Phase Balancing in Power Distribution Network with Data Center

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ABSTRACT

High degree of unbalance in electric distribution feeders can significantly affect power quality, damage electrical equipment, and result in tripping of protective devices. If not properly managed, integration of new data center and distributed energy resources into the power distribution network will exacerbate the problem. This paper proposes a new paradigm which coordinates the operation of data center and distributed energy resources to reduce phase unbalance and improve the reliability and efficiency of electric distribution networks. The coordination scheme is implemented within the framework of a distribution system operator managed electricity market. The proposed phase balancing algorithm with data center is validated using a modified IEEE distribution test feeder. The simulation results show the proposed data center and distributed energy resources coordination scheme not only significantly reduces the degree of unbalance of distribution feeders but also results in sizable reduction in data center electricity costs.

CCS CONCEPTS

• **Hardware → Enterprise level and data centers power issues**;

KEYWORDS

Demand response, phase balancing, data center, power distribution network

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NOMENCLATURE

Ar^p	Vector of requests arrival rate for virtual machines on phase p.
${a_0}^p, {a_1}^p$	Coefficients for the linear response time function of the servers on phase p.
a_i^m, b_i^m	Bid curve coefficients for flexible loads at node i with phase m.
Cg	Supply offer price at the reference bus.
$(DF_i^m)^p$	Phase p's delivery factor at node i with phase m.
E_q^g	Fictitious nodal demand at node q with phase p.
FP, FQ	Set of real and reactive power branch flows.
$GSFP_{ik\_q}^{p\_g}$	Generation shift factor for real power flow of the branch which connects node i and k with phase p when power injection is at node q with phase g.
N	Total number of nodes including the swing bus.
PD_q^g	Real power of total demand at node q with phase g.
P_{dc}^m	Real power demand of data center on phase m.
${P_{dyn}}_i^p$	Dynamic power demand of the i-th server on phase p.
${P_f}_i^m$	Flexible loads at node i with phase m.
PG^m	Real power of generation at the reference bus on phase m.
${P_{idle}}_i^p$	Idle power demand of the i-th server on phase p.
P_{loss}^p	Total real power losses on phase p.
$PLimit_{ik}^p$	Real power flow limit between node i and k with phase p.
${P_S}_i^p$	Real power demand of the i-th server on phase p.
R^{pr}	Vector of equivalent requests of moving VMs from the servers on phase p to phase r.
T_r^p	The response time of the servers on phase p.
T_{SLA}	The service level agreement limit.
γ	Power imbalance limit between phases.

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1 INTRODUCTION

With the cloud-computing industry experiencing exponential growth, data center is becoming a significant electricity consumption source. According to the U.S. data center energy usage report [12], an estimated 70 billion kWh of electricity is consumed by data center in 2014 representing about 1.8% of total U.S. electricity consumption. The electricity usage by data center is also expected to increase 4% annually in the next five to ten years. Electricity is also the fastest growing operational costs of a medium-scale or large-scale data center which pays millions of dollar in annual electricity bill. Therefore, it is imperative for data centers to improve operational efficiency and reduce electricity costs.

Data center typically receives electric power through a three-phase transformer connected to an unbalanced three-phase distribution feeder. In power distribution systems, unbalanced feeders with unbalanced electric loadings are very common. High degree of unbalance in distribution feeders can significantly affect power quality, damage electrical equipment and appliances [16], and result in highly unbalanced three-phase voltages. In addition, unbalanced systems are more likely to experience overloading on a phase wire or a neutral wire. The overloading will not only cause overheating but also lead to tripping of a protective device if there is large neutral current [3]. If not properly managed, the integration of a new data center into an existing distribution network can exacerbate the degree of unbalance in the distribution circuit and may require expensive distribution system upgrades.

This paper proposes a new paradigm which coordinates the operation of data center and distributed energy resources (DERs) to reduce phase unbalance and improve the reliability and operational efficiency of electric distribution networks. The coordination scheme is implemented within the framework of a distribution system operator (DSO) managed electricity market. The data center and DERs proactively participate in the resource dispatch and market price formation processes [19]. The electricity sold and purchased on three different phases are settled using the three-phase locational marginal prices (LMPs) [17]. The LMPs on the three phases serve as the coordination signals with high prices discouraging electricity consumption and low prices encouraging consumption on a phase wire.

The existing research in the field of green computing tries to improve data center energy efficiency at five different levels. At the processor level, dynamic voltage and frequency scaling (DVFS) techniques have been shown to be highly effective in improving the energy-efficiency [5, 8, 13]. At the server level, various scheduling policies have been designed to create opportunities for deep sleep [10, 11]. At the data center level, virtual machine migration and autoscaling techniques have been proposed to optimize energy consumption [9, 15, 20]. The trade-off between minimizing energy cost and maximizing cloud computing services for a data center was analyzed in [4]. At the transmission grid level, receding horizon control approach [7], game theoretic approach [14, 18], and distributed control approach [21] have been developed to coordinate the operations of data centers and distributed energy resources such as renewable generation and electric vehicles.

Our work differs from the existing research by exploring the ways to coordinate the operations of data centers and distributed energy resources at the electric power distribution system level. The existing work ignored the three-phase electrical wiring within a data center and modeled only balanced three-phase power systems. In this paper, we filled the knowledge gap by carefully modeling the realistic three-phase unbalanced electric power distribution network and the data center. We propose solving the distribution network phase balancing problem by shifting computational loads among the servers connected to three different phase wires in a data center.

The unique contributions of this paper are listed as follows.

• To the best of our knowledge, this is the first paper that solves the distribution network phase balancing problem by shifting computational loads among the servers connected to three different phase wires in a data center.

• This paper proposes an iterative scheme to coordinate the operations of data center and distributed energy resources within a DSO managed electricity market (Section 2).

• We also derived the three-phase Locational Marginal Prices (LMPs) sensitivities in a distribution market and embedded the price sensitivities into the data center's electricity cost minimization problem (Section 3).

• The operational coordination strategy for data center and DERs is very effective in reducing phase unbalance, improving distribution network operational efficiency and reliability. The simulation results show that the degree of unbalance of a distribution feeder can be reduced by up to 100% and the electricity cost of the data center decreases by more than 4.0% (Section 4).

The remainder of this paper is organized as follows. Section 2 provides an overview of the coordination scheme for the data center and DERs within the framework of a DSO managed electricity market. Section 3 formulates the DSO market clearing problem, derives the three-phase LMPs sensitivities, and presents the electricity cost minimization problem formulation for a data center. The numerical study results are shown in Section 4. The conclusions are stated in Section 5.

2 OVERALL FRAMEWORK

The overall framework of coordinating the operation of data center and DERs to reduce phase unbalance and improve operational efficiency of electric distribution networks is depicted in Fig. 1. The coordination framework involves interactions among three decision making entities in the DSO managed electricity market. They are the DSO, the DERs, and the data center.

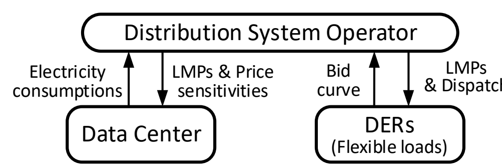

Figure 1: Overall coordination framework

2.1 Distribution System Operator

The DSO manages the distribution electricity market and adopts a transactive and iterative approach to coordinate the operations of DERs and data center. In each iteration of the market clearing process, the DSO tries to maximize the social welfare in the distribution circuit with the three-phase DC optimal power flow (DCOPF) algorithm. The inputs to the three-phase DCOPF algorithm include the price-sensitive energy bid curves from the DERs, the electricity consumption target from the data center, and forecast for fixed loads in the distribution feeder. The outputs of the three-phase DCOPF algorithm include the three-phase LMPs, the dispatch levels for the DERs, and the LMPs sensitivities. After the distribution electricity market is cleared, the DSO will send the LMPs and the dispatch operating points to the DERs, the LMPs and the prices sensitivities to the data center. The three-phase DCOPF algorithm and the derivation for three-phase LMPs sensitivities are described in Sections 3.1 and 3.2.

2.2 Distributed Energy Resources

The DERs proactively participate in the DSO managed distribution electricity market by submitting their single-phase price-sensitive energy bid curves on an hourly basis. The single-phase bid curves can be constructed based on the resource control model and the customers' preferences as described in [19]. If the DER is a load resource, then a price-sensitive demand bid curve will be submitted to the DSO. The demand bid curve is a graphical representation of the relationship between quantity of electricity demand and customer's willingness-to-pay. The demand bid curve must be monotonically decreasing in the price-quantity space.

2.3 Data Center

It is not straightforward to construct energy bid curves for data centers due to the electrical and computational coupling among the servers on the three individual phases. To illustrate this coupling effect, a simplified electrical wiring diagram of a typical data center is shown in Fig. 2. As shown in the figure, the computational load and electrical consumption can be partially shifted among servers connected to different phase wires by migrating computational services. However, the total electrical consumption and service level agreement constraints which link all three phases depend on the level of load shifting. Therefore, it is difficult for the data center operator to decompose its six dimensional bid curves with prices and loads of the three phases directly into three independent two-dimensional energy bid curves.

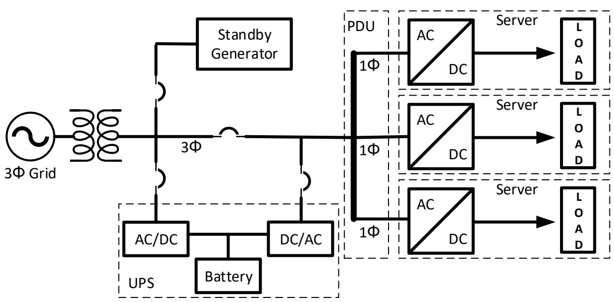

Figure 2: Electrical wiring diagram of a data center.

In order to enable the proactive participation of data center in distribution electricity market, an iterative approach is proposed to facilitate the negotiation between the DSO and the data center. In each iteration of the negotiation, the data center operator first determines the optimal load shifting plan with the latest three-phase LMPs and price sensitivities information. The data center operator then submits its electricity consumption targets for the three individual phases to the DSO. The DSO will clear the distribution electricity market and sends the updated three-phase LMPs and price sensitivities to the data center. The data center electricity cost minimization algorithm is described in Section 3.3.

3 PROBLEM FORMULATION

3.1 Distribution Network Optimization with Three-phase DCOPF

The DSO adopts a transactive approach to coordinate the operations of DERs and data center in a distribution electricity market. To clear the distribution electricity market, the DSO runs the three-phase DCOPF algorithm [17]. The objective of the three-phase DCOPF problem is to maximize the social welfare, which is the summation of the surplus of electricity customers and energy suppliers in a distribution system as shown in equation (1). The energy supplier from the transmission system is assumed to be submitting a supply offer from the reference bus to the DSO. The flexible loads are located at all other buses in the distribution network. The bid curves of DERs or flexible loads are assumed to be linear for simplicity. Hence, the customer's willingness-to-pay function at node i with phase m for electricity with the amount of ${P_f}_i^m$ is in a quadratic form. The operating constraints in the distribution system include the real power balance constraints (2), the distribution line thermal limit constraints (3), and the phase imbalance constraints (4).

$$\max_{P_f} \sum_{n=2}^{N} \sum_{m=1}^{3} (a_i^m ({P_f}_i^m)^2 + b_i^m {P_f}_i^m) - Cg \sum_{m=1}^{3} PG^m \tag{1}$$

subject to

$$PG^p - \sum_{i=2}^{N} \sum_{m=1}^{3} (DF_i^m)^p \cdot PD_i^m$$
$$+P_{Loss}^p(FP) - P_{Loss}^p(FQ) = 0, p = 1,2,3 \tag{2}$$

$$|\sum_{q=2}^{N} \sum_{g=1}^{3} GSFP_{ik\_q}^{p\_g} \cdot \left(-PD_q^g - E_q^g\right)| \le PLimit_{ik}^p,$$
$$\forall i,k \quad and \quad i \ne k \tag{3}$$

$$|\sum_{n=2}^{N} P_n^i - \sum_{n=2}^{N} P_n^j| \le \gamma,\ i,j = 1,2,3 \ \ and \ \ i \ne j \tag{4}$$

The three-phase generation shift factors $GSFP_{ik\_q}^{p\_g}$, fictitious nodal demands (FNDs) E_q^g, and delivery factors (DFs) $(DF_i^m)^p$ are derived from three-phase power flow equations and three-phase admittance matrix. The three-phase GSFs, FNDs, and DFs are very different from that of the single-phase systems. These differences arise from the mutual coupling among three phases of distribution system lines. The derivation details can be found in [17]. The iterative algorithm used to solve the FND-based three-phase DCOPF problem is described in Algorithm 1. The outputs of the iterative three-phase DCOPF algorithm include the dispatch for flexible loads, generation, the LMPs at each bus with all three phases.

The three-phase LMPs can be decomposed using the Lagrangian function of three-phase DCOPF problem. Define λ^p as the Lagrange multiplier of the constraints (2), μ_b^{p+} and μ_b^{p-} as the Lagrange multipliers of the constraints (3), and μ^{pm+} and μ^{pm-} as the Lagrange multipliers of the constraints (4). As shown in [17], the LMP of node i with phase g can be decomposed as

$$LMP_i^g = \sum_{p=1}^{3} \lambda^p (DF_i^g)^p + \sum_{b=1}^{B} \sum_{p=1}^{3} \mu_b^{p\prime} GSFP_{b\_i}^{p\_g} + \mu^{g\prime\prime} \tag{5}$$

Where $GSFP_{b\_i}^{p\_g}$ is generation shift factor for real power flow of the branch b with phase p when power injection is at node i with phase g. B is the set of total branches. The Lagrange multipliers $\mu_b^{p\prime}$ and $\mu^{g\prime\prime}$ are defined as

$$\mu_b^{p\prime} = \mu_b^{p+} - \mu_b^{p-}$$

$$\mu^{g\prime\prime} = \begin{cases} \mu^{12+} + \mu^{13+} - \mu_i^{12-} - \mu^{13-} & \text{if } g = 1; \\ -\mu^{12+} + \mu^{23+} + \mu_i^{12-} - \mu^{23-} & \text{if } g = 2; \\ -\mu^{13+} - \mu^{23+} + \mu_i^{13-} + \mu^{23-} & \text{if } g = 3. \end{cases}$$

$\mu_b^{p\prime}$ is the equivalent Lagrange multiplier of the thermal limit constraints (3) for branch b with phase p. $\mu^{g\prime\prime}$ is the equivalent Lagrange multiplier of the phase imbalance constraints (4) related to loading limit on phase g.

Algorithm 1 Iterative algorithm for three-phase DCOPF

Initialize DFs, FNDs, and power losses.
Solve linear optimization problem using (1)-(4).
while 1 **do**
  Update the values of FNDs, power losses, and DFs;
  Solve linear optimization problem using (1)-(4);
  if the difference of the dispatch of loads and generation between the current iteration and previous iteration's result is larger than the pre-defined tolerance **then**
    break;
  end if
end while

3.2 Three-phase LMPs Sensitivities

The sensitivities of single-phase LMPs in transmission electricity market can be derived using a perturbation approach [1]. In this section, we extend the derivation for sensitivities to three-phase LMPs in the distribution electricity market. In particular the three-phase LMPs sensitivities with respect to changes in bus demands are derived here.

Denote $h(\boldsymbol{x}, \boldsymbol{a})$ and $g(\boldsymbol{x}, \boldsymbol{a})$ as the set of equality and inequality constraints of three-phase OPF problem respectively. $\boldsymbol{x}$ represents the load and generation dispatch variables. $\boldsymbol{a}$ stands for the vector of electricity demands at all nodes.

Define z as

$$z = f(\boldsymbol{x}, \boldsymbol{a}) = Cg \sum_{m=1}^{3} PG^m - \sum_{n=2}^{N} \sum_{m=1}^{3} (a_i^m ({P_f}_i^m)^2 + b_i^m {P_f}_i^m) \quad (6)$$

The three-phase DCOPF can be written in compact form as

$$\min_x z = f(\boldsymbol{x}, \boldsymbol{a}) \quad (7)$$

subject to

$$h(\boldsymbol{x}, \boldsymbol{a}) = 0 \quad (8)$$

$$g(\boldsymbol{x}, \boldsymbol{a}) \leq 0 \quad (9)$$

By applying the perturbation technique on top of the Karush-Kuhn-Tucker first-order optimality conditions, we can obtain the sensitivities with respect to the electricity demands [2].

$$\left[d\boldsymbol{x}^T, d\boldsymbol{\lambda}^T, d\boldsymbol{\mu}^T, dz\right]^T / d\boldsymbol{a} = U^{-1}\boldsymbol{S} \quad (10)$$

where $\boldsymbol{\lambda}$ and $\boldsymbol{\mu}$ are the Lagrange multipliers vector for the equality and inequality constraints respectively. Matrix U and vector $\boldsymbol{S}$ can be derived as

$$U = \begin{bmatrix} F_x & 0 & 0 & -1 \\ F_{xx} & H_x^T & G_x^T & 0 \\ H_x & 0 & 0 & 0 \\ G_x & 0 & 0 & 0 \end{bmatrix} \quad (11)$$

$$\boldsymbol{S} = -\left[F_a^T, F_{xa}^T, H_a^T, G_a^T\right]^T \quad (12)$$

where F_x and F_a are the first order derivatives of the objective function with respect to $\boldsymbol{x}$ and $\boldsymbol{a}$. F_{xx} is the second order derivative of the Lagrange function with respect to $\boldsymbol{x}$. F_{xa} is the second order derivative of the Lagrange function with respect to $\boldsymbol{x}$ and then $\boldsymbol{a}$. H_x, H_a, G_x, and G_a are the first order derivatives of the equality and binding inequality constraints with respect to $\boldsymbol{x}$ and $\boldsymbol{a}$. The detailed derivation can be found in [2]. Taking derivatives on both sides of equation (5) with respect to fixed demand of node u phase v, we get the LMPs sensitivities of node i phase g with respect to P_u^v.

$$\frac{\partial LMP_i^g}{\partial P_u^v} = \sum_{p=1}^{3} \frac{\partial \lambda^p}{\partial P_u^v} (DF_i^g)^p + \sum_{b=1}^{B} \sum_{p=1}^{3} \frac{\partial \mu_b^{p\prime}}{\partial P_u^v} GSFP_{b\_i}^{p\_g} + \frac{\partial \mu^{g\prime\prime}}{\partial P_u^v} \quad (13)$$

Where the derivatives of Lagrange multipliers of non-binding inequality constraints with respect to P_u^v are zeros. Now, the derivatives of Lagrange multipliers in equation (10) can be substituted into (13) to calculate the three-phase LMPs sensitivities.

3.3 Data Center Electricity Cost Minimization

Since majority of the electrical appliances used in the data center cooling systems consume three-phase electrical power, they can not be leveraged to address phase balancing problem. Hence, they are not modeled in this paper. In this section, the data center electricity cost minimization problem only considers the electricity cost from the servers. Assume there are N_S^p servers on phase p. Define M^{rp} as a $N_0^r \times 1$ binary variable vector. If the i-th element of M^{rp} is 1, it means that the i-th VM is moving from a server on phase r to phase p. N_0^r denotes the number of VMs running on the servers of phase r initially.

The objective function of the data center is to minimize its electricity cost as shown in equation (14). The electricity costs of all servers equal to the dot product of updated LMPs vector after the VMs live migration and the vector of per phase electricity consumption of servers $\boldsymbol{P_{dc}}$. In the objective function, $\boldsymbol{LMP} = [LMP^1, LMP^2, LMP^3]^T$ denotes the LMPs for the three different phases at the data center bus and $\boldsymbol{P_{dc}} = [P_{dc}^1, P_{dc}^2, P_{dc}^3]^T$ denotes the electricity consumption from the servers on the three-phases. $\boldsymbol{P_{dc}^0}$ stands for the initial value of $\boldsymbol{P_{dc}}$. The updated LMPs vector after the VMs live migration is estimated by using the LMPs sensitivities $\partial \boldsymbol{LMP}/\partial \boldsymbol{P_{dc}}$, which is a 3×3 matrix. The LMPs sensitivities are introduced into the data center electricity cost minimization process to serve as a damping factor which prevents oscillation of computing load shifts in the data center and the DSO's negotiation process. Without the LMP sensitives, the data center will aggressively move its load from the phase with higher price to the phase with lower price without considering the impacts of the move on

LMPs. This could prevent the iterative negotiation process between the DSO and the data center from reaching an equilibrium point.

The electricity consumption of a particular phase equals to the sum of electricity consumptions from each individual server connected to the phase wire in equation (16). The electricity consumption of each server includes a dynamic component and an idle component as shown in equation (16) [4]. The dynamic component of the server electricity consumption is closely related to the server utilization rate which is modeled in equation (17). The utilization rate of server i on phase p, U_i^p, can be estimated based on the requests arrival rate for servers on phase p and the live migration of VMs [15]. During the migration period, computational loads increase on the servers which the VMs migrated to and from. After the migration period, computational loads increase/decrease on the servers which the VMs migrated to and from. For simplification purpose, a uniform utilization of servers is assumed for each phase.

$$\min_{M^{pr}, p, r=1,2,3, p\neq r} [\boldsymbol{LMP} + \frac{\partial \boldsymbol{LMP}}{\partial \boldsymbol{P_{dc}}}(\boldsymbol{P_{dc}} - \boldsymbol{P_{dc}^0})]^T \boldsymbol{P_{dc}} \tag{14}$$

subject to

$$P_{dc}^p = \sum_{i=1}^{N_S^p} P_{S_i}^p, \; p = 1,2,3 \tag{15}$$

$$P_{S_i}^p = P_{dyn_i}^p * U_i^p + P_{idle_i}^p, \quad \forall i, \; p = 1,2,3 \tag{16}$$

$$U_i^p = \Big\{(Ar^p)^T \cdot \mathbf{1} - \sum_{r=1, r\neq p}^3 \Big[(1 - \frac{T^{pr}}{T_{int}})(Ar^p)^T \cdot M^{pr}\Big] + \sum_{r=1, r\neq p}^3 \Big[(1 - \frac{T^{rp}}{T_{int}})(Ar^r)^T \cdot M^{rp}\Big] + \sum_{r=1, r\neq p}^3 R^{rp} M^{rp} + \sum_{r=1, r\neq p}^3 R^{pr} M^{pr}\Big\} / \sum_{i=1}^{N_S^p} R_{cap_i^p}, \quad \forall i, \; p = 1,2,3 \tag{17}$$

In equation (17), T^{pr} denotes the VM migration time from phase p to r and T_{int} is the market clearing time step. Ar^p denotes the vector of requests arrival rate of VMs on phase p before the migration. It has a dimension of $N_0^p \times 1$. $R_{cap_i^p}$ denotes the request processing capability of server i on phase p. $\mathbf{1}$ is a vector of ones with the same dimension as Ar^p.

The data center electricity cost minimization formulation also includes two sets of constraints related to the service level agreement (18) and (19).

During the live migration of VMs, the response time constraints in the service level agreement is modeled as

$$T_r^p = a_0{}^p + a_1{}^p \cdot \Big((Ar^p)^T \cdot \mathbf{1} + \sum_{r=1, r\neq p}^3 R^{rp} M^{rp} + \sum_{r=1, r\neq p}^3 R^{pr} M^{pr}\Big) / N_S^p \leq T_{SLA}, \quad p = 1,2,3 \tag{18}$$

After the migration of VMs, the response time constraints in the service level agreement is modeled as

$$T_r^p = a_0{}^p + a_1{}^p \left((Ar^p)^T (\mathbf{1} - \sum_{r=1, r\neq p}^3 M^{pr}) + \sum_{r=1, r\neq p}^3 (Ar^r)^T M^{rp}\right) \leq T_{SLA}, \quad p = 1,2,3 \tag{19}$$

4 NUMERICAL STUDY

4.1 Simulation Setup

The standard IEEE 4-bus distribution test feeder [6] is modified to validate the effectiveness of the proposed phase balancing algorithm with data center. The data center is located at node 2. The fixed loads and flexible loads are located at node 4. The transmission system is assumed to supply electric power to the distribution network through the distribution substation at a price of $0.6/kWh$. The total amount of fixed demands and price-sensitive demands are summarized in Table 1. Two simulation cases with different degree of unbalance are created. The distribution feeder is slightly unbalanced in case 1 and heavily unbalanced in case 2. The power imbalance limit between any two phases in the distribution feeder is set to be $60KW$.

The price-sensitive demand bid curves of flexible loads on the three phases are assumed to be linear functions as $Price^1 = 1 - P_f^1/200$, $Price^2 = 1 - P_f^2/250$, and $Price^3 = 1 - P_f^3/300$. $Price^1$, $Price^2$, and $Price^3$ are the bidding prices for the three phases. The price ranges of the three demand bid curves are from $0.1/kWh$ to $1/kWh$.

Table 1: Fixed and Flexible Load Profile

Node 4		Phase A	Phase B	Phase C
Fixed Load Capacity (KW)	Case 1	460	500	530
	Case 2	420	500	580
Flexible Load Capacity (KW)		180	225	270

In the simulation, the data center powers 400 servers on each of the three phase wires through a PDU. The maximum power rating of each server is assumed to be $500W$. The maximum dynamic power and idle power of each server are $400W$ and $100W$ respectively. The data center is operating in a homogeneous computing environment. It is assumed that a total of 1200 VMs are running on the servers and they are distributed uniformly on all three phases. Each VM processes 200 requests per second. It is also assumed that the servers on each phase can host up to 800 VMs. Live migration is implemented in the data center where a VM is moved from a server on one of the phases to another server on the other phase without the need to bring down the VM instance. It is assumed that live migration can be finished within 10 minutes. The computational cost of live migration of each VM is assumed to be equivalent to the processing time for 24 request/s. The average response time of each server is a linear function with respect to the number of requests as shown in equations (18) and (19) with parameters $a_0 = 0.2s$ and $a_1 = 8 \times 10^{-4} s/request$. The upper limit of response time is set at $500ms$ according to the service level agreement.

4.2 Simulation Results

The LMPs and electricity consumptions on all three phases of the data center are calculated with and without phase balancing for the two different unbalance cases. As shown in Table 2, in both cases electricity load shifts from servers on phase c which has higher price to phase a which has lower price. After phase balancing, the degree of unbalance of the distribution feeder is reduced which leads to smaller price difference between phase a and phase c. The amount of load shift and reduction in price difference is higher in the heavily unbalanced case than the slightly unbalanced case.

Table 2: LMPs and Data Center Electricity Consumption

Case	Phase balancing	Price & Power	Phase A	Phase B	Phase C
1	Without	($/KWh)	0.5048	0.6015	0.6988
		(KW)	120.0	120.0	120.0
	With	($/KWh)	0.5454	0.6015	0.6582
		(KW)	130.5	120.0	110.0
2	Without	($/KWh)	0.3241	0.6015	0.8795
		(KW)	120.0	120.0	120.0
	With	$(/KWh)	0.4543	0.6015	0.7494
		(KW)	153.8	120.0	87.9

The electricity cost of data center and total surpluses are reported in Table 3. As shown in the table, the phase balancing algorithm not only reduces the electricity bill of the data center but also increases the total surpluses of the flexible loads and the supplier. The savings in the heavily unbalanced case is much more significant than the slightly unbalanced case. In case 2, the phase balancing algorithm reduces the electricity bill of the data center by more than 4.0% and increases the total surpluses by 35%. Note that the savings also depend on the price elasticity of demand.

Table 3: Electricity Cost and Total Surpluses

	Case	Without Phase Balancing	With Phase Balancing
Electricity cost of data center ($)	1	216.6	215.8
	2	216.6	207.9
Total Surpluses ($)	1	57.3	58.9
	2	39.9	53.9

Instead of keeping the phase imbalance limit γ in equation (4) constant at 60 KW, we try to reduce the phase imbalance as much as possible without making the savings of the data center and the flexible loads worse. The simulation results show that the percentage reduction in phase imbalance varies with the electric power rating of the data center. In the simulation setup of case 2, the power rating of the data center is about 25% of the total feeder demand. In this case, our proposed algorithm can reduce the phase imbalance by 100%. In other words, by shifting computational loads in the data center, the electric loads on the distribution feeder can be completely balanced. By gradually reducing the size of the data center by 40%, 45%, and 50%, the reduction in phase imbalance also decreases to about 93%, 83%, and 73%.

5 CONCLUSIONS

This paper develops an iterative scheme to coordinate the operations of data center and DERs to tackle the electric distribution network phase balancing problem. We also derived the three-phase LMPs sensitivities in a distribution electricity market and integrated the price sensitivities into the data center's electricity cost minimization algorithm. Comprehensive simulations are conducted on a modified IEEE distribution test feeder to demonstrate the effectiveness of our proposed phase balancing algorithm. The simulation results showed the degree of unbalanced of a distribution feeder is decreased by up to 100%, and the electricity cost of a data center is reduced by more than 4.0%.

In the future, we plan to develop a scalable, decentralized, and distributed control architecture to coordinate the operations of a large number data centers and DERs.

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