Systems & Control Letters 61 (2012) 117–126

Contents lists available at SciVerse ScienceDirect

Systems & Control Letters

journal homepage: www.elsevier.com/locate/sysconle

Leader–follower swarm tracking for networked Lagrange systems[☆]

Ziyang Meng[a,*], Zongli Lin[b], Wei Ren[c]

[a] *Department of Automation, Shanghai Jiao Tong University, and Key Laboratory of System Control and Information Processing, Ministry of Education of China, Shanghai 200240, China*

[b] *Charles L. Brown Department of Electrical and Computer Engineering, University of Virginia, Charlottesville, VA 22904-4743, USA*

[c] *Department of Electrical Engineering, University of California, Riverside, CA 92521, USA*

ARTICLE INFO

Article history:
Received 13 March 2011
Received in revised form
14 June 2011
Accepted 6 September 2011
Available online 20 November 2011

Keywords:
Multiple leaders
Lagrange systems
Group cohesion
Group dispersion

ABSTRACT

In this paper, swarm tracking problems with group dispersion and cohesion behaviors are discussed for a group of Lagrange systems. The agent group is separated into two subgroups. One is called the leader group, whose members are encapsulated with the desired generalized coordinates and generalized coordinate derivatives. The other one, referred to as the follower group, is guided by the leader group. The objective is to guarantee distributed tracking of generalized coordinate derivatives for the followers and to drive the generalized coordinates of the followers close to the convex hull formed by those of the leaders. Both the case of constant leaders' generalized coordinate derivatives and the case of time-varying leaders' generalized coordinate derivatives are considered. The proposed control algorithms are shown to achieve velocity matching, connectivity maintenance and collision avoidance. In addition, the sum of the steady-state distances between the followers and the convex hull formed by the leaders is shown to be bounded and the bound is explicitly given. Simulation results are presented to validate the effectiveness of theoretical conclusions.

1. Introduction

Coordination of networked Lagrange systems has been receiving significant attention recently. This research interest is aroused in part by the rapid development of distributed control of multi-agent systems. We refer the readers to [1] and [2] for an overview of these research efforts. Traditionally, the system model is often simplified to that of a single-integrator kinematics or a double-integrator dynamics to highlight the interactions among different agents. But this simplification imposes an obvious limitation on the model's abilities to represent the real physical objects. On the other hand, a Lagrange model is often used to describe mechanical systems, such as mobile robots, autonomous vehicles, robotic manipulators, and rigid bodies. Indeed, coordination of networked Lagrange systems has numerous practical applications. One typical example of such applications is the relative attitude keeping problem in the context of deep space interferometry [3–5].

Much effort has been made toward coordination problems of networked Lagrange systems. For instance, with the attitude kinematic and dynamic equations transformed into the Lagrange model, the author of [6] solves the leader–follower cooperative attitude synchronization problem where there exists a time-varying leader. Global exponential stability and various communication topologies are considered in [7] and [8] for consensus tracking of a group of Lagrange systems, where the nonlinear contraction analysis is introduced. The cases of actuator saturation and unavailability of measurements of generalized coordinate derivatives are addressed in [9]. Communication delays and dynamic topologies are considered in [10], where collision avoidance behavior is highlighted. An adaptive approach is introduced in [11] to compensate for the unknown parameters in the Lagrange dynamic models. In addition, Ref. [12] takes into consideration delays, limited data rates and bounded disturbance input in the design of the control law. An ultimate boundedness result instead of an absolute tracking result is obtained.

Group dispersion and cohesion behaviors are often very important for coordination of multi-agent systems. Group dispersion is to ensure minimum safety distance between different agents and group cohesion is to maintain the connectivity once two agents are connected. A variable structure approach is taken in [13,14] to guarantee cooperative swarm tracking for a group of agents with or without a leader. Ref. [15] extends this result to the case of a general directed communication topology. In [16], the framework

☆ This work was supported in part by National Natural Science Foundation of China under Grant 60928008, Zhiyuan Professorship at Shanghai Jiao Tong University, China Postdoctoral Science Foundation under Grant 20110490067, and the National Science Foundation under Grant ECCS—1002393.

* Corresponding author. Tel.: +86 21 34205476.
E-mail addresses: panday227@gmail.com, mengzy06@mails.tsinghua.edu.cn (Z. Meng), zl5y@virginia.edu (Z. Lin), ren@ee.ucr.edu (W. Ren).

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doi:10.1016/j.sysconle.2011.09.006

of a leaderless and a leader-following flocking is given, where three behaviors, i.e., velocity matching, cohesion, and collision avoidance are established. Connectivity maintenance approaches are proposed in [17] and [18], where a bounded or an unbounded input function is introduced. Most of the existing works on flocking with a leader relies on the strict assumption that all the followers have access to the leader's information. In contrast, a variable structure approach is developed in [19] to address a swarm tracking problem with reduced interaction. For multiple Lagrange systems, coordination and collision avoidance are studied in [20], where the case of cooperative regulation is considered. The authors of [21] designed a so-called region-based shape control algorithm to force a group of mobile robots modeled by Lagrange dynamics to move into a desired region while maintaining a minimum distance among themselves. However, this algorithm relies on the strict assumptions that the minimum distance be small enough and all the followers have access to the information of the desired region.

Although there are many results on coordination of multi-agent systems, we note that the existing research often considers a leaderless or a one-leader case. The case of multiple leaders, where the leaders form a *cohesive inclusion* and the followers are guided by the leaders, is also of practical value, for example, in the analysis of collective behaviors of biological groups or in a rescue mission in a disaster area. Here, the term cohesive inclusion means a rigidly enclosed space formed by the leaders. With multiple leaders, the system robustness can be improved and the design flexibility can be increased. The concept of multi-leader was proposed in [22], which also gives a containment control algorithm to solve the multi-leader problem. Here, "containment", also referred to as "cohesive inclusion", means the containment of the leaders. Refs. [23,24] extend the results given in [22] to the case of switching communication topologies. In [22–24], the system model is simplified to that of a single-integrator kinematics. Ref. [25] extends this simplification to the case of attitude containment control for multiple rigid bodies. Finite-time attitude containment control problems are addressed in [26] for both cases of multiple stationary and dynamic leaders.

In this paper, we focus on the swarm tracking problem in the presence of multiple leaders and multiple followers. In particular, we establish the leader–follower swarm tracking framework with group dispersion and cohesion behaviors, where there exist multiple leaders and multiple followers. In our formulation, the system model is described as more realistic nonlinear Lagrange dynamics, instead of simpler single-integrator kinematics or double-integrator dynamics. The information interaction is assumed to be strictly distributed, i.e., the leaders' information is available to only a portion of the followers. This is a rather mild assumption compared with those in the existing works, such as [14,16,21], especially when the leaders' generalized coordinate derivatives are time varying. We further show that only a compromised result can be obtained when the group dispersion and cohesion behaviors and the containment objective are all considered together, i.e., the sum of the steady-state distances between the followers and the convex hull formed by the leaders might be bounded instead of approaching zero. In addition, we give an explicit description of the magnitude of this bound.

The remainder of this paper is organized as follows. In Section 2, we state the problem to be solved and present some relevant background materials. In Sections 3 and 4, we derive swarm tracking control algorithms for the followers when the leaders' generalized coordinate derivatives are, respectively, constant and time-varying, where the detailed analysis is given in Appendix. The proposed control algorithms are shown to achieve velocity matching, connectivity maintenance, collision avoidance and containment boundedness. Simulation results are presented in Section 5 to validate our control laws and Section 6 contains our conclusions.

2. Background and problem statement

2.1. Lagrange dynamics

Suppose that there are n follower Lagrange systems. The dynamics of the Lagrange systems are described as

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\,\dot{q}_i + g_i(q_i) = \tau_i, \quad i = 1, 2, \ldots, n, \tag{1}$$

where $q_i \in \mathbb{R}^p$ is the vector of generalized coordinates, $M_i(q_i)$ is the $p \times p$ inertia (symmetric) matrix, $C_i(q_i,\dot{q}_i)\dot{q}_i$ is the Coriolis and centrifugal terms, $g_i(q_i)$ is the vector of gravitational force, and τ_i is the control force. Note that the dynamics of a Lagrange system satisfies the following properties.

(1) There exist positive constants $k_{\underline{M}}, k_{\overline{M}}, k_{\overline{C}}, k_{\overline{g}}$ such that $k_{\underline{M}} I_p \le M_i(q_i) \le k_{\overline{M}} I_p$, $\|C_i(q_i,\dot{q}_i)\| \le k_{\overline{C}}\|\dot{q}_i\|$[1], and $\|g_i(q_i)\| \le k_{\overline{g}}$.
(2) $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew symmetric.
(3) The left-hand side of the dynamics can be parameterized, i.e., $M_i(q_i)x + C_i(q_i,\dot{q}_i)y + g_i(q_i) = Y_i(q_i,\dot{q}_i,x,y)\theta_i$, $\forall x, y \in \mathbb{R}^p$, where $Y_i \in \mathbb{R}^{p\times p_\theta}$ is a regression matrix with a constant parameter vector $\theta_i \in \mathbb{R}^{p_\theta}$.

From property 3, we know that the nominal dynamics satisfy

$$\widehat{M}_i(q_i)\ddot{q}_i + \widehat{C}_i(q_i,\dot{q}_i)\dot{q}_i + \widehat{g}_i(q_i) = Y_i(q_i,\dot{q}_i,\dot{q}_i,\ddot{q}_i)\widehat{\theta}_i, \tag{2}$$

where $\widehat{M}_i$, $\widehat{C}_i$, $\widehat{g}_i$, and $\widehat{\theta}_i$ are nominal dynamics terms. For later use, we define

$$\varphi_i(t) = \triangle M_i(q_i)\ddot{q}_i + \triangle C_i(q_i,\dot{q}_i)\dot{q}_i + \triangle g_i(q_i) = Y_i(q_i,\dot{q}_i,\dot{q}_i,\ddot{q}_i)\triangle\theta_i,$$

where $\triangle M_i(q_i) = M_i(q_i) - \widehat{M}_i(q_i)$, $\triangle C_i(q_i,\dot{q}_i) = C_i(q_i,\dot{q}_i) - \widehat{C}_i(q_i,\dot{q}_i)$, $\triangle g_i(q_i) = g_i(q_i) - \widehat{g}_i(q_i)$, and $\triangle\theta_i = \theta_i - \widehat{\theta}_i$.

Suppose that in addition to the n follower agents with Lagrange dynamics, there are m leader agents with the desired generalized coordinates and generalized coordinate derivatives. Our goal here is to drive the generalized coordinate derivatives of the followers to converge to those of the leaders and to force the generalized coordinates of the followers close to the cohesive inclusion formed by those of the leaders. Both the cases of constant and time-varying leaders' generalized coordinate derivatives will be discussed.

2.2. Graph theory

We will use graph theory to model the communication topology among agents (both followers and leaders). A directed graph $\mathcal{G}$ consists of a pair $(\mathcal{V}, \mathcal{E})$, where $\mathcal{V} = \{\nu_1, \nu_2, \ldots, \nu_{n+m}\}$ is a finite nonempty set of nodes and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is a set of ordered pairs of nodes. An edge (ν_i, ν_j) denotes that node ν_j obtains information from node ν_i.

All neighbors of node ν_i are denoted as $N_i := \{\nu_j | (\nu_j, \nu_i) \in \mathcal{E}\}$. An undirected graph is defined such that $(\nu_j, \nu_i) \in \mathcal{E}$ implies $(\nu_i, \nu_j) \in \mathcal{E}$. A directed path in a directed graph or an undirected path in an undirected graph is a sequence of edges of the form $(\nu_{i_1}, \nu_{i_2}), (\nu_{i_2}, \nu_{i_3}), \ldots$.

The adjacency matrix $\mathcal{A} = [a_{ij}] \in \mathbb{R}^{(m+n)\times(m+n)}$ associated with the directed graph $\mathcal{G}$ is defined such that a_{ij} is positive if $(\nu_j, \nu_i) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. For the undirected graph, we assume that $a_{ij} = a_{ji}$. In this paper, we assume that $a_{ii} = 0, \forall i$. The Laplacian matrix $\mathcal{L} = [l_{ij}] \in \mathbb{R}^{(m+n)\times(m+n)}$ associated with $\mathcal{A}$ is defined as $l_{ii} = \sum_{j\neq i} a_{ij}$ and $l_{ij} = -a_{ij}$, where $i \neq j$.

Definition 2.1. Suppose that there exist m leader nodes and n follower nodes. Without loss of generality, we let nodes ν_1 to ν_n

[1] $\|x\|$ denotes the Euclidean vector norm of $x \in \mathbb{R}^p$ in this paper, i.e., $\|x\| = \sqrt{x^T x}$.

represent the followers, and nodes ν_{n+1} to ν_{n+m} represent the leaders. The follower set and the leader set are denoted as, respectively, $F := \{\nu_1, \nu_2, \dots, \nu_n\}$ and $L := \{\nu_{n+1}, \nu_{n+2}, \dots, \nu_{n+m}\}$.

We also define the communication graphs for the generalized coordinates and the generalized coordinate derivatives, respectively. Consistent with [10] and [27], we assume that all the followers are equipped with communication units and sensing units. The sensing unit accounts for the measurements of relative generalized coordinates between different agents and the communication unit accounts for the measurements of relative generalized coordinate derivatives. In such case, we use, respectively, the sensing graph $\mathcal{G}^S := (\mathcal{V} = L \bigcup F, \mathcal{E}^S)$ (or the generalized coordinate graph) and the communication graph $\mathcal{G}^C := (\mathcal{V}, \mathcal{E}^C)$ (or the generalized coordinate derivative graph) to denote the information interaction between different agents, where $\mathcal{E}^S$ and $\mathcal{E}^C$ are defined in Definition 2.2 below.

Definition 2.2. The neighbors of the followers and the leaders in the generalized coordinate graph $\mathcal{G}^S$ are defined as $N_i^S := \{\nu_j | (\nu_j, \nu_i) \in \mathcal{E}^S\}$, where

$$\mathcal{E}^S := \begin{cases} \{(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \mid \|q_i - q_j\| \le r\}, & \forall i \in \mathcal{V}, \forall j \in F \\ \{(\nu_i, \nu_j) \in \emptyset\}, & \forall i \in \mathcal{V}, \forall j \in L. \end{cases}$$

Note that r denotes the sensing radius.

Definition 2.3. The neighbors of the followers and the leaders in the generalized coordinate derivative graph $\mathcal{G}^C$ are defined as $N_i^C := \{\nu_j | (\nu_j, \nu_i) \in \mathcal{E}^C\}$, where

$$\mathcal{E}^C := \begin{cases} \{(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \mid \nu_i \Leftrightarrow \nu_j\}, & \forall i \in F, \forall j \in F \\ \{(\nu_i, \nu_j) \in \mathcal{V} \times \mathcal{V} \mid \nu_i \Rightarrow \nu_j\}, & \forall i \in L, \forall j \in F \\ \{(\nu_i, \nu_j) \in \emptyset\}, \quad \forall i \in \mathcal{V}, \forall j \in L, \end{cases}$$

$\Leftrightarrow$ denotes unordered adjacency, and $\Rightarrow$ denotes ordered adjacency.

2.3. Graph connectivity assumptions and Laplacian matrix decomposition

In this paper, the following graph connectivity assumptions will be made to guarantee the necessary information sharing within the group.

Assumption 2.1. For each follower, there exists at least one leader that has a path to the follower at the initial time $t = 0$ in the sensing graph $\mathcal{G}^S$.

Assumption 2.2. The graph connectivity relationship is fixed and for each follower, there exists at least one leader that has a path to the follower in the communication graph $\mathcal{G}^C$.

By expanding the Kronecker product, we have that

$$(\mathcal{L} \otimes I_p) \begin{bmatrix} x_f \\ x_l \end{bmatrix} = \begin{bmatrix} \mathcal{T} \otimes I_p & \mathcal{T}_d \otimes I_p \\ 0_{pm \times pn} & 0_{pm \times pm} \end{bmatrix} \begin{bmatrix} x_f \\ x_l \end{bmatrix},$$

where $\mathcal{T} \in \mathbb{R}^{n \times n}$, $\mathcal{T}_d \in \mathbb{R}^{n \times m}$, $x_f = [x_1^T, x_2^T, \dots, x_n^T]^T \in \mathbb{R}^{pn}$, and $x_l = [x_{n+1}^T, x_{n+2}^T, \dots, x_{n+m}^T]^T \in \mathbb{R}^{pm}$. Also define $x_d = [x_{d1}^T, x_{d2}^T, \dots, x_{dn}^T]^T = -(\mathcal{T}^{-1} \otimes I_p)(\mathcal{T}_d \otimes I_p) x_l \in \mathbb{R}^{pn}$. Then we have that $\mathcal{T} \otimes I_p x_f + \mathcal{T}_d \otimes I_p x_l = \mathcal{T} \otimes I_p (x_f - x_d)$.

Definition 2.4. The convex hull $\text{co}\{X\}$ of the set X is defined as $\text{co}\{X\} = \{\sum_{i=1}^k \alpha_i x_i | x_i \in X, \alpha_i \in \mathbb{R}, \alpha_i \ge 0, \sum_{i=1}^k \alpha_i = 1\}$.

Lemma 2.1. *$\mathcal{T}$ is positive-definite if $\mathcal{G}^C$ satisfies Assumption 2.2. In addition, each entry of $-\mathcal{T}^{-1}\mathcal{T}_d$ is nonnegative and the sum of each row of $-\mathcal{T}^{-1}\mathcal{T}_d$ is equal to one. This further shows that $-\mathcal{T}^{-1}\mathcal{T}_d \otimes I_p x_l \in \text{co}\{x_j, j \in L\}$. Also note that when $m = 1$, $-\mathcal{T}^{-1}\mathcal{T}_d = 1$.*

Proof. See Lemma 4 in [26]. □

3. Followers' swarm tracking control when the leaders' generalized coordinate derivatives are constant

In this section, $\dot{q}_i \in \mathbb{R}^p$, $i \in L$, is assumed to be identical and constant. We let $\dot{q}_i = \dot{q}_d$, $i \in L$.

3.1. Followers' swarm tracking control

Since the leaders have the same generalized coordinate derivative $\dot{q}_d$, the relative generalized coordinates between different leaders remains unchanged. Thus, the leaders have formed a stable cohesive inclusion. The goal here is to drive the generalized coordinate derivatives of the followers to converge to $\dot{q}_d$ and the generalized coordinates of the followers close to the cohesive inclusion formed by the leaders. Note here that the leaders' information is available to only a portion of the followers and we use nominal parameters of the model. The proposed control law for the followers is given by

$$\tau_i = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})\widehat{\theta}_i - k_i s_i - \delta \left(\sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} + \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) \right), \quad i \in F, \tag{3}$$

where Y_i, V_{ij} and a_{ij} are defined in Section 2.1, Appendix A.1, and Appendix A.2, and k_i, $i \in F$, and δ are arbitrary positive constants. Note that the term $\sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i}$ is used to guarantee the group cohesion and the group dispersion and the term $\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j)$ is used to drive the generalized coordinates of the followers close to the cohesive inclusion formed by the leaders. In addition, the virtual reference trajectory, the adaptive control term, the leaders' generalized coordinate derivative estimator, and the sliding surface are, respectively, given by

$$\dot{q}_{ri} = \widehat{v}_i - \delta \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right), \quad i \in F, \tag{4}$$

$$\dot{\widehat{\theta}}_i = -Y_i^T s_i, \quad i \in F, \tag{5}$$

$$\dot{\widehat{v}}_i = -\sum_{j=1}^{n+m} b_{ij}(\widehat{v}_i - \widehat{v}_j) - \delta \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right), \quad i \in F, \tag{6}$$

and

$$s_i = \delta \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right) + (\dot{q}_i - \widehat{v}_i), \quad i \in F. \tag{7}$$

Here $\widehat{v}_j = \dot{q}_d, j \in L$, and b_{ij} denotes the (i, j)th entry of the adjacency matrix $\mathcal{A}^C = [b_{ij}]$ associated with $\mathcal{G}^C$ (defined in Section 2.2). It then follows from Property 3 in Section 2.1 that

$$M_i \ddot{q}_{ri} + C_i \dot{q}_{ri} + g_i(q_i) = Y_i(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})\theta_i, \quad \forall i \in F,$$

and

$$M_i \dot{s}_i + C_i s_i = -M_i \ddot{q}_{ri} - C_i \dot{q}_{ri} - g_i + \tau_i, \quad \forall i \in F.$$

Before proceeding on, we first give a definition for describing the distance between follower i and the convex hull formed by the leaders.

Definition 3.1. We use φ_i to describe the distance between follower i and the convex hull formed by the leaders, where

$\varphi_i = \inf \|q_i - y_i\|, \forall y_i \in \mathrm{co}\{q_j, j \in L\}$. The sum of the distances between all the followers and the convex hull formed by the leaders is then given by $\varphi = \sum_{i=1}^{n} \varphi_i$.

Theorem 3.1. *Assume that $\mathcal{G}^S$ satisfies Assumption 2.1 and $\mathcal{G}^C$ satisfies Assumption 2.2. Also assume that $\|q_i(0) - q_j(0)\| > d_1$ for all $i, j \in \mathcal{V}$, $i \neq j$. By using the proposed distributed control law (3) with (4)–(7) for followers' dynamics (1), we can conclude that*

(1) $N_i^S(0) \subseteq N_i^S(t)$ for all $i \in F$ and $t \geq 0$.
(2) $\dot{q}_i \to \dot{q}_d$, $\forall i \in F$.
(3) $\|q_i(t) - q_j(t)\| > d_1$ for all $i, j \in \mathcal{V}$, $i \neq j$.
(4) $\limsup_{t\to\infty} \varphi \leq \frac{n(n+m)}{\lambda_{\min}(\mathcal{T}^S)} \alpha^*$ for some $\alpha^* > 0$, where φ and $\mathcal{T}^S$ are as defined in Definition 3.1 and Section 2.3.

Proof. The proof involves four parts: connectivity maintenance analysis, velocity matching analysis, group dispersion analysis, and containment boundedness analysis. In the connectivity maintenance analysis, we show that no edge in $\mathcal{G}^S$ will be lost for $t \geq 0$ if the initial connectivity relationship for $\mathcal{G}^S$ satisfies Assumption 2.1. In the velocity matching analysis, we prove that the generalized coordinate derivatives for the followers will track those of the leaders. In the group dispersion analysis and the containment boundedness analysis, the group dispersion and cohesion behaviors within the group will be evaluated. The detailed proof can be found in Appendix A.3. □

Remark 3.1. As seen in the proof of Theorem 3.1 in Appendix A.3, the bound on the sum of the steady-state distances between the followers and the convex hull formed by the leaders is related to the sensing radius r, the minimum safety distance d_1, the cohesive radius d_2, the numbers of leaders and followers, $\lambda_{\min}(\mathcal{T}^S)$, and the system initial state $U(0)$.

Remark 3.2. The group dispersion result readily implies collision avoidance of different agents within the group. The extension to the case of avoidance of external obstacles, as considered in [16], is important in some applications and will be one of our future research directions.

Remark 3.3. In this paper, we assume that the leaders' generalized coordinate derivatives are identical. In such a situation, the formation of the leaders is fixed. The extension to the case of a time-varying leader formation is of interest and will need further consideration.

3.2. Extension to the case where the communication graph is replaced by the sensing graph

We note that the sensing graph and the communication graph are considered separately in Section 3.1. In addition, the communication graph is assumed to be connected for all time. In this section, we replace the communication graph with the sensing graph and only impose an assumption on the initial connectivity relationship. In such a case, the generalized coordinate derivative estimator is replaced by

$$\dot{\widehat{v}}_i = -\sum_{j=1}^{n+m} a_{ij}(q)(\widehat{v}_i - \widehat{v}_j) - \delta\left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i}\right), \quad i \in F, \tag{8}$$

where $\widehat{v}_j \triangleq \dot{q}_d, j \in L$.

Theorem 3.2. *Assume that $\mathcal{G}^S$ satisfies Assumption 2.1. Also assume that $\|q_i(0) - q_j(0)\| > d_1$ for all $i, j \in \mathcal{V}$, $i \neq j$. By using the proposed distributed control law (3) with (4), (5), (7), and (8) for the followers' dynamics (1), we can conclude that*

(1) $N_i^S(0) \subseteq N_i^S(t)$ for all $i \in F$ and $t \geq 0$.
(2) $\dot{q}_i \to \dot{q}_d$, $\forall i \in F$.
(3) $\|q_i(t) - q_j(t)\| > d_1$ for all $i, j \in \mathcal{V}$, $i \neq j$.
(4) $\limsup_{t\to\infty} \varphi \leq \frac{n(n+m)}{\lambda_{\min}(\mathcal{T}^S)} \alpha^*$ for some $\alpha^* > 0$.

Proof. The proof is similar to that of Theorem 3.1. Construct the same Lyapunov function as (15). It is easy to show that

$$\dot{U} = -\sum_{i=1}^{n} k_i s_i^2 - \sum_{i=1}^{n} \delta^2 \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i}\right)^2 + \sum_{i=1}^{n} (\widehat{v}_i - \dot{q}_d) \sum_{j=1}^{n+m} a_{ij}(q)(\widehat{v}_i - \widehat{v}_j).$$

Assume that $\mathcal{G}^S$ switches at t_k, $k = 1, 2, \ldots$. Then, following the same analysis as given in the proof of Theorem 3.1, we know that for $t \in [0, t_1)$, $U(t) \leq U(0)$. By the definition of V_{ij}, $\lim_{\|q_i - q_j\| \to r} V_{ij} = \infty$, and thus no edge will be lost at time t_1 for $i, j \in \mathcal{V}$. Therefore, new edges must be added for $\mathcal{G}^S$ at switching time t_1. The definition of V_{ij} also guarantees the boundness and continuity of U. Therefore, we can verify that $U(t_1)$ is bounded. Similar to the aforementioned analysis, it follows that no edge will be lost for $i, j \in \mathcal{V}$ and $t \in [t_{k-1}, t_k)$. Therefore $N_i^S(0) \subseteq N_i^S(t)$ for all $i \in F$ and $t \geq 0$.

The velocity matching analysis, group dispersion analysis and containment boundedness analysis are the same as those in the proof of Theorem 3.1. □

4. Followers' swarm tracking control when the leaders' generalized coordinate derivatives are time-varying

In this section, $\dot{q}_i \in \mathbb{R}^p$, $i \in L$, is assumed to be identical and time-varying. We let $\dot{q}_i = \dot{q}_d$, $i \in L$.

4.1. Followers' swarm tracking control

The control goal here is the same as the one in Section 3, i.e., to drive the generalized coordinate derivatives of the followers to converge to $\dot{q}_d$ and the generalized coordinates of the followers close to the cohesive inclusion formed by the leaders. We will use a variable structure approach instead of an adaptive control method to compensate for the model uncertainties. Thus, besides assuming that $\dot{q}_d$ and $\ddot{q}_d$ are bounded, we also assume that $\triangle\theta_i$, $i \in F$, is bounded, where $\triangle\theta$ is defined in Section 2.1.[2] The proposed control law for the followers is given by

$$\tau_i = \widehat{C}_i \dot{q}_i + \widehat{g}_i + \kappa_i - \widehat{M}_i \left(\frac{d}{dt}\left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j)\right) + \frac{d}{dt}\left(\sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i}\right) + K s_i\right), \quad i \in F, \tag{9}$$

where $\widehat{M}_i$, $\widehat{C}_i$ and $\widehat{g}_i$ represent $M_i(q_i, \widehat{\theta}_i)$, $C_i(q_i, \dot{q}_i, \widehat{\theta}_i)$ and $g_i(q_i, \widehat{\theta}_i)$, as given in Section 2.1, V_{ij} and a_{ij} are defined in Appendix A.1 and

[2] This is naturally satisfied for constant θ_i and $\widehat{\theta}_i$.

Appendix A.2, and K is any positive constant. The sliding surface is specified as

$$s_i = \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} + \sum_{j=1}^{n+m} b_{ij}(\dot{q}_i - \dot{q}_j), \quad i \in F, \tag{10}$$

and κ_i is given by

$$\kappa_i = -\mu_i Y_i \mathrm{sgn}\left(Y_i^T \widehat{M}_i^{-T} \sum_{j=1}^{n+m} b_{ij}(s_i - s_j)\right) - \rho_i \widehat{M}_i \mathrm{sgn}\left(\sum_{j=1}^{n+m} b_{ij}(s_i - s_j)\right), \quad i \in F, \tag{11}$$

where Y_i is as defined in Section 2.1, b_{ij} denotes the (i, j)th entry of the adjacency matrix $\mathcal{A}^C = [b_{ij}]$ associated with $\mathcal{G}^C$, defined in Section 2.2, $s_j = 0$, $\forall j \in L$, $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^T$ for $x = [x_1, x_2, \ldots, x_n]^T$, with sgn being the signum function, and μ_i and ρ_i are positive constants, whose values are to be specified.

Applying the control law (9) to the followers' dynamics (1) leads to

$$\widehat{M}_i \frac{d}{dt}(\dot{q}_i - \dot{q}_d) = -\widehat{M}_i \ddot{q}_d - Y(q_i, \dot{q}_i, \ddot{q}_i)\triangle\theta_i - \widehat{M}_i \times \left(\frac{d}{dt}\left(\sum_{j=1}^{n} \frac{\partial V_{ij}}{\partial q_i}\right) + \frac{d}{dt}\left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j)\right) + K s_i\right) + \kappa_i, \quad i \in F. \tag{12}$$

Theorem 4.1. *Assume that $\mathcal{G}^S$ satisfies Assumption 2.1 and $\mathcal{G}^C$ satisfies Assumption 2.2. Also assume that $\|q_i(0) - q_j(0)\| > d_1$ for all $i, j \in \mathcal{V}$, $i \neq j$. If $\mu_i > \|\triangle\theta_i\|_\infty$ and $\rho_i > \|\ddot{q}_d\|_\infty$, $\forall i \in F$, then, by using the proposed distributed control law (9) with (10) and (11) for the followers' dynamics (1), we can conclude that*

(1) *$N_i^S(0) \subseteq N_i^S(t)$ for all $i \in F$ and $t \geq 0$.*
(2) *$\dot{q}_i \to \dot{q}_d$, $\forall i \in F$.*
(3) *$\|q_i(t) - q_j(t)\| > d_1$ for all $i, j \in \mathcal{V}$, $i \neq j$.*
(4) *$\limsup_{t\to\infty} \varphi \leq \frac{n(n+m)}{\lambda_{\min}(\mathcal{T}^S)}\alpha^*$ for some $\alpha^* > 0$.*

Proof. See Appendix A.4. □

Remark 4.1. Different from Section 3, we do not use the generalized coordinate derivative estimator to obtain the leaders' generalized coordinate derivatives in this section. Thus, a large amount of calculation is avoided.

4.2. Extension to containment control

In Section 4.1, the bound on the sum of the steady-state distances between the followers and the convex hull formed by the leaders might not be zero when we have other requirements on group cohesion and dispersion. In this section, group cohesion and dispersion behaviors are not considered in the control law. In such a case, we will show that the followers will converge into the convex hull formed by the leaders, i.e., the bound on the sum of the steady-state distances between the followers and the convex hull formed by the leaders will converge to zero. The proposed control law for the followers is given by

$$\tau_i = \widehat{C}_i \dot{q}_i + \widehat{g}_i - \widehat{M}_i \frac{d}{dt}\left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j)\right) - K\widehat{M}_i s_i + \kappa_i, \quad i \in F, \tag{13}$$

where the sliding mode is defined as

$$s_i = \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} b_{ij}(\dot{q}_i - \dot{q}_j), \quad i \in F, \tag{14}$$

and κ_i is given in (11) in Section 4.1.

Theorem 4.2. *Assume that $\mathcal{G}^S$ satisfies Assumption 2.1 and $\mathcal{G}^C$ satisfies Assumption 2.2. If $\mu_i > \|\triangle\theta_i\|_\infty$ and $\rho_i > \|\ddot{q}_d\|_\infty$, $\forall i \in F$, then, by using the proposed distributed control law (13) with (11) and (14) for the followers' dynamics (1), we can conclude that*

1. $\dot{q}_i \to \dot{q}_d$, $\forall i \in F$.
2. $q_i \to \mathrm{co}\{q_j, j \in L\}$, $\forall i \in F$.

Proof. Following the similar analysis in the proof of Theorem 4.1, we construct a Lyapunov function as:

$$U = \frac{1}{2}\sum_{i=1}^{n} s_i^T s_i + \frac{\alpha}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} + \alpha \sum_{i=1}^{n}\sum_{j=1+n}^{m+n} Q_{ij}.$$

It is easy to show that $\dot{U} \leq 0$ when $0 < \alpha < 2\lambda_{\min}(\mathcal{T}^C)\sqrt{K}$, $\mu_i > \|\triangle\theta_i\|_\infty$ and $\rho_i > \|\ddot{q}_d\|_\infty$, $\forall i \in F$. Therefore, the connectivity maintenance analysis follows from the proof of Theorem 3.1. Note that the connectivity maintenance result guarantees that $\lambda_{\min}(T^S) > 0$. Similar to the proof of Theorem 4.1, it follows that $s_i \to 0$ and $\dot{q}_i \to \dot{q}_d$, $\forall i \in F$, as $t \to \infty$. On the sliding surface, by Lemma 2.1, $\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) = 0$ implies $q_i \to \mathrm{co}\{q_j, j \in L\}$, $\forall i \in F$. This completes the proof. □

Corollary 4.1. *If there is only one leader in the leader set, the convex hull formed by the leaders will reduce to a single point, i.e., with the proposed control law in this section, the final generalized coordinates of the followers will track that of the leader exactly.*

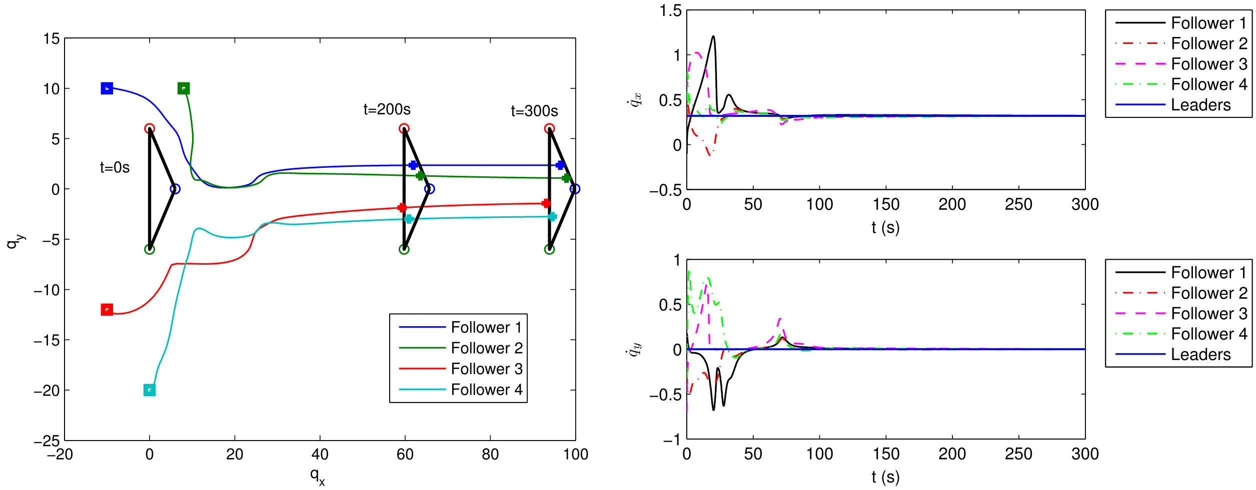
Remark 4.2. A containment control algorithm for networked Lagrange systems is also proposed in [26] when the leaders' generalized coordinate derivatives are time-varying. However, the model uncertainties are not considered and the design of the sliding mode estimator may increase the complexity of the algorithm in [26]. In contrast, the control law (13) of this paper is easier to implement and the control parameters only rely on the local information.

5. Simulation results

In this section, numerical simulation results are given to validate the effectiveness of the theoretical results obtained in this paper. We assume that there exist four followers ($n = 4$) and three leaders ($m = 3$) in the group. The system dynamics is given by [11,28]

$$\begin{bmatrix} M_{11} & M_{12} \\ M_{21} & M_{22} \end{bmatrix}\begin{bmatrix} \ddot{q}_{ix} \\ \ddot{q}_{iy} \end{bmatrix} + \begin{bmatrix} -b\dot{q}_{iy} & -b(\dot{q}_{ix} + \dot{q}_{iy}) \\ b\dot{q}_{ix} & 0 \end{bmatrix}\begin{bmatrix} \dot{q}_{ix} \\ \dot{q}_{iy} \end{bmatrix} = \begin{bmatrix} \tau_{ix} \\ \tau_{iy} \end{bmatrix}, \quad i = 1, 2, 3, 4,$$

where $M_{11} = a_1 + 2a_3 \cos q_{iy} + 2a_4 \sin q_{iy}$, $M_{12} = M_{21} = a_2 + a_3 \cos q_{iy} + a_4 \sin q_{iy}$, $M_{22} = a_2$, and $b = a_3 \sin q_{iy} - a_4 \cos q_{iy}$. In addition, we choose $a_1 = 8$, and $a_2 = a_3 = a_4 = 1$.



(a) The generalized coordinates of the leaders and the followers. The circles denote the leaders and the big triangle is the convex hull spanned by the leaders. The squares and the crosses denote, respectively, the generalized coordinates of the followers at, respectively, $t = 0$ s, $t = 200$ s, and $t = 300$ s. The lines between the squares and crosses are the trajectories of the followers.

(b) The generalized coordinate derivatives of the followers and the leaders.

Fig. 1. Trajectories of the leaders and the followers under control law (3) with (4), (5), (6) and (7) for the followers' dynamics (1).

The adjacency matrix $\mathcal{A}^C$ of the generalized coordinate derivatives associated with $\mathcal{G}^C$ is chosen to be

$$\mathcal{A}^C = \begin{bmatrix} 0 & 2 & 0 & 4 & 0 & 1 & 9 \\ 2 & 0 & 1 & 0 & 2 & 4 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 4 & 0 & 1 & 0 & 0 & 1 & 1.6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

For the case of the leaders' generalized coordinate derivative being constant, the initial states of the followers are chosen as $q_1(0) = [-10, 10]^T$, $q_2(0) = [8, 10]^T$, $q_3(0) = [-10, -12]^T$, $q_4(0) = [0, -20]^T$, $\dot{q}_1(0) = [-0.1, 0.1]^T$, $\dot{q}_2(0) = [0.2, -0.2]^T$, $\dot{q}_3(0) = [0.7, -0.7]^T$ and $\dot{q}_4(0) = [0.4, -0.4]^T$. The trajectory of the leaders is chosen as $q_5(0) = [6, 0]^T$, $q_6(0) = [0, -6]^T$, $q_7(0) = [0, 6]^T$, $\dot{q}_d(0) = [0.32, 0]^T$ and $\ddot{q}_d(0) = [0, 0]^T$. The adjacency matrix $\mathcal{A}^S$ of generalized coordinates associated with $\mathcal{G}^S$ can be calculated by the initial states of the followers and the leaders. The initial setup for the generalized coordinate derivative estimators (6) is chosen as $\widehat{v}_1(0) = [0.1, -0.1]^T$, $\widehat{v}_2(0) = [-0.1, 0.1]^T$, $\widehat{v}_3(0) = [0.3, -0.3]^T$ and $\widehat{v}_4(0) = [-0.2, 0.2]^T$. The control parameters are chosen as $r = 20$, $d = 1$, $d_2 = 4$ and $\delta = 0.05$ for $i = 1, 2, 3, 4$, $k_i = 1, \forall i \in F$.

Fig. 1(a) shows the generalized coordinates of the leaders and the followers under control law (3) with (4), (5), (6) and (7). It can be seen that the generalized coordinate derivatives of the followers are close to the convex hull formed by the leaders. However, there exists a non-zero bound on the distance of the followers and the convex hull formed by the leaders. Fig. 1(b) shows that the generalized coordinate derivatives of the followers converge to those of the leaders.

Fig. 2(a) shows the generalized coordinates of the leaders and the followers under control law (3) with (4), (5), (7) and (8) for the followers' dynamics (1). It can be seen that the generalized coordinate derivatives of the followers are close to the convex hull formed by the leaders even if only the sensing information is available. Fig. 2(b) shows that the generalized coordinate derivatives of the followers converge to those of the leaders.

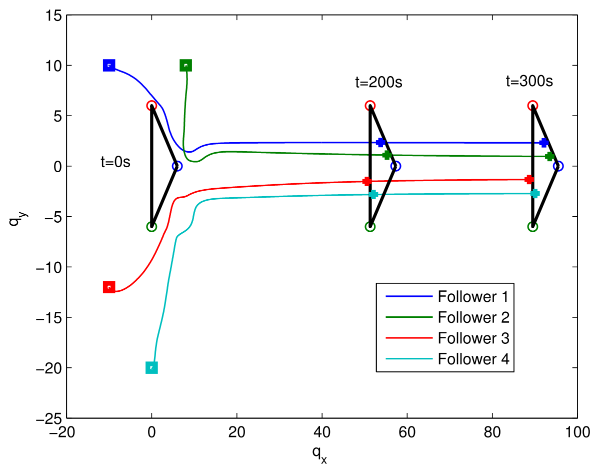
For the case of the leaders' generalized coordinate derivative being time-varying, the initial states of followers are chosen as $q_1(0) = [-10, 5]^T$, $q_2(0) = [8, 7]^T$, $q_3(0) = [-10, -10]^T$, $q_4(0) = [5, -10]^T$, $\dot{q}_1(0) = [-0.1, 0.1]^T$, $\dot{q}_2(0) = [0.2, -0.2]^T$, $\dot{q}_3(0) = [0.7, -0.7]^T$ and $\dot{q}_4(0) = [0.4, -0.4]^T$. The initial states of the leaders are chosen as $q_5(0) = [6, 0]^T$, $q_6(0) = [0, -6]^T$, $q_7(0) = [0, 6]^T$, $\dot{q}_d(0) = [0.5, -0.5]^T$, and $\ddot{q}_d(0) = [-\frac{t-40}{300}, -\frac{t-20}{300}]^T$. The adjacency matrix $\mathcal{A}^S$ of generalized coordinates associated with $\mathcal{G}^S$ can be calculated by the initial states of the followers and the leaders. The control parameters are chosen as $r = 20$, $d = 1$, $d_2 = 4$, $K = 1$, and $\rho_i = 1, \forall i \in F$.

Fig. 3(a) shows the generalized coordinates of the leaders and the followers under control law (9) with (10) and (11) for the followers' dynamics (1). It can be seen that the generalized coordinate derivatives of the followers are close to the convex hull formed by the leaders when the leaders' generalized coordinate derivatives are time-varying. Fig. 3 shows that the generalized coordinate derivatives of the followers converge to those of the leaders.

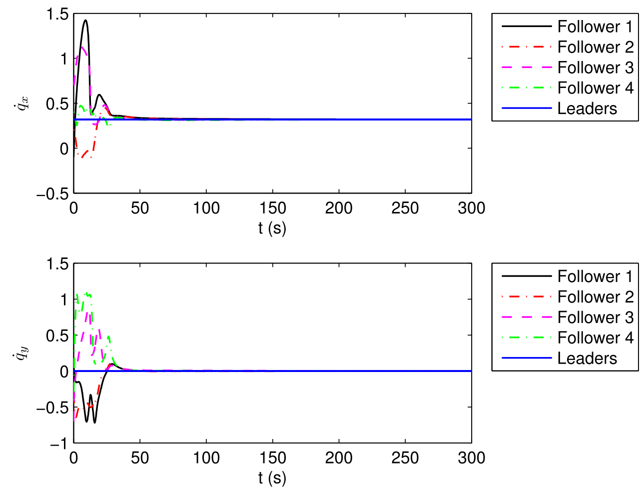
Fig. 4(a) shows the generalized coordinates of the leaders and the followers under control law (13) with (11) and (14) for the followers' dynamics (1). It can be seen that the generalized coordinate derivatives of the followers converge into the convex hull formed by the leaders. Fig. 4(b) shows that the generalized coordinate derivatives of the followers converge to those of the leaders.

6. Conclusions

In this paper, the leader–follower swarm tracking control with group dispersion and cohesion behaviors was studied for a group of Lagrange systems. Both the cases of leaders' generalized coordinate derivatives being constant and time-varying were considered. The proposed control algorithms were shown to achieve velocity matching, connectivity maintenance, collision avoidance and the followers were driven close to the cohesive inclusion formed by the leaders. In addition, the bound on the sum of the steady-state distances between the followers and the convex hull formed by the leaders was shown to be bounded and the bound was explicitly given. Numerical simulation verified these theoretical results. One interesting future research direction is the swarm tracking problem of multiple non-holonomic mobile agents.

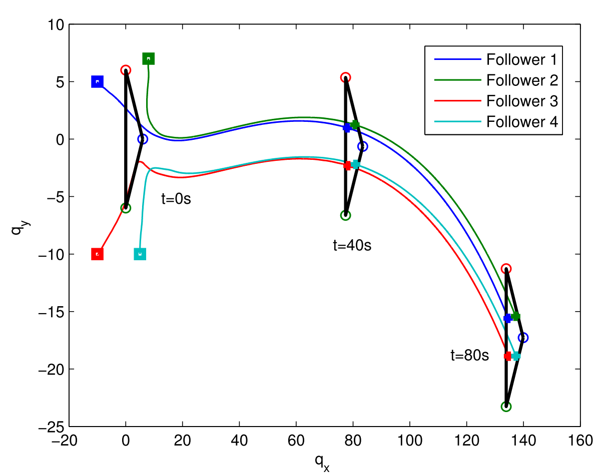


(a) The generalized coordinates of the leaders and the followers. The circles denote the leaders and the big triangle is the convex hull spanned by the leaders. The squares and the crosses denote, respectively, the generalized coordinates of the followers at, respectively, $t = 0$ s, $t = 200$ s, and $t = 300$ s. The lines between the squares and crosses are the trajectories of the followers.

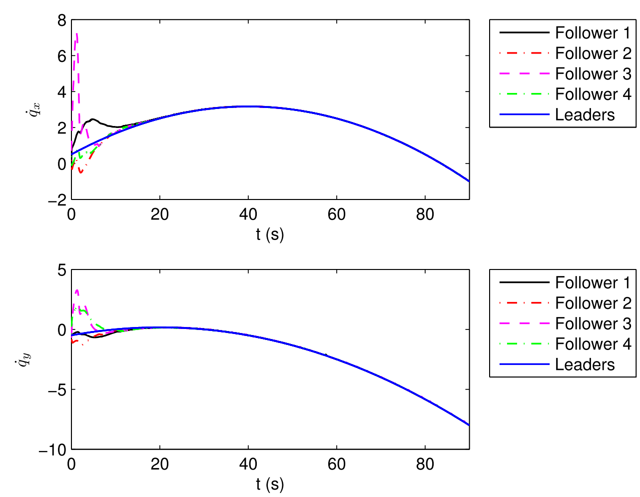


(b) The generalized coordinate derivatives of the followers and the leaders.

Fig. 2. Trajectories of the leaders and the followers under control law (3) with (4), (5), (7) and (8) for the followers' dynamics (1).



(a) The generalized coordinates of the leaders and the followers. The circles denote the leaders and the big triangle is the convex hull spanned by the leaders. The squares and the crosses denote, respectively, the generalized coordinates of the followers at, respectively, $t = 0$ s, $t = 40$ s, and $t = 60$ s. The lines between the squares and crosses are the trajectories of the followers.



(b) The generalized coordinate derivatives of the followers and the leaders.

Fig. 3. Trajectories of the leaders and the followers under control law (9) with (10) and (11) for the followers' dynamics (1).

Appendix

A.1. Potential function for group cohesion and dispersion behaviors

In this paper, V_{ij} is chosen as follows: for the case of $\|q_i - q_j\| \geq r$ when $t = 0$, V_{ij} is given by

$$V_{ij} = \begin{cases} \dfrac{(r^2 - \|q_i - q_j\|^2)^3}{(\|q_i - q_j\|^2 - d_1^2) r^4}, & d_1 < \|q_i - q_j\| \leq r, \\ 0, & \|q_i - q_j\| > r, \end{cases}$$

for which,

$$\frac{\partial V_{ij}}{\partial q_i} = \begin{cases} -\dfrac{2(r^2 - \|q_i - q_j\|^2)^2 (2\|q_i - q_j\|^2 + r^2 - 3d_1^2)}{(\|q_i - q_j\|^2 - d_1^2)^2 r^4}(q_i - q_j), \\ \quad d_1 < \|q_i - q_j\| \leq r, \\ 0, \quad \|q_i - q_j\| > r, \end{cases}$$

and

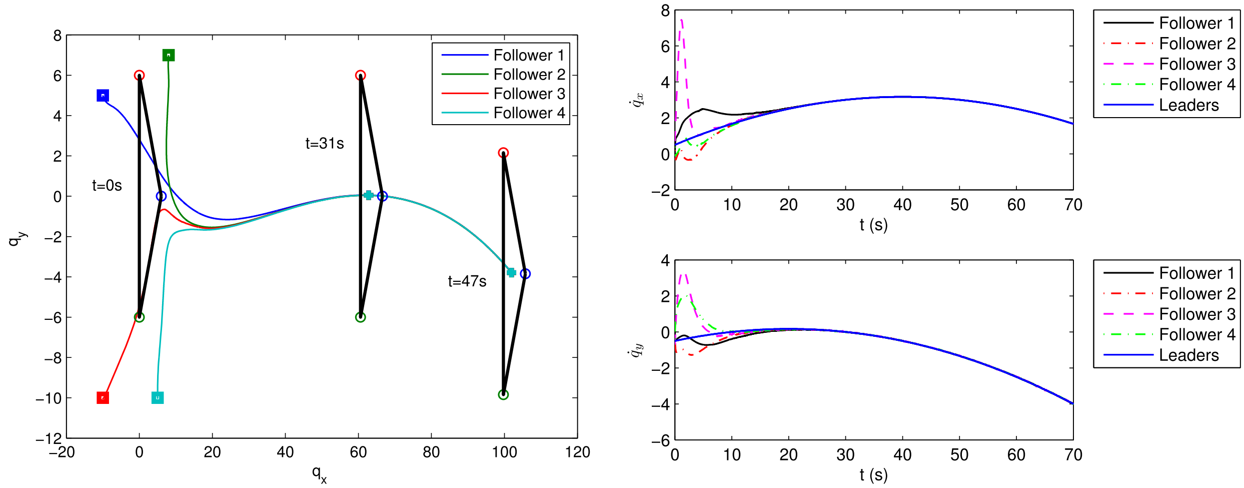
$$\frac{\partial^2 V_{ij}}{\partial q_i^2} = \begin{cases} -2(r^2 - \|q_i - q_j\|^2)\left[(2\|q_i - q_j\|^2 + r^2 - 3d_1^2)\right. \\ \quad \times(-\|q_i - q_j\|^4 - 3\|q_i - q_j\|^2 r^2 + 5\|q_i - q_j\|^2 d_1^2 \\ \quad - d_1^2 r^2) + 4(\|q_i - q_j\|^2 - d_1^2)(r^2 - \|q_i - q_j\|^2) \\ \quad \left.\|q_i - q_j\|^2\right] / [(\|q_i - q_j\|^2 - d_1^2)^3 r^4], \\ \quad d_1 < \|q_i - q_j\| \leq r, \\ 0, \quad \|q_i - q_j\| > r. \end{cases}$$

For the case of $d_1 < \|q_i - q_j\| < r$ when $t = 0$, V_{ij} is given by

$$V_{ij} = \frac{1}{2(r^2 - \|q_i - q_j\|^2)} + \frac{d_2^2}{2} \frac{1}{\|q_i - q_j\|^2 - d_1^2},$$

for which,

$$\frac{\partial V_{ij}}{\partial q_i} = \left(\frac{1}{(r^2 - \|q_i - q_j\|^2)^2} - \frac{d_2^2}{(\|q_i - q_j\|^2 - d_1^2)^2} \right)(q_i - q_j),$$



(a) The generalized coordinates of the leaders and the followers. The circles denote the leaders and the big triangle is the convex hull spanned by the leaders. The squares and the crosses denote, respectively, the generalized coordinates of the followers at, respectively, $t = 0$ s, $t = 31$ s, and $t = 47$ s. The lines between the squares and crosses are the trajectories of the followers.

(b) The generalized coordinate derivatives of the followers and the leaders.

Fig. 4. Trajectories of the leaders and the followers under control law (13) with (11) and (14) for the followers' dynamics (1).

and

$$\frac{\partial^2 V_{ij}}{\partial q_i^2} = \frac{r^2 + 3\|q_i - q_j\|^2}{(r^2 - \|q_i - q_j\|^2)^3} + d_2^2 \frac{d_1^2 + 3\|q_i - q_j\|^2}{(\|q_i - q_j\|^2 - d_1^2)^3}.$$

We also assume that $\|q_i - q_j\| > d_1$ when $t = 0$. It will be shown in Appendix A.3 that $\|q_i(t) - q_j(t)\| > d_1$ for $t \geq 0, \forall i, j \in \mathcal{V}, i \neq j$, if $\|q_i(0) - q_j(0)\| > d_1$ by using the proposed control laws. Note that V_{ij} achieves its local minimum when $\|q_i - q_j\| = \sqrt{\frac{d_2 R^2 + d_1^2}{1 + d_2}}$, where d_2 is used to adjust the minimum value of V_{ij}. Also note that $\frac{d}{dt}\left(\sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i}\right) = \sum_{j=1}^{n+m} (\dot{q}_i - \dot{q}_j) \frac{\partial^2 V_{ij}}{\partial q_i^2}$.

A.2. The adjacency matrix for the sensing graph

In order to design a smooth control law, we give a proper definition for the adjacency matrix $\mathcal{A}^S = [a_{ij}(q)]$ associated with $\mathcal{G}^S$, where $q = [q_1^T, q_2^T, \ldots, q_{n+m}^T]^T \in \mathbb{R}^{p(n+m)}$. Let

$$Q_{ij} = \begin{cases} \frac{(r^2 - \|q_i - q_j\|^2)^3}{6r^4}, & 0 < \|q_i - q_j\| \leq r, \\ 0, \ \|q_i - q_j\| > r. \end{cases}$$

a_{ij} is defined as

$$a_{ij}(q) = \begin{cases} \frac{(r^2 - \|q_i - q_j\|^2)^2}{r^4}, & 0 < \|q_i - q_j\| \leq r, \\ 0, & \|q_i - q_j\| > r. \end{cases}$$

Note that $\frac{\partial Q_{ij}}{\partial q_i} = a_{ij}(q)(q_i - q_j)$. Each element $a_{ij}(q)$ of $\mathcal{A}$ is nonnegative, differentiable and a function of $\|q_i - q_j\|$. Also note that the boundedness of Q_{ij} guarantees the boundedness $a_{ij}(q)$ and $\frac{\partial a_{ij}}{\partial q_i}$.

A.3. Proof of Theorem 3.1

Proof. 1) Connectivity maintenance analysis

Motivated by [29], [30], and [31], we construct a Lyapunov function candidate as

$$U = \frac{1}{2}\sum_{i=1}^{n} s_i^T M_i s_i + \frac{\delta}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} V_{ij} + \delta \sum_{i=1}^{n}\sum_{j=n+1}^{n+m} V_{ij} + \frac{\delta}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} + \delta \sum_{i=1}^{n}\sum_{j=1+n}^{m+n} Q_{ij} + \frac{1}{2}\sum_{i=1}^{n}\|\widehat{v}_i - \dot{q}_d\|^2 + \frac{1}{2}\sum_{i=1}^{n}\|\Delta\theta_i\|^2, \quad (15)$$

where $\triangle\theta_i$ is defined in Section 2.1. Taking the derivative of U, we have

$$\begin{aligned}
\dot{U} = {} & \sum_{i=1}^{n} s_i^T \left(Y_i \Delta\theta_i - k_i s_i - \delta \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) - \delta \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right) \\
& - \sum_{i=1}^{n} \Delta\theta_i^T Y_i^T s_i + \delta \sum_{i=1}^{n} \dot{q}_i^T \left(\sum_{j=1}^{n} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n} \frac{\partial V_{ij}}{\partial q_i} \right) \\
& + \delta \sum_{i=1}^{n} (\dot{q}_i - \dot{q}_d)^T \left(\sum_{j=1+n}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1+n}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right) \\
& + \sum_{i=1}^{n} (\widehat{v}_i - \dot{q}_d)^T \dot{\widehat{v}}_i = -\sum_{i=1}^{n} k_i s_i^2 \\
& + \sum_{i=1}^{n} \left(\dot{q}_i - \dot{q}_d + \dot{q}_d - \widehat{v}_i + \delta \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) \right. \\
& \left. + \delta \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right)^T \left(-\delta \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) - \delta \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right) \\
& + \delta \sum_{i=1}^{n} (\dot{q}_i - \dot{q}_d)^T \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right) \\
& + \sum_{i=1}^{n} (\widehat{v}_i - \dot{q}_d)^T \dot{\widehat{v}}_i = -\sum_{i=1}^{n} k_i s_i^2 - \sum_{i=1}^{n} \delta^2 \\
& \times \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right)^2 \\
& - \sum_{i=1}^{n} (\widehat{v}_i - \dot{q}_d)^T \sum_{j=1}^{n+m} b_{ij}(\widehat{v}_i - \widehat{v}_j),
\end{aligned}$$

where we have used the facts that $\frac{\partial V_{ij}}{\partial q_i} = \frac{\partial V_{ij}}{\partial (q_i - q_d)}$, $\forall i \in F, j \in L$, $\dot{q}_d \sum_{i=1}^n \sum_{j=1}^n \left(a_{ij}(q)(q_i - q_j) + \frac{\partial V_{ij}}{\partial q_i} \right) = 0$ and $\frac{\partial V_{ij}}{\partial q_i} = -\frac{\partial V_{ij}}{\partial q_j}$, $\forall i, j \in F$.

The fact that $\mathcal{G}^C$ satisfies Assumption 2.2 implies that $\lambda_{\min}(\mathcal{T}^C) > 0$, where $\mathcal{T}^C$ is as defined in Section 2.3 associated with the graph $\mathcal{G}^C$. Therefore, we have

$$\dot{U} \le -\sum_{i=1}^n k_i s_i^2 - \delta^2 \sum_{i=1}^n \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right)^2 - \lambda_{\min}(\mathcal{T}^C) \sum_{i=1}^n \|\widehat{v}_i - \dot{q}_d\|^2 \le 0.$$

This implies that $U(t)$ is bounded for $t \ge 0$ and hence $\|q_i - q_j\|$ is bounded for all $i, j \in \mathcal{V}$ and $t \ge 0$. On the other hand, the definition of V_{ij} implies that $\lim_{\|q_i - q_j\| \to r} V_{ij} = \infty$. Thus, we know that no edge will be lost at switching times, which implies that $N_i^S(0) \subseteq N_i^S(t)$ for all $i \in F$ and $t \ge 0$.

2) Velocity matching analysis

From the fact that $U(t)$ is bounded, we know that s_i, $\triangle\theta_i$, $\widehat{v}_i - \dot{q}_d$, V_{ij} and Q_{ij} are bounded. Since the boundedness of V_{ij} and Q_{ij} guarantee the boundedness $\frac{\partial V_{ij}}{\partial q_i}$ and $\frac{\partial Q_{ij}}{\partial q_i}$, we know that $\sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} + \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j)$ is bounded and further know that $\dot{q}_i - \widehat{v}_i$ is bounded in view of (7). In view of (6), it follows that $\dot{\widehat{v}}_i$ is bounded from the fact that $\dot{q}_d$ is bounded. Since the boundedness of V_{ij} and Q_{ij} also guarantees the boundedness of $\frac{\partial^2 V_{ij}}{\partial q_i^2}$ and $\frac{\partial a_{ij}}{\partial q_i}$, it is easy to show that $\frac{d}{dt}\left(\sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} + \sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) \right)$ is bounded. Thus, we know that $\dot{q}_{ri}$ is bounded and $\ddot{q}_{ri}$ is bounded. Then, from the closed-loop dynamics

$$M_i(q_i)\dot{s}_i + C_i(q_i, \dot{q}_i)s_i = Y(q_i, \dot{q}_i, \dot{q}_{ri}, \ddot{q}_{ri})\triangle\theta_i - k_i s_i - \delta \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right), \quad i \in F,$$

we know that $\dot{s}_i$ is bounded. This implies that $\ddot{U}$ is bounded. Then, by the Barbalat's lemma, we have $\dot{U} \to 0$ as $t \to \infty$. Therefore, we know that $s_i \to 0$ and $\dot{q}_i \to \widehat{v}_i \to \dot{q}_d$, $\forall i \in F$, as $t \to \infty$. This shows that the velocity matching is achieved for each follower.

3) Group dispersion analysis

Because $U(t)$ is bounded, it is easy to show that $\|q_i - q_j\|$ is bounded for all $i, j \in \mathcal{V}$ and $t \ge 0$. We also know that $\lim_{\|q_i - q_j\| \to d_1} V_{ij} = \infty$. Therefore, it follows that $\|q_i(t) - q_j(t)\| > d_1$ for all $i, j \in \mathcal{V}, i \ne j$.

4) Containment boundedness analysis

Since $s_i \to 0$ and $\dot{q}_i \to \widehat{v}_i \to \dot{q}_d$, $\forall i \in F$, as $t \to \infty$, we know that $\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} = 0$ as $t \to \infty$. Thus, we have that $\varphi_i = \inf \|q_i - y_i\|$, $\forall y_i \in \partial \text{co}\{q_j, j \in L\}$ (defined in Definition 3.1) is bounded by

$$\varphi_i \le \left\| q_i - \frac{1}{\sum_{j=1}^{n+m} a_{ij}} \sum_{j=1}^{n+m} a_{ij} q_j \right\| \le \frac{1}{\lambda_{\min}(\mathcal{T}^S)} \left\| \sum_{j=1}^{n+m} a_{ij}(q_i - q_j) \right\|$$
$$= \frac{1}{\lambda_{\min}(\mathcal{T}^S)} \left\| \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right\| \le \frac{1}{\lambda_{\min}(\mathcal{T}^S)} \sum_{j=1}^{n+m} \left\| \frac{\partial V_{ij}}{\partial q_i} \right\|,$$

where we have used the fact that $\frac{1}{\sum_{j=1}^{n+m} a_{ij}} \sum_{j=1}^{n+m} a_{ij} q_j$ is in the convex hull formed by the leaders because $-\mathcal{T}^{-1}\mathcal{T}_d \otimes I_p q_l \in \text{co}\{q_j, j \in L\}$, with $q_l = [q_{n+1}^T, q_{n+2}^T, \ldots, q_{n+m}^T]^T \in \mathbb{R}^{pm}$ (see Section 2.3). Note that the connectivity maintenance result guarantees that $\lambda_{\min}(\mathcal{T}^S) > 0$.

Consider any $i \in F$ and $j \in \mathcal{V}$. If $\|q_i(0) - q_j(0)\| > r$, we know that $V_{ij}(0) = 0$ and $\frac{\partial V_{ij}}{\partial q_i}(0) = 0$. Let $l_{1(i,j)} > 0$ be such that $\frac{(r^2 - l_1^2)^3}{(l_{1(i,j)}^2 - d_1^2)} \frac{1}{r^4} = U(0)$. Then, based on the fact that $U(t) \le U(0)$ and monotonicity of the function $\frac{(r^2 - \|q_i - q_j\|^2)^3}{\|q_i - q_j\|^2 - d_1^2} \frac{1}{r^4}$ with respect to $\|q_i - q_j\|$, we have that $\|q_i - q_j\| \ge l_{1*} = \min_{i,j}\{l_{1(i,j)}\}$. Note that $\frac{\partial V_{ij}}{\partial q_i} = 0$ for $\|q_i - q_j\| > r$. Therefore, for the case of $\|q_i(0) - q_j(0)\| > r$, we have that

$$\left\| \frac{\partial V_{ij}}{\partial q_i} \right\| \le \frac{6(r^2 - l_{1*}^2)^2 (r^2 - d_1^2)}{(l_{1*}^2 - d_1^2)^2 r^3}.$$

Similarly, if $d_1 < \|q_i(0) - q_j(0)\| \le r$, let $l_{2(i,j)} > 0$ be such that $\frac{1}{2} \frac{1}{(r^2 - l_{2(i,j)}^2) r^4} = U(0)$. Then, based on the fact that $U(t) \le U(0)$ and monotonicity of the function $\frac{1}{2} \frac{1}{(r^2 - \|q_i - q_j\|^2) r^4}$ with respect to $\|q_i - q_j\|$, we have that $\|q_i - q_j\| \le l_{2*} = \max_{i,j}\{l_{2(i,j)}\}$. Also let $l_{3(i,j)} > 0$ be such that $\frac{d_2^2}{2} \frac{1}{(l_{3(i,j)}^2 - d_1^2) r^4} = U(0)$. Then based on the fact that $U(t) \le U(0)$ and monotonicity of the function $\frac{d_2^2}{2} \frac{1}{(\|q_i - q_j\|^2 - d_1^2) r^4}$ with respect to $\|q_i - q_j\|$, we have that $\|q_i - q_j\| \ge l_{3*} = \min_{i,j}\{l_3\}$. Therefore, for the case of $d_1 < \|q_i(0) - q_j(0)\| \le r$, we have that

$$\left\| \frac{\partial V_{ij}}{\partial q_i} \right\| \le \left(\frac{1}{(r^2 - l_{2*}^2)^2} + \frac{d_2^2}{(l_{3*}^2 - d_1^2)^2} \right) \frac{l_{2*}}{r^4}.$$

In all cases, we have that

$$\left\| \frac{\partial V_{ij}}{\partial q_i} \right\| \le \alpha^* \triangleq \max \left\{ \frac{6(r^2 - l_{1*}^2)^2 (r^2 - d_1^2)}{(l_{12*}^2 - d_1^2)^2 r^3}, \left(\frac{1}{(r^2 - l_{2*}^2)^2} + \frac{d_2^2}{(l_{3*}^2 - d_1^2)^2} \right) \frac{l_{2*}}{r^4} \right\},$$

and $\varphi \le \frac{n(n+m)}{\lambda_{\min}(\mathcal{T}^S)} \alpha^*$.

A.4. Proof of Theorem 4.1

Proof. Motivated by [32], we construct a Lyapunov function candidate as

$$U = \frac{1}{2} \sum_{i=1}^n s_i^T s_i + \frac{\alpha}{2} \sum_{i=1}^n \sum_{j=1}^n V_{ij} + \alpha \sum_{i=1}^n \sum_{j=1+n}^{m+n} V_{ij} + \frac{\alpha}{2} \sum_{i=1}^n \sum_{j=1}^n Q_{ij} + \alpha \sum_{i=1}^n \sum_{j=1+n}^{m+n} Q_{ij},$$

where $0 < \alpha < 2\lambda_{\min}(\mathcal{T}^C)\sqrt{K}$. Taking the derivative of U, we have

$$\dot{U} = \sum_{i=1}^n \sum_{j=1}^n T_{ij}^C s_j^T \left(\widehat{M}_i^{-1} Y_i \triangle\theta_i - \ddot{q}_d - \rho_i \text{sgn}\left(\sum_{j=1}^n T_{ij}^C s_j \right) - \mu_i \widehat{M}_i^{-1} Y_i \text{sgn}\left(Y_i^T \widehat{M}_i^{-T} \sum_{j=1}^n T_{ij}^C s_j \right) - K s_i \right)$$
$$+ \alpha \sum_{i=1}^n (\dot{q}_i - \dot{q}_d)^T \left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right)$$
$$= -\sum_{i=1}^n (\mu_i - \|\triangle\theta_i\|_\infty) \|Y_i^T \widehat{M}_i^{-T} \sum_{j=1}^n T_{ij}^C s_j\|_1$$

$$
\begin{aligned}
&- \sum_{i=1}^{n}(\rho_i - \|\ddot{q}_d\|_\infty)\| \sum_{j=1}^{n} T_{ij}^C s_j \|_1 - K\lambda_{\min}(\mathcal{T}^C) \sum_{i=1}^{n} s_i^2 \\
&+ \alpha \sum_{i=1}^{n} (\dot{q}_i - \dot{q}_d)^T \left(s_i - \sum_{j=1}^{n+m} b_{ij}(\dot{q}_i - \dot{q}_j) \right) \\
\le & -K\lambda_{\min}(\mathcal{T}^C) \sum_{i=1}^{n} s_i^2 + \alpha \sum_{i=1}^{n} (\dot{q}_i - \dot{q}_d)^T s_i \\
&- \lambda_{\min}(\mathcal{T}^C) \sum_{i=1}^{n} (\dot{q}_i - \dot{q}_d)^2,
\end{aligned}
$$

where T_{ij}^C is the (i, j)th entry of matrix $\mathcal{T}^C$ (defined in Section 2.3) associated with $\mathcal{G}^C$, and we have used the facts that $\mu_i > \|\triangle\theta_i\|_\infty$, $\rho_i > \|\ddot{q}_d\|_\infty$, $\forall i \in F$, $\frac{\partial V_{ij}}{\partial q_i} = \frac{\partial V_{ij}}{\partial (q_i - q_d)}$, $\forall i \in F, j \in L$, $\dot{q}_d \sum_{i=1}^{n} \sum_{j=1}^{n} \left(a_{ij}(q)(q_i - q_j) + \frac{\partial V_{ij}}{\partial q_i} \right) = 0$ and $\frac{\partial V_{ij}}{\partial q_i} = -\frac{\partial V_{ij}}{\partial q_j}$, $\forall i, j \in F$.

Then, if α is selected as $0 < \alpha < 2\lambda_{\min}(\mathcal{T}^C)\sqrt{K}$, we have that $\dot{U} \le 0$. Therefore, the connectivity maintenance analysis follows from Theorem 3.1.

Similarly to the analysis given in Appendix A.3, we know that s_i, $\triangle\theta_i$, and $\frac{d}{dt}\left(\sum_{j=1}^{n+m} a_{ij}(q)(q_i - q_j) + \sum_{j=1}^{n+m} \frac{\partial V_{ij}}{\partial q_i} \right)$ are bounded. It follows from the closed-loop dynamics (12) that $\ddot{q}_i$ is bounded. This implies that $\ddot{U}$ is bounded when $\sum_{j=1}^{n} T_{ij}^C s_j \neq 0, \forall i \in F$. It is easy to show that $\ddot{U}$ is also bounded when $\sum_{j=1}^{n} T_{ij}^C s_j = 0, i \in F$. Then, by the Barbalat's lemma, we have $\dot{U} \to 0$ as $t \to \infty$. Therefore, it follows that $s_i \to 0$ and $\dot{q}_i \to \dot{q}_d$, $\forall i \in F$, as $t \to \infty$. Then, the velocity matching analysis, group dispersion analysis and containment boundedness analysis all follow from the proof of Theorem 3.1.

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