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Surrounding control in cooperative agent networks☆

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ABSTRACT

In this paper, two surrounding control problems are proposed, where a team of followers is used to surround a team of leaders. The problems are solved under a decentralized estimation-and-control framework. Using tools from algebraic graph theory and dynamical systems theory, it is shown that the two teams, involving a team of leaders and a team of followers, preserve some desired convergence properties, even if the geometric center of the leaders can only be obtained from estimators. A simulation example is presented to verify the validity of the derived results.

1. Introduction

The explosion in computation and communication capabilities has made it possible to coordinate large numbers of autonomous vehicles communicating through a network to perform a variety of challenging tasks, which are beyond the ability of a single vehicle. This gives rise to a new research arena, cooperative control of multiagent systems. Recent years have witnessed an intensive and growing interest in this area. In particular, consensus [1–5], formation control [6], swarming [7], flocking problems [8], and synchronization [9] have received significant attention. For a multiagent system, regardless of the cooperative task it performs, a graph is a natural choice to describe the information flows of the system, and graph connectivity is reported to be critical to ensure system stability. The combination of algebraic graph theory and dynamical systems theory has revived a broad interest in the analysis of multiagent systems. In particular, some elegant results on the distribution of the eigenvalues of Laplacian matrices have been derived, and have been used in related areas, like synchronization of chaotic oscillators [10,11]. In this paper, motivated by the previous research on multiagent coordination, we propose a new cooperative control problem. We start describing this problem by an example.

Suppose a team of unmanned ground vehicles (UGVs) is sent to detect and establish a corridor through a hostile terrain.[1] To protect the UGVs from potential threats, another team of armed robotic vehicles (ARVs) is dispatched to provide ground coverage for the UGVs. That is, ARVs must surround the UGVs. What kind of algorithms can the ARVs use to achieve this purpose? Can they still be decentralized?

In this paper, this problem, referred to as a surrounding control problem, is formulated under a leader–follower framework, where a team of followers is used to surround a team of leaders. To design controllers for the followers, the geometric center of the leaders should be known. Generally, this piece of information cannot be obtained directly since each follower might have access to only a subset of the leaders. Therefore, a decentralized estimator is constructed at each follower to estimate the geometric center of the leaders. Then, the estimated center is used to design controllers for the followers, which results in a system with coupled estimation and control. The framework of multiagent coordination by simultaneous decentralized estimation and control is proposed in [13]. In this paper, we first assume that each follower has the knowledge of the geometric center of the leaders, and propose a control law to achieve some desired convergence properties. We then construct a distributed estimator and show that when using the estimated information, the resulting system still has the desired convergence properties if the control parameters are chosen appropriately.

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[1] This example is motivated by [12].

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The surrounding control problem can be considered an inverse containment control problem, where a group of agents are driven to be contained in a particular area specified by another group during their transportation. The containment control problem was proposed and studied in [14] under an undirected network topology and was extended to a directed network topology in [15] and to incorporate swarming behavior in [16]. As shown in [14,15], a decentralized consensus-like protocol can be used to solve the containment control problem. However, for the surrounding control problem, the situation is quite different. To solve this problem, some global information, e.g., the geometric center of the leaders, needs to be known. To keep the decentralized nature of the controller, an estimator is used in the controller, which brings some difficulties in stability analysis of the resulting system.

The surrounding control problem is also closely related to the target enclosing problem [17,18], where there is only one leader involved. In [17], the task of capturing a moving object is divided into two problems: the enclosure problem and the grasping problem. It is assumed that each robot can recognize up to two robots as its neighbors and the neighborhood is defined by the angles of the robots. A similar problem is investigated in [18] by using the cyclic pursuit strategy. Simple feedback control laws are designed to achieve the desired global behavior. It is assumed that the position information of the leader is available to all the agents. The problem of steering a group of unicycles to form a collective uniform circular motion around a fixed target with equal angular distances is studied in [19]. The problem is divided into two subproblems. One is to achieve a coordinated motion if the agents are close to the target, while the other is to navigate the agents closer to the target if they are far away from the target. In [20], the authors study a model of self-propelled particles that move at a constant speed on the surface of a sphere. Lie group representation is used to identify circular formations. Shape control laws are proposed to isolate the circular formations of the particles arranged in symmetric patterns. Note that in the surrounding control problem, multiple leaders need to be surrounded, which is more difficult than the single leader case, especially when only local information can be used. In addition, we consider more general communication topologies, rather than a special topology such as the cyclic pursuit topology. Finally, we do not assume that the position information of the leaders is available to all the followers.

The rest of the paper is organized as follows: in Section 2, the notation and terminology used throughout this paper are introduced. The surrounding control problem and the balanced surrounding control problem are formulated in Section 3. They are solved in Sections 4 and 5, respectively. A simulation example is given in Section 6. Finally, Section 7 summarizes the main conclusions.

2. Mathematical preliminaries

Let $\mathbb{R}^d$ denote the d-dimensional Euclidean space. The identity matrix is denoted by I, $\mathbf{0}$ is the vector with all zeros, and $\mathbf{1}$ is the vector with all ones. Unless otherwise stated, the norm used throughout this paper is the Euclidean norm. For a set S, $|S|$ denotes the number of elements in S. For $x \in \mathbb{R}^d$ and $S \subseteq \mathbb{R}^d$, define

$$\|x - S\| \triangleq \inf_{y \in S} \|x - y\|.$$

An undirected graph of order n is denoted by $G \triangleq (V, E)$ comprising a set $V \triangleq \{1, 2, \dots, n\}$ of nodes and a set $E \triangleq \{(i, j) | i \sim j\}$ of edges. If there is an edge from node i to node j, then node i and node j are neighbors of each other. The set of all neighbors of node i is denoted by $N_i \triangleq \{j \in V | i \sim j\}$. A path in a graph G is a sequence of nodes such that from each of its nodes there is an edge to the next node in the sequence. A graph is called connected if every pair of distinct nodes in the graph can be connected through some path. For a graph $G = (V, E)$, let $A = [a_{ij}]$ be the adjacency matrix where $a_{ij} = 1$ if $(i, j) \in E$ and $a_{ij} = 0$ otherwise. In addition, let $D \triangleq \operatorname{diag}(d_1, d_2, \dots, d_n)$ be the degree matrix with $d_i = \sum_{j \in N_i} a_{ij}$, where d_i is called the degree of node i. The Laplacian matrix of the graph G is given by $L \triangleq D - A$. Here L is symmetric positive semi-definite, and L has a simple zero eigenvalue if and only if the graph G is connected [21]. In this paper, the eigenvalues of a Laplacian matrix L is renumbered in the following way:

$$0 = \lambda_{\min}(L) \triangleq \lambda_1(L) \leq \lambda_2(L) \leq \cdots \leq \lambda_n(L) \triangleq \lambda_{\max}(L).$$

3. Problem description

Suppose that there are a number of agents which behave either as leaders or followers. The set of the leaders is denoted by V_L, while the set of the followers is denoted by V_F. Both V_L and V_F are assumed to be nonempty. In particular, let $N > 0$ be the number of the leaders, and $n > 0$ be the number of the followers. Let $x_i \in \mathbb{R}^d$ denote the position of agent i that moves in $\mathbb{R}^d$. In this paper, the leaders are assumed to be stationary. In addition, assume that the followers obey to the single-integrator dynamics, i.e.,

$$\dot{x}_i = u_i, \quad i \in V_F. \tag{1}$$

If the control input u_i makes the followers surround the leaders eventually, i.e.,

$$\lim_{t \to \infty} \|x_j(t) - co(V_F)\| = 0 \tag{2}$$

for all $j \in V_L$, where $co(V_F)$ denotes the convex hull formed by the positions of the followers, then we say that a *surrounding control problem* is solved. If an additional condition that the final configuration of all the followers forms a regular polytope centered at the geometric center of the leaders is imposed, we say that a *balanced surrounding control problem* is solved. Of course, a balanced surrounding control problem is more difficult than a surrounding control problem because it requires an additional condition on the final configuration of the followers.

We assume that each follower is equipped with a sensing device and a communication device. Let the sensing radius of each follower be r. An undirected graph $G_s \triangleq (V_F, E_s)$ is used to describe the sensing relationships between the followers, where

$$E_s \triangleq \{(i, j) | \|x_i - x_j\| < r\}.$$

Let $N_i^s \triangleq \{j \in V_F | (i, j) \in E_s\}$, which is the set of all followers who are within the sensing radius of follower i. An undirected graph $G_c \triangleq (V_F, E_c)$ is used to describe the communication relationships between the followers, where

$$E_c \triangleq \{(i, j) | \text{ followers } i, j \text{ can communicate with each other}\}.$$

In addition, let $N_i^c \triangleq \{j \in V_F | (i, j) \in E_c\}$, which is the set of all the followers who can communicate with follower i.

The position information exchange between the leaders and the followers is achieved via communication. An undirected graph $\bar{G} \triangleq (V_L \cup V_F, \bar{E})$ is used to describe the information flows between the leaders and the followers, where $(i, j) \in \bar{E}$ for $i \in V_L$ and $j \in V_F$ if and only if position information can be communicated between leader i and follower j. For leader i, let $\bar{N}_i^F$ denote the set of the followers who can communicate with leader i. For follower i, let $\bar{N}_i^L$ denote the set of the leaders who can communicate with follower i.

Before designing the controllers, four assumptions are in order.

Assumption 1. The number of the followers satisfies $n \geq d + 1$, where d is the dimension of the moving space.

Remark 1. It can be verified that Assumption 1 is also necessary for the purpose of surrounding control. For example, when $d = 1$, if $n = d$, then there is only one follower, which cannot surround multiple leaders.

Assumption 2. Initially, there are no d followers with positions x_{p_i}, $i = 1, \ldots, d$, $p_i \in V_F$, such that

$$x_i = \sum_{j=1}^{d} a_j x_{p_j}$$

for all $i \in V_F$ where $a_j \geq 0$ and $\sum_{j=1}^{d} a_j = 1$.

Remark 2. When $d = 1$, Assumption 2 means that the set of the initial positions of all the followers should not be a singleton. When $d = 2$, Assumption 2 means that, initially, the followers should not be on a line.

Assumption 3. Initially, the followers are placed within a circle with radius $\frac{r}{2}$.

Assumption 4.
- The graph G_c is assumed to be fixed and connected.
- In the graph $\bar{G}$, a leader can communicate with at least one follower initially.

4. Surrounding control problem

In this section, the surrounding control problem is studied. In the following, we first focus on the one-dimensional case ($d = 1$), and then extend the results to the higher-dimensional case ($d > 1$).

4.1. $\bar{x}$ is available

In this subsection, it is assumed that $\bar{x} \triangleq \frac{1}{n}\sum_{j\in V_L} x_j$ is available to all the followers. This assumption holds when the positions of all the leaders can be obtained by any follower. If $\bar{x}$ is available to all the followers, then the control law is designed for each follower as

$$u_i = \kappa_1 \sum_{j\in N_i^s}(x_i - x_j) + \kappa_2\left(\bar{x} + \xi \operatorname{sgn}\left\{\sum_{j\in N_i^s(0)}[x_i(0) - x_j(0)]\right\} - x_i\right) \tag{3}$$

where κ_1, κ_2, and ξ are positive constants, and $\operatorname{sgn}(\cdot)$ is the signum function.

Remark 3. In the control input, the first term is a repulsive force between follower i and its neighbors. This term is used to enlarge the convex hull $co(V_F)$ formed by the followers. The second term is an attractive force, which is used to drive the followers to $\bar{x} + \xi \operatorname{sgn}\left\{\sum_{j\in N_i^s(0)}[x_i(0) - x_j(0)]\right\}$.

It is first shown that the algorithm (3) has the ability of collision avoidance.

Lemma 1. *Suppose that Assumptions 1–3 hold for the system (1) with the control input (3). If $x_i(0) > x_j(0)$, $i, j \in V_F$, then $x_i(t) > x_j(t)$ for all $t \geq 0$.*

Proof. For notational convenience, define $\xi_i \triangleq \xi \operatorname{sgn}\{\sum_{j\in N_i^s}[x_i(0) - x_j(0)]\}$. Because Assumption 1 holds, the number of the followers satisfies $n \geq 2$. From Assumption 3, we know that the sensing graph $G_s(0)$ is a complete graph initially. In addition, due to Assumption 2, we know that if $x_i(0) > x_j(0)$, then $\xi_i > \xi_j$. Suppose that, at time t_1, x_j is still less than but sufficiently close to x_i such that $N_i^s \setminus \{j\} = N_j^s \setminus \{i\}$ and $x_i - x_j < \min_{i,j\in V_F,\xi_i\neq\xi_j}|\xi_i - \xi_j|$. Then, in the following period of time whose length is supposed to be T, one has

$$\dot{x}_i(t) - \dot{x}_j(t) = \kappa_1\sum_{k\in N_i^s}(x_i - x_k) + \kappa_2(\bar{x} + \xi_i - x_i) - \kappa_1\sum_{k\in N_j^s}(x_j - x_k) - \kappa_2(\bar{x} + \xi_j - x_j).$$

Because $N_i^s \setminus \{j\} = N_j^s \setminus \{i\}$ and $x_i > x_j$, one knows that

$$\dot{x}_i(t) - \dot{x}_j(t) \geq \kappa_2((\xi_i - \xi_j) - (x_i - x_j)).$$

Since $\xi_i - \xi_j \geq \min_{k,l\in V_F,\xi_k\neq\xi_l}|\xi_k - \xi_l|$ and $x_i - x_j < \min_{k,l\in V_F,\xi_k\neq\xi_l}|\xi_k - \xi_l|$, one has $\dot{x}_i(t) - \dot{x}_j(t) > 0$. Therefore, at time $t_1 + T$,

$$\begin{aligned} x_i(t_1 + T) - x_j(t_1 + T) &= x_i(t_1) - x_j(t_1) + \int_{t_1}^{t_1+T}(\dot{x}_i(\tau) - \dot{x}_j(\tau))d\tau \\ &> x_i(t_1) - x_j(t_1), \end{aligned}$$

which indicates that the distance between x_i and x_j will become larger. Thus x_j will never catch up with x_i, i.e. $x_j(t) < x_i(t)$ for all $t \geq 0$. □

Remark 4. This lemma indicates that if there is no collision initially, then collision avoidance between followers is guaranteed in the following time interval.

Remark 5. It can be drawn from this lemma that if $x_q(0) = \max_{j\in V_F} x_j(0)$ and $x_p(0) = \min_{j\in V_F} x_j(0)$, then $x_q(t) = \max_{j\in V_F} x_j(t)$ and $x_p(t) = \min_{j\in V_F} x_j(t)$ for all $t \geq 0$.

In the following, the equilibrium points of the system (1) is analyzed.

Lemma 2. *Suppose that Assumptions 1–3 hold. If the right hand side of (3) equals to 0 for all $i \in V_F$, then $[\bar{x} - \xi, \bar{x} + \xi] \subseteq [\min_{j\in V_F} x_j, \max_{j\in V_F} x_j]$.*

Proof. If $[\bar{x} - \xi, \bar{x} + \xi] \subseteq [\min_{j\in V_F} x_j, \max_{j\in V_F} x_j]$ does not hold, then either $\bar{x} - \xi < \min_{j\in V_F} x_j$ or $\bar{x} + \xi > \max_{j\in V_F} x_j$.

Suppose $\bar{x} - \xi < \min_{j\in V_F} x_j$. Due to Lemma 1, the set $M \triangleq \{i | i \in V_F, x_i = \min_{j\in V_F} x_j\}$ is nonempty and $M \subset V_F$. For $p \in M$, one has

$$\dot{x}_p = \kappa_1\sum_{j\in N_p^s}(x_p - x_j) + \kappa_2\left(\bar{x} + \xi \operatorname{sgn} \times \left\{\sum_{k\in N_p^s(0)}[x_p(0) - x_k(0)]\right\} - x_p\right). \tag{4}$$

Since $x_p = \min_{j\in V_F} x_j$, one knows that

$$\sum_{j\in N_p^s}(x_p - x_j) \leq 0. \tag{5}$$

Because $\xi \operatorname{sgn}\left(\sum_{k\in N_p^s(0)}(x_p(0) - x_k(0))\right) = -\xi$, one knows that

$$\bar{x} + \xi \operatorname{sgn}\left\{\sum_{k\in N_p^s(0)}[x_p(0) - x_k(0)]\right\} - x_p = \bar{x} - \xi - x_p < 0. \tag{6}$$

Then, (4)–(6) yield $\dot{x}_p < 0$, which is a contradiction.

Similarly, it can be proved that $\bar{x} + \xi > \max_{j\in V_F} x_j$ will also lead to a contradiction. □

The next lemma shows that $u_i = 0$ will be achieved asymptotically.

Lemma 3. *For the system (1) with the control input (3), if $\frac{\kappa_2}{\kappa_1} > 2(n-1)$, then $\lim_{t\to\infty} u_i = 0$ for all $i \in V_F$.*

Proof. Let $x \triangleq [x_1, \ldots, x_n]^T$. Define a Lyapunov function candidate

$$V = \frac{1}{2}\dot{x}^T\dot{x}. \tag{7}$$

Because

$$\ddot{x}_i = \dot{u}_i = \kappa_1 \sum_{j \in N_i^s} (\dot{x}_i - \dot{x}_j) - \kappa_2 \dot{x}_i,$$

one has

$$\ddot{x} = \kappa_1 L_s(t)\dot{x} - \kappa_2 \dot{x}, \tag{8}$$

with $L_s(t)$ the Laplacian matrix of the graph G_s at time t, which yields

$$\begin{aligned}\dot{V} &= \dot{x}^T \ddot{x} \\ &= \dot{x}^T (\kappa_1 L_s(t)\dot{x} - \kappa_2 \dot{x}) \\ &= \dot{x}^T (\kappa_1 L_s(t) - \kappa_2 I)\dot{x} \\ &\le \{\kappa_1 \lambda_{\max}[L_s(t)] - \kappa_2\} \|\dot{x}\|^2.\end{aligned}$$

From Gershgorin Disc Theorem, it can be proved that $\lambda_{\max}(L_s(t)) \le 2(n-1)$. If $\frac{\kappa_2}{\kappa_1} > 2(n-1)$, then $\dot{V} < 0$ for $\dot{x} \neq 0$, which leads to the result that $\lim_{t\to\infty} \dot{x}_i(t) = 0$ for all $i \in V_F$. □

Remark 6. The condition $\frac{\kappa_2}{\kappa_1} > 2(n-1)$ can be replaced with $\lambda_{\max}(L_s(t)) < \frac{\kappa_2}{\kappa_1}$. Because $\lambda_{\max}(L_s(t))$ is determined by the topology of the graph G_s, an interesting problem is to investigate the relationships between the topology of G_s and the dynamics of the system (1) with the control input (3) when κ_1 and κ_2 are both fixed. This problem becomes even more interesting when the graph G_s is generated by some complex network models, e.g., small-world model [22] or scale-free model [23]. The control of complex networks has received intensive attention from various disciplines, see, for example, [24] and [25].

By using Lemmas 2 and 3, one can prove the following result.

Theorem 1. *Suppose that Assumptions 1–3 hold for the system (1) with the control input (3). If $\frac{\kappa_2}{\kappa_1} > 2(n-1)$ and $\xi \ge \max_{i\in V_L} \|x_i - \bar{x}\|$, then the surrounding control problem is solved asymptotically.*

Proof. From Lemma 3, one knows that $\lim_{t\to\infty} u_i = 0$ for all $i \in V_F$. Because Assumptions 1–3 hold, we know that Lemma 2 holds. Then, by Lemma 2, one has

$$[\bar{x} - \xi, \bar{x} + \xi] \subseteq [\min_{j\in V_F} x_j, \max_{j\in V_F} x_j]$$

as $t \to \infty$. Because $\xi \ge \max_{i\in V_L} \|x_i - \bar{x}\|$, one knows that

$$x_i \in [\bar{x} - \xi, \bar{x} + \xi] \subseteq [\min_{j\in V_F} x_j, \max_{j\in V_F} x_j]$$

for all $i \in V_L$ as $t \to \infty$, which indicates

$$\lim_{t\to\infty} \|x_i - co(V_F)\| = 0$$

for all $i \in V_L$. □

4.2. $\bar{x}$ is not available

In this subsection, it is not assumed that $\bar{x}$ is available to all the followers. Therefore, a decentralized estimator needs to be constructed at each follower to estimate $\bar{x}$. The followers exchange their estimates via the communication graph G_c. The estimator is given as follows:

$$\dot{y}_i = \kappa \sum_{j\in N_i^c} (y_j - y_i), \tag{9}$$

where y_i is follower i's estimate of $\bar{x}$, $\kappa > 0$ is a constant, and the initial condition of y_i satisfies

$$y_i(0) = \frac{n}{N} \sum_{j\in \bar{N}_i^L(0)} \frac{1}{|\bar{N}_j^F(0)|} x_j(0) \tag{10}$$

for all $i \in V_F$. Recall from Section 3 that for leader j, $\bar{N}_j^F$ denotes the set of the followers who can communicate with leader j, while for follower i, $\bar{N}_i^L$ denotes the set of the leaders who can communicate with follower i.

Remark 7. Note that to implement the estimator, the absolute positions of the leaders at the initial time are required.

The next lemma shows that the estimator (9) is globally asymptotically stable.

Lemma 4. *Suppose that Assumption 4 holds. Using the estimator (9) with the initial conditions (10), one has $\lim_{t\to\infty} \|y_i - \bar{x}\| = 0, \forall i \in V_F$.*

Proof. Define $e_i \triangleq y_i - \bar{x}$, then

$$\dot{e}_i = \dot{y}_i = \kappa \sum_{j\in N_i^c} (e_j - e_i).$$

Let

$$e \triangleq [e_1, \ldots, e_n]^T \tag{11}$$

and $V(e) \triangleq \frac{1}{2} e^T e$. Then one has

$$\dot{V} = e^T \dot{e} = \kappa e^T (-L_c e). \tag{12}$$

Due to Assumption 4, we know that initially, a leader can communicate with at least a follower. Therefore, it can be verified that $e\mathbf{1} = \mathbf{0}$, which yields that

$$\dot{V} \le -\kappa \lambda_2(L_c) \|e\|^2. \tag{13}$$

Since the graph G_c is fixed and connected, one has $\lambda_2(L_c) > 0$ [21], which leads to $\dot{V} < 0$ for $\|e\| \neq 0$. □

Then, one can design the following new control law

$$u_i = y_i + \xi \operatorname{sgn}\left\{ \sum_{k\in N_i^s(0)} [x_i(0) - x_k(0)] \right\} - x_i, \tag{14}$$

where y_i is given by (9) and (10).

Although the estimator is globally asymptotically stable, when $\bar{x}$ is replaced with the estimator (9) with the initial conditions (10), it is not clear whether the resulting system of (1) using (14) is still stable. Therefore, stability analysis is needed. Before that, a lemma is in order.

Lemma 5. *Suppose that Assumptions 1–3 hold for the system (1) with the control input (14). Define $M \triangleq \{i \in V_F | x_i = \min_{j\in V_F} x_j(0)\}$ and $\bar{M} \triangleq \{i \in V_F | x_i = \max_{j\in V_F} x_j(0)\}$. Let $p \in M$ and $q \in \bar{M}$. If $u_i = 0$ for all $i \in V_F$, then*

$$[y_p - \xi, y_q + \xi] \subseteq [\min_{j\in V_F} x_j, \max_{j\in V_F} x_j].$$

Proof. The proof is similar to that of Lemma 2, and hence omitted here. □

Theorem 2. *Suppose that Assumptions 1–4 hold for the system (1) with the control input (14). If $\kappa < 2$, and $\xi \ge \max_{i\in V_L} \|x_i - \bar{x}\|$, then the surrounding control problem is solved asymptotically.*

Proof. Define

$$V \triangleq \frac{1}{2}\dot{x}^T \dot{x} + \frac{1}{2} a e^T e \tag{15}$$

where e is defined in (11) and $a > \frac{2(\max_{i\in V_F} d_i^c)^2}{\lambda_2(L_c)}$ with d_i^c being the degree of node i in G_c. It can be obtained that

$$\ddot{x}_i = \dot{u}_i = \kappa \sum_{j\in N_i^c} (y_j - y_i) - \dot{x}_i,$$

which can be rewritten in a matrix form as

$$\ddot{x} = \kappa(-L_c y) - \dot{x}.$$

Then one has

$$\dot{V} = \dot{x}^T \ddot{x} + a e^T \dot{e} = -\dot{x}^T \dot{x} + \kappa \dot{x}^T (-L_c y) + a e^T \dot{e}. \tag{16}$$

By using the fact that $L_c \mathbf{1} = 0$, we know that $L_c y = L_c(e + \bar{x}\mathbf{1}) = L_c e$, which yields that

$$\kappa \dot{x}^T (-L_c y) = \kappa \dot{x}^T (-L_c e).$$

According to Cauchy Schwartz inequality, one has

$$\begin{aligned} -\dot{x}^T L_c e &\le |\dot{x}^T L_c e| \\ &\le \|\dot{x}\| \, \|L_c e\| \\ &\le \frac{1}{2}(\dot{x}^T \dot{x} + (L_c e)^T (L_c e)) \\ &= \frac{1}{2}(\dot{x}^T \dot{x} + e^T L_c^2 e). \end{aligned}$$

By using the Gershgorin Disc Theorem, $\lambda_{\max}(L_c) < 2\max_{i \in V_F} d_i$. Because an eigenvalue λ of L_c corresponds to an eigenvalue λ^2 of L_c^2, one knows that

$$\lambda_{\max}(L_c^2) \le 4(\max_{i \in V_F} d_i)^2,$$

which yields

$$-\dot{x}^T L_c e \le \frac{1}{2}\dot{x}^T \dot{x} + 2(\max_{i \in V_F} d_i)^2 e^T e. \tag{17}$$

Using (13) and (17), (16) can be further reduced to

$$\begin{aligned} \dot{V} &\le -\dot{x}^T \dot{x} + \frac{1}{2}\kappa \dot{x}^T \dot{x} + 2\kappa(\max_{i \in V_F} d_i^c)^2 e^T e - a\kappa\lambda_2(L_c)\|e\|^2 \\ &\le \dot{x}^T \left(-1 + \frac{\kappa}{2}\right)\dot{x} + \left(2\kappa(\max_{i \in V_F} d_i^c)^2 - a\kappa\lambda_2(L_c)\right)\|e\|^2. \end{aligned} \tag{18}$$

Because $\kappa < 2$ and $a > \frac{2(\max_{i \in V_F} d_i^c)^2}{\lambda_2(L_c)}$, one has $\dot{V} < 0$ whenever $\|\dot{x}\| \neq 0$ or $\|e\| \neq 0$, which yields

$$\lim_{t \to \infty} \dot{x} = 0.$$

By using Assumptions 1–3, we know that Lemma 5 holds, which yields

$$[y_p - \xi, y_q + \xi] \subseteq [\min_{j \in V_F} x_j, \max_{j \in V_F} x_j],$$

as $t \to \infty$. In addition, by using Assumption 4, we know Lemma 4 hold. Then one can conclude

$$[\bar{x} - \xi, \bar{x} + \xi] \subseteq [\min_{j \in V_F} x_j, \max_{j \in V_F} x_j], \tag{19}$$

as $t \to \infty$. Because $\xi \ge \max_{i \in V_L} \|x_i - \bar{x}\|$, one knows that

$$x_i \in [\bar{x} - \xi, \bar{x} + \xi] \subseteq [\min_{j \in V_F} x_j, \max_{j \in V_F} x_j]$$

for all $i \in V_L$ as $t \to \infty$, which indicates

$$\lim_{t \to \infty} \|x_i - co(V_F)\| = 0$$

for all $i \in V_L$. □

Remark 8. When $\bar{x}$ is not available, we cannot guarantee collision avoidance due to the fact that the followers have different estimates of $\bar{x}$. However, the followers can be equipped with local collision avoidance capabilities and mechanisms. For example, sonars or infrared sensors can be installed on the followers such that collision avoidance is guaranteed when the followers are close to each other or close to a leader.

Remark 9. To solve the surrounding control problem, we need some global information, i.e., $\bar{x}$, $\max_{i \in V_L} |x_i - \bar{x}|$, and n. The geometric center of the leaders $\bar{x}$ is obtained by using a decentralized estimator. Therefore, $\max_{i \in V_L} |x_i - \bar{x}|$ can also be calculated in a decentralized way. Moreover, the number of the followers n can be obtained by using some decentralized communication protocols in computer science. In other words, the control laws that we designed are decentralized.

The results in Theorem 2 can be generalized to the case of higher dimensions, provided that some initial conditions are met.

Theorem 3. *For the case of higher dimensions, i.e., $d > 1$, suppose that Assumptions 1–4 hold for the system (1) with the control input (14), and initially $co(V_F)$ is a hyperrectangle, which is the generalization of a rectangle for higher dimensions. If $\kappa < 2$ and $\xi \ge \max_{i \in V_L} \|x_i - \bar{x}\|$, then the surrounding control problem is solved asymptotically.*

Proof. The proof is similar to that of Theorem 2. □

5. Balanced surrounding control problem

In this section, the balanced surrounding control problem is considered. That is, (2) holds and all followers converge to a configuration that forms a regular polytope with a geometric center at $\bar{x}$. Here, we assume that the geometric center of the leaders is available to all the followers. If it is not available, the estimator presented in the last section can be used, and the stability analysis can be done under the estimation-and-control framework as presented in the last section. We hence assume that all agents share a common coordinate system centered at $\bar{x}$. In this section, we only consider the two-dimensional case, but the derived results can be extended to the case of a higher dimension. To simplify the analysis, the polar coordinate system is used, where r_i and θ_i are, respectively, the radius and angle of follower i. Suppose that

$$\dot{r}_i = \eta_i \tag{20}$$

$$\dot{\theta}_i = \omega_i. \tag{21}$$

In the new coordinate system, we design η_i and ω_i. Note that, once η_i and ω_i are designed, the controllers u_i for the original system (1) can be specified as follows:

$$u_i = \begin{bmatrix} \eta_i \cos\theta_i - r_i\omega_i \sin\theta_i \\ \eta_i \sin\theta_i + r_i\omega_i \cos\theta_i \end{bmatrix}.$$

We first design ω_i. Define

$$\theta_{ij} \triangleq \min\{(\theta_i - \theta_j) \bmod 2\pi, (\theta_j - \theta_i) \bmod 2\pi\},$$

which measures the distance between θ_i and θ_j. If $\theta_{ij} < \frac{2\pi}{n}$, there are two possible cases: either $0 \le |\theta_i - \theta_j| < \frac{2\pi}{n}$ or $2\pi - \frac{2\pi}{n} < |\theta_i - \theta_j| \le 2\pi$. We use an undirected graph $G_\theta \triangleq (V_F, E_\theta)$ to denote the angle relationships between the followers, where $E_\theta \triangleq \{(i,j) | \theta_{ij} < \frac{2\pi}{n}\}$. The set of the neighbors of follower i in G_θ is denoted by N_i^θ.

The requirement that the followers form a regular polytope indicates that the angles of the followers should distribute in a balanced way, which is defined as follows.

Definition 1 (*Balanced Distribution*)**.** The angle vector $\theta \triangleq [\theta_1, \ldots, \theta_n]^T$ is said to be a balanced distribution if and only if $\theta_{ij} \bmod \frac{2\pi}{n} = 0$ and $\theta_i \neq \theta_j$ for all $i \neq j$, $i \in V_F$, and $j \in V_F$.

$$r_{ij} \triangleq \begin{cases} \dfrac{1}{a_1(\theta_i-\theta_j)^2+a_2(\theta_i-\theta_j)^4}, & 0 \le |\theta_i-\theta_j| < \dfrac{2\pi}{n}, \\ \dfrac{1}{a_1\left(\frac{2\pi}{n}\right)^2+a_2\left(\frac{2\pi}{n}\right)^4}, & \dfrac{2\pi}{n} \le |\theta_i-\theta_j| \le 2\pi-\dfrac{2\pi}{n}, \\ \dfrac{1}{a_3((2\pi)^2-(\theta_i-\theta_j)^2)+a_4((2\pi)^2-(\theta_i-\theta_j)^2)^2}, & 2\pi-\dfrac{2\pi}{n} < |\theta_i-\theta_j| \le 2\pi, \end{cases}$$

where $a_1 = -2a_2(\frac{2\pi}{n})^2$, $a_2 = \frac{((2\pi)^2-(2\pi-\frac{2\pi}{n})^2)^2}{(\frac{2\pi}{n})^4}a_4$, $a_3 = -2a_4((2\pi)^2-(2\pi-\frac{2\pi}{n})^2)$, and $a_4 < 0$.

Box I.

To design ω_i, we first define a potential function r_{ij}. Then ω_i is designed such that the angles of the followers change along the negative gradient of $\sum_{j\in N_i^\theta} r_{ij}$. The function r_{ij} should have the following properties: (i) it is a repulsive force when $0 \le |\theta_i-\theta_j| < \frac{2\pi}{n}$, and (ii) it becomes an attractive force when $2\pi-\frac{2\pi}{n} < |\theta_i-\theta_j| \le 2\pi$. In this way, a balanced distribution will be achieved (see Box I).

Lemma 6. *The following properties hold for the function r_{ij}:*

1. $r_{ij} \ge 0$,
2. $\frac{\partial r_{ij}}{\partial(\theta_i-\theta_j)^2} < 0$ *for* $0 < |\theta_i-\theta_j| < \frac{2\pi}{n}$ *and* $\frac{\partial r_{ij}}{\partial(\theta_i-\theta_j)^2} > 0$ *for* $2\pi-\frac{2\pi}{n} < |\theta_i-\theta_j| < 2\pi$,
3. r_{ij} *is continuously differentiable.*

Proof. This can be shown by some simple calculations, and is hence omitted. □

Then, the control law ω_i is defined as

$$\omega_i = -\sum_{j\in N_i^\theta}\frac{\partial r_{ij}}{\partial\theta_i}, \quad \forall i \in V_F. \tag{22}$$

To implement (22), we need the following assumption.

Assumption 5. The edge set of G_θ is a subset of the edge set of the communication graph G_c, i.e., $E_\theta \subseteq E_c$.

This assumption can be achieved by choosing appropriate initial values of $\theta(0)$. For example, we can choose $\theta(0)$ such that $G_\theta(0)$ is a subgraph of a linear cyclic pursuit graph [26]. Then let E_c be the linear pursuit graph, we can have $E_\theta(t) \subseteq E_c$ for all $t \ge 0$. Therefore, the followers can exchange their information through the communication graph G_c. Because of Assumptions 1 and 5, we know that the controller (22) can be implemented.

Lemma 7. *Suppose that Assumptions 1 and 5 hold. The set $\Theta \triangleq \{\theta | 0 < |\theta_i-\theta_j| < 2\pi\}$ is positively invariant for the system (21) using (22).*

Proof. Define

$$V \triangleq \frac{1}{2}\sum_{i\in V_F, j\in V_F, i\neq j} r_{ij}. \tag{23}$$

It can be verified that (21) using (22) becomes

$$\dot{\theta}_i = -\frac{\partial V}{\partial\theta_i}$$

which yields $\dot{V} \le 0$. By noting that $|\theta_i-\theta_j| = 0$ or $|\theta_i-\theta_j| = 2\pi$ lead to $V = \infty$ and

$$V(t) \le V(0) < \infty, \tag{24}$$

one can conclude that Θ is positively invariant. □

Theorem 4. *Suppose that Assumptions 1 and 5 hold. For the system (21) using (22) with $\theta(0) \in \Theta$, θ will achieve a balanced distribution asymptotically.*

Proof. By using Lemma 7 and LaSalle's invariant principle [27], we know that the system will converge to the largest invariant set $S \triangleq \{\theta | \dot{V}(\theta) = 0\}$. Because $\dot{V} = -\sum_i\left(\frac{\partial V}{\partial\theta_i}\right)^2 = -\sum_i \dot{\theta}_i^2$, it follows that $\dot{V} = 0$ implies that $\dot{\theta}_i = 0$. Therefore, it is straightforward to obtain that $\lim_{t\to\infty}\dot{\theta}(t) = 0$. That is, the angles of all the followers will stop changing eventually.

Renumber the followers such that

$$\theta_{p_1} \le \theta_{p_2} \le \cdots \le \theta_{p_n}.$$

For notational convenience, define $\theta_{p_{n+1}} = \theta_{p_1}$ and $\theta_{p_0} = \theta_{p_n}$. It can be verified that

$$\sum_{i\in V_F}\theta_{p_i,p_{i+1}} = 2\pi. \tag{25}$$

If θ is not a balanced distribution, one knows that the set $\bar{\Delta} \triangleq \{i \in V_F | \theta_{p_i,p_{i+1}} \ge \frac{2\pi}{n}\}$ is not empty. In addition, it can be proved that $\bar{\Delta} \cap \underline{\Delta}$ is not empty where $\underline{\Delta} \triangleq \{i \in V_F | \theta_{p_{i-1},p_i} < \frac{2\pi}{n}\}$. This is shown by contradiction. Suppose $\bar{\Delta} \cap \underline{\Delta}$ is empty, then for all $p_i \in \bar{\Delta}$, one has $\theta_{p_{i-1},p_i} \ge \frac{2\pi}{n}$, which indicates that $p_{i-1} \in \bar{\Delta}$ for all $p_i \in \bar{\Delta}$. By repeating the above statements, one can obtain $\bar{\Delta} = V_F$, which yields $\theta_{p_i,p_{i+1}} \ge \frac{2\pi}{n}$ for all $i \in V_F$. Because θ is not balanced, one knows that there is at least one follower $i \in V_F$ such that $\theta_{p_i,p_{i+1}} > \frac{2\pi}{n}$, which leads to

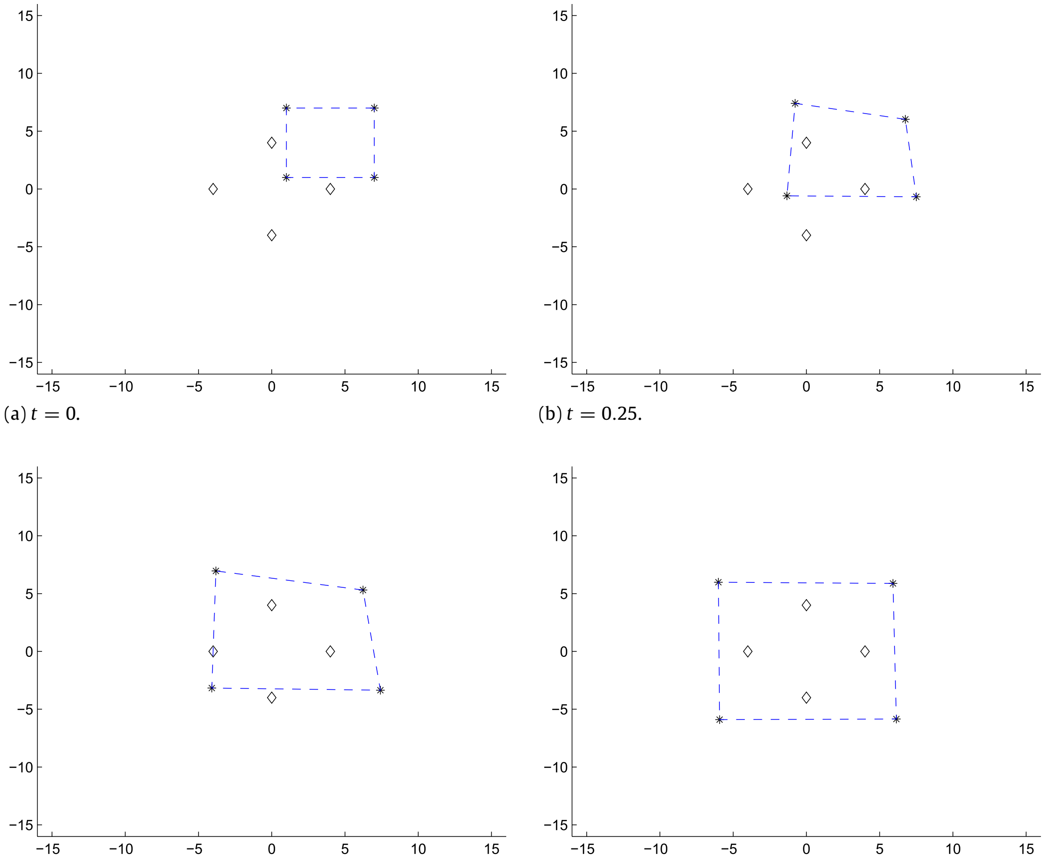
$$\sum_{i\in V_F}\theta_{p_i,p_{i+1}} > 2\pi. \tag{26}$$

This contradicts with (25). Therefore, one can conclude that $\bar{\Delta} \cap \underline{\Delta}$ is not empty.

For $p_i \in \bar{\Delta} \cap \underline{\Delta}$, one has

$$\begin{aligned}\dot{\theta}_{p_i} &= -\sum_{j\neq p_i, \theta_{p_i,j}<\frac{2\pi}{n}}\frac{\partial r_{ij}}{\partial\theta_{p_i}} \\ &= -2\sum_{j\neq p_i, \theta_{p_i,j}<\frac{2\pi}{n}}\frac{\partial r_{ij}}{\partial(\theta_{p_i}-\theta_j)^2}(\theta_{p_i}-\theta_j).\end{aligned}$$

If $j \in \{p_1, p_2, \ldots, p_{i-1}\}$, one knows $0 < |\theta_{p_i}-\theta_j| < \frac{2\pi}{n}$, which yields $\frac{\partial r_{ij}}{\partial(\theta_{p_i}-\theta_j)^2}(\theta_{p_i}-\theta_j) < 0$ by using Lemma 6. Similarly, it can be proved that $\frac{\partial r_{ij}}{\partial(\theta_{p_i}-\theta_j)^2}(\theta_{p_i}-\theta_j) < 0$ holds also for $j \notin \{p_1, p_2, \ldots, p_{i-1}\}$. Then, one can conclude $\dot{\theta}_{p_i} > 0$, which contradicts with $\dot{\theta} = 0$. □



(a) $t = 0$. (b) $t = 0.25$.

(c) $t = 1.25$. (d) $t = 5$.

Fig. 1. Surrounding control of mobile networks. The leaders are denoted by "$\diamond$", while the followers are represented by "$*$".

We next design η_i. The main purpose of η_i is to drive all agents to a circle centered at $\bar{x}$ and with a radius ξ. The controller for η_i is of the following form:

$$\eta_i = -\beta \operatorname{sgn}\left[r_i - \sqrt{\xi^2 - \bar{r}^2 \sin^2(\theta_i - \bar{\theta})} - \bar{r}\cos(\theta_i - \bar{\theta})\right], \quad (27)$$

where $\beta > 0$ is a constant, and $\bar{r}$ and $\bar{\theta}$ are, respectively, the radius and angle of the geometric center of the leaders.

From Assumptions 1 and 5, we know that Theorem 4 holds. Therefore, θ will achieve a balanced distribution asymptotically. In the following, it is shown that $\dot{\theta}_i$ is bounded for all $i \in V_F$. If $0 \le |\theta_i - \theta_j| < \frac{2\pi}{n}$, let $r_{ij}(r_a) = V(0)$, where $V(0)$ is the value of the function V, defined by (23), at time 0. In addition, one has that

$$r_{ij}(|\theta_i - \theta_j|) \le V(t) \le V(0) = r_{ij}(r_a),$$

which yields that

$$r_a \le |\theta_i - \theta_j| < \frac{2\pi}{n}.$$

It thus follows that

$$\left|\frac{\partial r_{ij}}{\partial \theta_i}\right| \le 4a_2\left(\frac{2\pi}{n}\right)\left[a_2 r_a^2\left(\frac{2\pi}{n}\right)^2\right]^{-2}\left[r_a^2 - \left(\frac{2\pi}{n}\right)^2\right]. \quad (28)$$

If $2\pi - \frac{2\pi}{n} < |\theta_i - \theta_j| \le 2\pi$, let $r_{ij}(r_b) = V(0)$. Then one has that $2\pi - \frac{2\pi}{n} < |\theta_i - \theta_j| \le r_b$, which yields that

$$\left|\frac{\partial r_{ij}}{\partial \theta_i}\right| \le 4a_4^2 r_b\left[(2\pi)^2 - r_b^2\right]\left[(2\pi)^2 - \left(2\pi - \frac{2\pi}{n}\right)^2\right] \times \left[r_b^2 - \left(2\pi - \frac{2\pi}{n}\right)^2\right]. \quad (29)$$

Define

$$r_{\max} \triangleq \max\left(4a_2\left(\frac{2\pi}{n}\right)\left[a_2 r_a^2\left(\frac{2\pi}{n}\right)^2\right]^{-2}\left[r_a^2 - \left(\frac{2\pi}{n}\right)^2\right], \times 4a_4^2 r_b[(2\pi)^2 - r_b^2]\left[(2\pi)^2 - \left(2\pi - \frac{2\pi}{n}\right)^2\right] \times \left[r_b^2 - \left(2\pi - \frac{2\pi}{n}\right)^2\right]\right), \quad (30)$$

one has that

$$|\dot{\theta}_i| \le (n-1) r_{\max}. \quad (31)$$

Theorem 5. *Suppose that Assumptions 1 and 5 hold. For the system (21) using (22) and (20) using (27), if $\beta > \bar{r}(n-1)r_{\max}(\bar{r}(\xi^2 - \bar{r}^2)^{-\frac{1}{2}} + 1)$, $\xi > \max(\bar{r}, \max_{i \in V_L} \|x_i - \bar{x}\| / \cos(\pi/n))$, and $\theta(0) \in \Theta$, then the balanced surrounding control problem is solved asymptotically. Here $r_{\max}$ is a constant defined by (30).*

Proof. Define

$$V_i = \frac{1}{2}\left[r_i - \sqrt{\xi^2 - \bar{r}^2 \sin^2(\theta_i - \bar{\theta})} - \bar{r}\cos(\theta_i - \bar{\theta})\right]^2.$$

Then one has

$$\dot{V}_i \le \left|r_i - \sqrt{\xi^2 - \bar{r}^2 \sin^2(\theta_i - \bar{\theta})} - \bar{r}\cos(\theta_i - \bar{\theta})\right| \times \left\{\bar{r}|\dot{\theta}_i|[\bar{r}(\xi^2 - \bar{r}^2)^{-\frac{1}{2}} + 1] - \beta\right\}.$$

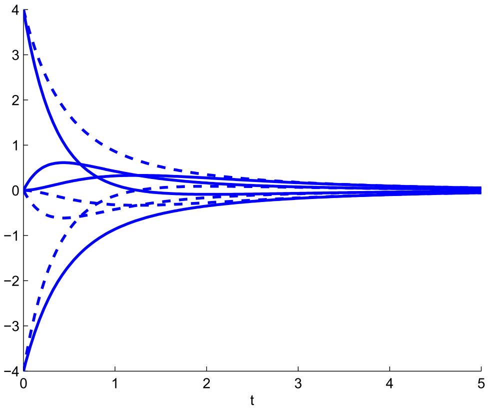


Fig. 2. The estimates of the geometric center of the leaders.

Because $\beta > \bar{r}(n-1)r_{\max}(\bar{r}(\xi^2-\bar{r}^2)^{-\frac{1}{2}}+1)$, one knows $\dot{V}_i < 0$. Therefore, the system will converge to

$$\lim_{t\to\infty} r_i(t) = \sqrt{\xi^2 - \bar{r}^2\sin^2(\theta_i-\bar{\theta})} - \bar{r}\cos(\theta_i-\bar{\theta}). \quad (32)$$

Thus, as $t\to\infty$,

$$(r_i - \bar{r}\cos(\theta_i-\bar{\theta}))^2 \to \xi^2 - \bar{r}^2\sin^2(\theta_i-\bar{\theta}),$$

which yields

$$r_i^2 + \bar{r}^2 - 2r_i\bar{r}\cos(\theta_i-\bar{\theta}) \to \xi^2.$$

This further leads to $r_i^2\cos^2\theta_i - 2r_i\bar{r}\cos\theta_i\cos\bar{\theta} + \bar{r}^2\cos^2\bar{\theta} + r_i^2\sin^2\theta_i - 2r_i\bar{r}\sin\theta_i\sin\bar{\theta} + \bar{r}^2\sin^2\bar{\theta} \to \xi^2$. Then one has

$$(r_i\cos\theta_i - \bar{r}\cos\bar{\theta})^2 + (r_i\sin\theta_i - \bar{r}\sin\bar{\theta})^2 \to \xi^2.$$

Therefore, $C(\bar{x},\xi)$ will be the circumcircle of the regular polytope formed by all the followers, where $C(\bar{x},\xi)$ denotes the circle centered at $\bar{x}$ and with the radius ξ. Because the radius of the incircle of the regular polytope is

$$r_{in} = \xi\cdot\cos(\pi/n).$$

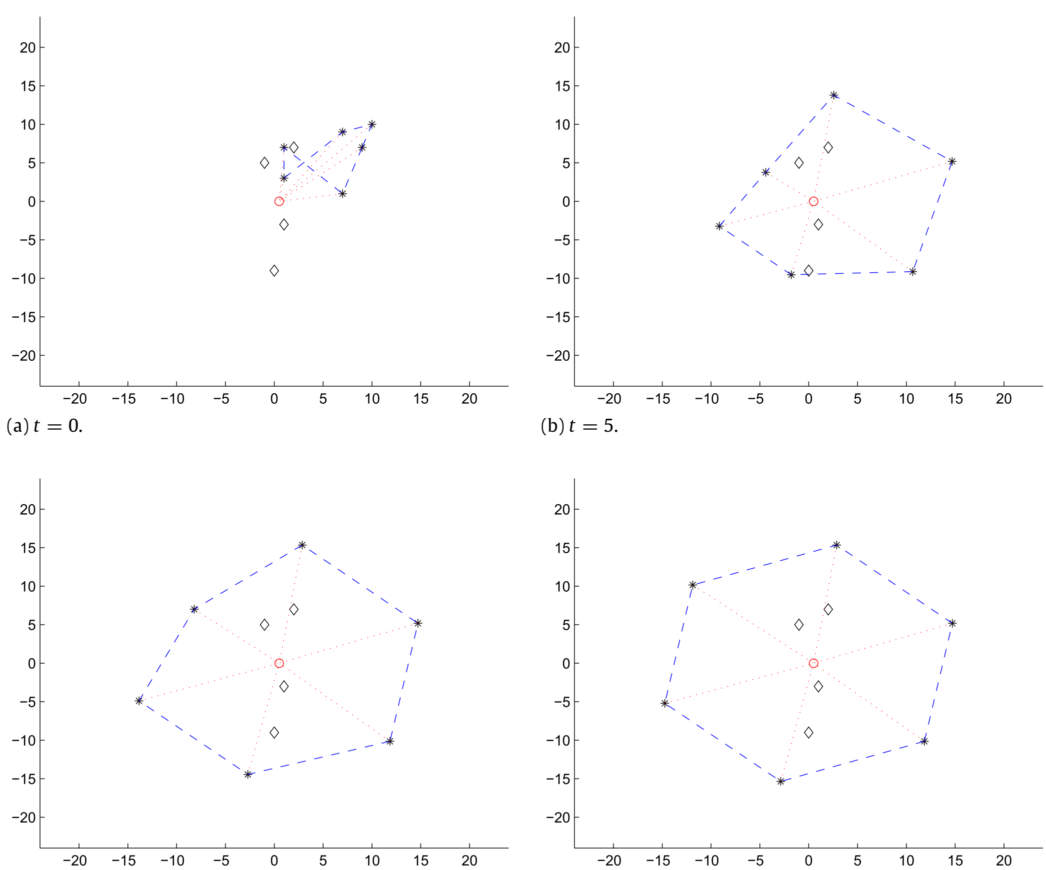
Since $\xi \geq \max_{i\in V_L}\|x_i-\bar{x}\|/\cos(\pi/n)$, one knows that

$$x_i \in B(\bar{x}, r_{in}) \subseteq co(V_F), \quad \forall i\in V_L, \quad (33)$$

where $B(\bar{x}, r_{in}) \triangleq \{x\in\mathbb{R}^2 \mid \|x-\bar{x}\| \leq r_{in}\}$. $\square$

6. Simulations

In this section, a simulation example as shown in Fig. 1 is first presented to illustrate Theorem 3. This example includes four leaders and four followers. The control parameters are specified as follows: $\kappa = 1$, $r = 8$, and $\xi = \max_{i\in V_f}\|x_i-\bar{x}\|$. The edge set E_c of the communication graph G_c is defined to be $E_c \triangleq \{(i, i+1) \mid i = 1,\ldots,|V_f|-1\}$. Initially, each leader can communicate with a follower, and the convex hull of the followers is set to be a rectangle, where the convex hull is formed by the dashed lines (Fig. 1(a)). When the followers are moving, the rectangle might not be kept because the followers have different estimates of the geometric center of the leaders $\bar{x}$ (Fig. 1(b) and (c)). However, as the estimates converge to $\bar{x}$, the rectangle is recovered and the followers can surround the leaders eventually (Fig. 1(d)). Fig. 2 shows the estimates of the geometric center of the leaders, which is [0, 0] in this case. The dashed lines denote the followers' estimates of the x dimension of the geometric center, while the solid lines represent the estimates of the y dimension of the geometric center.



(a) $t = 0$. (b) $t = 5$. (c) $t = 15$. (d) $t = 100$.

Fig. 3. Balanced surrounding control of mobile networks.

In the following, we present another example as shown in Fig. 3 to illustrate Theorem 5. This example includes 4 leaders and 6 followers. Initially, the angles of the followers are generated such that $0 < |\theta_i - \theta_j| < 2\pi$ for all $i \neq j$, $i, j \in V_F$ (See Fig. 3(a)). Note that in this example, $co(V_F)$ is not required to be a hyperrectangle initially. The control parameters are chosen to satisfy $\beta > \bar{r}(n-1)r_{\max}(\bar{r}(\xi^2 - \bar{r}^2)^{-\frac{1}{2}} + 1)$ and $\xi > \max(\bar{r}, \max_{i\in V_L} \|x_i - \bar{x}\|/\cos(\pi/n))$, and the communication graph G_c is generated to satisfy Assumption 5. The convex hull of the followers is indicated by dashed lines. The geometric center of the leaders is denoted by a circle. As can be seen from Fig. 3, not only can the followers surround the leaders, but also the final configuration of the followers forms a regular polytope centered at the geometric center of the leaders. It can be shown that the distances, denoted by dotted lines, from the followers to the geometric center are of the same values finally.

7. Conclusions

In this paper, a new cooperative control problem, surrounding control, is proposed, and partially solved. To solve this problem, some global information is needed, i.e., the geometric center of the leaders. To this aim, a decentralized estimator is constructed to estimate the geometric center, and is used in the controller. By using tools from graph theory and dynamical systems theory, we show that our controllers guarantee that the followers can surround the leaders eventually.

This paper focuses on the stationary leader case and a fixed communication graph. The model we proposed is of course very simple, but it serves as a natural starting point for the study of more complicated models, for instance, when the communication graph is time varying. Another future direction is to extend the current paper to consider moving leaders. The most challenging part in the extension might be to design an estimator that can track the geometric center of the moving leaders. In addition, the stability analysis could also be more involved. As stated in the introduction, the algorithm proposed in this paper may ultimately find applications in real life. But before this can happen, some compelling issues such as the effects of time delays and disturbances should be satisfactorily addressed.

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