# An Efficient Surrogate Subgradient Method within Lagrangian Relaxation for the Payment Cost Minimization Problem

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***Abstract***[1]**-Studies have shown that for a given set of bids, Payment Cost Minimization leads to lower customer payments as compared to Bid Cost Minimization. In order to provide a thorough analysis of the two mechanisms an efficient solution methodology is required. It has previously been shown that the surrogate optimization within the Lagrangian relaxation framework can lead to savings in the CPU time while ensuring a high-quality solution. This paper develops an efficient methodology to solve Payment Cost Minimization using the surrogate optimization framework and the branch-and-cut method. In the presented methodology the problem structure is exploited using Lagrangian relaxation and the relaxation of the integrality constraints is exploited using branch-and-cut. The resulting method is further improved by using additional cutting planes that reduce the search space and by the advanced start to reinitialize the decision variables at each iteration. For large Payment Cost Minimization problems, the method can find significantly better feasible solutions within less CPU time than that obtained by standard branch-and-cut methods implemented in commercial MIP solver. The methodology developed in this paper is generic and can be used for solving other optimization problems.**

## I. Introduction.

PRESENTLY, most Independent System Operators (ISOs) in the United States minimize the total bid cost by using the Bid Cost Minimization (BCM) auction mechanism. After the total bid cost is minimized, the customer payments are determined by a different mechanism that assigns locational marginal prices (LMPs). The total customer payment cost based on locational marginal prices is typically higher than the total minimized bid cost [10], [15]. An alternative auction mechanism discussed in the literature (i.e., in [10]) to determine the total customer payment cost is the Payment Cost Minimization (PCM).

Studies have shown that both the PCM and the BCM problems are NP-hard, however, the BCM problem can be solved more efficiently by commercial MIP solvers than the PCM problem [9]. The principal difference between the PCM and the BCM problems is the method for defining the prices. In the PCM problem, the market-clearing price (MCP) is determined by the marginal units. Therefore, the MCP constraints couple individual offers in addition to the system demand and the start-up cost constraints. The presence of such system-wide constraints makes the PCM problem hard to separate the problem into individual subproblems within the Lagrangian relaxation framework.

Since the Payment Cost Minimization is mixed-integer and can be efficiently linearized, it can be solved by the optimization methods that were specifically designed for solving linear mixed-integer problems (e.g., branch-and-cut). Within branch-and-cut [4], [12], and [14], the cutting planes that are valid across all constraints (especially the MCP constraint) of the PCM problem are hard to obtain. This results in difficulties of obtaining feasible solutions, and providing a tight lower bound. Branching operations, which are needed to tighten the bounds on the optimal value, increase drastically when the size of the problem increases. Therefore, most standard optimization methods (e.g., branch-and-cut, Lagrangian relaxation) become inefficient and the new efficient solution methodology is required.

The surrogate subgradient method [6], [7], [10], and [16] is a variation of the subgradient method that aims at improving convergence of the Lagrangian relaxation method. The gist of the surrogate optimization approach is to find an optimal (or good suboptimal) dual solution quickly in order to obtain a feasible solution by removing the infeasibilities. Since the dual function and the subgradient are hard to obtain, in order to simplify the optimization process, the relaxed problem is not optimized exactly. Rather, a surrogate dual and the surrogate subgradient are obtained. Under simple conditions, the surrogate directions form an acute angle with the direction toward the optimal multipliers.

The Lagrangian relaxation of the linearized PCM problem is a linear mixed-integer problem and can be optimized by the branch-and-cut method. In the method, the integrality constraints are relaxed and the integral solution is found by adding valid cuts. The difficulty that the method is facing (similarly to the original problem) is the presence of the MCP constraint. Therefore, in order to obtain surrogate

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subgradient directions efficiently, additional inequalities that better define integral solutions are required.

The literature review is presented in Section II. The linearized PCM problem formulation is presented in Section III. In the PCM mechanism, the total payment cost is minimized. The objective function consists of two terms, namely, the total payment cost and the total start-up cost. The optimization is performed subject to the system demand, the start-up cost, the MCP, and the generation capacity constraints. The market clearing prices (MCPs) are determined by the marginal units. The coupling of start-up cost, system demand and MCP constraints make the PCM problem hard to separate into smaller subproblems.

The solution methodology is presented in Section IV. In PCM the system demand constraints are relaxed. The surrogate optimization condition, which is used to ensure that the surrogate subgradient directions are proper, can also serve as a valid cut to reduce the search space of the algorithm, thus leading to higher computational efficiency. Additional valid cuts and constraints help to find the surrogate subgradient direction quickly by reducing the search space of the algorithm. The relaxed PCM problem can be optimized more efficiently by using the combination of the surrogate optimization and standard branch-and-cut methods.

Numerical results, presented in Section V, illustrate the effectiveness of the approach. When implemented in CPLEX, the Lagrangian relaxation and surrogate optimization approach handles large problems more efficiently than the standard optimization methods of CPLEX.

## II. Literature Review

Several approaches have been developed and used for solving optimization programming problems, linear integer and mixed-integer programming problems in particular. They are Lagrangian relaxation and subgradient optimization [2], [3], [5], [11], Lagrangian relaxation and surrogate subgradient optimization [6], [7], [16], and branch-and-cut [4], [12] and [14].

Lagrangian relaxation is frequently used together with the gradient optimization method for differentiable problems and with the subgradient optimization for non-differentiable problems. It has been shown that the gradient method converges with constant step size for differentiable problems under the Lipschitz condition [5] and that the subgradient method converges for the non-differentiable programming problems with diminishing and dynamic step size [11]. Perhaps one of the most recent and exhaustive surveys on the subgradient methods for convex optimization is [2].

The surrogate subgradient approach was specifically developed to optimize large and complex dual functions within the Lagrangian relaxation framework [16]. In the surrogate subgradient method, a proper subgradient direction toward the optimal $\lambda^*$ can be obtained without obtaining the exact optimum of the relaxed problem at every iteration. In other words, a good subgradient direction is obtained without optimizing the relaxed problem over the entire feasible set. An approximate solution of the relaxed problem has to satisfy the surrogate optimization condition. This condition ensures that a surrogate subgradient direction forms an acute angle with the direction toward the optimal multipliers [16] provided the optimal dual value is used to update the stepsize. This condition also guarantees the lower bound property of the surrogate Lagrangian dual. After an approximate solution of the relaxed problem is found, the Lagrange multipliers are updated. The computational effort required to obtain an approximate solution is much less compared to that required to obtain the exact optimum at each iteration. Therefore, the method's efficiency enables solving very large and complex problems [6], [16]. It was shown that for separable problems, not all subproblems need to be solved optimally to find a proper surrogate subgradient direction [16]. In addition, the surrogate subgradient method can find good directions compared to the subgradient methods that require much computational effort leading to fast convergence. These directions are smooth from one iteration to the next, thus the zigzagging is avoided.

The Lagrangian relaxation and surrogate subgradient optimization approach was specifically treated in [16] and [8]. The former paper develops the surrogate subgradient methodology and proves convergence of the method. Compared to subgradient and gradient methods, the surrogate subgradient approach finds better and smoother directions within shorter CPU time. The latter paper extends the methodology to solving coupled problems. Convergence of the surrogate optimization within Lagrangian framework approach has been proved under certain conditions, namely, the upper bound on the step size and the surrogate optimization condition [16]. In addition, to ensure that the surrogate Lagrangian dual function provides a lower bound property, the surrogate optimization approach requires the optimal dual value.

There have been several attempts to deal with the difficulty of obtaining the optimal dual value. An approximation of the optimal dual value is introduced in [13]. Studies indicate that when the approximation is an underestimate of $L^*$, the algorithm converges to this approximate value rather than to $L^*$. When the approximation is an overestimate, the algorithm does not converge. Since it is not known beforehand whether the approximate value is an underestimate or an overestimate, it takes several iterations to establish that. For large and complex problems taking several additional iterations is very costly computationally.

One of the recent attempts to avoid estimating the optimal dual value, while ensuring convergence of the surrogate optimization method, was undertaken in [6] by introducing a series of subgradient approximations converging to an *a fortiori* good subgradient direction. Theoretical insights and good preliminary results have been obtained.

The branch-and-cut method has been of great interest for solving linear mixed-integer programming problems, i.e., problems with linear objective function and linear constraints [4], [12] and [14]. In the branch-and-cut method, the valid cuts, i.e., cuts that reduce the feasible region of the integer-relaxed problem without cutting off feasible points of the original problem, are added after relaxing the integrality constraints. Branching is then frequently used to decompose the problem into two or more subproblems while searching for the global optimum. The branch-and-cut method requires the global validity of the cuts in order to efficiently split the problem at a given node of the branch tree. Therefore when the facet-defining inequalities are not easily obtainable the performance of the branch-and-cut significantly deteriorates, and the computational efficiency of the commercial software that utilizes branch-and-cut is low.

## III. Problem Formulation

This subsection presents the linearized version of the general definition of the Payment Cost Minimization problem and demonstrates how the surrogate optimization framework and branch-and-cut can be used to solve Payment Cost Minimization.

In the Payment Cost Minimization problem, the total customer payment cost is minimized subject to the system demand, start-up cost and the market-clearing price constraints. Standard simplifying assumptions, which are usually made in the existing literature (e.g. in [10]) are used. The system demand $P_d(t)$ is deterministic at each hour $t$ and is known. Transmission and ramp-rate constraints are omitted for simplicity and are not considered in this paper. Start-up costs are given and fully compensated, and participants submit a single-block offer with start-up cost, offer price, and maximum and minimum generation levels for each hour $t$.

Consider a market with $I$ participants (sometimes referred in the literature to as offers or units) enumerated by $i=1,2,\ldots,I$. For each offer $i$, $p_i(t)$ denotes the offer generation level at time $t$, $S_i(t)$ denotes the start-up cost at time $t$, which is incurred if and only if an offer $i$ is turned "on" at time $t$, after being "off" at time $t-1$.

The PCM problem can be formulated in the following way [10]:

$$\min_{\{MCP(t),u_i(t)\}} \sum_{t=1}^{T}\left(MCP(t)P_d(t)+\sum_{i=1}^{I}S_i(t)u_i(t)\right) \tag{1}$$

The decision variables of the problem are $MCP(t)$, $u_i(t)$, $x_i(t)$ and $p_i(t)$ for all $i=1,\ldots,I$ and $t=1,\ldots,T$. The latter two, though not included into the objective function, enter start-up cost and system demand constraints respectively as shown below.

The problem is optimized subject to the following constraints:

**1. System demand constraints:**

$$P_d(t)=\sum_{i=1}^{I}p_i(t),\forall(t=1,2,\ldots,T) \tag{2}$$

The system demand has to be satisfied exactly by the selected offers at each time $t$.

**2. Start-up cost constraints:**

$$u_i(t)\geq\left(x_i(t)-x_i(t-1)\right),\forall(i\in I,t=1,2,\ldots,T) \tag{3}$$

The start-up cost binary decision variable $u_i(t)$ equals $1$ at time $t$ if and only if an offer $i$ is turned "on", which corresponds to $x_i(t)=1$, from an "off" state at time $t-1$ (this corresponds to $x_i(t-1)=0$), and 0 otherwise.

**3. MCP constraints:**

Currently, the MCP is determined by a price which is set by the marginal unit, i.e. a unit that generates power at neither minimum nor maximum generation level. Mathematically, the relations between the MCP and binary decision variables $x$ are as follows

$$MCP(t)\geq c_i y_i(t),\forall(i\in I,t=1,2,\ldots,T) \tag{4a}$$

$$y_i(t)\geq\frac{p_i(t)-p_i^{\min}(t)}{p_i^{\max}(t)-p_i^{\min}(t)},\forall(i\in I,t=1,2,\ldots,T) \tag{4b}$$

$$y_i(t)\geq\frac{\sum_{j\in I}\left(p_j^{\max}(t)x_j(t)-p_j(t)\right)-p_i^{\min}(t)+\varepsilon}{M}+\left(x_i(t)-1\right),\forall(i\in I,t=1,2,\ldots,T) \tag{4c}$$

here $c_i$ denotes the offer price of an offer $i$ and for simplicity is assumed to be constant for all $t$.

Constraints (3), (4a) and (4c) are coupling with respect to the binary variables $x_i(t)$. The constraints (3) couple $x_i(t)$ in time $t$, (4a) and (4c) couple $x_i(t)$ by offers $i$.

As discussed in the Introduction section, the facet-defining inequalities that define convex hulls for the decision variables $x_i(t)$ need to be valid across all offers and all time periods and are hard to obtain.

**4. Generation capacity constraints:**

$$x_i(t)p_{i,\min}(t)\leq p_i(t)\leq x_i(t)p_{i,\max}(t),\forall(i\in I,t=1,2,\ldots,T) \tag{5}$$

where $p_{i,\min}(t)/\,p_{i,\max}(t)$ are minimum/maximum generation levels of a unit $i$ at time $t$;

Generation levels of selected offers should be within the bounds specified by $p_{i,min}(t)$ and $p_{i,max}(t)$ at each hour. If an offer $i$ is "off" at time $t$, then the generation level is zero. It has to be noted that the feasible region of power generation levels is discontiguous since $p_i(t)$ exhibits a jump from 0 to $p_{i,min}(t)$.

## IV. Solution Methodology

### *A. Surrogate Subgradient Method*

In terms of the PCM problem, the relaxed problem is written as follows:

$$L(\lambda(t);MCP(t),p(t),u(t))=\sum_{t=1}^{T}\left(MCP(t)P_d(t)+\sum_{i=1}^{I}S_i(t)u_i(t)\right)+\sum_{t=1}^{T}\lambda(t)\left(\sum_{i=1}^{I}p_i(t)-P_d(t)\right) \tag{6}$$

The relaxed problem (6) is to be minimized subject to the constraints (2)-(5) and the following surrogate optimization condition

$$L\left(\lambda^{k+1}(t);MCP(t),p(t),u(t)\right)-L\left(\lambda^{k+1}(t);MCP(t),\tilde{p}^{k}(t),u^{k}(t)\right)<0. \quad (7)$$

The multipliers are updated in the following fashion:

$$\lambda^{k+1}(t)=\lambda^{k}(t)+c^{k}\tilde{g}\left(p^{k}(t)\right), \quad (8)$$

where $\tilde{g}\left(p^{k}(t)\right)$ is the surrogate subgradient defined by

$$\tilde{g}\left(p^{k}(t)\right)=\sum_{i=1}^{I}\tilde{p}_{i}^{k}(t)-P_{d}(t), \quad (9)$$

such that $\tilde{p}_{i}^{k}(t)$ satisfy (7), and the stepsize satisfies

$$c^{k}<\frac{L^{*}-L\left(\lambda^{k}(t),MCP^{k}(t),\tilde{p}^{k}(t),u^{k}(t)\right)}{\left\|\tilde{g}\left(p^{k}(t)\right)\right\|^{2}}. \quad (10)$$

It was shown in [16], when the conditions (7) and (10) are satisfied, the surrogate Lagrangian dual function provides the lower bound for the feasible cost, that is

$$L\left(\lambda^{k}(t),MCP^{k}(t),\tilde{p}^{k}(t),u^{k}(t)\right)<L^{*}. \quad (11)$$

The stepsize defined in (10) requires the optimal dual value. When the optimal dual value is unknown, the upper bound on the stepsize $c^k$ is unknown, therefore convergence cannot be guaranteed and the lower bound property (11) is lost.

As shown in [7], the multipliers converge to the optimum without invoking the optimal dual value. The necessary condition for convergence is achieved by decreasing the distance between consecutive multipliers, which mathematically is expressed as

$$\left\|\lambda^{k+1}-\lambda^{k}\right\|=\alpha\left\|\lambda^{k}-\lambda^{k-1}\right\|,\ 0<\overline{\alpha}<\alpha<1. \quad (12)$$

The stepsize that satisfies (12) is

$$c^{k}(\alpha)=\alpha\frac{c^{k-1}\left\|\tilde{g}\left(x^{k-1}\right)\right\|}{\left\|\tilde{g}\left(x^{k}\right)\right\|}. \quad (13)$$

The sufficient condition for convergence is established by establishing that the stepsize is never "too small" so that the multipliers do not get "stuck."

Mathematically, $\forall K$, $\exists k \geq K$, and the stepsize satisfies

$$c^{k}(\alpha)>\frac{L^{*}-\tilde{L}^{k}}{\left\|\tilde{g}\left(x^{k}\right)\right\|^{2}}. \quad (14)$$

In other words, (14) occurs periodically but infinitely often.

While the value of $\alpha$ cannot be specified beforehand, it was shown in [7] that there exists $\alpha_i$ from any increasing sequence $\{\alpha_i\}$ bounded by 1 from above such that the multipliers converge to the optimum when the stepsize $c^k(\alpha_i)$ defined in (13) is used.

In other words, for any sequence $\{\alpha_i\}$ such that $0<\alpha_1<\alpha_2<\ldots<1$, there always exists the limit $\overline{\lambda}_i \equiv \lim_{k\to\infty}\left(\lambda^{k}+c^{k}(\alpha_i)\tilde{g}\left(x^{k}\right)\right)$, moreover, $\lim_{\alpha_i\to 1^-}\overline{\lambda}_i=\lambda^{*}$. In other words, by choosing $\alpha$ to close to 1, the multipliers converge to $\lambda^*$.

Since no intrinsic properties of a particular problem are required to establish convergence of the surrogate subgradient method, this method can be used to solve a broad class of linear mixed-integer problems.

*B. Valid Cuts and Additional Constraints*

As mentioned above, the problem (6) subject to constraints (2)-(5) is NP-hard and the problem of obtaining the subgradient is hard. In order to alleviate the computational burden, the surrogate subgradient satisfying condition (7) is obtained instead of the subgradient. Very often a feasible solution that satisfies condition (7) is difficult to find. Therefore, additional cuts that define integral solutions are required. The idea behind using the additional cuts and constraints is to make the complexity of obtaining the surrogate subgradient low regardless of the size of the problem.

While additional valid cuts largely depend on the structure of the problem under consideration, the surrogate subgradient condition can always be used as a valid cut. In terms of the PCM, the following cut is valid

$$L\left(\lambda^{k+1}(t);MCP(t),p(t),u(t)\right)-L\left(\lambda^{k+1}(t);MCP(t),\tilde{p}^{k}(t),u^{k}(t)\right)\leq 0. \quad (15)$$

This cut can significantly reduce the search space of the algorithm, and help to define the integral hull. This often obviates the need to perform branching operations and increases the overall efficiency of the algorithm.

In terms of the PCM problem, additional valid cuts can be found using the start-up cost constraints (3) and the market-clearing price (MCP) constraints. In order to increase the efficiency of branch-and-cut, the valid cuts have to be globally valid across all offers and all time periods.

Each constraint in (3) relates only neighboring time periods ($t$ and $t$-1), and the standard cuts used in branch-and-cut handle these constraints locally. Since the global optimality of the sought-for solution is desired, global validity if the cuts across all time periods is necessary.

Consider

$$u_i(1)\geq\left(x_i(1)-x_i(0)\right),\forall(i\in I) \quad (3a)$$

and

$$u_i(2)\geq\left(x_i(2)-x_i(1)\right),\forall(i\in I). \quad (3b)$$

Adding the constraints (3a) and (3b) yields

$$u_i(2)+u_i(1)\geq\left(x_i(2)-x_i(0)\right),\forall(i\in I). \quad (3c)$$

The latter constraint is "more" globally valid than (3). A more general expression of (3c) is

$$\sum_{t=\tau-\xi+1}^{\tau}u_i(t)\geq\left(x_i(\tau)-x_i(\tau-\xi)\right),\forall(i\in I,\tau=\xi,\ldots,T,\xi=1,\ldots,T). \quad (3d)$$

The family of constraints (3d) generates valid cuts that are handled globally within the branch-and-cut framework. Therefore, the lower bound becomes tighter and the global search within the branching tree can be organized more efficiently.

Like constraints (3), while constraints (4a) are system-wide, they are handled locally using branch-and-cut since the MCP decision variable relates only each offer at a time. In order to handle such constraints globally, the aggregated form of (4a) is preferable.

$$MCP(t) \geq \frac{1}{I}\sum_{i=1}^{I} c_i y_i(t), \forall (t = 1,2,...,T) \qquad (4a.1)$$

In (4a.1), the MCP decision variables are related to all offers explicitly and are valid across all offers, therefore, constrains (4a.1) define valid cuts that are globally valid within the branch-and-cut framework.

Since the exact optimality of the relaxed problem within the surrogate optimization framework is not required, a feasible solution satisfying (7) can be found by first cutting off other feasible solutions and then by optimizing over the remaining space. Thus, while additional constraints may not be valid in the classical sense, they further reduce the search space of the algorithm and help obtain the surrogate subgradient fast.

For example, by introducing $\Delta MCP(t)$, the additional cuts can be specified at the iteration $k$+1 in the following way

$$MCP^k(t)-\Delta MCP(t) \leq MCP^{k+1}(t) \leq MCP^k(t)+\Delta MCP(t), \qquad (16)$$

for $t=1,\ldots,T$. The inequalities in (16) ensure that the search space is small enough so that the surrogate direction can be obtained quickly. Here $MCP^k(t)$ is the MCP decision variable value at time $t$ computed at the iteration $k$.

In addition, the initialization of the decision variables values that are *a priori* close to optimal values can save computational effort, and reinitialization of the decision variables can further improve the convergence speed.

## V. Numerical Testing

In this section, three examples are presented. Example 1 demonstrates the convergence aspect of the surrogate subgradient method. A small instance of the PCM problem is tested. Convergence of the method is tested against that of the surrogate optimization method with the stepsize defined in (10). The method is then used for solving large PCM problems. Example 2 tests the impact of the reinitialization of the decision variables on the performance of the method. A medium sized problem is generated and tested. Example 3 demonstrates the performance of the method for large-scale PCM problems and provides the robustness results. All medium and large-scale problems are generated using the perturbed data from the ISO-NE for May 1999.

The algorithm presented is tested using CPLEX on Intel® Core™ i7 CPU Q 720 @ 1.60 GHz and 4.00 GB RAM.

CPLEX is used for solving the relaxed problem within the Lagrangian relaxation framework. In addition, the solution obtained in CPLEX using branch-and-cut can serve as a benchmark for evaluating the effectiveness of the approach.

*Example 1: Small Payment Cost Minimization problem.* Consider the following 5-hour 4-offer PCM problem as described in Tables I and II.

TABLE I
Supply Offer Parameters for Example 1

| | Min MW | Max MW | \$/MW | Start Up Cost (\$) |
|---|---|---|---|---|
| Offer 1 | 5 | 45 | 10 | 0 |
| Offer 2 | 5 | 45 | 20 | 0 |
| Offer 3 | 0 | 12 | 100 | 50 |
| Offer 4 | 5 | 80 | 30 | 1200 |

TABLE II
Demand Parameters for Example 1

| | Hour 1 | Hour 2 | Hour 3 | Hour 4 | Hour 5 |
|---|---|---|---|---|---|
| System Demand (MW) | 100 | 95 | 110 | 120 | 115 |

Small problems like the one described in this Example, can be solved analytically. Therefore, the optimal dual value can be easily verified to be equal to 16650.

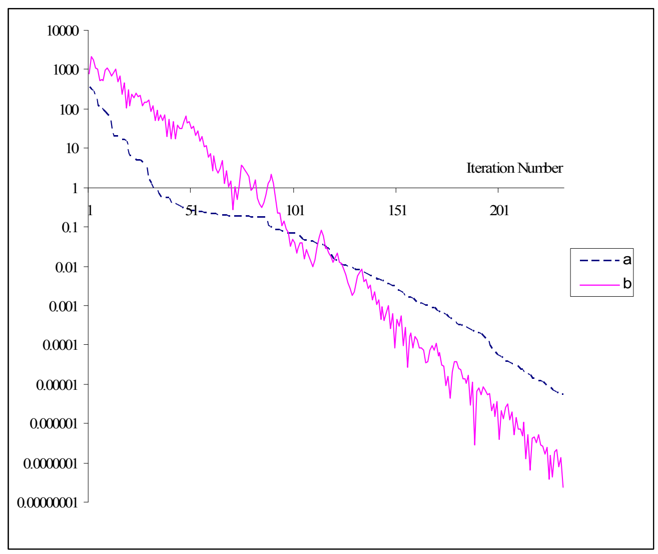


**Fig. 1:** Performance comparison of the surrogate subgradient method with different stepsizes. The dashed line represents $\left\|\lambda_a^k - \lambda^*\right\|^2$ and the solid line represents $\left\|\lambda_b^k - \lambda^*\right\|^2$. Here $\lambda_a^k$ is updated using (10) and $\lambda_b^k$ is updated using (13).

Figure 1 illustrates the comparison of the two methods by comparing the Euclidean distances between the current multipliers and the optimal multiplier. The stepsizes are using the stepsizes defined in (10) and (13), where $\alpha$ is chosen to be equal to 0.975.

While the multipliers move closer to the optimum as every iteration when the formula (10) for the stepsize is used, the multipliers converge faster to the optimum when the stepsize satisfying relation (13) is used.

*Example 2: Medium-sized Payment Cost Minimization problem.* Consider a 25-offer 24-hour PCM problem. Since the relaxed PCM problem is NP-hard, the complexity of the problem of obtaining the surrogate subgradient drastically increases as the size of the problem increases. Therefore, while the surrogate subgradient that satisfies the surrogate optimization condition (7) can be obtained quickly for small problems, in order to obtain the surrogate subgradient quickly for the problems of a medium size, additional constraints that define integral solutions (not necessarily optimal) and reinitialization of the decision variables are required. Constraints (16) accomplish both tasks. By restricting the values for the MCP variables, the complexity of the relaxed problem decreases. In addition, boundaries of the feasible region specified by the additional constraints in (16) provide good starting points for the branch-and-cut method.

The results are summarized in the Table III.

TABLE III
RESULTS FOR EXAMPLE 2

| Method | Lower bound ($) | Upper bound ($) | Gap (%) | CPU time (s) |
|---|---|---|---|---|
| Surrogate optimization with (16) | 3,667,052 | 3,886,965 | 5.65 | 300 |
| Surrogate optimization without (16) | 3,643,081 | 4,029,185 | 9.58 | 300 |

The surrogate subgradient is obtained quicker when the additional constraints are used. Therefore, the method converges faster and provides a tighter lower bound. In addition, simple local search heuristics can obtain a better feasible solution based on a better dual solution.

*Example 3: Large scale Payment Cost Minimization problem*. This example demonstrates how the surrogate subgradient method with the stepsize satisfying relation (13) can be used to solve the large scale Payment Cost Minimization problems. The robustness of the method is tested.

When solving the PCM problems of such size using CPLEX, the CPU time and the duality gap are unreasonably high as indicated in the Table VI. This confirms the difficulties described in Section I. In contrast, when the surrogate optimization within the Lagrangian relaxation framework is used subject to (16), these difficulties are overcome. Lagrangian relaxation is first used to relax the system demand constraints, after that the relaxed problem (6) is optimized subject to the surrogate optimization condition (7). Even though the resulting problem is still NP-hard, it can be solved efficiently after imposing conveniently chosen set of constraints (16), which reduce the search space. The multipliers are updated iteratively according to (8). At each iteration, heuristics are used to remove infeasibilities that result from relaxing the systems demand constraint to create a feasible solution. The quality of this solution is quantified by the dual cost.

The results presented in the Table IV illustrate the method performance gain over the standard branch-and-cut method. Such a drastic improvement of performance is largely due to saving the effort of finding the surrogate subgradient directions and a warm start procedure that utilizes the best available information about the decision variables from previous iterations.

TABLE IV
RESULTS OBTAINED BY THE TWO METHODS

| Method | Lower bound ($) | Upper bound ($) | Gap (%) | CPU time (s) |
|---|---|---|---|---|
| B&C and SO | 55,244,634 | 56,174,579 | 1.66 | 600 |
| B&C | 39,703,846 | 87,671,446 | 54.71 | 18000 |

Results presented in the Table V demonstrate that initialization of the MCP variables plays a significant role in finding good feasible solution and obtaining a tight lower bound. In practical scenarios, the MCP variables can be initialized based on the values available from previous planning horizons.

TABLE V
RESULTS OBTAINED BY THE SURROGATE OPTIMIZATION WITH AND WITHOUT THE INITIALIZATION

| Method | Lower bound ($) | Upper bound ($) | Gap (%) | CPU time (s) |
|---|---|---|---|---|
| B&C and SO w/MCP initialization | 55,244,634 | 56,174,579 | 1.66 | 600 |
| Branch-and-cut wo/MCP initialization | 55,140,679 | 72,217,885 | 23.64 | 600 |

Results presented in the Table VI show that heuristics can obtain good feasible solutions with quantifiable quality. The comparison is made between the local search heuristic, and the heuristic that assigns the maximum offer generation to the least expensive offer in terms of the offer and the start-up costs. That is, the offers with the smallest values (similarly to [10])

$$c_i + S_i(t) / p_i^{\max}(t), \qquad (17)$$

are turned on one by one until the leftover system demand is less than the maximum generation level of the next expensive offer, which is chosen to set the initial approximation of the MCP.

TABLE VI
RESULTS OBTAINED BY THE SURROGATE OPTIMIZATION USING DIFFERENT HEURISTICS

| Method | Lower bound ($) | Upper bound ($) | Gap (%) | CPU time (s) |
|---|---|---|---|---|
| B&C and SO w/local search | 54,695,043 | 55,704,278 | 1.81 | 600 |
| Branch-and-cut w/(17) | 54,677,055 | 57,554,213 | 4.99 | 600 |

Both heuristics are capable of obtaining good feasible solutions. Since the local search heuristic is more expensive computationally, the heuristic based on (17) is preferable.

In order to assess the algorithm's robustness, twenty 24-hour problems with 300-offer problems were generated. The feasible solution is determined using the heuristic (17). The computation results of selected instances are summarized in Table VII.

TABLE VII
RESULTS FOR EXAMPLE 3

| **Instance** | **Gap (%)** | **LB ($)** | **UB ($)** | **CPU time (s)** |
|---|---|---|---|---|
| 1 | 1.66 | 55,244,634 | 56,174,579 | 600 |
| 2 | 2.44 | 57,266,074 | 58,701,285 | 600 |
| 3 | 1.81 | 54,695,043 | 55,704,278 | 600 |
| **Mean** | **2.01** | | | |
| **St. Dev.** | **0.42** | | | |

Standard MIP optimization software (here CPLEX) can handle small PCM problems with relative ease. However, large problems pose much bigger computational challenges for MIP solvers such as CPLEX since the facet-defining cuts cannot be easily obtained and the number of branching operations increases dramatically as the number of offers increases. The combination of the surrogate optimization approach and branch-and-cut becomes superior to the standard branch-and-cut method for large problems.

## VI. CONCLUSION

The problem that minimizes the customer payments directly (PCM) is shown to possess several desirable economic properties. The investigation of other properties it may possess and the comparison with other mechanisms that compute customer payments is inhibited when an efficient solution methodology in unavailable. Current research was prompted by the fact that existing optimization methods are inefficient for solving PCM as pointed out in the introduction. The difficulties that arise when using known optimization methods are overcome by a successful "marriage" of the surrogate optimization with the branch-and-cut method. The methodology developed in this paper is generic, and can be used for solving linear mixed-integer problems and is efficiently implemented in commercial MIP solvers.

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