

A Distributed Algorithm for Distribution Network Reconfiguration

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Abstract— Distribution network reconfiguration algorithms change the status of sectionalizing and tie switches to reduce network line losses, relieve network overloads, minimize loss of load, or increase hosting capacity for distributed energy resources. Most of the existing work adopt centralized or hierarchical approaches to solve the network reconfiguration problem. This paper proposes a distributed scheme for network reconfiguration. In the distributed approach, the switches are represented by edge computing agents who communicate and work collectively with their neighbors to find the optimal reconfiguration solution. This scheme breaks the complex computation tasks required by centralized algorithms into much smaller ones. It also relieves the communication and data sharing burden via neighbor-to-neighbor communication. Simulation results on a 16-bus distribution test feeder demonstrate that the quality of the distributed solution is comparable to that of the centralized approach.

Index Terms— Distribution network reconfiguration, distributed optimization, mixed-integer programming.

I. INTRODUCTION

BY changing the status of switches, distribution network reconfiguration algorithms improve the system performance under both normal and abnormal operating conditions. The objectives of distribution network reconfiguration range from loss, overload, and outage reduction to hosting capacity enhancement. The distribution network reconfiguration functionality is especially valuable when a larger number of distributed renewable resources (DERs) have been or are expected to be installed [1]. Both Federal sponsored programs and market forces are facilitating the wide-spread adoption of smart grid technologies such as the advanced metering infrastructure and remote controllable switches [2]. These two technologies enabled remote data collection and

actuation of loads and switches which are critical to the implementation of distribution network reconfiguration.

The distribution network reconfiguration outcomes need satisfy two set of operational constraints. First, the system operating limits such as the line flow limit must not be violated. Second, the radiality constraint must be enforced on distribution network, that is, no loop should be formed.

The existing literature on distribution network reconfiguration can be divided into two groups based on the solution methodology. The first group adopts heuristic methods within which there are two approaches. The first approach starts with a meshed network and then open the switch that will contribute the most to the objective function [3, 4, 7, 8]. The procedure continues until a radial network is achieved. The second approach starts from a radial network and selects a pair of closed and open switches and exchange their status [5, 6]. Selecting such a pair requires an accurate estimation of loss reduction due to the exchange.

The second group of literature formulates the distribution network reconfiguration problem as a mixed-integer program or a combinatorial optimization problem. The optimization problem is solved by either general-purpose metaheuristic algorithms or deterministic ones. Metaheuristic algorithms such as simulated annealing [9], genetic algorithm [10], and ant colony algorithm [11] have been used to solve the network reconfiguration problem. The deterministic algorithms work by linearizing or convexifying the original problem and converting it to a mixed-integer linear or convex optimization problem. Then mixed-integer linear or mix-integer convex optimization algorithms are adopted to solve the problem [1, 12, 13, 14]. The deterministic approach has a number of advantages such as the repeatability of solutions, guarantees of global optimality, and ease of implementation thanks to the optimization solvers.

Most of the existing methods followed the centralized control framework within which all network data are collected and sent back to the control center to determine the reconfiguration solution. The switch control signals are then sent from the control center via the communication network for switch actuation. Although centralized approaches have shown good numerical performance on some distribution test feeders,

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they usually result in high latency and communication bottleneck in the system. The distributed approaches on the other hand have great potential in reducing the communication burden, improving cybersecurity and preserving the privacy of smart meter data [15].

In this paper, we propose a distributed algorithm to overcome the communication bottleneck problem by distributing the computation task among the network switches (agents) with only neighbor-to-neighbor communications. Our contributions are as follows. First, a novel decomposed formulation of the distribution network reconfiguration problem is developed. Second, an alternating direction method of multipliers (ADMM) release and fix algorithm is adopted to solve the problem in a distributed manner. Third, we introduce a distributed approximated Newton's method to speed up the distributed optimization algorithm.

The rest of the paper is organized as follows. Section 2 formulates the distribution network reconfiguration problem. Section 3 presents the distributed algorithm. Section 4 shows the simulation results. Section 5 provides the conclusion.

II. PROBLEM FORMULATION

A. Overview

One of the most commonly used objectives of the network reconfiguration problem is the minimization of line losses. The constraints of the optimization problem include the operating limits such as the line flow limits, the power flow constraints, and the network radiality constraint. The power flow constraints ensure that the steady-state operating conditions are consistent with the electric loads, distributed generations and the physics of the distribution network. The network radiality constraints require that every primary feeder of the distribution network have a radial topology. The goal of network reconfiguration is to find on/off status for all switches that minimize the network losses while satisfying all operating constraints. In this work, the distribution network is assumed to be reasonably balanced so that the single-phase representation of the three-phase network is acceptable. It is also assumed that each line segment has a switch installed which can be remotely controlled for network reconfiguration.

The objective function of line loss minimization is given by:

$$\min \sum_{ij \in E} r_{ij} l_{ij}^2 \quad (1)$$

where r_{ij} is the resistance of line ij . l_{ij}^2 denotes the squared magnitude of current flowing on line ij . E is the set of all lines in the network. Note that each line ij has a reference direction $i \rightarrow j$ associated with it. Throughout the paper we use ij to denote a line if the reference direction is needed; otherwise we will simply use ℓ in place of ij .

Two operating limits will be considered in the problem

formulation, namely the nodal voltage magnitude limit (2) and branch flow limit (3):

$$V^{2min} \leq v_i^2 \leq V^{2max} \quad \forall i \in N \setminus N^0 \quad (2)$$

$$l_{ij}^2 \leq \alpha_\ell l^{2max} \quad \forall ij \in E \quad (3)$$

where v_i^2 is the squared nodal voltage magnitude of node i ; $\alpha_\ell \in \{0,1\}$ is a binary variable representing the close ($\alpha_\ell = 1$) and open ($\alpha_\ell = 0$) status of each switch; N denotes the set of all nodes in the distribution network; N^0 is the set of substation nodes (reference nodes).

B. Power Flow Model

The DistFlow equations [14] are adopted to capture the power flow constraints.

$$P_i = \sum_{ij \in E} p_{ij} - \sum_{ki \in E} (p_{ki} - r_{ki} l_{ki}^2) + g_i v_i^2 \quad \forall i \in N \quad (4)$$

$$Q_i = \sum_{ij \in E} q_{ij} - \sum_{ki \in E} (q_{ki} - x_{ki} l_{ki}^2) + b_i v_i^2 \quad \forall i \in N \quad (5)$$

$$v_j^2 = v_i^2 - 2(r_{ij} p_{ij} + x_{ij} q_{ij}) + (r_{ij}^2 + x_{ij}^2) l_{ij}^2 \quad \forall ij \in E \quad (6)$$

$$l_{ij}^2 = \frac{p_{ij}^2 + q_{ij}^2}{v_i^2} \quad \forall ij \in E \quad (7)$$

$$v_i^2 = v^{ref2} \quad \forall i \in N^0 \quad (8)$$

where $P_i + jQ_i$ is the complex net power injection at node i ; $p_{ij} + jq_{ij}$ is the complex branch power flow of line ij ; $r_{ij} + jx_{ij}$ is the impedance of line ij ; $g_i + jb_i$ is the shunt admittance from bus i to ground. It has been shown [16] that for practical radial networks, the system of equations Eq.(4-8) has a unique solution near the flat voltage solution. Therefore, they are suffice for the reconfiguration application.

Since equation (7) defines a non-convex feasible set, the relaxation is typically applied [14]:

$$l_{ij}^2 \geq \frac{p_{ij}^2 + q_{ij}^2}{v_i^2} \quad \forall ij \in E \quad (9)$$

Note that Eq.(9) defines a quadratic cone and can be handled by many optimization solvers.

C. Radiality Constraint

To enforce the network radiality in the reconfiguration problem, the method proposed in [1, 12] is adopted:

$$\beta_{ij} + \beta_{ji} = \alpha_\ell \quad \forall \ell \in E \quad (10)$$

$$\sum_{j \in N(i)} \beta_{ij} = 1 \quad \forall i \in N \setminus N^0 \quad (11)$$

$$\beta_{ij} = 0 \quad \forall i \in N^0 \quad (12)$$

$$\beta_{ij} \in \{0,1\} \quad \forall i \in N \setminus N^0, j \in N(i) \quad (13)$$

$$0 \leq \alpha_\ell \leq 1 \quad \forall \ell \in E \quad (14)$$

where $\beta_{ij}, \beta_{ji}, \alpha_\ell$ are variables associated with each line ij ; $N(i)$ is the set of neighbor nodes of i . It has been shown [12] that Eq.(10-14) are sufficient for the radiality for each graph component that is connected to one of the reference nodes. Therefore Eq.(10-14) together with Eq.(4-8) define a radial

network for practical loads.

We summarize the final optimization problem as:

$$\begin{aligned}
 & \min && \text{Network loss: Eq.(1)} \\
 & \text{s.t.} && \text{Operating limits: Eq.(2,3)} \\
 & && \text{Power flow: Eq.(4,5,6,8,9)} \\
 & && \text{Network radiality: Eq.(10,11,12,13,14)}
 \end{aligned} \quad (\text{OP1})$$

The decision variables are $p_{ij}, q_{ij}, l_{ij}^2, \beta_{ij}, \beta_{ji}, \alpha_\ell \forall ij \in E; v_i^2 \forall i \in N$. Problem (OP1) is a mixed-integer conic programming problem and can be solved by existing solvers in a centralized manner.

III. DISTRIBUTED SOLUTION METHODOLOGY

In this section, we propose a distributed solution to problem (OP1). First, (OP1) will be decomposed into a collection of coupled sub-problems. Second, each of the sub-problems is solved by an agent (switch) via local computation and neighbor-to-neighbor communication. In the following, we first define the agents and their communication graph, then we present the two-step distributed algorithm.

A. Definition of Agents and Communication Graph

We assume each switch has computing capability and can communicate with its neighbors. The agents are defined as the switches in the network. It is assume that each line has a switch. Hence, we do not distinguish the concept of switch, line, and agent and refer to them as agent in the rest of the paper.

We define the neighbors $E(ij)$ of each agent ij as the agents that have a node in common with agent ij . ij itself is not in $E(ij)$. In other words, let $G = (N, E)$ be the graph representing the distribution network, then the communication graph is $G^c = (E, M)$ where if $\ell \in E$ and $m \in E$, then $\ell m \in M$ if ℓ and m are incident in N . In graph-theoretic terms, G^c is called the line graph of G .

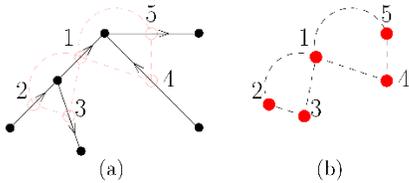


Fig. 1. Example of agents and their communications

An agent ij 's set of neighbors $E(ij)$ is partitioned into four subsets based on their and agent ij 's reference directions. Denote $E_i^{from}(ij)$ as the neighbors that connect to node i with their "from" nodes being i . $E_i^{to}(ij)$, $E_j^{from}(ij)$, and $E_j^{to}(ij)$ are defined in a similar manner. Fig.1 shows an example illustrating the concepts mentioned in this section. There are five switch agents in Fig.1(a). The arrows denote the reference directions. Fig.1(b) shows the communication graph of the network. According to the reference direction, $E(1) = \{2,3,4,5\}$

can be partitioned into $E_i^{to}(1) = \{2\}$, $E_i^{from}(1) = \{3\}$, $E_j^{to}(1) = \{4\}$, and $E_j^{from}(1) = \{5\}$.

B. ADMM Release-and-Fix

This subsection describes the distributed solution to problem (OP1). The state vector of agent ij is defined as $\mathbf{x}_{ij} = [p_{ij}, q_{ij}, l_{ij}^2, v_i^{2(ij)}, v_j^{2(ij)}, \beta_{ij}, \beta_{ji}, \alpha_\ell]^T$. Superscripts (ij) are introduced on the voltage variables. This is because the same voltage variable v_i^2 is shared by all agents that are incident to node i and must be distinguished. As a result, agents must agree on the value of shared voltage variables:

$$v_i^{2(ij)} = v_i^{2(ik)} \quad \forall ij \in E, \forall ik \in E_i^{from}(ij) \quad (15a)$$

$$v_i^{2(ij)} = v_i^{2(ki)} \quad \forall ij \in E, \forall ki \in E_i^{to}(ij) \quad (15b)$$

$$v_j^{2(ij)} = v_j^{2(jk)} \quad \forall ij \in E, \forall jk \in E_j^{from}(ij) \quad (15c)$$

$$v_j^{2(ij)} = v_j^{2(kj)} \quad \forall ij \in E, \forall kj \in E_j^{to}(ij) \quad (15d)$$

Using the definition of \mathbf{x}_{ij} , problem (OP1) can be written as:

$$\begin{aligned}
 & \min_{\mathbf{x}_{ij}, ij \in E} && \sum_{ij \in E} \mathbf{c}_{ij}^T \mathbf{x}_{ij} \\
 & \text{s.t.} && \mathbf{x}_{ij} \in \mathbb{X}_{ij} \quad \forall ij \in E \\
 & && \sum_{j \in N(i)} \mathbf{A}_{ij} \mathbf{x}_{ij} = \mathbf{b}_i \quad \forall i \in N
 \end{aligned} \quad (\text{OP2})$$

where \mathbb{X}_{ij} is a local mixed-integer set whose continuous relaxation is convex; $N(i)$ is the set of neighbor nodes of i ; the matrices \mathbf{A}_{ij} and vectors \mathbf{b}_i are identified through problem (OP1) as well as Eq.(15). We refer the first set of constraints in (OP2) as local constraints and the second the coupling constraints. To solve problem (OP2) in a distributed manner, we derive the augmented Lagrangian function by absorbing the coupling constraints into the objective function:

$$\begin{aligned}
 & \min_{\mathbf{x}_{ij}, ij \in E} L_\rho = && \sum_{ij \in E} \mathbf{c}_{ij}^T \mathbf{x}_{ij} + \\
 & && \sum_{i \in N} \mu_i^T \left(\sum_{j \in N(i)} \mathbf{A}_{ij} \mathbf{x}_{ij} - \mathbf{b}_i \right) + \\
 & && \frac{\rho}{2} \sum_{i \in N} \left\| \sum_{j \in N(i)} \mathbf{A}_{ij} \mathbf{x}_{ij} - \mathbf{b}_i \right\|_2^2 \\
 & \text{s.t.} && \mathbf{x}_{ij} \in \mathbb{X}_{ij} \quad \forall ij \in E
 \end{aligned} \quad (\text{OP3})$$

where $\rho > 0$ is called the penalty parameter. Problem (OP3) may be solved in a distributed manner by the alternating direction method of multipliers (ADMM) [17, 18, 19].

The ADMM algorithm can be used to solve convex separable problems. However, the presence of binary variables β_{ij}, β_{ji} in (OP3) destroys the convexity. A heuristic remedy to handle binary variables was introduced in [17] and the resulting modified algorithm is called ADMM Release-and-Fix. The modified algorithm proceeds by iterating between two stages. The first stage (ADMM-Release) is identical to conventional ADMM with the exception of the presence of binary variables. The goal of ADMM-Release is to search for feasible binary solutions, which are "stable" across multiple runs of

ADMM-Release. In order to encourage exploration of new binary solutions, the penalty parameter ρ will be gradually decreased after a feasible solution is found. In order to force convergence to a “stable” solution ρ will gradually increase. After a new stable binary solution is found, the second stage (ADMM-Fix) fixes the binary solution from ADMM-Release and solves the simplified optimization problem with only continuous variables. These two stages will alternate until the stopping criteria is met.

The ADMM-Fix step converges slowly. To speed up the distributed computation we propose an approximated Newton’s method to replace the ADMM-Fix step.

C. Approximated Newton’s Method

After a feasible binary solution is found by ADMM-Release, the ADMM-Fix problem becomes the same as solving the DistFlow equations Eq.(4-8) in a distributed manner with a given network configuration. To solve the problem, we first linearize Eq.(7) of the DistFlow equations as:

$$\begin{aligned} 2p_{ij}^v \tilde{p}_{ij} + 2q_{ij}^v \tilde{q}_{ij} - v_i^{2v} \tilde{l}_{ij}^2 - l_{ij}^{2v} \tilde{v}_i^2 \\ = p_{ij}^{2v} + q_{ij}^{2v} - l_{ij}^{2v} v_i^{2v} \quad \forall ij \in E \end{aligned} \quad (16)$$

where v is the iteration number and variables with a tilde \sim denotes the increment, e.g., $\tilde{p}_{ij} = p_{ij}^v - p_{ij}$. The resulting system of linear equations, namely Eq.(4,5,6,8,16), is denoted as $\mathbf{Ax} = \mathbf{b}$. Next, we propose a distributed algorithm which solve this linear system in an iterative manner.

We define a new vector $\mathbf{x}_{ij}^c = [p_{ij}, q_{ij}, l_{ij}^2, v_i^{2(ij)}, v_j^{2(ij)}]^\top$ with continuous variables only. Solving $\mathbf{Ax} = \mathbf{b}$ is equivalent to solving the following unconstrained optimization problem [20]:

$$\min_{\mathbf{x}_{ij}^c, ij \in E} f = \frac{1}{2} \sum_{ij \in E} \|\mathbf{A}_{ij}^c \mathbf{x}_{ij}^c - \mathbf{b}_{ij}^c\|_2^2 \quad (\text{OP4})$$

where \mathbf{A}_{ij}^c and \mathbf{b}_{ij}^c are identified from \mathbf{A} and \mathbf{b} by rearranging equations and variables accordingly and appending Eq.(15) to enforce voltage constraints. Note that \mathbf{A}_{ij}^c is a constant matrix while \mathbf{b}_{ij}^c depends linearly on \mathbf{x}_m^c for all neighbors, m , of agent ij (excluding ij itself). We would like to solve problem (OP4) using Newton’s iteration where the gradient and Hessian matrix can be derived as follows:

$$\begin{aligned} \nabla_{\ell} f &= \mathbf{A}_{\ell}^{c\top} (\mathbf{A}_{\ell}^c \mathbf{x}_{\ell}^c - \mathbf{b}_{\ell}^c) \quad \forall \ell \in E \\ \mathbf{H}_{\ell\ell} &= \frac{\partial}{\partial \mathbf{x}_{\ell}^c} \nabla_{\ell} f = \mathbf{A}_{\ell}^{c\top} \mathbf{A}_{\ell}^c \quad \forall \ell \in E \\ \mathbf{H}_{\ell m} &= \frac{\partial}{\partial \mathbf{x}_m^c} \nabla_{\ell} f = -\mathbf{A}_{\ell}^{c\top} \frac{\partial}{\partial \mathbf{x}_m^c} \mathbf{b}_{\ell}^c \quad \forall \ell, m \in E, m \in E(\ell) \\ \mathbf{H}_{\ell m} &= \mathbf{0} \quad \forall \ell, m \in E, m \notin E(\ell) \end{aligned} \quad (17)$$

However, it is challenging to invert the Hessian matrix \mathbf{H} of the objective function in a distributed manner. Therefore, an method to approximate the inverse Hessian is needed. To do so, let’s define two new matrices: $\mathbf{D} = \text{diag}(\mathbf{D}_1, \mathbf{D}_2, \dots, \mathbf{D}_{|E|})$ where $\mathbf{D}_{\ell} = \gamma \mathbf{H}_{\ell\ell}$ and \mathbf{B} where

$$\mathbf{B}_{\ell m} = \begin{cases} (1 - \gamma) \mathbf{H}_{\ell\ell} & \text{if } \ell = m \\ \mathbf{H}_{\ell m} & \text{otherwise} \end{cases} \quad (18)$$

Now we can approximate \mathbf{H}^{-1} as follows [20]:

$$\begin{aligned} \mathbf{H}^{-1} &= (\mathbf{D} + \mathbf{B})^{-1} \\ &= \mathbf{D}^{-\frac{1}{2}} \left(\mathbf{I} + \mathbf{D}^{-\frac{1}{2}} \mathbf{B} \mathbf{D}^{-\frac{1}{2}} \right)^{-1} \mathbf{D}^{-\frac{1}{2}} \\ &\approx \mathbf{D}^{-\frac{1}{2}} \left(\mathbf{I} - \mathbf{D}^{-\frac{1}{2}} \mathbf{B} \mathbf{D}^{-\frac{1}{2}} \right) \mathbf{D}^{-\frac{1}{2}} \\ &= \mathbf{D}^{-1} - \mathbf{D}^{-1} \mathbf{B} \mathbf{D}^{-1} \end{aligned} \quad (19)$$

where the third equation with the approximation sign is analogous to the first order Taylor series expansion $\frac{1}{1+x} \approx 1 - x$ near $x = 0$. Note than Eq.(19) enables the computation of \mathbf{H}^{-1} and the update of local variables to be carried out (approximately) locally as follows:

$$\mathbf{x}_{ij}^c \leftarrow \mathbf{x}_{ij}^c - \mathbf{D}_{\ell}^{-1} \nabla_{\ell} f + \mathbf{D}_{\ell}^{-1} \sum_{m \in E(ij) \cup ij} \mathbf{B}_{\ell m} \mathbf{D}_m^{-1} \nabla_m f \quad (20)$$

Since $\mathbf{B}_{\ell m} = \mathbf{0}$ if $m \neq \ell, m \notin E(\ell)$, the computation of each term in Eq.(20) requires only the information of agent ij and its neighbors.

In summary, the approximated Newton’s method has two levels of iterations. In the outer iteration, problem (OP4) is formed by obtaining \mathbf{A}_{ij}^c , \mathbf{b}_{ij}^c , and \mathbf{x}_{ij}^c from previous iteration; in the inner iteration, variables are updated using Eq.(20).

IV. SIMULATION RESULTS

This section presents a simulation study to validate the proposed distributed algorithm for network reconfiguration. We first describe the test system and then discuss the results from both centralized and the proposed distributed algorithm. In particular, the computation speed of the ADMM algorithm and our proposed approximated Newton’s method is compared.

The distribution test feeder described in [21] is used in the simulation. The distribution test feeder is shown in Fig.2

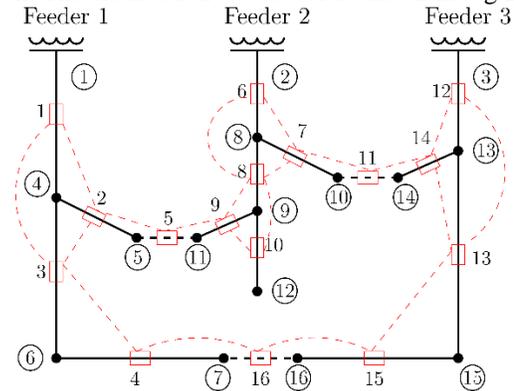


Fig. 2. 16-bus test feeder

where the dots represent load buses; the solid lines represent sectionalizing switches and dashed lines represent tie switches. Agents are represented by a red box. The edges of the communication graph are represented by red dashed lines.

Initially, all sectionalizing switches are closed and all tie switches are open. The global optimum solution found by the centralized algorithm was reported in [21]. The network

reconfiguration results of the proposed distributed algorithm and the centralized one are shown in Table 1. It can be seen that the proposed method found the same global optimum solution as that of the centralized algorithm.

Table.1 Numerical results

	Original configuration	Centralized Method (MICP)	The proposed method
Opened switches	5, 11, 16	7, 9, 16	7, 9, 16
Power loss (kW)	511.4	466.1	466.1
Loss reduction	-	8.86%	8.86%
Voltage magnitude (p.u.)	Vmax=1.000 (Bus 1,2,3)	Vmax=1.000 (Bus 1,2,3)	Vmax=1.000 (Bus 1,2,3)
	Vmin=0.969 (Bus 12)	Vmin=0.972 (Bus 12)	Vmin=0.972 (Bus 12)

In order to evaluate the computation speed of the ADMM algorithm and our proposed approximated Newton's method, we conducted testing using 20 different radial network configurations of the test system. The tunable parameters are $\rho = 1$ for ADMM and $\gamma = 1.5$ for the approximated Newton's method. To make a fair comparison, both of the algorithms terminate when the solutions reach the same level of accuracy. The computation time are reported in Table 2. As shown in the table, the proposed approximated Newton's method achieves roughly 5 times speed up compared to the ADMM.

Table.2 Speed comparison of ADMM and approximated Newton's method

	ADMM	Approximated Newton
Min (second)	3.87	0.64
Max (second)	18.32	7.66
Average (second)	10.41	2.05

V. CONCLUSIONS

This paper proposes a distributed algorithm to solve the distribution network reconfiguration problem. The proposed algorithm can be implemented on a group of switch agents in the distribution network, which work collaboratively via neighbor-to-neighbor communication to find the optimum network reconfiguration. The simulation results show that the distributed algorithm correctly finds the global optimum solution on a 16-bus distribution test system. In addition, the proposed approximated Newton's method dramatically improves the computation speed of the distributed algorithm.

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