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Distributed attitude alignment in spacecraft formation flying

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SUMMARY

In this paper, we consider a distributed attitude alignment problem for a team of deep space formation flying spacecraft through local information exchange. We propose control laws for three different cases. In the first case, multiple spacecraft converge to their (possibly time-varying) desired attitudes while maintaining the same attitude or given relative attitudes during formation maneuvers under an undirected communication graph. In the second case, multiple spacecraft converge to the same rotation rate while aligning their attitudes during formation maneuvers under an undirected communication graph. In the third case, attitude alignment under a directed information exchange graph is addressed. Simulation results for attitude alignment among six spacecraft demonstrate the effectiveness of our approach. Copyright © 2006 John Wiley & Sons, Ltd.

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KEY WORDS: attitude alignment; spacecraft formation flying; cooperative control; multi-vehicle systems

1. INTRODUCTION

Autonomous vehicle systems are expected to find potential applications in military operations, search and rescue, environment monitoring, commercial cleaning, material handling, and homeland security. While single vehicles performing solo missions will yield some benefits, greater benefits will come from the cooperation of teams of vehicles. One motivation for multi-vehicle systems is to achieve the same gains for mechanically controlled systems as has been gained in distributed computation. Rather than having a single monolithic (and therefore expensive and complicated) machine do everything, the hope is that many inexpensive, simple machines, can achieve the same or enhanced functionality, through cooperation.

Advances in networking and distributed computing make possible numerous applications for multi-vehicle systems. One example is space-based interferometry, where a formation of networked spacecraft could be used to synthesize a space-based interferometer with

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base-lines reaching tens to hundreds of kilometers as an alternative to traditional monolithic spacecraft. The research of spacecraft formation flying has received significant attention in the literature. Recent results have been reported in References [1–8], to name a few.

In interferometry applications, it is essential that spacecraft maintain relative or the same attitudes during formation maneuvers. In the following we use the term *attitude alignment* to refer to the case that multiple spacecraft maintain the same relative attitudes to their reference frames, respectively. Leader–follower and behavioural approaches are two techniques to tackle the attitude alignment problem. In the leader–follower approach [1, 2] each follower in the team simply tracks the attitude of a designated leader. This approach has the advantage that the formation flying problem reduces to well-studied tracking problems. However, no information feedback is introduced from the followers to the leader and the leader becomes a single point of failure. As a comparison, in the behavioural approach [7, 8], the control torque for each spacecraft is a function of the attitudes and angular velocities of two adjacent neighbours. As a result, group feedback is introduced in the team through coupled dynamics between spacecraft.

Related to the behavioural approach [7, 8] are the consensus type problems in cooperative control of mobile autonomous agents, where each agent in a team updates its information state based on the information states of its local neighbours in such a way that the final information state of each agent converges to a common value. The research on consensus algorithms has been reported in References [9–13], to name a few. Those algorithms take the form of single integrator dynamics. Extensions to double integrator dynamics are discussed in References [14–18].

The main contribution of this paper is to extend the previous synchronized spacecraft rotation results reported in References [7, 8] to a more general scenario. Rather than requiring a restricted bidirectional ring communication graph, we show that under certain conditions multiple networked spacecraft can approach their (possibly time-varying) desired attitudes while maintaining the same attitude or given relative attitudes during formation maneuvers as long as the undirected communication graph is connected (Theorem 3.1, Corollaries 3.2, 3.3). As a result, there is no need for each spacecraft to explicitly identify its two adjacent neighbours in the team to form a bidirectional ring communication graph. In addition, we generalize the synchronized rotation results in References [7, 8] to the case that attitude is aligned with non-zero final angular velocities (Theorem 3.4). This may be appropriate for applications where multiple spacecraft are required to rotate at the same rate and maintain the same attitude simultaneously. Furthermore, attitude alignment with a unidirectional information exchange graph is also discussed (Theorem 3.5). It is worthwhile to mention that although we use PD-like control laws for attitude alignment, existing formation control results developed for double integrator dynamics are not directly applicable to spacecraft attitude dynamics due to the inherent non-linearity in quaternion kinematics. The extension from double integrator dynamics to spacecraft attitude dynamics is non-trivial.

The remainder of this paper is organized as follows. In Section 2 we introduce background information and preliminary notations. In Section 3 we propose distributed control laws for attitude alignment among multiple networked spacecraft. In Section 4 we show simulation results for attitude alignment among six spacecraft using the control laws proposed in Section 3. Section 5 contains our conclusion.

2. BACKGROUND AND PRELIMINARIES

2.1. Matrix theory

Let $\mathbf{1}$ denote the $n \times 1$ column vector of all ones. Let I_n denote the $n \times n$ identity matrix. Given a real scalar γ, we use $\gamma > 0$ to denote that γ is positive. Given an $n \times n$ real matrix P, we use $P > 0$ to denote that matrix P is symmetric positive definite. In the following, a lower case symbol denotes a scalar or vector while an upper case symbol denotes a matrix.

2.2. Spacecraft attitude dynamics

We use unit quaternions to represent spacecraft attitudes in this paper. A unit quaternion is defined as $q = [\hat{q}^{\mathrm{T}}, \bar{q}]^{\mathrm{T}} \in \mathbb{R}^4$, where $\hat{q} = a \cdot \sin(\phi/2) \in \mathbb{R}^3$ denotes the vector part and $\bar{q} = \cos(\phi/2) \in \mathbb{R}$ denotes the scalar part of the unit quaternion. In this notation, $a \in \mathbb{R}^3$ is a unit vector, known as the Euler axis, and $\phi \in \mathbb{R}$ is the rotation angle about a, called the Euler angle. Note that $q^{\mathrm{T}}q = 1$ by definition. A unit quaternion is not unique since q and $-q$ represent the same attitude. However, uniqueness can be achieved by restricting ϕ to the range $0 \leqslant \phi \leqslant \pi$ so that $\bar{q} \geqslant 0$ [19].

The product of two unit quaternions p and q is defined by

$$qp = \begin{bmatrix} \bar{q}\hat{p} + \bar{p}\hat{q} + \hat{q} \times \hat{p} \\ \bar{q}\bar{p} - \hat{q}^{\mathrm{T}}\hat{p} \end{bmatrix}$$

which is also a unit quaternion. The conjugate of the unit quaternion q is defined by $q^* = [-\hat{q}^{\mathrm{T}}, \bar{q}]^{\mathrm{T}}$. The conjugate of qp is given by $(qp)^* = p^* q^*$. The multiplicative identity quaternion is denoted by $\mathbf{q}_I = [0, 0, 0, 1]^{\mathrm{T}}$, where $qq^* = q^*q = \mathbf{q}_I$ and $q\mathbf{q}_I = \mathbf{q}_I q = q$ [19].

Spacecraft attitude dynamics are given by

$$\begin{aligned} \dot{\widehat{q_i}} &= -\tfrac{1}{2}\omega_i \times \widehat{q_i} + \tfrac{1}{2}\overline{q_i}\omega_i, \quad \dot{\overline{q_i}} = -\tfrac{1}{2}\omega_i \cdot \widehat{q_i} \\ J_i\dot{\omega}_i &= -\omega_i \times (J_i\omega_i) + \tau_i, \quad i = 1, \ldots, n \end{aligned} \tag{1}$$

where n is the total number of spacecraft in the team, $\widehat{q_i} \in \mathbb{R}^3$ and $\overline{q_i} \in \mathbb{R}$ are vector and scalar parts of the unit quaternion of the ith spacecraft, $\omega_i \in \mathbb{R}^3$ is the angular velocity, and $J_i \in \mathbb{R}^{3\times 3}$ and $\tau_i \in \mathbb{R}^3$ are inertia tensor and control torque [19].

2.3. Graph theory

It is natural to model information exchange between spacecraft by directed/undirected graphs. A digraph (directed graph) consists of a pair $(\mathcal{N}, \mathcal{E})$, where $\mathcal{N}$ is a finite non-empty set of nodes and $\mathcal{E} \in \mathcal{N}^2$ is a set of ordered pairs of nodes, called edges. As a comparison, the pairs of nodes in an undirected graph are unordered. If there is a directed edge from node v_i to node v_j, then v_i is defined as the parent node and v_j is defined as the child node. A directed path is a sequence of ordered edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots$, where $v_{i_j} \in \mathcal{N}$, in a digraph. An undirected path in an undirected graph is defined accordingly. A digraph is called strongly connected if there is a directed path from every node to every other node. An undirected graph is called connected if there is a path between any distinct pair of nodes. A directed tree is a digraph, where every node, except the root, has exactly one parent. A directed spanning tree of a digraph is a directed tree formed by graph edges that connect all the nodes of the graph. We say that a digraph has (or contains) a directed spanning tree if there exists a directed spanning tree being a subset of the

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graph. In the case of undirected graphs, a tree is a graph in which any two nodes are connected by exactly one path. Note that the condition that a digraph has a directed spanning tree is equivalent to the case that there exists at least one node having a directed path to all the other nodes. In the case of undirected graphs, having an undirected spanning tree is equivalent to being connected. However, in the case of directed graphs, having a directed spanning tree is a weaker condition than being strongly connected.

The adjacency matrix $G = [g_{ij}] \in \mathbb{R}^{n\times n}$ of a graph is defined as $g_{ii} = 0$ and $g_{ij} = 1$ if $(j, i) \in \mathscr{E}$ where $i \neq j$. For a weighted graph, G is defined as $g_{ii} = 0$ and $g_{ij} > 0$ if $(j, i) \in \mathscr{E}$ where $i \neq j$. Note that the adjacency matrix of an undirected graph is symmetric since $(j, i) \in \mathscr{E}$ implies $(i, j) \in \mathscr{E}$. However, the adjacency matrix of a digraph does not have this property. Let matrix $L = [\ell_{ij}] \in \mathbb{R}^{n\times n}$ be defined as $\ell_{ii} = \sum_{j\neq i} g_{ij}$ and $\ell_{ij} = -g_{ij}$ where $i \neq j$. Matrix L satisfies the following conditions:

$$\ell_{ij} \leqslant 0, \quad i \neq j$$

$$\sum_{j=1}^{n} \ell_{ij} = 0, \quad i = 1, \ldots, n$$

For an undirected graph, L is called the Laplacian matrix [20], which is symmetric positive semi-definite. However, matrix L for a digraph does not have this property.‡

In the case of an undirected interaction graph, matrix L has a simple zero eigenvalue with an associated eigenvector $\mathbf{1}$ if and only if the graph is connected [20]. In the case of a directed interaction graph, matrix L has a simple zero eigenvalue with an associated eigenvector $\mathbf{1}$ if and only if the digraph has a directed spanning tree [21]. Let $x = [x_1, \ldots, x_n]^{\mathrm{T}}$, where $x_j \in \mathbb{R}$, $j = 1, \ldots, n$, and $y = [y_1^{\mathrm{T}}, \ldots, y_n^{\mathrm{T}}]^{\mathrm{T}}$, where $y_j \in \mathbb{R}^m$, $j = 1, \ldots, n$. Under the conditions of both cases, $Lx = 0$ implies that $x = \alpha\mathbf{1}$ (i.e. $x_1 = \cdots = x_n$), where $\alpha \in \mathbb{R}$, and $(L \otimes I_m)y = 0$, where $\otimes$ is the Kronecker product, implies that $y = \mathbf{1} \otimes \beta$ (i.e. $y_1 = \cdots = y_n$), where $\beta \in \mathbb{R}^m$.

The digraph of an $n \times n$ real matrix $S = [s_{ij}]$, denoted by $\Gamma(S)$, is the digraph on n nodes such that there is a directed edge in $\Gamma(S)$ from v_j to v_i if and only if $s_{ij} \neq 0$ (c.f. Reference [22]).

3. ATTITUDE ALIGNMENT IN SPACECRAFT FORMATION FLYING

In this section, we propose distributed control laws for attitude alignment among n networked spacecraft. All proposed control laws are distributed in the sense that each spacecraft in the team only exchanges information with its local neighbours. In the following we assume that all the vectors in each control law have been appropriately transformed and represented in the same co-ordinate frame.

We will consider three cases. In Case 1, multiple spacecraft converge to their (possibly time-varying) desired attitudes while maintaining the same attitude or given relative attitudes during formation maneuvers under an undirected communication graph. In Case 2, multiple spacecraft converge to the same rotation rate while aligning their attitudes during formation maneuvers under an undirected communication graph. In Case 3, attitude alignment under a directed information exchange graph is addressed.

Before moving on, we need the following lemma for our main results.

‡For a digraph, matrix L is sometimes called the digraph Laplacian or non-symmetric Laplacian in the literature.

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Lemma 3.1
If the unit quaternion and angular velocity pairs (q_k, ω_k) and (q_ℓ, ω_ℓ) satisfy the quaternion kinematics defined by the first two equations in Equation (1), then the unit quaternion and angular velocity pair $(q_\ell^* q_k, \omega_k - \omega_\ell)$ also satisfies the quaternion kinematics. In addition, if $V_q = \|q_\ell^* q_k - \mathbf{q}_I\|^2$, then $\dot{V}_q = (\omega_k - \omega_\ell)^{\mathrm{T}} \widehat{q_\ell^* q_k}$, where $\hat{p}$ denotes the vector part of quaternion p.

Proof
See Reference [23]. □

3.1. Case 1

We first show the basic results of attitude alignment with zero final angular velocities and then extend these results to other scenarios.

3.1.1. Basic results. In this section, we consider the case that multiple spacecraft align their attitudes during formation maneuvers and their angular velocities approach zero under an undirected communication graph. The control torque to the ith spacecraft is proposed as

$$\tau_i = -k_G \widehat{q^{d*} q_i} - D_{Gi}\omega_i - \sum_{j=1}^{n} g_{ij}[a_{ij}\widehat{q_j^* q_i} + b_{ij}(\omega_i - \omega_j)] \tag{2}$$

where $k_G \in \mathbb{R} \geqslant 0$, $D_{Gi} \in \mathbb{R}^{3\times 3} > 0$, $q^d \in \mathbb{R}^4$ denotes the desired constant attitude for each spacecraft, $a_{ij} = a_{ji} \in \mathbb{R} > 0$, $b_{ij} = b_{ji} \in \mathbb{R} > 0$, $g_{ii} \stackrel{\triangle}{=} 0$, and g_{ij} is 1 if spacecraft i receives information from spacecraft j and 0 otherwise. In Equation (2), k_G, D_{Gi}, a_{ij}, and b_{ij} are control gains while g_{ij} is an entry of the adjacency matrix denoting the information flow between spacecraft. Note that control law (2) is model independent (i.e. no J_i). Also note that although certain torque feedback can be chosen to linearize the last equation in Equation (1), the quaternion kinematics represented by the first two equations in Equation (1) are inherently non-linear. This feature makes the spacecraft attitude alignment problem more complicated than formation control problems for systems modelled by single or double integrator dynamics.

Compared to the control law in Reference [8], where a bidirectional ring graph is assumed, control law (2) does not require each spacecraft in the team to identify its two adjacent neighbours. Each spacecraft simply communicates with all the other spacecraft that are in its communication range.

We have the following theorem for attitude alignment among multiple networked spacecraft with control torque (2).

Theorem 3.1
Assume that the control torque is given by Equation (2) and the undirected communication graph is connected. Let $\mathscr{E}$ denote the edge set of unordered pairs of spacecraft, where an edge $(k,\ell) \in \mathscr{E}$ implies that $g_{k\ell} = g_{\ell k} = 1$.[§] Also let $|\mathscr{E}|$ denote the cardinality of $\mathscr{E}$. If $k_G > 2\sum_{j=1}^{n} g_{ij}a_{ij}$, then $q_i \to q_j \to q^d$ and $\omega_i \to \omega_j \to 0$ asymptotically, $\forall i \neq j$. If $k_G = 0$ and $|\mathscr{E}| \leqslant n$, then $q_i \to q_j$ and $\omega_i \to \omega_j \to 0$ asymptotically, $\forall i \neq j$.

[§]Note that (k,ℓ) and (ℓ,k) denote the same element in $\mathscr{E}$ in the case of undirected graphs. In the following we assume that $k < \ell$ without loss of generality.

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Proof
Consider a Lyapunov function candidate:

$$V = k_G \sum_{i=1}^{n} \|q^{d*} q_i - \mathbf{q}_I\|^2 + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}a_{ij}\|q_j^* q_i - \mathbf{q}_I\|^2 + \frac{1}{2}\sum_{i=1}^{n}(\omega_i^{\mathrm{T}} J_i \omega_i)$$

Applying Lemma 3.1, the derivative of V is

$$\dot{V} = k_G \sum_{i=1}^{n} \omega_i^{\mathrm{T}} \widehat{q^{d*} q_i} + \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}a_{ij}(\omega_i - \omega_j)^{\mathrm{T}} \widehat{q_j^* q_i} + \sum_{i=1}^{n} \omega_i^{\mathrm{T}}(\tau_i - \omega_i \times J_i\omega_i) \tag{3}$$

Note that $\omega_i^{\mathrm{T}}(\omega_i \times J_i\omega_i) = 0$ and

$$\begin{aligned}
&\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}a_{ij}(\omega_i - \omega_j)^{\mathrm{T}} \widehat{q_j^* q_i}\\
&= \frac{1}{2}\sum_{i=1}^{n}\omega_i^{\mathrm{T}}\left(\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i}\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}a_{ij}\omega_j^{\mathrm{T}}\widehat{q_j^* q_i}\\
&= \frac{1}{2}\sum_{i=1}^{n}\omega_i^{\mathrm{T}}\left(\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i}\right) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ji}a_{ji}\omega_j^{\mathrm{T}}\widehat{q_j^* q_i}\\
&= \frac{1}{2}\sum_{i=1}^{n}\omega_i^{\mathrm{T}}\left(\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i}\right) + \frac{1}{2}\sum_{j=1}^{n}\sum_{i=1}^{n} g_{ji}a_{ji}\omega_j^{\mathrm{T}}\widehat{q_i^* q_j}\\
&= \frac{1}{2}\sum_{i=1}^{n}\omega_i^{\mathrm{T}}\left(\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i}\right) + \frac{1}{2}\sum_{j=1}^{n}\omega_j^{\mathrm{T}}\left(\sum_{i=1}^{n} g_{ji}a_{ji}\widehat{q_i^* q_j}\right)\\
&= \sum_{i=1}^{n}\omega_i^{\mathrm{T}}\left(\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i}\right)
\end{aligned}$$

where we have used the fact that $g_{ij} = g_{ji}$ and $a_{ij} = a_{ji}$ to obtain the second equality, and we have switched the order of the summation signs and have used the fact that $\widehat{q_j^* q_i} = -\widehat{q_i^* q_j}$ to obtain the third equality.

As a result, Equation (3) becomes

$$\dot{V} = \sum_{i=1}^{n}\omega_i^{\mathrm{T}}\left(k_G\widehat{q^{d*} q_i} + \sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i} + \tau_i\right)$$

With control law (2), the derivative of V becomes

$$\dot{V} = -\sum_{i=1}^{n}(\omega_i^{\mathrm{T}} D_{Gi}\omega_i) - \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}b_{ij}\|\omega_i - \omega_j\|^2 \leqslant 0$$

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where we have used the fact that

$$\sum_{i=1}^{n} \omega_i^{\mathrm{T}} \sum_{j=1}^{n} g_{ij} b_{ij} (\omega_i - \omega_j) = \frac{1}{2} \sum_{i=1}^{n} \sum_{j=1}^{n} g_{ij} b_{ij} \|\omega_i - \omega_j\|^2$$

We consider the following two subcases:

Case A: $k_G > 2\sum_{j=1}^{n} g_{ij} a_{ij}$ *and* $D_{Gi} > 0$.

Let $\Omega = \{q^{d*} q_i - \mathbf{q}_I, \omega_i | \dot{V} = 0\}$. Also let $\bar{\Omega}$ be the largest invariant set in Ω. On $\bar{\Omega}$, $\dot{V} \equiv 0$, which implies that $\omega_i \equiv 0$, $i = 1, \ldots, n$. Because $\omega_i \equiv 0$, we know that

$$k_G \widehat{q^{d*} q_i} + \sum_{j=1}^{n} g_{ij} a_{ij} \widehat{q_j^* q_i} = 0, \quad i = 1, \ldots, n \tag{4}$$

from Equations (1) and (2).

Noting that $q_j^* q_i = q_j^* (q^d q^{d*}) q_i = (q_j^* q^d)(q^{d*} q_i)$, we rewrite Equation (4) as

$$\widehat{p_i^* q_i} = 0 \tag{5}$$

where

$$p_i = k_G \mathbf{q}_I + \sum_{j=1}^{n} g_{ij} a_{ij} q^{d*} q_j \tag{6}$$

Also note that Equation (5) is equivalent to

$$-\overline{q^{d*} q_i} \widehat{p}_i + \overline{p}_i \widehat{q^{d*} q_i} + \widehat{q^{d*} q_i} \times \widehat{p}_i = 0 \tag{7}$$

Motivated by Reference [7], we multiply Equation (7) by $(\widehat{q^{d*} q_i} \times \widehat{p}_i)^{\mathrm{T}}$ and get

$$\|\widehat{q^{d*} q_i} \times \widehat{p}_i\|^2 = 0 \tag{8}$$

Combining Equations (7) and (8), gives

$$-\overline{q^{d*} q_i} \widehat{p}_i + \overline{p}_i \widehat{q^{d*} q_i} = 0 \tag{9}$$

Using Equation (6), we rewrite Equation (9) as

$$\begin{aligned} &- \overline{q^{d*} q_i} \sum_{j=1}^{n} g_{ij} a_{ij} \widehat{q^{d*} q_j} \\ &+ \left(k_G + \sum_{j=1}^{n} g_{ij} a_{ij} \overline{q^{d*} q_j} \right) \widehat{q^{d*} q_i} = 0, \quad i = 1, \ldots, n \end{aligned} \tag{10}$$

Note that Equation (10) can be written in matrix form as

$$(P(t) \otimes I_3) \widehat{q}_s = 0$$

where $\otimes$ is the Kronecker product, I_3 is the 3×3 identity matrix, $\widehat{q}_s \in \mathbb{R}^{3n}$ is a column vector stack composed of $\widehat{q^{d*} q_\ell}$, $\ell = 1, \ldots, n$, and $P(t) = [p_{ij}(t)] \in \mathbb{R}^{n \times n}$ is given by $p_{ii}(t) = k_G + \sum_{j=1}^{n} g_{ij} a_{ij} \overline{q^{d*} q_j}$ and $p_{ij}(t) = -g_{ij} a_{ij} \overline{q^{d*} q_i}$.

Noting that $|\overline{q^{d*} q_j}| \leqslant 1$, $j = 1, \ldots, n$, and $k_G > 2\sum_{j=1}^{n} g_{ij} a_{ij}$, we see that $P(t)$ is strictly diagonally dominant and therefore has full rank, which in turn implies that $\widehat{q}_s = 0$. Thus, we see that $\widehat{q^{d*} q_i} = 0$, $i = 1, \ldots, n$, which implies that $q_i = q^d$.

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Therefore, by LaSalle's invariance principle $q^{d*}q_i - \mathbf{q}_I \to 0$ and $\omega_i \to \omega_j \to 0$ asymptotically. Equivalently, we know that $q_i \to q_j \to q^d$, $\forall i \neq j$, and $\omega_i \to \omega_j \to 0$, $i = 1, \ldots, n$.

Case B: $k_G = 0$, $D_{Gi} > 0$, *and* $|\mathscr{E}| \leqslant n$.

In this subcase, let $\Omega = \{(q_j^* q_i - \mathbf{q}_I, \omega_i | \dot{V} = 0\}$. Also let $\bar{\Omega}$ be the largest invariant set in Ω. On $\bar{\Omega}$, $\dot{V} \equiv 0$, which implies that $\omega_i \equiv 0$, $i = 1, \ldots, n$. Because $\omega_i \equiv 0$, we know that

$$\sum_{j=1}^{n} g_{ij} a_{ij} \widehat{q_j^* q_i} = 0, \quad i = 1, \ldots, n \tag{11}$$

from Equations (1) and (2).

Let $\widehat{q_j^* q_i}$ be a variable associated with an edge $(i,j) \in \mathscr{E}$, where $i<j$. Noting that the undirected communication graph is connected and $|\mathscr{E}| \leqslant n$, we know that $|\mathscr{E}| = n-1$ or $|\mathscr{E}| = n$, which implies that there are $n-1$ or n variables associated with edge set $\mathscr{E}$. Let $\widehat{q_u}$ be a column vector stack composed of all $\widehat{q_j^* q_i}$, $\forall (i,j) \in \mathscr{E}$, where $i<j$. By noting that $\widehat{q_i^* q_j} = -\widehat{q_j^* q_i}$, Equation (11) can be rewritten as

$$(Q \otimes I_3)\widehat{q_u} = 0 \tag{12}$$

where $Q \in \mathbb{R}^{n \times n-1}$ and $\widehat{q_u} \in \mathbb{R}^{3(n-1)}$ if $|\mathscr{E}| = n-1$, and $Q \in \mathbb{R}^{n \times n}$ and $\widehat{q_u} \in \mathbb{R}^{3n}$ if $|\mathscr{E}| = n$.

Consider a system given by $Q\tilde{x} = 0$, where $\tilde{x}$ is a column vector stack composed of $x_{ij} = x_i - x_j$, $\forall (i,j) \in \mathscr{E}$, where $i<j$ and $x_k \in \mathbb{R}$, $k = 1, \ldots, n$. Note that $Q\tilde{x} = 0$ can be written as $Lx = 0$, where $x = [x_1, \ldots, x_n]^{\mathrm{T}}$ and L is the graph Laplacian matrix. Noting that the undirected communication graph is connected, we know that $x_1 = \cdots = x_n$, which in turn implies that $\tilde{x} = 0$. As a result, we know that Q can be transformed to a row echelon form to show that $Q\tilde{x} = 0$ implies that $x_{ij} = 0$, $\forall (i,j) \in \mathscr{E}$, where $i<j$. The same transformation procedure can be used to Equation (12) to show that $(Q \otimes I_3)\widehat{q_u} = 0$ implies that $\widehat{q_j^* q_i} = 0$, $\forall (i,j) \in \mathscr{E}$, where $i<j$. Thus, we see that $q_j^* q_i = \mathbf{q}_I$, $\forall i \neq j$.

Therefore, by LaSalle's invariance principle $q_j^* q_i - \mathbf{q}_I \to 0$ and $\omega_i \to 0$ asymptotically. Equivalently, we know that $q_i \to q_j$, $\forall i \neq j$, and $\omega_i \to \omega_j \to 0$, $i = 1, \ldots, n$. □

3.1.2. Extensions. As an extension to Theorem 3.1, if it is required that the spacecraft maintain given relative attitudes during formation maneuvers, we propose the control torque to the ith spacecraft as

$$\tau_i = -k_G \widehat{q^{d*} q_i q_{\delta_i}} - D_{Gi}\omega_i - \sum_{j=1}^{n} g_{ij}[a_{ij}\widehat{(q_j q_{\delta_j})^* q_i q_{\delta_i}} + b_{ij}(\omega_i - \omega_j)] \tag{13}$$

where $q_{\delta_\ell} \in \mathbb{R}^4$, $\ell = 1, \ldots, n$, are constant quaternions defining the relative attitudes between the ℓth spacecraft and the desired attitude, and k_G, $D_{Gi}, a_{ij}, b_{ij}, g_{ij}$ are defined as in Equation (2).

Note that the ith spacecraft defines q_{δ_i}. Also note that product $q_{\delta_j} q_{\delta_i}^*$ defines the relative attitudes between the ith spacecraft and the jth spacecraft. As a result, relative attitudes between the spacecraft can be achieved by appropriately choosing q_{δ_i}, $i = 1, \ldots, n$.

Corollary 3.2

Assume that the control torque is given by Equation (13) and the undirected communication graph is connected. Let $\mathscr{E}$ be defined as in Theorem 3.1. If $k_G > 2\sum_{j=1}^{n} g_{ij} a_{ij}$, then $q_i \to q^d q_{\delta_i}^*$,

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$i = 1, \ldots, n$, and $\omega_i \to \omega_j \to 0$ asymptotically, $\forall i \neq j$. If $k_G = 0$ and $|\mathscr{E}| \leqslant n$, then $q_j^* q_i \to q_{\delta_j} q_{\delta_i}^*$ and $\omega_i \to \omega_j \to 0$ asymptotically, $\forall i \neq j$.

Proof
Replacing each q_i by $q_i q_{\delta_i}$ in the proof of Theorem 3.1, we know that $q_i q_{\delta_i} \to q_j q_{\delta_j} \to q^d$, that is, $q_i \to q^d q_{\delta_i}^*$, $i = 1, \ldots, n$, if $k_G > 2\sum_{j=1}^n g_{ij} a_{ij}$ and $q_i q_{\delta_i} \to q_j q_{\delta_j}$, that is, $q_j^* q_i \to q_{\delta_j} q_{\delta_i}^*$, if $k_G = 0$ and $|\mathscr{E}| \leqslant n$. □

As another extension to Theorem 3.1, if it is required that $q_i \to q_j \to q^d(t)$ and $\omega_i \to \omega_j \to \omega^d(t)$, $\forall i \neq j$, where $q^d(t) \in \mathbb{R}^4$ and $\omega^d(t) \in \mathbb{R}^3$ denote the desired (time-varying) attitude and angular velocity for each spacecraft, we propose the control torque to the ith spacecraft as

$$\tau_i = \omega_i \times J_i \omega_i + J_i \dot{\omega}^d - k_G \widehat{q^{d*} q_i} - D_{Gi}(\omega_i - \omega^d) - \sum_{j=1}^n g_{ij}[a_{ij} \widehat{q_j^* q_i} + b_{ij}(\omega_i - \omega_j)] \tag{14}$$

where $k_G, D_{Gi}, a_{ij}, b_{ij}, g_{ij}$ are defined as in Equation (2).

Corollary 3.3
Assume that the control torque is given by Equation (14) and the undirected communication graph is connected. Also assume that $q^d(t)$ and $\omega^d(t)$ satisfy the quaternion kinematics. Let $\mathscr{E}$ be defined as in Theorem 3.1. If $k_G > 2\sum_{j=1}^n g_{ij} a_{ij}$, then $q_i \to q_j \to q^d(t)$ and $\omega_i \to \omega_j \to \omega^d(t)$ asymptotically, $\forall i \neq j$. If $k_G = 0$ and $|\mathscr{E}| \leqslant n$, then $q_i \to q_j$ and $\omega_i \to \omega_j \to \omega^d(t)$ asymptotically, $\forall i \neq j$. □

Proof
If $k_G > 2\sum_{j=1}^n g_{ij} a_{ij}$, we let $\tilde{q}_i = q^{d*} q_i$ and $\tilde{\omega}_i = \omega_i - \omega^d$. Note that $\tilde{q}_i$ and $\tilde{\omega}_i$ also satisfy the quaternion kinematics. With $\tilde{q}_i$, $\tilde{\omega}_i$, and $\mathbf{q}_I$ playing the role of q_i, ω_i, and q^d in Equation (2), we see from the proof of Theorem 3.1 that $\tilde{q}_i \to \tilde{q}_j \to \mathbf{q}_I$ and $\tilde{\omega}_i \to \tilde{\omega}_j \to 0$, that is, $q_i \to q_j \to q^d(t)$ and $\omega_i \to \omega_j \to \omega^d(t)$ asymptotically, $\forall i \neq j$. If $k_G = 0$ and $|\mathscr{E}| \leqslant n$, we see that $\tilde{q}_i \to \tilde{q}_j$ and $\tilde{\omega}_i \to \tilde{\omega}_j \to 0$, that is, $q_i \to q_j$ and $\omega_i \to \omega_j \to \omega^d(t)$ asymptotically, $\forall i \neq j$. □

3.2. Case 2

In this section, we consider the case that multiple spacecraft align their attitudes but with (possibly) non-zero final angular velocities under an undirected communication graph. We propose the following control torque:

$$\tau_i = \omega_i \times J_i \omega_i - J_i \sum_{j=1}^n g_{ij}[a_{ij} \widehat{q_j^* q_i} + b_{ij}(\omega_i - \omega_j)] \tag{15}$$

where a_{ij}, b_{ij}, g_{ij} are defined as in Equation (2). Note that control law (15) is model dependent in the sense that J_i is required to be known.

Theorem 3.4
With control torque given by Equation (15), if the undirected communication graph is connected and $|\mathscr{E}| \leqslant n$, where $\mathscr{E}$ is defined as in Theorem 3.1, then $q_i \to q_j$, and $\omega_i \to \omega_j$ asymptotically, $\forall i \neq j$.

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Proof
Consider a Lyapunov function candidate:

$$V = \frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}a_{ij}\|q_j^* q_i - \mathbf{q}_I\|^2 + \frac{1}{2}\sum_{i=1}^{n}\omega_i^{\mathrm{T}}\omega_i$$

Following a similar procedure to that of Theorem 3.1, we obtain

$$\dot{V} = -\frac{1}{2}\sum_{i=1}^{n}\sum_{j=1}^{n} g_{ij}b_{ij}\|\omega_i - \omega_j\|^2 \leqslant 0$$

Let $\Omega = \{(q_j^* q_i - \mathbf{q}_I, \omega_i | \dot{V} = 0\}$. Also let $\bar{\Omega}$ be the largest invariant set in Ω. On $\bar{\Omega}, \dot{V} \equiv 0$, which implies that $\omega_i \equiv \omega_j$, $\forall i \neq j$, since the undirected communication graph is connected. Therefore, we see that $\dot{\omega}_i \equiv \dot{\omega}_j$. As a result, we know that $\dot{\omega} \in \mathrm{span}\{\mathbf{1} \otimes \eta\}$, where $\dot{\omega} = [\dot{\omega}_1^{\mathrm{T}}, \ldots, \dot{\omega}_n^{\mathrm{T}}]^{\mathrm{T}}$ and η is some 3×1 vector.

Because $\omega_i \equiv \omega_j$, $\forall i \neq j$, we know that

$$\dot{\omega}_i = -\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i} \quad i = 1, \ldots, n \tag{16}$$

from Equations (1) and (15). We also know that

$$\sum_{i=1}^{n} \eta^{\mathrm{T}}\dot{\omega}_i = -\sum_{i=1}^{n}\eta^{\mathrm{T}}\left(\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i}\right) = 0$$

where we have used the fact that $\widehat{q_j^* q_i} = -\widehat{q_i^* q_j}$. As a result, we see that $\dot{\omega}$ is orthogonal to $\mathrm{span}\{\mathbf{1} \otimes \eta\}$. Therefore, we conclude that $\dot{\omega} \equiv 0$. From Equation (16), we know that

$$\sum_{j=1}^{n} g_{ij}a_{ij}\widehat{q_j^* q_i} = 0, \quad i = 1, \ldots, n$$

With the assumption that the undirected communication graph is connected and $|\mathscr{E}| \leqslant n$, by following the proof for Case B in Theorem 3.1, we see that $q_i = q_j$, $\forall i \neq j$. By LaSalle's invariance principle $q_i \to q_j$ and $\omega_i \to \omega_j$ asymptotically, $\forall i \neq j$. □

3.3. Case 3

In this section, we consider the case that multiple spacecraft align their attitudes under a directed information flow graph.

Let $\mathscr{J}_i$ denote the set of spacecraft whose information is available to the ith spacecraft. Let $|\mathscr{J}_i|$ denote the cardinality of $\mathscr{J}_i$. We assume that index i is not in set $\mathscr{J}_i$. Note that $j \in \mathscr{J}_i$ does not imply $i \in \mathscr{J}_j$ under the directed information flow graph.

Suppose that $|\mathscr{J}_i| \geqslant 1$. Define

$$q_i^r = q_i\left(\prod_{j \in \mathscr{J}_i}(q_j^* q_i)\right)^*$$

and

$$\omega_i^r = \omega_i - \sum_{j \in \mathscr{J}_i}(\omega_i - \omega_j)$$

Note that $q_i^r \in \mathbb{R}^4$ and $\omega_i^r \in \mathbb{R}^3$ also satisfy the quaternion kinematics in this case. The control torque to the ith spacecraft is defined as

$$\tau_i = \omega_i \times J_i \omega_i + \frac{1}{|\mathscr{J}_i|}\left[J_i \sum_{j \in \mathscr{J}_i} \dot{\omega}_j - k_{qi}\widehat{q_i^{r*} q_i} - K_{\omega i}(\omega_i - \omega_i^r)\right] \tag{17}$$

where $k_{qi} \in \mathbb{R} > 0$ and $K_{\omega i} \in \mathbb{R}^{3\times 3} > 0$.

Theorem 3.5

With control torque (17), attitude alignment is achieved among a team of spacecraft only if the directed information flow graph contains a directed spanning tree. If the information flow graph contains a directed spanning tree, then $\widehat{q_{\pi i}} \to 0$, where $q_{\pi i} = \prod_{j \in \mathscr{J}_i}(q_j^* q_i)$, and $\omega_i \to \omega_j$ asymptotically, $\forall i \neq j$.

Proof

For the first statement, if the directed information flow graph does not contain a directed spanning tree, there exist either multiple separated groups or multiple leaders. In the former case, there is no interaction between the separated subgroups, which implies that attitude alignment cannot be achieved between these subgroups. In the later case, the attitudes of the multiple leaders are not affected by any other spacecraft in the team, which imply that attitude alignment cannot be achieved between these leaders.

For the second statement, note that with control torque (17) Equation (1) can be written as $J_i\dot{\omega}_i = J_i\dot{\omega}_i^r - k_{qi}\widehat{q_i^{r*} q_i} - K_{\omega i}(\omega_i - \omega_i^r)$, which implies that $\widehat{q_i^{r*} q_i} \to 0$ and $\omega_i \to \omega_i^r$, $i = 1, \ldots, n$, according to Reference [2]. Noting that $q_i^{r*} q_i = q_{\pi i}$, we know that $\widehat{q_{\pi i}} \to 0$. Also note that $\omega_i \to \omega_i^r$ implies that $\sum_{j \in \mathscr{J}_i}(\omega_i - \omega_j) \to 0$, $i = 1, \ldots, n$, asymptotically. Note that $\sum_{j \in \mathscr{J}_i}(\omega_i - \omega_j) \to 0$, $i = 1, \ldots, n$, can be written in matrix form as $(L_\omega \otimes I_3)\omega \to 0$, where L_ω satisfies the conditions for matrix L in Section 2.3 and $\omega = [\omega_1^{\mathrm{T}}, \ldots, \omega_n^{\mathrm{T}}]^{\mathrm{T}}$. Therefore, if the information flow graph contains a directed spanning tree, we know that $\omega_i \to \omega_j$, $\forall i \neq j$, from Section 2.3. □

Corollary 3.6

With control torque (17), if the information flow graph is a unidirectional ring, then $q_i \to q_j$ and $\omega_i \to \omega_j$ asymptotically, $\forall i \neq j$.

Proof

If the information flow graph is a unidirectional ring, we number an arbitrary spacecraft in the team as spacecraft S_1 and number the other spacecraft consecutively as spacecraft S_2 to S_n following the order of information flow. As a result, we see that $q_1^r = q_n$ and $q_j^r = q_{j-1}$, where $j = 2, \ldots, n$. With control torque (17), we see that $q_1 \to q_1^r = q_n$ and $q_j \to q_j^r = q_{j-1}, j = 2, \ldots, n$, which implies that $q_i \to q_j$ and $\omega_i \to \omega_j$, $\forall i \neq j$. □

The implication of $\widehat{q_{\pi i}} \to 0$ in the case of a general unidirectional information flow graph will be a topic of future research.

3.4. Discussion

Note that if the undirected graph is not connected, then the conclusions in Sections 3.1 and 3.2 are still valid for each connected subgroup.

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Also note that in the proof of Theorem 3.1, we do not require that $\overline{q_i(t)} > 0$ or $\overline{q^{d*}(t)q_i(t)} \geqslant 0$, $\forall t \geqslant 0$. As a comparison, the results in Reference [7] rely on a region of attraction that ensures that $\overline{q_i(0)} > 0$ implies that $\overline{q_i(t)} > 0$, $\forall t \geqslant 0$, while the results in Reference [8] rely on the assumption that $\overline{q^{d*}(t)q_i(t)} \geqslant 0$, $\forall t \geqslant 0$.

4. SIMULATION RESULTS

In this section, we simulate a scenario where six spacecraft align their attitudes through local information exchange. We will consider the three different cases discussed in Section 3. In particular, Case 1 contains two subcases denoted as Case 1-A and Case 1-B, respectively. Cases 1 and 2 correspond to an undirected communication graph shown by Figure 1 while Case 3 corresponds to a directed information flow graph shown by Figure 2. Note that Figure 2 contains a directed spanning tree.

The spacecraft specifications are shown in Table I. The control parameters and control laws used for each case are shown in Table II. In the following, we let $q^d = [0, 0, 0, 1]^T$ and choose $q_i(0) \in \mathbb{R}^4$ and $\omega_i(0) \in \mathbb{R}^3$ randomly. We also assume that the control torque of each spacecraft satisfies $|\tau_i^{(j)}| \leqslant 1$ Nm, where $j = 1, 2, 3$ denotes each component of the control torque. In the following, we use a superscript (j) to denote the jth component of a quaternion or a vector.

Figures 3 and 4 show, respectively, the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 1-A. Note that each spacecraft converges to its desired attitude while aligning their

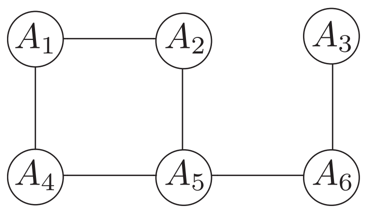

Figure 1. Communication graph for Cases 1 and 2.

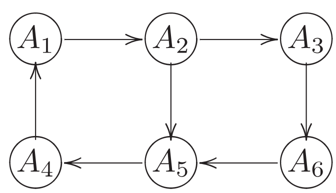

Figure 2. Information flow graph for Case 3.

Table I. Spacecraft specifications.

J_1	[1 0.1 0.1; 0.1 0.1 0.1; 0.1 0.1 0.9] kg m^2
J_2	[1.5 0.2 0.3; 0.2 0.9 0.4; 0.3 0.4 2.0] kg m^2
J_3	[0.8 0.1 0.2; 0.1 0.7 0.3; 0.2 0.3 1.1] kg m^2
J_4	[1.2 0.3 0.7; 0.3 0.9 0.2; 0.7 0.2 1.4] kg m^2
J_5	[0.9 0.15 0.3; 0.15 1.2 0.4; 0.3 0.4 1.2] kg m^2
J_6	[1.1 0.35 0.45; 0.35 1.0 0.5; 0.45 0.5 1.3] kg m^2

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Table II. Control parameters for different cases.

Case 1	Control law (2)	
	Case 1-A	$k_G = 1, D_{Gi} = 2I_3, a_{ij} = 5, b_{ij} = 10$
	Case 1-B	$k_G = 0, D_{Gi} = 2I_3, a_{ij} = 5, b_{ij} = 10$
Case 2	Control law (15)	$a_{ij} = 5, b_{ij} = 10$
Case 3	Control law (17)	$k_{qi} = 5, K_{\omega i} = 10I_3$

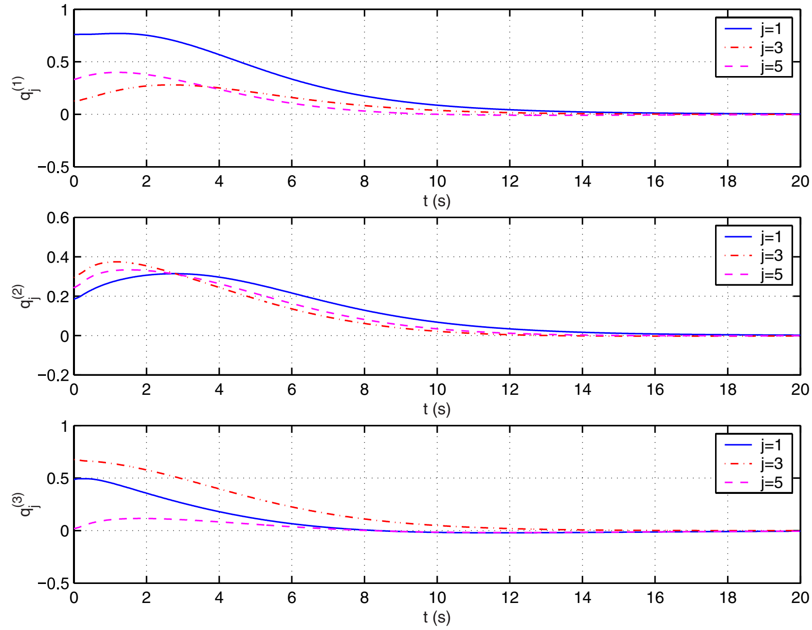

Figure 3. Spacecraft attitudes in Case 1-A.

attitudes during the transition. Also note that their angular velocities converge to zero. Figure 5 shows the control torques of spacecraft 1, 3, and 5 in Case 1-A.

Figures 6 and 7 show, respectively, the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 1-B. Note that each spacecraft converges to the same attitude and the angular velocities of each spacecraft converge to zero. Hereafter, we omit the plot for the control torques due to space limitation.

Figures 8 and 9 show, respectively, the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 2. Note that each spacecraft converges to the same attitude and the same (possibly non-zero) angular velocity.

In Case 3, with control torque (17), we see that $\widehat{q_{\pi i}} \to 0$, $i = 1, \ldots, 6$, and $\omega_i \to \omega_j$, $\forall i \neq j$. With the information flow graph given by Figure 2, it is straightforward to verify that $\widehat{q_{\pi i}} \to 0$ implies that $q_i \to q_j$, $\forall i \neq j$. Figures 10 and 11 show, respectively, the attitudes and angular velocities of spacecraft 1, 3, and 5 in Case 3. Note that each spacecraft converges to the same attitude and the same angular velocity as desired.

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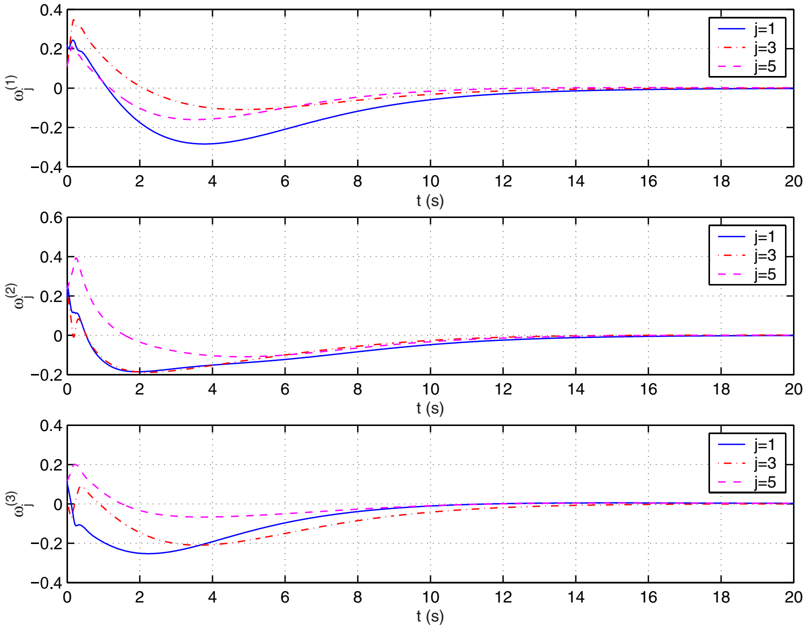

Figure 4. Spacecraft angular velocities in Case 1-A.

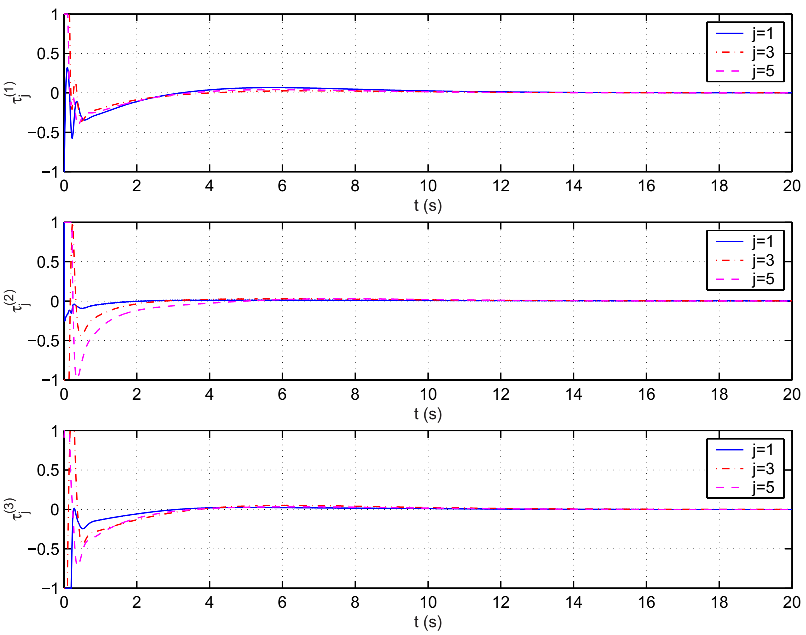

Figure 5. Spacecraft control torques in Case 1-A.

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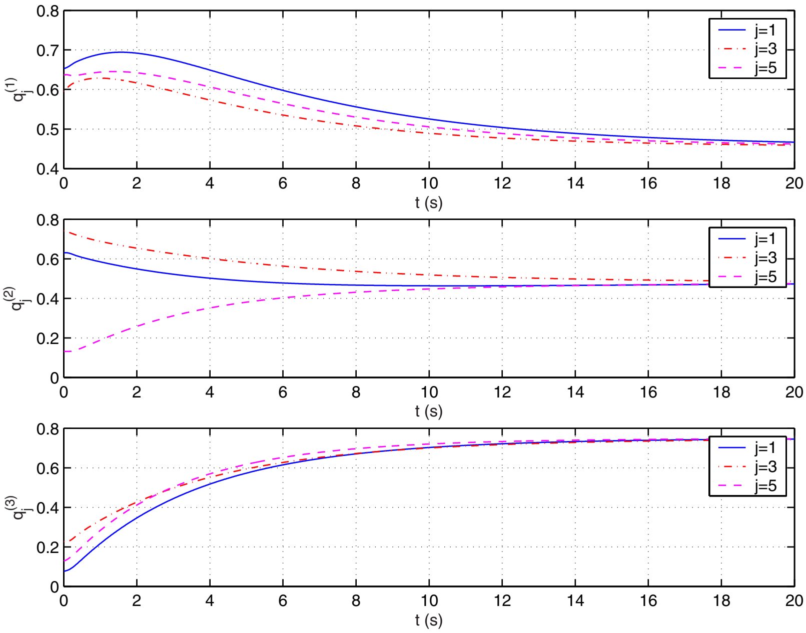

Figure 6. Spacecraft attitudes in Case 1-B.

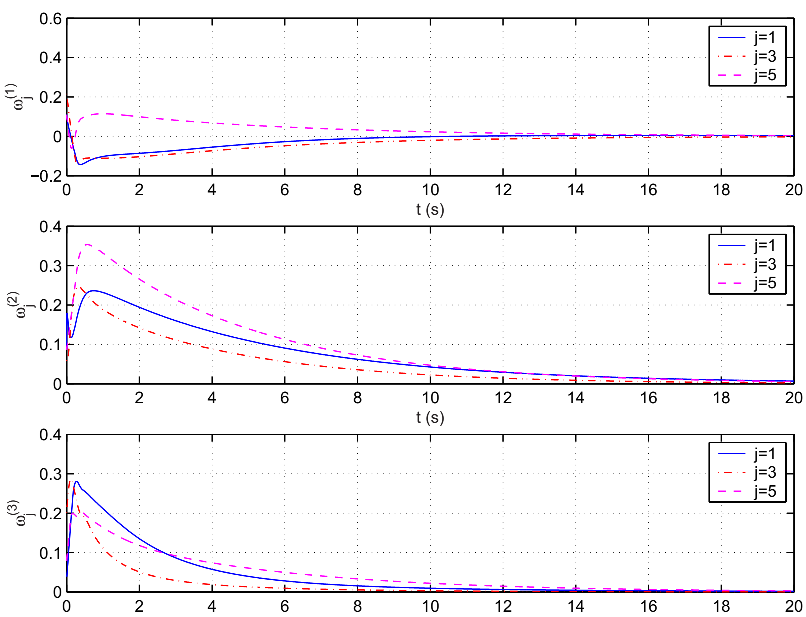

Figure 7. Spacecraft angular velocities in Case 1-B.

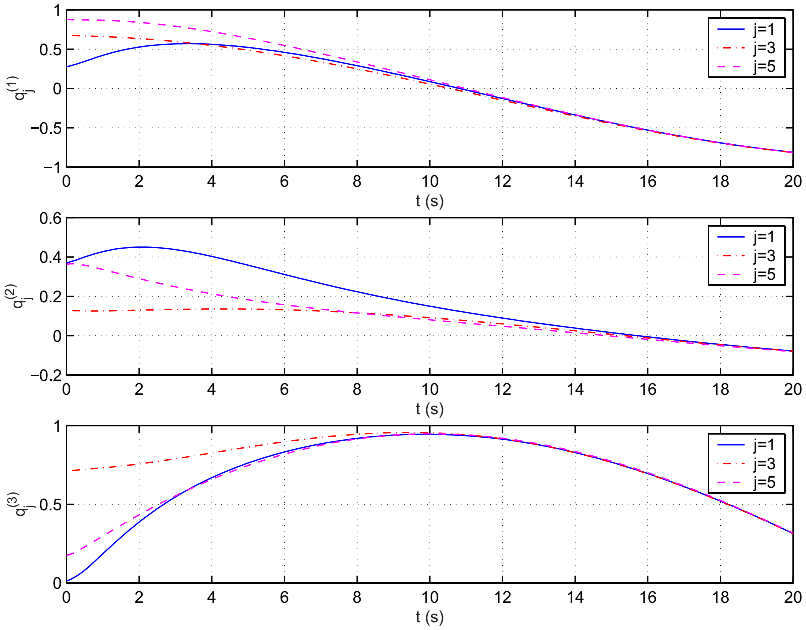

Figure 8. Spacecraft attitudes in Case 2.

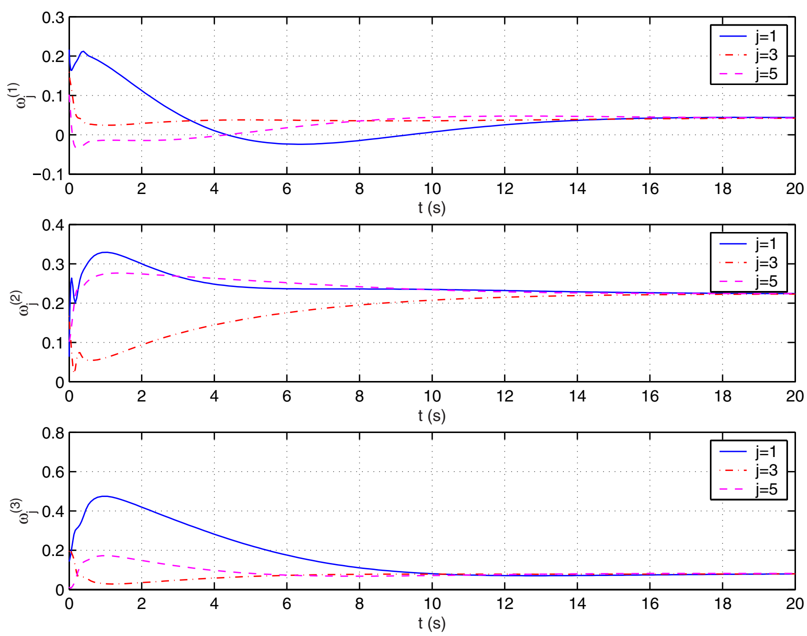

Figure 9. Spacecraft angular velocities in Case 2.

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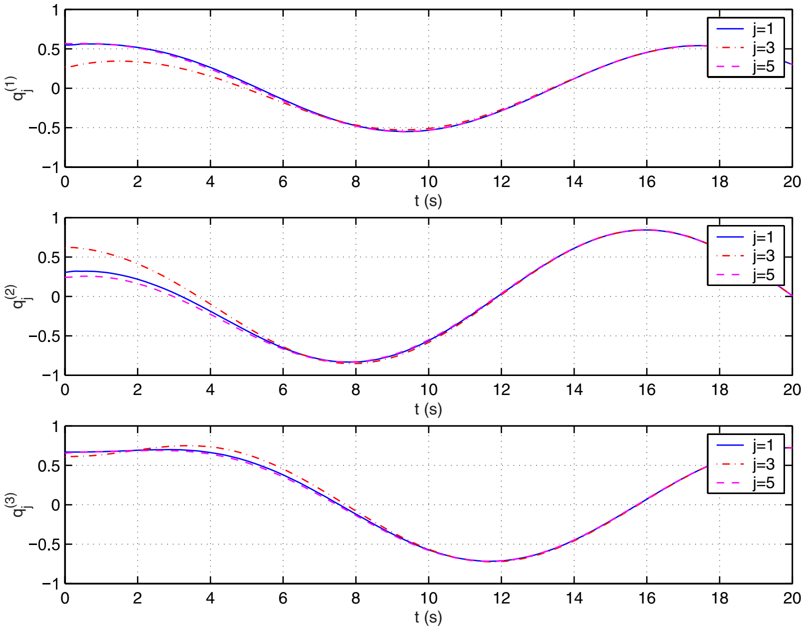

Figure 10. Spacecraft attitudes in Case 3.

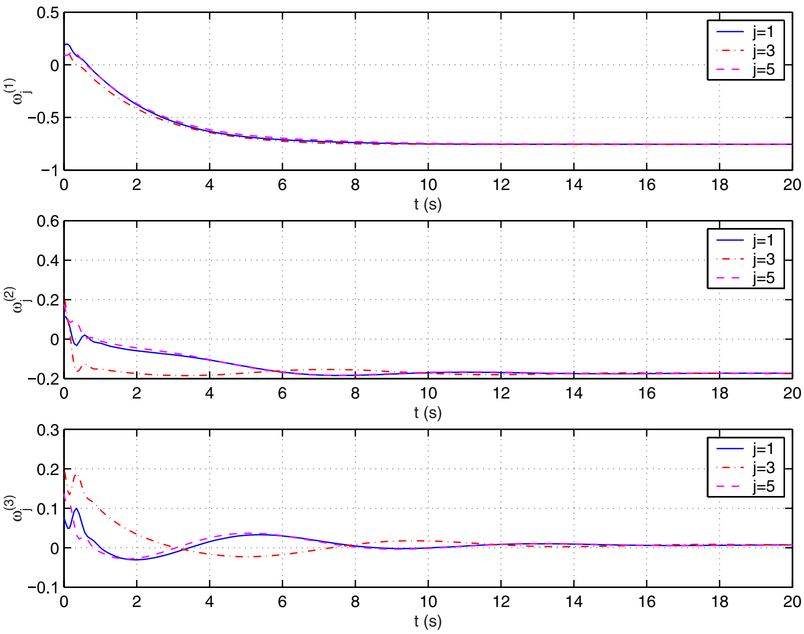

Figure 11. Spacecraft angular velocities in Case 3.

5. CONCLUSION AND FUTURE WORK

We have considered the distributed attitude alignment problem among a team of spacecraft. We have proposed control laws and shown conditions under which attitudes are aligned with zero or non-zero final angular velocities under an undirected communication graph. The case of a directed information flow graph is also discussed. Simulation results have shown a scenario where six spacecraft align their attitudes through local information exchange. Future work will address attitude alignment under a general (possibly switching) directed information flow graph. In addition, the extension of the current work to an orbital environment will also be a topic of future research.

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