Distribution System Bad Data Detection Using Graph Signal Processing

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***Abstract*—As advanced metering infrastructure becomes increasingly prevalent, new data-driven techniques for distribution system control and monitoring have emerged. The efficacy of these techniques, however, can be compromised by erroneous data. This paper develops a graph signal processing-based bad data detection algorithm. First, a physics-based graph construction algorithm is proposed for three-phase power distribution systems. Then, we introduce a data-driven algorithm for detecting erroneous data points by clustering the low-dimensional representations of the graph Fourier transforms of voltage signals. The numerical study results using the IEEE 13-bus test feeder and real-world smart meter time series data show that our proposed algorithm achieves high F1 and accuracy scores.**

***Index Terms*—Bad data detection, graph signal processing, power distribution systems, smart meter.**

I. Introduction

Reduction in the costs of advanced metering infrastructure has contributed to smart meters being increasingly installed for residential and commercial customers, leading to an influx of data on power distribution systems. This data has opened new avenues for key data-driven applications [1], such as state estimation, volt-var control, and theft detection. However, these applications rely on clean data. Anomalous data may be recorded by smart meters for a variety of reasons, such as meter malfunction, false data injection attacks, or interruptions in the communication channel between the meter and the utility. All of these potential issues compromise the assumption that the recorded smart meter data is actually clean. Therefore, an erroneous data detection algorithm for smart meter data is in critical need.

In this paper, we propose a method for identifying erroneous voltage measurements in distribution systems. The proposed method is based on graph signal processing, and particularly, clustering of voltage signals in the frequency domain. The main contributions of this paper are as follows. First, we propose a physics-based approach to construct a weight matrix and perform graph signal processing for three-phase power distribution systems. Second, we propose a data-driven method for detecting bad data by clustering spectral voltage signals in a lower-dimensional space.

Graph signal processing is an emerging field based on exploiting signal processing in the context of graph theory. In particular, it leverages the connectivity of a system and elements of classical signal processing. The use of graph signal processing has been extensively explored in areas such as transportation networks, meteorological systems, and social networks. However, graph signal processing techniques have not been thoroughly studied in three-phase power systems. To this end, we propose a physically-inspired weight matrix construction tailored to analyze voltage signals.

The remainder of this paper is structured as follows. Section II reviews works in the field of bad data detection in power distribution systems. Section III introduces the basics of graph signal processing and the proposed method for bad data detection in three-phase distribution networks. Section IV describes the experimental setup and performance of the proposed bad data detection algorithm. Section V states the conclusions.

II. Related Work

There have been several proposed methods for detecting anomalies in the context of power, both in transmission and distribution systems. In distribution systems, bad data detection is often considered in the context of state estimation [2], [3]. However, these methods rely on redundant measurements. Although deployment of advanced metering infrastructure is increasing, redundant measurements are not always available.

Others have proposed approaches to bad data detection using neural networks [4]–[6]. However, these approaches require a model to be trained, with the assumption that clean training data is available. [7] uses a PCA-based approach to detect errors using a residual in the reduced space. The approach is purely data-driven, with no accounting for the physical parameters of the system. As a result, a topology change may be flagged by the algorithm as an error. A physics-based data-driven model is developed to detect electricity theft in power distribution systems [8]. Although the algorithm achieves great performance, it relies on an accurate smart meter to transformer mapping model, which may not be available.

The use of graph signal processing as it relates to distribution systems has been limited. [9], [10] cover the use of graph signal processing for non-intrusive load disaggregation. [11], [12] use graph signal processing to detect false data injection attacks and abnormal events in transmission systems, where the network model is approximated by its single phase representation. Graph signal processing for unbalanced three-phase distribution systems remains untouched. There remains

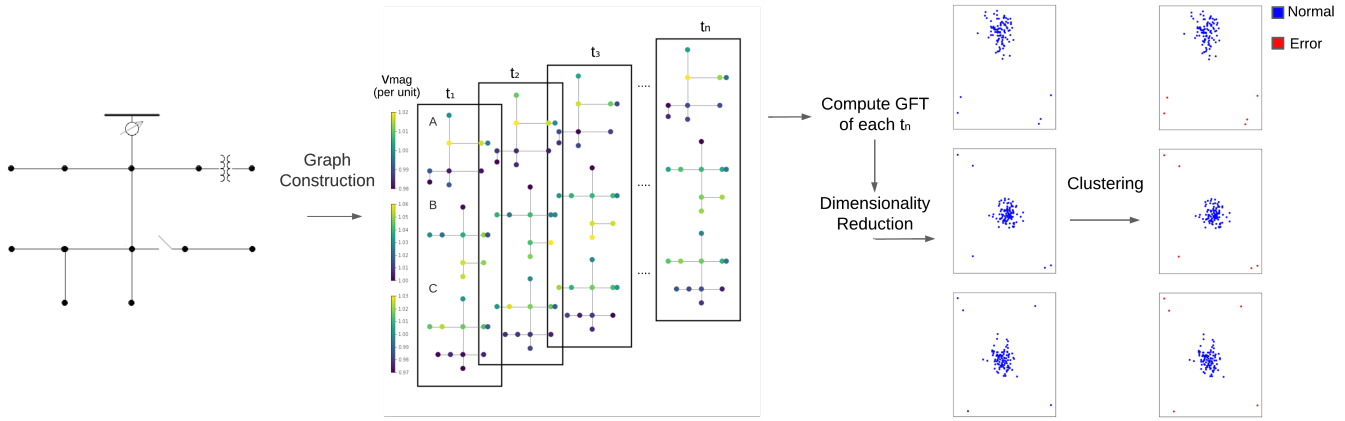

Fig. 1. Overall framework of the proposed bad data detection algorithm for power distribution systems

a need for a bad data detection algorithm which does not rely on training data and does not require measurement redundancy.

III. Technical Method

A. Overall Framework

The proposed framework is based on graph signal processing, and in particular the transformation of a signal comprised of voltage measurements into the frequency domain using a physically-inspired graph Laplacian. Such signals should have similar spectral components among normal measurements. Thus, by clustering these signals based on their spectral components, signals with erroneous measurements can be separated from those without.

The following section introduces graph signal processing and, in particular, transforming a nodal signal into a spectral signal by the graph Fourier transform. Then, a graph weight matrix based on voltage drop is introduced to facilitate this transformation for the task of nodal voltage error detection. This section then concludes by introducing the clustering method by which anomalous signals may be separated from normal ones.

B. Review of Graph Signal Processing

A graph is defined by a set of N nodes or vertices $V \in [v_1, ..., v_N]$ connected by M edges $E \in [e_1, \ldots, e_M]$. The adjacency matrix A of a graph is a symmetric matrix with the ijth element $a_{ij} = 1$ if e_{ij} exists and 0 otherwise. In general, these connections may or may not be directed. That is, a_{ij} may or may not equal a_{ji}. An extension of the adjacency matrix is the weight matrix W, in which the w_{ij} may assume some value relating to the strength of the connection between nodes i and j. The degree matrix D is a diagonal matrix wherein the entries d_{ii} are defined as the sum of all weights related to node i: $d_{ii} = \sum_{j=1}^{N} e_{ij}$. As the basic building block of graph signal processing, the graph Laplacian can then be defined as $L = D - W$.

Graph signal processing extends traditional graph theory by defining a signal on a graph as a set of nodal measurements $\mathbf{x} = \{x_1, ..., x_N\}$. The graph Laplacian may be decomposed by eigenvalue decomposition. The decomposition is obtained as:

$$L = U\Lambda U \tag{1}$$

where the columns of U are the eigenvectors of L and Λ is a diagonal matrix comprised of the eigenvalues, with the eigenvalues sorted in ascending order. The Laplacian has at least one zero eigenvalue [13]. The magnitude of the eigenvalues corresponds to the frequency component of the signal, with eigenvectors corresponding to small eigenvalues being part of the low-frequency component.

Using the eigenvalue decomposition of the Laplacian, the graph Fourier transform (GFT) $\mathbf{X}$ of signal $\mathbf{x}$, also referred to as the spectral signal, is defined as

$$\mathbf{X} = U^{-1}\mathbf{x} \tag{2}$$

Similar to the Fourier transform of a classical signal, the GFT takes the signal from the vertex domain to the frequency domain. Arising from this transformation is a notion of 'smoothness', a measure of the overall variation across the graph. A signal is smooth if it varies slowly in the frequency domain, and thus has mostly low-frequency components.

C. GSP for Power Distribution System Measurements

We treat smart meters and SCADA measurement points as the nodes in the graph. The nodal signals are the voltage magnitude measurements. In power distribution systems, most buses are served by more than one phase wire, and accordingly there can be more than one voltage magnitude measurement at every bus. To address this issue, a distribution grid is split into multiple graphs, each corresponding to a certain phase connection. The manner in which the phase connection is defined in each graph depends on the phases across which the nodal voltage magnitude is measured. If all measurements are phase-to-neutral, then the grid may be separated into three graphs, each corresponding to a particular phasing $\phi_{i_1} - \phi_{i_2} \in [A-N, B-N, C-N]$. If all measurements are phase-to-phase or some combination of phase-to-phase and phase-to-neutral, then the grid may be separated into three graphs corresponding to phasings $\phi_{i_1} - \phi_{i_2} \in [A-B, B-C, C-A]$.

Note that in terms of voltage magnitude, phasing $A-B$ is equivalent to $B-A$, as they differ only in voltage angle by 180 degrees. Our proposed graph construction strategy allows for combinations of measurements, wherein voltage is measured as line-to-neutral at some buses but line-to-line at others. Distribution systems often measure line-to-line voltages, but there is the possibility that a bus is served by only one phase wire, and thus only line-to-neutral measurement is possible. In such a case, the line-to-neutral measurement is included in each graph containing that phase, assuming that phasing is present on the wires connecting the bus to the source. For example, a certain bus voltage may be measured on phase $C-N$, but served by a distribution line which at some point only carries phases B and C, but not A. In this scenario, the bus will only appear in the graph of phase $B-C$. Note that we assume voltage magnitude measurements are available at each bus on each phasing served. With the nodes defined, it remains to define the weights between these nodes.

Physical systems often present intuitive means of constructing the adjacency matrix and weight matrix. For example, the weight matrix can be constructed using the Euclidean distance between nodes or the similarity between nodal measurements. For power distribution systems, it is natural to leverage physical line parameters. The edge weights should model the similarities between voltage measurements. Nodes that are not physically connected, intuitively, should have an edge weight of 0. Nodes connected by lines with small voltage drops should have more similar voltage measurements than nodes connected by lines with large voltage drops.

1) Weight Matrix Definition: The weight matrix of a particular phasing of the distribution system graph is constructed as follows. If a line connecting node i and j exists, the weight between node i with phasing $\phi_{i_1}-\phi_{i_2}$ and node j with phasing $\phi_{j_1}-\phi_{j_2}$ can be defined as:

$$w_{i,j}=\begin{cases}(vdrop_{\phi_{i_1}})^{-1}, & \text{if } \phi_{i_2} \text{ or } \phi_{j_2}=N \\ & \text{and } \phi_{i_1}=\phi_{j_1} \text{ or } \phi_{j_2} \\ (vdrop_{\phi_{i_2}})^{-1}, & \text{if } \phi_{j_2}=N \text{ and} \\ & \phi_{i_2}=\phi_{j_1} \\ (vdrop_{\phi_{i_1}}-vdrop_{\phi_{i_2}})^{-1}, & \text{if } \phi_{i_2}=\phi_{j_2}\neq N\end{cases} \tag{3}$$

where the per-phase voltage drop with 1 per unit current flow on phase $\Phi\in[A,B,C]$ can be calculated as:

$$vdrop_{\Phi}=Z^{\Phi}\cdot Iline_{phase} \tag{4}$$

where $Z^{\Phi}=\begin{bmatrix}z^{\Phi A} & z^{\Phi B} & z^{\Phi C}\end{bmatrix}$ denotes the row of the 3 by 3 line impedance matrix that corresponds to phase Φ. $Iline_{phase}=\begin{bmatrix}1\angle 120^\circ & 1\angle 0^\circ & 1\angle -120^\circ\end{bmatrix}$ denotes the per unit balanced current vector.

As previously discussed, the nodes in any given graph are restricted to carrying at least one of the same phase. The weight matrix definition in (3) accounts for voltage drop, and is applicable both for voltages measured line-to-neutral and line-to-line. The first case in (3) corresponds to the weight between two line-to-neutral nodes, as well as the weight between a line-to-line node and a line-to-neutral node. The second case also corresponds to the weight between a line-to-line node and a line-to-neutral node, where the difference to the first case is due to phase ordering. The third case pertains to the weight between two line-to-line nodes. The per-phase voltage drop is an approximation of the voltage drop, assuming a nearly balanced distribution system, not including a scale factor for current magnitude.

The weight matrix definition presented above results in weights which are complex valued. Typically, weight matrices in graph signal processing are restricted to be real-valued. Accordingly, we define weight matrices corresponding to the real part $W_r=real(W)$ and imaginary part $W_i=imag(W)$.

D. Spectral Clustering Method

A common method for error detection in graph signal processing relies on a smoothness score $s=X^T diag(\Lambda)$. This score is larger when the signal contains more high frequency components. Using this smoothness score, a threshold may be defined such that if the smoothness exceeds the threshold, a bad data point is flagged at that time step. This method is effective. However, the selection of a threshold is often a tricky task in the absence of labelled erroneous data points. Instead of using the smoothness score, which is a specific engineered feature based on the spectral signals, we propose to detect bad data in a two step process. First, the dimensionality of the spectral signal is reduced using a nonlinear low-dimensional embedding technique. Then, we group the reduced-dimension components into the normal and abnormal data points using a density-based clustering approach.

The reasoning behind the adoption of a clustering approach to detect abnormal data points is as follows. Spatial and temporal fluctuations in power consumptions cause variations in nodal voltage measurements. Due to Kirchhoff's laws, the voltage signals should be similar at connected nodes. Thus, the corresponding spectral signal should be dominated by low-frequency components. With a weight matrix which properly accounts for nodal voltage correlations, the normal signals will have similar spectral components. However, if one or more meters report bad voltage data point(s), the frequency-domain signal will be altered. This results in faulty signals having spectral components different from that of the normal signals.

This method also allows for the use of more than one weight matrix. As previously mentioned, the complex weight matrix may be split into real and imaginary parts. Each signal is converted into two spectral signals corresponding to the real and imaginary parts, which can be concatenated to form a hybrid signal. This construction is more robust than relying on the voltage magnitude or the real part of the voltage phasor alone as it properly accounts for both the real and imaginary parts of the impedance's contributions to voltage drop. The real and imaginary parts of impedance do not contribute equally to voltage drop. Thus, taking the magnitude of impedance somewhat obscures the individual footprints of the two signals.

1) Feature Extraction and Dimensionality Reduction: The number of components of the spectral signal on each graph is equal to the number of nodes on the graph. Taking the

GFT using the real and imaginary parts of the weight matrix separately doubles the number of components in the hybrid signal. However, not all of the components contribute equally to the task of bad voltage data detection. Certain components, especially those corresponding to the low frequency variations on the graph, may not be good indicators of faulty measurements. As such, it is necessary to perform feature extraction and dimensionality reduction in order to successfully separate erroneous data points from normal ones through clustering.

For the task at hand, it is desirable to obtain visually interpretable results. In the real-world application of the proposed method, it is unlikely that a sufficient number of labelled erroneous data would be available for tuning hyperparameters of the clustering method. Visualization of the embedding allows for effective hyperparameter tuning.

T-distributed Stochastic Neighbour Embedding (t-SNE) is chosen to perform nonlinear dimensionality reduction [14], for its ability to model similar points in the high-dimensional space as clusters in the embedded space, as well as provide visualizations of high-dimensional data. t-SNE defines two probability matrices, one in the original high-dimensional space and one in the low-dimensional space, wherein high similarity between points corresponds to higher probability. Then, the positions of points in the embedding are found by minimizing the Kullback-Leibler divergence between the two matrices with a gradient descent method. If normal data outnumber the abnormal ones, t-SNE should create an embedding which shows normal measurements in large clusters, and erroneous measurements as smaller clusters or outliers.

2) Clustering: In order to identify the erroneous data, we apply a clustering algorithm on the low-dimensional embedding of the spectral signal. With a sufficient number of data points, the normal measurements should form high density regions and the erroneous ones should be separated from these regions by areas of low density. The clusters in the embedded space may have an irregular shape. Thus, a density-based clustering method, Density-based spatial clustering of applications with noise (DBSCAN), is selected as the clustering algorithm.

DBSCAN forms clusters by identifying points having a sufficient density to be considered a cluster [15], based on two parameters. $MinPts$ defines the minimum number of points to be considered a cluster. ϵ pertains to the distance between points. A cluster begins with an initial group of points with $MinPts$ within the ϵ-neighbourhood. The cluster then expands to include all points within the ϵ-neighbourhood of those points also satisfying the density constraint, and all other points satisfying the density constraint which can be connected by a chain of points back to the initial cluster. At the edges of the cluster are points not satisfying the density constraint, but being within the ϵ-neighbourhood of a point which does.

Assuming the probability of bad data is relatively low, the clusters of faulty signals are identifiable by the relative size of the clusters. Error labels are assigned by the cardinality of the clusters of each point. Outliers and groups with cardinality less than a threshold are flagged as errors.

IV. Experimental Validation

A. Experimental Setup

To validate the proposed method, we use the standard IEEE 13-bus distribution test feeder, coupled with a time-series dataset constructed with the Commission for Energy Regulation smart meter data from the Irish Social Science Data Archive [16]. This dataset consists of time-stamped real power consumptions in 30-minute windows over approximately one year. To represent the load of each node in the test feeder, 100 customers are chosen from this original smart meter dataset at random. These aggregated loads are then scaled such that the average consumption for each node is equal to the loads specified by the original IEEE test feeder. Then, three-phase load flow is performed to obtain the nodal voltages. To emulate random measurement noise, Gaussian noise is added to the voltage measurements with mean 0 and variance of 0.003% of the voltage. These nodal voltages are then corrupted with synthetic bad measurements. At each node and each time step, with a 0.1% probability a measurement error with magnitude chosen uniformly at random between 3% and 5% of the raw measurement is added or subtracted, with equal probability, to the original measurements.

Two test scenarios are created based on the test feeder. In the first scenario, it is assumed that the smart meter at each node measures line-to-neutral voltage. In the second scenario, it is assumed that smart meters measure line-to-line voltage, except at the buses served by single-phase lines. One modification is made to the topology of the IEEE 13-bus feeder to remove the switch which incurs no loss. In the context of GSP, two nodes separated by a switch are effectively the same node if that switch is closed. The voltages on the two sides of the switch are identical. Thus, the two nodes are merged together.

T-SNE has several hyperparameters, with the most important being the perplexity p, which has a significant impact on the distribution of the points in the low dimensional space. In the test scenarios, varying the perplexity between 30 and 80 results in similar performance. The best performance for each phasing in both test scenarios can be achieved with $p = 50$. Tuning this parameter may be done without labelled erroneous data by looking for the value which maximizes the space between groups. Because t-SNE is selected for the dimensionality reduction, the embedded spectral signals easily show interpretable clusters. The plots of points in the low-dimensional space can be used to select appropriate values of ϵ and $MinPts$ for DBSCAN. In this study, $\epsilon = 2.5$ and $MinPts = 3$ are used. The cardinality threshold used for classifying clusters is 25.

B. Numerical Results

We apply the proposed data-driven bad data detection algorithm on the two test scenarios and evaluate its performance with two metrics: F1 score and accuracy. F1 score is the harmonic mean of precision and recall: $F_1 = 2 \cdot \frac{Precision \cdot Recall}{Precision + Recall}$, where $Precision$ is calculated as the ratio between true positive and the summation of true positive and false positive.

TABLE I
PERFORMANCE OF GSP-BASED BAD DATA DETECTION ALGORITHM

	Phase	F1 Score	Accuracy
Scenario 1	AN	0.921	0.982
	BN	0.945	0.965
	CN	0.948	0.988
Scenario 2	AB	0.946	0.984
	BC	0.939	0.981
	CA	0.900	0.964

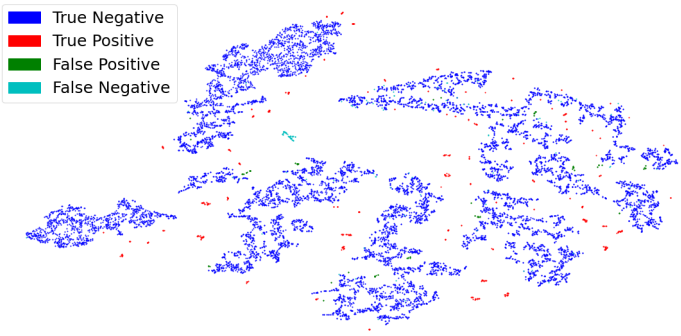

Fig. 2. Visualization of the spectral signals in the low-dimensional space with bad data detection results on phase $A - N$.

Recall is the ratio between true positive and the summation of true positive and false negative.

In the test scenarios, the number of normal measurements far outnumbers the number of erroneous data points. The F1 score essentially weights the false positive rate and false negative rate equally, but a higher false positive rate results in a much larger number of misclassified points than a higher false negative rate. Accordingly, accuracy is also reported. As shown in Table I, the GSP-based bad data detection algorithm achieves F_1 scores above 0.9 and approaching 0.95 for several phase connections. The accuracy scores are higher than 0.96 for all phase connections. The proposed method performs well for both the phase-to-neutral configuration of the first scenario and the hybrid configurations of the second scenario.

Figure 2 shows the spectral signals for the phase A-N graph in the embedded space. Normal signals are referred to as negatives and errors as positives. As shown in the figure, the combination of GFT and t-SNE results in good separation between normal signals and bad data. Normal signals form a small number of high-density clusters. Erroneous signals, for the most part, form small clusters separated from these larger clusters, or are outliers. A number of normal signals form small clusters with cardinality indistinguishable from clusters of errors, resulting in misclassification of those clusters. Similarly, some erroneous data points form clusters large enough that they are misclassified as normal data. A small number of bad data are grouped with normal clusters in the embedded space, resulting in incorrectly labelling those errors as normal.

V. CONCLUSION

In this paper, we introduce a novel graph signal processing-based approach to detect bad data in power distribution systems. We propose a physics-based graph construction and edge weights calculation methodology for three-phase distribution networks. We also develop an innovative approach, which combines feature extraction and density-based clustering to identify bad data by their spectral footprint. The proposed method is validated using the IEEE 13-bus test feeder and augmented real-world smart meter data. The results show that our proposed algorithm yields highly accurate abnormal data detection results for both phase-to-neutral and phase-to-phase configurations and all phase connections.

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