the followers. For simplicity, we use $\mathcal{L}$ to replace $\mathcal{L}_{n+m}$ later in this paper.

Definition 3 (*Cao & Ren, 2009*)**.** Let X be a set in a real vector space $V \subseteq \mathbb{R}^p$. The convex hull $\text{Co}(X)$ of the set X is defined as $\text{Co}(X) := \{\sum_{i=1}^k \alpha_i x_i \mid x_i \in X, \alpha_i \in \mathbb{R}, \alpha_i \geq 0, \sum_{i=1}^k \alpha_i = 1, k = 1, 2, \ldots\}$.

Definition 4 (*Bhat & Bernstein, 1997* and *Hong et al., 2002*)**.** Consider the following system

$$\dot{x} = f(x), \quad f(0) = 0, \quad x(0) = x_0, \quad x \in \mathbb{R}^p, \tag{3}$$

where $f : U_0 \mapsto \mathbb{R}^p$ is continuous on an open neighborhood U_0 of the origin. Let $(r_1, \ldots, r_p) \in \mathbb{R}^p$ with $r_g > 0, g = 1, \ldots, p$. Also let $f(x) = [f_1(x), \ldots, f_p(x)]^T$ be a continuous vector field. $f(x)$ is said to be *homogeneous of degree* $\kappa \in \mathbb{R}$ with respect to $(r_1, \ldots, r_p)$ if, for any given $\varepsilon > 0$, $f_g(\varepsilon^{r_1}x_1, \ldots, \varepsilon^{r_p}x_p) = \varepsilon^{\kappa+r_g} f_g(x)$, $g = 1, \ldots, p$, $\forall x \in \mathbb{R}^p$. System (3) is said to be homogeneous if $f(x)$ is homogeneous.

2.6. Lemmas

Lemma 1 (*Graham, 1981*)**.** *Given matrices A and B with compatible sizes, $(A \otimes B)^T = A^T \otimes B^T$, $(A \otimes I_p)(B \otimes I_p) = (AB) \otimes I_p$, where $\otimes$ denotes the Kronecker product.*

Lemma 2 (*Yu et al., 2005*)**.** $\frac{\mathrm{d}|x|^{\alpha+1}}{\mathrm{d}t} = (\alpha+1)\text{sig}(x)^\alpha \dot{x}$, *and* $\frac{\mathrm{d}[\text{sig}(x)^{\alpha+1}]}{\mathrm{d}t} = (\alpha+1)|x|^\alpha \dot{x}$, *where $x \in \mathbb{R}$ and $\alpha \in \mathbb{R}$.*

Lemma 3 (*Hardy, Littlewood, & Plya, 1952*)**.** *Let $x_1, \ldots, x_p \geq 0$ and $0 < \alpha \leq 1$. Then $\sum_{i=1}^p x_i^\alpha \geq (\sum_{i=1}^p x_i)^\alpha$.*

If Assumption 1 holds, by proper decomposition, we have that $(\mathcal{L} \otimes I_3)\begin{bmatrix} x_f \\ x_l \end{bmatrix} = \begin{bmatrix} \mathcal{T} \otimes I_3 & \mathcal{T}_d \otimes I_3 \\ 0_{3m\times 3n} & 0_{3m\times 3m} \end{bmatrix}\begin{bmatrix} x_f \\ x_l \end{bmatrix}$, where $\mathcal{T} \in \mathbb{R}^{n\times n}$, $\mathcal{T}_d \in \mathbb{R}^{n\times m}$, $x_f = [x_1^T, \ldots, x_n^T]^T \in \mathbb{R}^{3n}$, $x_l = [x_{n+1}^T, \ldots, x_{n+m}^T]^T \in \mathbb{R}^{3m}$, and $x_i \in \mathbb{R}^3$, $i = 1, \ldots, n+m$. Under Assumption 1, we know that $\mathcal{T}$ is symmetric since the communication between different followers are bidirectional. Under Assumption 1, we also know that $\mathcal{T}_d$ has at least one negative entry, which implies that at least one diagonal entry of $\mathcal{T}$ is strictly larger than the absolute value sum of the non-diagonal entries of that row. Following a similar analysis as given in Lemma 4 in Hu and Hong (2007), we know that $\mathcal{T}$ has eigenvalues equal to or larger than zero and at least one Gersgorin circle does not pass through the origin according to the Gersgorin disc theorem (Horn & Johnson, 1985, Page 344). Then, by the better theorem (Horn & Johnson, 1985, Page 356), the fact that zero is an eigenvalue of $\mathcal{T}$ implies that every Gersgorin circle passes through zero, which leads to a contradiction. Therefore, we can verify that $\mathcal{T}$ is symmetric positive definite. Define $x_d := -(\mathcal{T}^{-1} \otimes I_3)(\mathcal{T}_d \otimes I_3)x_l$ and $\bar{x}_f := x_f - x_d$. According to Lemma 1, $x_d = -(\mathcal{T}^{-1}\mathcal{T}_d) \otimes I_3 x_l$. Rewrite x_d as $x_d := [x_{d1}^T, \ldots, x_{dn}^T]^T$, where $x_{di} \in \mathbb{R}^3$, $i = 1, \ldots, n$. Then we have that $\mathcal{T} \otimes I_3 x_f + \mathcal{T}_d \otimes I_3 x_l = \mathcal{T} \otimes I_3 \bar{x}_f$.

Lemma 4. *Each entry of $-\mathcal{T}^{-1}\mathcal{T}_d$ is nonnegative and each row sum of $-\mathcal{T}^{-1}\mathcal{T}_d$ is equal to one.*

Proof. In mathematics, $\mathcal{T}$ belongs to a class of well-known matrices, called "nonsingular M-matrices" (Miroslav, 2009). One property of such matrices is that $\mathcal{T}^{-1}$ is a nonnegative matrix. Thus, because each entry of $\mathcal{T}_d$ is nonpositive, it follows that $-\mathcal{T}^{-1}\mathcal{T}_d$ is a nonnegative matrix.

We next show each row sum of $-\mathcal{T}^{-1}\mathcal{T}_d$ is equal to one. Note that $\begin{bmatrix} \mathcal{T} & \mathcal{T}_d \end{bmatrix}\begin{bmatrix} \mathbf{1}_n \\ \mathbf{1}_m \end{bmatrix} = 0$. It thus follows that $\mathcal{T}\mathbf{1}_n = -\mathcal{T}_d\mathbf{1}_m$, which implies that $-\mathcal{T}^{-1}\mathcal{T}_d\mathbf{1}_m = \mathbf{1}_n$. Thus each row sum of $-\mathcal{T}^{-1}\mathcal{T}_d$ is equal to one. □

Remark 1. Lemma 4 implies that if $x_f \to x_d$, then x_i, $i \in F$, converge to the convex hull $\text{Co}\{x_j, j \in L\}$ according to Definition 3.

Lemma 5 (*Hong et al., 2002*)**.** *Consider the following system*

$$\dot{x} = f(x) + \widehat{f}(x), \quad f(0) = 0, \quad x \in \mathbb{R}^p, \tag{4}$$

where $f(x)$ is a continuous homogeneous vector field of degree $\kappa < 0$ with respect to $(r_1, \ldots, r_p)$, and $\widehat{f}$ satisfies $\widehat{f}(0) = 0$. Assume $x = 0$ is an asymptotically stable equilibrium of the system $\dot{x} = f(x)$. Then $x = 0$ is a globally finite-time stable equilibrium of system (4) if $\lim_{\varepsilon\to 0} \frac{\widehat{f}_g(\varepsilon^{r_1}x_1,\ldots,\varepsilon^{r_p}x_p)}{\varepsilon^{\kappa+r_g}} = 0$, $g = 1, \ldots, p$, $\forall x \neq 0$, and the stable equilibrium $x = 0$ of the original system (4) is globally asymptotically stable.

Proof. The Lemma 5 follows from a combination of Lemma 3 and Remark 1 in Hong et al. (2002). □

Lemma 6 (*Bhat & Bernstein, 1998, Hong, Wang, & Cheng, 2006* and *Tang, 1998*)**.** *Consider the following system $\dot{x} = f(x, t)$, where $f(0, t) = 0$, $x \in U_0 \subset \mathbb{R}^p$, and $f : U_0 \times \mathbb{R}^+ \mapsto \mathbb{R}^n$ is continuous with respect to x on an open neighborhood U_0 of the origin $x = 0$. Suppose that there are a C^1 positive-definite functions $V(x, t)$ (defined on $\widehat{U} \times \mathbb{R}^+$, where $\widehat{U} \subset U_0 \subset \mathbb{R}^p$ is a neighborhood of the origin), real number $c > 0$ and $0 < \alpha < 1$, such that $\dot{V}(x, t) + cV^\alpha(x, t)$ is negative semi-definite (along the trajectory) on $\widehat{U}$. Then $V(x, t)$ approaches 0 in finite time. In addition, the finite settling time T satisfies that $T \leq \frac{V(x(t_0),t_0)^{1-\alpha}}{c(1-\alpha)}$.*

3. Finite-time attitude containment control with multiple stationary leaders

In this section, we suppose that there are n followers with dynamics (2) and m stationary leaders with the constant attitudes σ_i, $i \in L$. The control goal here is to guarantee that the attitudes of the followers converge to the stationary convex hull formed by those of the leaders in finite time. We propose the following containment control law for each follower as

$$\tau_i = -G^T(\sigma_i)\left\{p \sum_{j\in L\bigcup F} a_{ij}\left\{\text{sig}\left[\sum_{k\in L\bigcup F} a_{ik}(\sigma_i - \sigma_k)\right]^{\alpha_1} - \text{sig}\left[\sum_{k\in L\bigcup F} a_{jk}\left(\sigma_j - \sigma_k\right)\right]^{\alpha_1}\right\}\right.$$

$$+q\sum_{j\in L\bigcup F}a_{ij}\left\{\mathrm{sig}\left[\sum_{k\in L\bigcup F}a_{ik}\left(\dot{\sigma}_i-\dot{\sigma}_k\right)\right]^{\alpha_2}-\mathrm{sig}\left[\sum_{k\in L\bigcup F}a_{jk}\left(\dot{\sigma}_j-\dot{\sigma}_k\right)\right]^{\alpha_2}\right\}\Bigg\},\quad i\in F, \tag{5}$$

where a_{ij} is the (i,j)th entry of the adjacency matrix $\mathcal{A}_{n+m}$ associated with the graph $\mathcal{G}_{n+m}$ defined in Definition 1, p and q are positive constants, $0<\alpha_2<1$, and $\alpha_1=\frac{\alpha_2}{2-\alpha_2}$. Because the leaders have no neighbors, it follows that $\mathrm{sig}[\sum_{k\in L\bigcup F}a_{jk}(\sigma_j-\sigma_k)]^{\alpha_1}=0$ and $\mathrm{sig}[\sum_{k\in L\bigcup F}a_{jk}(\dot{\sigma}_j-\dot{\sigma}_k)]^{\alpha_2}=0, j\in L$. Note that in (5) each follower needs to use information from its neighbors' neighbors.

Define $\sigma_d:=-(\mathcal{T}^{-1}\otimes\mathcal{T}_d)\otimes I_3\sigma_l$ and rewrite it as $\sigma_d=[\sigma_{d1}^T,\ldots,\sigma_{dn}^T]^T$, where $\sigma_l=[\sigma_{n+1}^T,\ldots,\sigma_{n+m}^T]\in\mathbb{R}^{3m}$, $\sigma_{di}\in\mathbb{R}^3$, $i=1,\ldots,n$, and $\mathcal{T}$ and $\mathcal{T}_d$ are defined after Lemma 3. Also define $x_i:=\sigma_i-\sigma_{di}$ and $y_i:=\dot{x}_i=\dot{\sigma}_i$. Note that $\sum_{j\in L\bigcup F}a_{ij}\{\mathrm{sig}[\sum_{k\in L\bigcup F}a_{ik}(\sigma_i-\sigma_k)]^{\alpha_1}-\mathrm{sig}[\sum_{k\in L\bigcup F}a_{jk}(\sigma_j-\sigma_k)]^{\alpha_1}\}=\sum_{j=1}^n T_{ij}\mathrm{sig}[\sum_{k\in L\bigcup F}a_{jk}(\sigma_j-\sigma_k)]^{\alpha_1}=\sum_{j=1}^n T_{ij}\mathrm{sig}(\sum_{k=1}^n T_{jk}x_k)^{\alpha_1}$. By using (5), original system (2) can be transformed to

$$\dot{x}_i=y_i$$
$$M_i(x_i+\sigma_{di})\dot{y}_i+C_i(x_i+\sigma_{di},y_i)y_i=G^{-T}(x_i+\sigma_{di})\tau_i,\quad i\in F, \tag{6}$$

where $\tau_i=-G^T(x_i+\sigma_{di})[p\sum_{j=1}^n T_{ij}\mathrm{sig}(\sum_{k=1}^n T_{jk}x_k)^{\alpha_1}+q\sum_{j=1}^n T_{ij}\mathrm{sig}(\sum_{k=1}^n T_{jk}y_k)^{\alpha_2}]$, $i\in F$ and T_{ij} denotes the (i,j)th entry of the matrix $\mathcal{T}$.

Theorem 1. *Using (5) for (2), $\sigma_i\to\mathrm{Co}\{\sigma_j,j\in L\}$ and $\dot{\sigma}_i\to 0, i\in F$, in finite time if Assumption 1 holds. More specifically, $\sigma_i\to\sigma_{di}$.*

Proof. We know that (6) can be written in matrix expression as

$$\dot{x}=y$$
$$\dot{y}=-M^{-1}(\sigma_d)\left[p\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3x\right)^{\alpha_1}+q\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3y\right)^{\alpha_2}\right]+\widehat{f}(x,y) \tag{7}$$

where $x=[x_1^T,\ldots,x_n^T]^T\in\mathbb{R}^{3n}$, $y=[y_1^T,\ldots,y_n^T]^T\in\mathbb{R}^{3n}$, $M^{-1}(\sigma_d)=\mathrm{diag}[M_1^{-1}(\sigma_{d1}),\ldots,M_n^{-1}(\sigma_{dn})]$, and $\widehat{f}(x,y)$ is given by $\widehat{f}(x,y)=-[M^{-1}(x+\sigma_d)-M^{-1}(\sigma_d)][p(\mathcal{T}\otimes I_3)\mathrm{sig}(\mathcal{T}\otimes I_3x)^{\alpha_1}+q(\mathcal{T}\otimes I_3)\mathrm{sig}(\mathcal{T}\otimes I_3y)^{\alpha_2}]-M^{-1}(x+\sigma_d)C(x+\sigma_d,y)y$, where $C(x+\sigma_d,y)=\mathrm{diag}[C_1(\sigma_1,\dot{\sigma}_1),\ldots,C_n(\sigma_n,\dot{\sigma}_n)]$. Consider also the following reduced system of (7)

$$\dot{x}=y$$
$$\dot{y}=-M^{-1}(\sigma_d)\left[p\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3x\right)^{\alpha_1}+q\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3y\right)^{\alpha_2}\right]. \tag{8}$$

We first show that the original system (7) is globally asymptotically stable. Then we prove that the system (8) is also asymptotically stable and homogeneous. We finally show that $\lim_{\varepsilon\to 0}\frac{\widehat{f}(\varepsilon^{r_1}x,\varepsilon^{r_2}y)}{\varepsilon^{k+r_2}}=0$, where $\kappa<0$ and $r_1,r_2>0$. Then the globally finite-time stability of the equilibrium $x_i=y_i=0, i\in F$ can be concluded by using Lemma 5.

First, consider the Lyapunov function candidate $V=\sum_{\nu=1}^3\sum_{i=1}^n(\frac{p}{1+\alpha_1}|\sum_{j=1}^n T_{ij}x_{j(\nu)}|^{1+\alpha_1})+\frac{1}{2}y^TM(x+\sigma_d)y$ for system (7), where $x_{j(\nu)}$ denotes the νth entry of $x_j\in\mathbb{R}^3$. Note that V is positive definite with respect to x_i and y_i. Taking the derivative of V gives

$$\begin{aligned}\dot{V}&=\sum_{\nu=1}^3\sum_{i=1}^n p\mathrm{sig}\left(\sum_{j=1}^n T_{ij}x_{j(\nu)}\right)^{\alpha_1}\sum_{j=1}^n T_{ij}\dot{x}_{j(\nu)}\\&\quad+y^T\left[-p\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3x\right)^{\alpha_1}-q\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3y\right)^{\alpha_2}\right]\\&=\sum_{\nu=1}^3\sum_{i=1}^n p\mathrm{sig}\left(\sum_{j=1}^n T_{ij}x_{j(\nu)}\right)^{\alpha_1}\sum_{j=1}^n T_{ij}\dot{x}_{j(\nu)}\\&\quad-\sum_{\nu=1}^3\sum_{i=1}^n p\sum_{j=1}^n T_{ij}y_{j(\nu)}\sum_{j=1}^n\mathrm{sig}\left(T_{ij}x_{j(\nu)}\right)^{\alpha_1}\\&\quad-qy^T\left(\mathcal{T}\otimes I_3\right)\mathrm{sig}\left(\mathcal{T}\otimes I_3y\right)^{\alpha_2}\\&=-\sum_{\nu=1}^3\sum_{i=1}^n q\left|\sum_{j=1}^n T_{ij}y_{j(\nu)}\right|^{1+\alpha_2}\le 0,\end{aligned}$$

where $y_{j(\nu)}$ denotes the νth entry of $y_j\in\mathbb{R}^3$. Here we have used Lemma 2 and the facts that $\mathcal{T}$ is symmetric and $\dot{M}-2C$ is skew-symmetric. Let $\Omega=\{(x_i^T,y_i^T)|\dot{V}=0\}$ and $\overline{\Omega}$ be the largest invariant set in Ω. On $\overline{\Omega}$, we know that $\dot{V}\equiv 0$. Thus, it follows that $(\mathcal{T}\otimes I_3)y\equiv 0$, which implies that $y_i\equiv 0$, $i\in F$, because $\mathcal{T}$ is symmetric positive-definite. It thus follows from (7) that $(\mathcal{T}\otimes I_3)\mathrm{sig}(\mathcal{T}\otimes I_3x)^{\alpha_1}\equiv 0$, which implies that $x_i\equiv 0, i\in F$. Therefore, by LaSalle's invariant principle, $x_i\to 0$ and $y_i\to 0$ globally and asymptotically for the original system (7). Similarly, by using the Lyapunov function candidate $V=\sum_{\nu=1}^3\sum_{i=1}^n(\frac{p}{1+\alpha_1}|\sum_{j=1}^n T_{ij}x_{j(\nu)}|^{1+\alpha_1})+\frac{1}{2}y^TM(\sigma_d)y$, we can also prove that system (8) is globally asymptotically stable by noting that $M(\sigma_d)$ is constant.

Next, we show that the system (8) is homogeneous. We know that $0<\alpha_2<1$ and $\alpha_1=\frac{\alpha_2}{2-\alpha_2}$. By following the analysis in Bhat and Bernstein (1997), we know that (8) is a homogeneous function of degree $\kappa=\alpha_2-1<0$ with respect to $(r_1\mathbf{1}_{3n}^T,r_2\mathbf{1}_{3n}^T)$, where $r_1=2-\alpha_2$ and $r_2=1$.

Finally, we show that $\lim_{\varepsilon\to 0}\frac{\widehat{f}(\varepsilon^{r_1}x,\varepsilon^{r_2}y)}{\varepsilon^{\kappa+r_2}}=0$. By noting that $M^{-1}(\varepsilon^{r_1}x+\sigma_d)-M^{-1}(\sigma_d)=O(\varepsilon^{r_1})$ from the mean value inequality and following a similar analysis to that in Hong et al. (2002), we can get that

$$\begin{aligned}\lim_{\varepsilon\to 0}\frac{\widehat{f}(\varepsilon^{r_1}x,\varepsilon^{r_2}y)}{\varepsilon^{\kappa+r_2}}&=-\lim_{\varepsilon\to 0}\frac{\left[M^{-1}(\varepsilon^{r_1}x+\sigma_d)-M^{-1}(\sigma_d)\right]\Psi}{\varepsilon^{\kappa+r_2}}\\&\quad-\lim_{\varepsilon\to 0}\frac{M^{-1}(\varepsilon^{r_1}x+\sigma_d)C(\varepsilon^{r_1}x+\sigma_d,\varepsilon^{r_2}y)\varepsilon^{r_2}y}{\varepsilon^{\kappa+r_2}}\\&=-\lim_{\varepsilon\to 0}O\left(\varepsilon^{-2\kappa}\right)-M^{-1}(\sigma_d)C(\sigma_d,0)\lim_{\varepsilon\to 0}\varepsilon^{-\kappa}=0,\end{aligned}$$

where $\Psi=p(\mathcal{T}\otimes I_3)\mathrm{sig}(\mathcal{T}\otimes I_3x)^{\alpha_1}+q(\mathcal{T}\otimes I_3)\mathrm{sig}(\mathcal{T}\otimes I_3y)^{\alpha_2}$. Thus the finite-time stability of the equilibrium $x_i=y_i=0$ is proved by using Lemma 5. Then based on Lemma 4, we know that $\sigma_i\to\mathrm{Co}\{\sigma_j,j\in L\}$ and $\dot{\sigma}_i\to 0, i\in F$, in finite time. More specifically, $\sigma_i\to\sigma_{di}$. □

Remark 2. Note that (5) requires information from both the neighbors (one-hop neighbors) and the neighbors' neighbors (two-hop neighbors). Control law (5) is motivated by Hong et al. (2002), where regulation of a single robotic manipulator was considered. In contrast, (5) deals with distributed attitude containment control for multiple rigid bodies.

Remark 3. The results in Theorem 1 still hold if we use a control law with only self angular velocity damping of the form $\tau_i=-G^T(\sigma_i)\{p\sum_{j\in L\bigcup F}a_{ij}\{\mathrm{sig}[\sum_{k\in L\bigcup F}a_{ik}(\sigma_i-\sigma_k)]^{\alpha_1}-\mathrm{sig}[\sum_{k\in L\bigcup F}a_{jk}(\sigma_j-\sigma_k)]^{\alpha_1}\}+q\mathrm{sig}(\dot{\sigma}_i)^{\alpha_2}\}, i\in F$. In contrast, the relative angular velocity damping term in (5) introduces relative damping between neighboring rigid bodies.

3.1. Discussion on the special case of a single stationary leader

We note that control law (5) relies on not only the one-hop neighbors' information, but also the two-hop neighbors' information. Research indicates that the requirement on two-hop neighbors' information can be removed when all leaders have the same stationary attitudes, i.e., the case of a single stationary leader.

Suppose that there are n followers with dynamics (2) and one stationary leader (i.e., $m = 1$) with the constant attitude σ_{n+1}. The control goal here is to guarantee that the followers regulate their attitudes to that of the leader in finite time. The following cooperative attitude regulation control law is proposed for each follower as

$$\tau_i = -G^T(\sigma_i)\left[\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}\left(\sigma_i-\sigma_j\right)^{\alpha_1} + q_i\mathrm{sig}(\dot{\sigma}_i)^{\alpha_2}\right], \quad i \in F, \tag{9}$$

where a_{ij} is the (i,j)th entry of the adjacency matrix $\mathcal{A}_{n+1}$ associated with the graph $\mathcal{G}_{n+1}$ defined in Definition 1, q_i is a positive constant, $0 < \alpha_2 < 1$, and $\alpha_1 = \frac{\alpha_2}{2-\alpha_2}$. Define $x_i := \sigma_i - \sigma_{n+1}$ and $y_i := \dot{x}_i = \dot{\sigma}_i, i \in F$. Then the original system (2) using (9) can be transformed to the following second-order nonlinear system as

$$\begin{aligned}&\dot{x}_i = y_i\\ &M_i\left(x_i+\sigma_{n+1}\right)\dot{y}_i + C_i\left(x_i+\sigma_{n+1}, y_i\right)y_i = G^{-T}\left(x_i+\sigma_{n+1}\right)\tau_i,\\ &\quad i \in F,\end{aligned} \tag{10}$$

where τ_i is given by (9).

Main Result: Using (9) for (2), $\sigma_i \to \sigma_{n+1}$ and $\dot{\sigma}_i \to 0, i \in F$, in finite time if Assumption 1 holds.

Proof. It can be seen that (10) can be rewritten as

$$\begin{aligned}\dot{x}_i &= y_i\\ \dot{y}_i &= -M_i^{-1}(\sigma_{n+1})\left[\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}\left(x_i-x_j\right)^{\alpha_1} + q_i\mathrm{sig}(y_i)^{\alpha_2}\right]\\ &\quad + \widehat{f}_i(x,y), \quad i \in F,\end{aligned} \tag{11}$$

where $x_{n+1} = 0$, $\widehat{f}_i(x,y) = -[M_i^{-1}(x_i+\sigma_{n+1}) - M_i^{-1}(\sigma_{n+1})][\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}(x_i-x_j)^{\alpha_1} + q_i\mathrm{sig}(y_i)^{\alpha_2}] - M_i^{-1}(x_i+\sigma_{n+1})C_i(x_i+\sigma_{n+1}, y_i)y_i$, $x = [x_1^T, \ldots, x_n^T]^T$ and $y = [y_1^T, \ldots, y_n^T]^T$. Consider also the following reduced system of (11)

$$\begin{aligned}\dot{x}_i &= y_i\\ \dot{y}_i &= -M_i^{-1}(\sigma_{n+1})\left[\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}\left(x_i-x_j\right)^{\alpha_1} + q_i\mathrm{sig}(y_i)^{\alpha_2}\right],\\ &\quad i \in F.\end{aligned} \tag{12}$$

Similar to the analysis in Theorem 1, we first show that the original system (11) is globally asymptotically stable. Then we prove that the system (12) is asymptotically stable and homogeneous. Finally we show that $\lim_{\varepsilon\to 0}\frac{\widehat{f}_i(\varepsilon^{r_1}x,\varepsilon^{r_2}y)}{\varepsilon^{\kappa+r_2}} = 0$, where $\kappa < 0$ and $r_1, r_2 > 0$. Then the globally finite-time stability of the equilibrium $x_i = y_i = 0$, $i \in F$, can be obtained according to Lemma 5.

First, consider the Lyapunov function candidate $V = \sum_{\nu=1}^{3}\sum_{i=1}^{n}\frac{1}{1+\alpha_1}(\sum_{j=1}^{n} a_{ij}|x_{i(\nu)}-x_{j(\nu)}|^{1+\alpha_1}) + \sum_{\nu=1}^{3}\sum_{i=1}^{n}\frac{2a_{i(n+1)}}{1+\alpha_1}|x_{i(\nu)}|^{1+\alpha_1} + y^TM(x+(\mathbf{1}_n\otimes I_3)\sigma_{n+1})y$ for system (11), where $x_{i(\nu)}$, $\nu = 1, 2, 3$, is the νth entry of $x_i \in \mathbb{R}^3$, and $M = \mathrm{diag}[M_1(x_1+\sigma_{n+1}), \ldots, M_n(x_n+\sigma_{n+1})]$. Note that V is positive definite with respect to x_i and y_i. Taking the derivative of V gives

$$\begin{aligned}\dot{V} &= \sum_{\nu=1}^{3}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}\mathrm{sig}\left(x_{i(\nu)}-x_{j(\nu)}\right)^{\alpha_1}\left(\dot{x}_{i(\nu)}-\dot{x}_{j(\nu)}\right)\\ &\quad + \sum_{\nu=1}^{3}\sum_{i=1}^{n} 2a_{i(n+1)}\mathrm{sig}(x_{i(\nu)})^{\alpha_1}\dot{x}_{i(\nu)} + y^T\left(\dot{M}-2C\right)y\\ &\quad - \sum_{i=1}^{n} 2y_i^T\left[\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}\left(x_i-x_j\right)^{\alpha_1} + q_i\mathrm{sig}(y_i)^{\alpha_2}\right]\\ &= -2\sum_{\nu=1}^{3}\sum_{i=1}^{n} q_i|y_{i(\nu)}|^{\alpha_2+1} \le 0,\end{aligned}$$

where $y_{i(\nu)}$, $\nu = 1, 2, 3$, is the νth entry of $y_i \in \mathbb{R}^3$, $y_{n+1} = \dot{\sigma}_{n+1} = 0$, and $C = \mathrm{diag}[C_1(\sigma_1, \dot{\sigma}_1), \ldots, C_n(\sigma_n, \dot{\sigma}_n)]$. Here we have used Lemma 2 and the facts that $(\dot{M}-2C)$ is skew-symmetric and $\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(\dot{x}_i-\dot{x}_j)^T\mathrm{sig}(x_i-x_j)^{\alpha_1} = 2\sum_{i=1}^{n}\dot{x}_i^T\sum_{j=1}^{n} a_{ij}\mathrm{sig}(x_i-x_j)^{\alpha_1}$ by following a similar analysis to that of Lemma 3.1 in Ren (2008). Let $\Omega = \{(x_i^T, y_i^T)|\dot{V} = 0\}$ and $\overline{\Omega}$ be the largest invariant set in Ω. On $\overline{\Omega}$, we know that $\dot{V} \equiv 0$. Thus, it follows that $y_i \equiv 0$, $i \in F$. Then we know from (11) that $\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}(x_i-x_j)^{\alpha_1} \equiv 0$. It follows that $\sum_{i=1}^{n} x_i^T\sum_{j=1}^{n+1} a_{ij}\mathrm{sig}(x_i-x_j)^{\alpha_1} \equiv 0$, which implies that $\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}(x_i-x_j)^T\mathrm{sig}(x_i-x_j)^{\alpha_1} + \sum_{i=1}^{n} a_{i(n+1)}x_i^T\mathrm{sig}(x_i)^{\alpha_1} \equiv 0$. Because $\mathrm{sig}(x_i-x_j)^{\alpha_1}$ and $\mathrm{sig}(x_i)^{\alpha_1}$ are odd functions, we have that $a_{ij}(x_i-x_j)^T\mathrm{sig}(x_i-x_j)^{\alpha_1} \ge 0$ and $a_{i(n+1)}x_i^T\mathrm{sig}(x_i)^{\alpha_1} \ge 0$. Thus it follows that $a_{ij}(x_i-x_j)^T\mathrm{sig}(x_i-x_j)^{\alpha_1} \equiv 0$ and $a_{i(n+1)}x_i^T\mathrm{sig}(x_i)^{\alpha_1} \equiv 0, \forall i, j \in F$. Then Assumption 1 implies that $x_i \equiv 0$, $i \in F$. Therefore, by LaSalle's invariant principle, $x_i \to 0$ and $y_i \to 0$ globally and asymptotically for the original system (11). Similarly, by using the Lyapunov function candidate $V = \sum_{\nu=1}^{3}\sum_{i=1}^{n}\frac{1}{1+\alpha_1}(\sum_{j=1}^{n} a_{ij}|x_{i(\nu)}-x_{j(\nu)}|^{1+\alpha_1}) + \sum_{\nu=1}^{3}\sum_{i=1}^{n}\frac{2a_{i(n+1)}}{1+\alpha_1}|x_{i(\nu)}|^{1+\alpha_1} + y^TM(\sigma_{n+1})y$ for system (12), we can also prove that the system (12) is globally asymptotically stable by noting that $M(\sigma_{n+1})$ is a constant.

Then by following a similar analysis to that of Theorem 1, we can show that (12) is homogeneous and $\lim_{\varepsilon\to 0}\frac{\widehat{f}_i(\varepsilon^{r_1}x,\varepsilon^{r_2}y)}{\varepsilon^{\kappa+r_2}} = 0$, where κ, r_1 and r_2 are defined as in Theorem 1. Thus it follows that $\sigma_i \to \sigma_{n+1}$ and $\dot{\sigma}_i \to 0, i \in F$, in finite time by using Lemma 5. □

Remark 4. Although control law (9) uses only one-hop neighbors' information, it is only applicable to the case of a single stationary leader, leading all followers to regulate their attitudes to that of the leader, which can be thought of as a special case of the convex hull of multiple leaders. Control law (9) is motivated by Hong et al. (2002) and Jiang and Wang (2009). However, Hong et al. (2002) only discussed regulation of a single robotic manipulator while Jiang and Wang (2009) considered a finite-time leaderless consensus algorithm for single-integrator dynamics.

4. Finite-time attitude containment control with multiple dynamic leaders

In this section, the states of the leaders are assumed to be dynamic given by σ_i, $\dot{\sigma}_i$ and $\ddot{\sigma}_i$, $i \in L$. We also assume that $\dot{\sigma}_i$ and $\ddot{\sigma}_i$, $i \in L$ are bounded. The control goal is to guarantee that the states of the followers converge to the dynamic convex hull formed by those of the dynamic leaders in finite time. We first propose a distributed sliding-mode estimator to obtain the accurate estimates of the weighted average of the leaders' angular velocities in finite time. Then a control torque is proposed to guarantee that a distributed sliding surface is driven to zero in finite time. Finally, on the sliding surface, attitude containment control is achieved in finite time.

The distributed sliding-mode estimator is proposed as

$$\dot{\widehat{x}}_{fi} = -\beta\,\mathrm{sgn}\left\{\sum_{j\in L\bigcup F} a_{ij}\left[\widehat{x}_{fi}-\widehat{x}_{fj}\right]\right\}, \quad i \in F \tag{13}$$

where $\widehat{x}_{fj} = \dot{\sigma}_j, j \in L$, β is positive constant, a_{ij} is the (i,j)th entry of the adjacency matrix $\mathcal{A}_{n+m}$ associated with the graph $\mathcal{G}_{n+m}$ defined in Definition 1, and the initial states $\widehat{x}_{fi}(0)=0$, $i\in F$. When $t\le T_1$ (T_1 will be defined later), $\tau_i=-k_{p_i}\sigma_i-k_{d_i}\dot{\sigma}_i$, $i\in F$, where k_{p_i} and k_{d_i} are positive constants. When $t>T_1$, we propose the following control law for each follower

$$\tau_i = G^T(\sigma_i)\left(C_i(\sigma_i,\dot{\sigma}_i)\dot{\sigma}_i + m_i+\upsilon_i+e_i\right), \quad i\in F \tag{14}$$

where $m_i=-\alpha^{-1}b^{-1}M_i(\sigma_i)\mathrm{sig}(\dot{\sigma}_i-\widehat{x}_{fi})^{2-\alpha}$, $\upsilon_i=-\mu M_i(\sigma_i)\mathrm{sgn}(\sum_{j=1}^n T_{ij}s_j)$, $e_i=-\eta M_i(\sigma_i)\mathrm{sig}(\sum_{j=1}^n T_{ij}s_j)^\gamma$, and $s_i=\sum_{j\in L\bigcup F} a_{ij}(\sigma_i-\sigma_j)+b\sum_{j=1}^n T_{ij}\mathrm{sig}(\dot{\sigma}_j-\widehat{x}_{fj})^\alpha$. Here T_{ij} is the (i,j) entry of the matrix $\mathcal{T}$ defined in Lemma 4, $1<\alpha<2$, $0<\gamma<1$, and μ, η and b are positive constants.

Theorem 2. *Assume that $\dot{\sigma}_i$ and $\ddot{\sigma}_i$, $i\in L$ are bounded. Using (13) and (14) for (2), $\sigma_i\to \mathrm{Co}\{\sigma_j, j\in L\}$ and $\dot{\sigma}_i\to\mathrm{Co}\{\dot{\sigma}_j, j\in L\}$, $i\in F$, in finite time with a settling time T_3 (defined later) if $\mu>\beta>\sup_{i\in L,\nu=1,2,3}|\ddot{\sigma}_{i(\nu)}|$, and Assumption 1 holds, where $\ddot{\sigma}_{i(\nu)}$ denotes the νth entry of $\ddot{\sigma}_i\in\mathbb{R}^3$, $i\in L$. Specifically, $\sigma_i\to\sigma_{di}$ and $\dot{\sigma}_i\to\dot{\sigma}_{di}$ in finite time, where σ_{di} is defined as in Section 3.*

Proof. We first prove that $\widehat{x}_{fi}\to\dot{\sigma}_{di}$, $i\in F$ in finite time. Consider the Lyapunov function candidate $V_1=\frac{1}{2}\bar{\widehat{x}}_f^T(\mathcal{T}\otimes I_3)\bar{\widehat{x}}_f$, where $\bar{\widehat{x}}_f=\widehat{x}_f-\dot{\sigma}_d$, $\widehat{x}_f=[\widehat{x}_{f1}^T,\ldots,\widehat{x}_{fn}^T]^T$, $\dot{\sigma}_d=[\dot{\sigma}_{d1}^T,\ldots,\dot{\sigma}_{dn}^T]^T$, and $\mathcal{T}$ is defined as in Lemma 4. Taking the derivative of V_1 gives

$$\begin{aligned}\dot{V}_1 &= \bar{\widehat{x}}_f^T(\mathcal{T}\otimes I_3)\left[-\beta\mathrm{sgn}\left(\mathcal{T}\otimes I_3\bar{\widehat{x}}_f\right)-\ddot{\sigma}_d\right]\\ &\le -(\beta-\eta_1)\|\mathcal{T}\otimes I_3\bar{\widehat{x}}_f\|_1\le-(\beta-\eta_1)\|\mathcal{T}\otimes I_3\bar{\widehat{x}}_f\|_2\\ &\le -(\beta-\eta_1)\sqrt{\bar{\widehat{x}}_f^T(\mathcal{T}\otimes I_3)^2\bar{\widehat{x}}_f}\\ &\le -(\beta-\eta_1)\lambda_{\min}(\mathcal{T})\|\bar{\widehat{x}}_f\|_2\le-(\beta-\eta_1)\frac{\sqrt{2}\lambda_{\min}(\mathcal{T})}{\sqrt{\lambda_{\max}(\mathcal{T})}}V_1^{\frac{1}{2}},\end{aligned}$$

where $\eta_1=\|\ddot{\sigma}_d\|_\infty$. Here we have used Holder's inequality $|x^Ty|\le\|x\|_1\|y\|_\infty$ to obtain the first inequality. Therefore, it follows from Lemmas 4 and 6 that $\bar{\widehat{x}}_f\to 0$ (i.e., $\widehat{x}_{fi}\to\dot{\sigma}_{di}$, $i\in F$) in finite time, with a settling time $T_1=\frac{\sqrt{3n}\eta_0\lambda_{\max}(\mathcal{T})}{(\beta-\eta_1)\lambda_{\min}(\mathcal{T})}$, where $\eta_0=\|\dot{\sigma}_d\|_\infty$, if $\beta>\sup_{i\in L,\nu=1,2,3}|\ddot{\sigma}_{i(\nu)}|$. Thus $\dot{\sigma}_{di}$ can be used to replace $\widehat{x}_{fi}$ when $t\ge T_1$.

We next show that under the control of $\tau_i=-k_{p_i}\sigma_i-k_{d_i}\dot{\sigma}_i$, $i\in F$, the states σ_i, $\dot{\sigma}_i$ of (2) will not diverge to infinity in $t\in[0,T_1]$. By using the Lyapunov function candidate $V_{1i}=\frac{1}{2}k_{p_i}\sigma_i^T\sigma_i+\frac{1}{2}\dot{\sigma}_iM_i(\sigma_i)\dot{\sigma}_i$, $\forall i\in F$, we know that $\dot{V}_{1i}=-k_{d_i}\dot{\sigma}_i^T\dot{\sigma}_i\le 0$ from the fact that $\dot{M}_i(\sigma_i)-2C_i(\sigma_i,\dot{\sigma}_i)$ is a skew-symmetric matrix. This implies that $\dot{\sigma}_i$ and σ_i, $i\in F$, are bounded in $t\in[0,T_1]$ from the fact that $M_i(\sigma_i)$ is bounded (Wong et al., 2001).

We then show that $s_i\to 0$ in finite time. Define $\bar{\sigma}_i=\sigma_i-\sigma_{di}$. Consider the Lyapunov function candidate $V_2=\sum_{i=1}^n V_{2i}$, where $V_{2i}=\frac{1}{2}s_i^Ts_i$. Taking the derivative of V_{2i} when $t\ge T_1$ gives

$$\begin{aligned}\dot{V}_{2i} &= s_i^T\dot{s}_i = s_i^T\Big[\sum_{j\in L\bigcup F}a_{ij}\left(\dot{\sigma}_i-\dot{\sigma}_j\right)\\ &\quad + b\alpha\sum_{j=1}^n T_{ij}\mathrm{diag}\left(|\dot{\bar{\sigma}}_j|^{\alpha-1}\right)\left(\ddot{\sigma}_j-\dot{\widehat{x}}_{fj}\right)\Big]\\ &= s_i^T\Big\{\sum_{j=1}^n T_{ij}\dot{\bar{\sigma}}_j + b\alpha\sum_{j=1}^n T_{ij}\mathrm{diag}\left(|\dot{\bar{\sigma}}_j|^{\alpha-1}\right)M_j^{-1}(\sigma_j)\\ &\quad\times\left[-\alpha^{-1}M_j(\sigma_j)b^{-1}\mathrm{sig}(\dot{\bar{\sigma}}_j)^{2-\alpha}+\upsilon_j+e_j-M_j(\sigma_j)\dot{\widehat{x}}_{fj}\right]\Big\}\\ &= \alpha b s_i^T\Big[\sum_{j=1}^n T_{ij}\mathrm{diag}\left(|\dot{\bar{\sigma}}_j|^{\alpha-1}\right)M_j^{-1}(\sigma_j)\left(\upsilon_j+e_j-M_j(\sigma_j)\dot{\widehat{x}}_{fj}\right)\Big],\end{aligned}$$

where we have used Lemma 2 and the fact that when $t\ge T_1$, $M_j(\sigma_j)\left(\ddot{\sigma}_j-\dot{\widehat{x}}_{fj}\right)=m_j+\upsilon_j+e_j-M_j(\sigma_j)\dot{\widehat{x}}_{fj}$, $j\in F$.

Then, we know that

$$\begin{aligned}\dot{V}_2 &= \sum_{i=1}^n\dot{V}_{2i}=\alpha b\sum_{i=1}^n\left(\sum_{j=1}^n T_{ij}s_j^T\right)\\ &\quad\times\left[\mathrm{diag}\left(|\dot{\bar{\sigma}}_i|^{\alpha-1}\right)M_i^{-1}(\sigma_i)\left(\upsilon_i+e_i-M_i(\sigma_i)\dot{\widehat{x}}_{fi}\right)\right]\\ &= \alpha b\sum_{i=1}^n\left(\sum_{j=1}^n T_{ij}s_j^T\right)\mathrm{diag}\left(|\dot{\bar{\sigma}}_i|^{\alpha-1}\right)\\ &\quad\times\left\{-\dot{\widehat{x}}_{fi}-\mu\mathrm{sgn}\left(\sum_{j=1}^n T_{ij}s_j\right)\right\}\\ &\quad+\alpha b\sum_{i=1}^n\left(\sum_{j=1}^n T_{ij}s_j^T\right)\mathrm{diag}\left(|\dot{\bar{\sigma}}_i|^{\alpha-1}\right)M_i^{-1}(\sigma_i)e_i\\ &\le -\alpha b(\mu-\beta)\sum_{\nu=1}^3\sum_{i=1}^n|\dot{\bar{\sigma}}_{i(\nu)}|^{\alpha-1}\left|\sum_{j=1}^n T_{ij}s_{j(\nu)}\right|\\ &\quad-\eta\alpha b\sum_{i=1}^n\left(\sum_{j=1}^n T_{ij}s_j\right)^T\mathrm{diag}\left(|\dot{\bar{\sigma}}_i|^{\alpha-1}\right)\mathrm{sig}\left(\sum_{j=1}^n T_{ij}s_j\right)^\gamma\\ &\le -\eta\alpha b\sum_{\nu=1}^3\sum_{i=1}^n|\dot{\bar{\sigma}}_{i(\nu)}|^{\alpha-1}\left|\sum_{j=1}^n T_{ij}s_{j(\nu)}\right|^{1+\gamma}\\ &\le -\eta\alpha bk_{\min}\left[\sum_{\nu=1}^3\sum_{i=1}^n\left|\sum_{j=1}^n T_{ij}s_{j(\nu)}\right|^2\right]^{\frac{1+\gamma}{2}}\\ &\le -\eta\alpha bk_{\min}\lambda_{\min}^{1+\gamma}(\mathcal{T})\left[\sum_{\nu=1}^3\sum_{i=1}^n s_{i(\nu)}^2\right]^{\frac{1+\gamma}{2}} = -\chi V_2^{\frac{1+\gamma}{2}},\end{aligned} \tag{15}$$

where $k_{\min}\triangleq\inf_{i\in F,\nu=1,2,3}|\dot{\bar{\sigma}}_{i(\nu)}|^{\alpha-1}$ is positive when $\dot{\bar{\sigma}}_{i(\nu)}\neq 0$, $\forall i$, $\forall\nu$, $s_{j(\nu)}$ and $\dot{\bar{\sigma}}_{i(\nu)}$ denotes, respectively, the νth entry of s_j and $\dot{\bar{\sigma}}_i$, and $\chi\triangleq\eta\alpha bk_{\min}\lambda_{\min}^{1+\gamma}(\mathcal{T})2^{\frac{1+\gamma}{2}}$. Here we have used the facts that $\mathcal{T}$ is symmetric, $\mu>\beta$, Eq. (13), and Lemma 3.

If $\dot{\bar{\sigma}}_{i(\nu)}\neq 0$, $\forall i$, $\forall\nu$, we can obtain that $s_i\to 0$ in finite time, with a settling time $T_2=T_1+\frac{2V_2^{\frac{1-\gamma}{2}}(T_1)}{(1-\gamma)\eta\alpha bk_{\min}\lambda_{\min}^{1+\gamma}(\mathcal{T})2^{\frac{1+\gamma}{2}}}$, by using Lemma 6. From (15), we note that in the approaching phase, $\dot{\bar{\sigma}}_{i(\nu)}=0$ may hinder the finite-time reachability of V_2. We next show that $\dot{\bar{\sigma}}_{i(\nu)}=0$ cannot hold for a nonzero time interval when $\sum_{j=1}^n T_{ij}s_{j(\nu)}\neq 0$. When $t\ge T_1$, by substituting (14) into (2), we have that

$$\begin{aligned}\ddot{\bar{\sigma}}_i &= -\ddot{\sigma}_{di}-\alpha^{-1}b^{-1}\mathrm{sig}\left(\dot{\sigma}_i-\widehat{x}_{fi}\right)^{2-\alpha}\\ &\quad-\mu\mathrm{sgn}\left(\sum_{j=1}^n T_{ij}s_j\right)-\eta\mathrm{sig}\left(\sum_{j=1}^n T_{ij}s_j\right)^\gamma.\end{aligned}$$

If $\dot{\bar{\sigma}}_{i(\nu)}=0$, we have that

$$\ddot{\bar{\sigma}}_{i(\nu)}=-\ddot{\sigma}_{di(\nu)}-\mu\mathrm{sgn}\left(\sum_{j=1}^n T_{ij}s_{j(\nu)}\right)-\eta\mathrm{sig}\left(\sum_{j=1}^n T_{ij}s_{j(\nu)}\right)^\gamma,$$

where $\ddot{\sigma}_{di(\nu)}$ denotes the νth entry of $\ddot{\sigma}_{di}$. Because $\mu>\beta>\sup_{i\in L,\nu=1,2,3}|\ddot{\sigma}_{i(\nu)}|$ and $\mathrm{sgn}(\sum_{j=1}^n T_{ij}s_{j(\nu)})$ and $\mathrm{sig}(\sum_{j=1}^n T_{ij}s_{j(\nu)})^\gamma$

have the same sign, we have that $\ddot{\overline{\sigma}}_{i(\nu)} \neq 0$ when $\dot{\overline{\sigma}}_{i(\nu)} = 0$ and $\sum_{j=1}^{n} T_{ij} s_{j(\nu)} \neq 0$. This implies that $\dot{\overline{\sigma}}_{i(\nu)} = 0$ is not an attractor (Feng, Yu, & Man, 2002) and cannot hold in the reaching phase. Thus, $\dot{\overline{\sigma}}_{i(\nu)} = 0$ will not hinder the finite-time reachability of V_2.

We finally show that σ_i, $i \in F$, converge to the convex hull $\text{Co}\{\sigma_j, j \in L\}$ in finite time. When $t \geq T_2$, $s_i = 0$, i.e., $\sum_{j=1}^{n} T_{ij} \overline{\sigma}_j = -b \sum_{j=1}^{n} T_{ij} \text{sig}(\dot{\overline{\sigma}}_j)^{\alpha}$ for each $i \in F$. This implies that $\overline{\sigma}_i + b\text{sig}(\dot{\overline{\sigma}}_i)^{\alpha} = 0, \forall i \in F$, from the fact that $\mathcal{T}$ is symmetric positive definite. We can then easily show that $\overline{\sigma}_i + b\text{sig}(\dot{\overline{\sigma}}_i)^{\alpha} = 0$ is equivalent to the sliding surface $\dot{\overline{\sigma}}_i + b'\text{sig}(\overline{\sigma}_i)^{\alpha'} = 0$ by following a similar analysis to that in Yu et al. (2005). Here $\alpha' = \frac{1}{\alpha}$ and $b' = b^{-1/\alpha}$. Then, we know that $\dot{\overline{\sigma}} = -b'\text{sig}(\overline{\sigma})^{\alpha'}$, where $\overline{\sigma} = [\overline{\sigma}_1^T, \ldots, \overline{\sigma}_n^T]^T$. Similar to the approach given in Xiao, Wang, and Jia (2008), we can obtain the finite-time stability of the equilibrium $\overline{\sigma}_i = 0$ by using the Lyapunov function candidate $V_3 = \sum_{\nu=1}^{3} \sum_{i=1}^{n} \frac{1}{1+\alpha'} |\overline{\sigma}_{i(\nu)}|^{1+\alpha'}$, where $\overline{\sigma}_{i(\nu)}$ denotes the νth entry of $\overline{\sigma}_i \in \mathbb{R}^3$. Taking the derivative of V_3 gives

$$\dot{V}_3 = -b' \left[\text{sig}(\overline{\sigma})^{\alpha'}\right]^T \text{sig}(\overline{\sigma})^{\alpha'} = -b' \sum_{\nu=1}^{3} \sum_{i=1}^{n} \left[\text{sig}(\overline{\sigma}_{i(\nu)})^{\alpha'}\right]^2$$
$$= -b' \sum_{\nu=1}^{3} \sum_{i=1}^{n} |\overline{\sigma}_{i(\nu)}|^{2\alpha'},$$

where Lemma 2 has been used. We also know that

$$V_3^{\frac{2\alpha'}{1+\alpha'}} = \left(\sum_{\nu=1}^{3} \sum_{i=1}^{n} \frac{1}{1+\alpha'} |\overline{\sigma}_{i(\nu)}|^{1+\alpha'} \right)^{\frac{2\alpha'}{1+\alpha'}}$$
$$\leq \left(\frac{1}{1+\alpha'} \right)^{\frac{2\alpha'}{1+\alpha'}} \sum_{\nu=1}^{3} \sum_{i=1}^{n} |\overline{\sigma}_{i(\nu)}|^{2\alpha'},$$

where we have used Lemma 3. Thus, we have $\dot{V}_3 \leq -b'/[(\frac{1}{1+\alpha'})^{\frac{2\alpha'}{1+\alpha'}}] V_3^{\frac{2\alpha'}{1+\alpha'}}$. This implies that $\overline{\sigma} \to 0$ in finite time, with a settling time $T_3 = T_2 + \frac{[(\frac{1}{1+\alpha'})^{\frac{2\alpha'}{1+\alpha'}}](1+\alpha') V_3^{\frac{1-\alpha'}{1+\alpha'}}(T_2)}{b'(1-\alpha')}$, by using Lemma 6. Thus, $\sigma_i \to \text{Co}\{\sigma_j, j \in L\}$ and $\dot{\sigma}_i \to \text{Co}\{\dot{\sigma}_j, j \in L\}, i \in F$, in finite time, with a settling time T_3 by using Lemma 4. Specifically, $\sigma_i \to \sigma_{di}$ and $\dot{\sigma}_i \to \dot{\sigma}_{di}$. □

Remark 5. A sliding surface of the form $s_i = \dot{\overline{\sigma}}_i + b'\overline{\sigma}_i^{\alpha'}$ was proposed in Tang (1998) and Man, Paplinski, and Wu (1994) for a single robotic manipulator. However, this sliding surface might lead the control torque to contain the term $\text{diag}(\overline{\sigma}_i^{\alpha'-1})\dot{\overline{\sigma}}_i$ and hence results in a singularity when $\overline{\sigma}_i = 0$ but $\dot{\overline{\sigma}}_i \neq 0$ (Feng et al., 2002). In contrast, the sliding surface proposed here is non-singular by a proper design and introduction of absolute value and signum operators. Control law (14) is motivated by Feng et al. (2002), Tang (1998), Xiao et al. (2008) and Yu et al. (2005). However, Feng et al. (2002), Tang (1998) and Yu et al. (2005) only considered the tracking problem for a single robotic manipulator while Xiao et al. (2008) discussed finite-time leaderless consensus for single-integrator dynamics.

Remark 6. Note that control law (14) for multiple dynamic leaders is model-dependent due to the fact that M_i and C_i (and hence J_i) are required. In contrast, control laws (5) and (9) for stationary leaders are model-independent. Here the term "model" is mainly used to refer to the knowledge of the inertia tensor J_i. Also note that control law (14) introduces discontinuous signum operators to counteract the impact of the lack of acceleration measurements, which makes the control torque discontinuous. In contrast, control laws (5) and (9) are continuous.

Remark 7. Note that the switching time T_1 depends on global information, i.e., the maximum and minimum eigenvalues of the matrix $\mathcal{T}$ (a variant of the Laplacian matrix). How to obtain the maximum and minimum eigenvalues of $\mathcal{T}$ (and hence the switching time T_1) in a distributed way deserves further investigation. The controller $\tau_i = -k_{p_i}\sigma_i - k_{d_i}\dot{\sigma}_i$, $i \in F$ is used to guarantee the boundedness of the closed-loop system when $t \leq T_1$, but it may have an adverse effect on the tracking of $\dot{\sigma}_{di}$ and σ_{di}. Its influence can be alleviated by selecting small k_{p_i} and k_{d_i}.

Remark 8. If a saturation function is used to approximate the signum operator in (13) and (14), the tracking error s_i will be within a bounded set rather than approaching zero. In addition, the bounded set will be related to the control parameters, the width of the boundary layer of the saturation function, and the (nonsymmetric) Laplacian matrix $\mathcal{L}_{m+n}$.

5. Concluding remarks

In this paper, we proposed distributed attitude containment control laws for multiple rigid bodies with multiple stationary and dynamic leaders. A model-independent control law was proposed for attitude containment control with stationary leaders. A model-dependent control law was proposed for attitude containment control with dynamic leaders. The attitudes of the followers were shown to be driven to the convex hull formed by those of the leaders in finite time for both cases. Future work includes extending the results in this paper to cases where the graph is directed and/or time varying and there exist multiple communication and input time delays.

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