Charge sensitivity of single-electron transistor with superconducting electrodes

ALEXANDER N. KOROTKOV
Department of Physics, State University of New York, Stony Brook, NY 11794-3800, U.S.A.
and Nuclear Physics Institute, Moscow State University, Moscow 119899 Russia

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The noise-limited charge sensitivity of a single-electron transistor with superconducting electrodes operating near the threshold of quasiparticle tunneling, can be considerably higher than that of a similar transistor made of normal metals or semiconductors. The reason is that the superconducting energy gap, in contrast to the Coulomb blockade, is not smeared by the finite temperature. The same reason leads to the increase of the maximum operation temperature due to superconductivity.

Key words: single-electron transistor, superconductivity, noise properties.

The simplest and most thoroughly studied single-electron [1] circuit is the Single Electron Transistor [2] (SET) which consists of two tunnel junctions connected in series. At low temperatures ($T \ll e^2/C_\Sigma$, $C_\Sigma = C_1 + C_2$ where $C_1$ and $C_2$ are the junction capacitances) the current through this structure depends on the background charge $Q_0$ of the central electrode (the dependence is periodical with a period equal to the electron charge $e$). Hence, by controlling $Q_0$ (for example, by a capacitive gate) it is possible to control the current $I$ through the circuit. The possibility for use of the SET as a highly-sensitive electrometer has been confirmed in numerous experiments. It has been noticed [3–5] that the superconductivity of electrodes improves the performance of the SET (operating near the threshold of quasiparticle tunneling) as an electrometer in comparison with the normal-state operation. This issue will be a subject of the quantitative analysis in the present paper (see also Ref. [6]).

There are two major characteristics of the SET operation as an electrometer. The first one is the amplitude of the output signal modulation for $Q_0$ variations larger than $e$. It was found experimentally [4] that the use of superconducting electrodes increases the modulation amplitude of current $I$ (for fixed bias voltage $V$), especially at temperatures comparable to $e^2/C_\Sigma$, thus increasing the maximum temperature. The theoretical results of the present paper confirm this statement for both NISIN and SISIS structures.

The other, even more important characteristic of the SET operation is the noise-limited sensitivity (ability to detect variations of $Q_0$ much smaller than $e$). In the present-day technology the sensitivity is typically limited by $1/f$ noise which is most likely caused by random trapping-escape processes in nearby impurities. However, with technological improvement one can expect the reduction of the noise due to impurities. Then the charge sensitivity of the SET would achieve the limit determined by the intrinsic noise [7, 8] of the
device caused by random electron jumps through tunnel junctions (this ‘white’ noise has been recently measured in experiment [9]). Though the theory of the ‘classical’ thermal/shot intrinsic noise of the SET is applicable to the general case of one-particle tunneling (normal metals, semiconductors, quasiparticle current in superconductors, etc.), most numerical results in Refs [7] and [8] as well as in a number of subsequent papers on this subject (see, e.g. Refs [10–13]) were obtained only for SETs made of normal metals. (Recently some generalization was done [14] to include the possibility of two-particle tunneling which can be important in the superconducting case. Let us also mention Ref. [5] in which the noise in NISIN SET was briefly considered.)

In the present paper we apply the theory of Refs [7] and [8] to the cases of capacitively coupled superconducting SISIS and NISIN SETs (the analysis of a resistively coupled SET can be done in a similar way—see Ref. [7]). We show that the noise-limited sensitivity of a SET-electrometer can be considerably improved by the use of superconducting electrodes.

We consider only the quasiparticle tunneling, neglecting the Josephson current, resonant tunneling of Cooper pairs, Andreev reflection, and cotunneling. This assumption is appropriate when the Josephson coupling is negligible and the normal state resistances $R_1$ and $R_2$ of tunnel junctions are well above the resistance quantum $R_Q = \pi \hbar / 2e^2$. We use the ‘orthodox’ theory [1, 2] of the SET and the BCS theory [15] for the calculation of the tunneling rates.

Figure 1 shows the $I$–$V$ curves at different temperatures for (A) the normal metal NININ case, (B) NISIN case (which is equivalent to SINIS case), and (C) and (D) SISIS case. SETs with $C_1 = C_2$ and $R_1 = R_2 = R_\Sigma/2$ are chosen, and we neglect the gate capacitance $C_g$ because it can always be formally distributed between $C_1$ and $C_2$ (see, e.g. Ref. [16]). Three curves in each set represent $Q_0 = 0, e/4, \text{and } e/2$, respectively. Temperature increase decreases the superconducting energy gap $\Delta(T)$ (which is assumed to be equal in all S-electrodes) leading to the noticeable shift to the left of the positions of the current jumps in Fig. 1C and D. The pure BCS theory would lead to the abrupt jumps of the current in SISIS case. To take into account the
unavoidable smoothing of the jumps in reality, we assume additionally the inhomogeneous broadening of $\Delta(0)$ with Gaussian distribution characterized by the dispersion $w_0$. This phenomenological parameter is chosen as $w_0 = 0.05 \Delta(0)$ in Fig. 1C and D (for finite temperatures $w(T) = w_0[\Delta(T)/\Delta(0) - (T/\Delta(0))(d\Delta(T)/dT)]$ was used).

One can see that in the normal metal case the current $I$ can be considerably modulated $(I_{\text{max}}/I_{\text{min}} \gtrsim 2)$ by $Q_0$ ($V$ is fixed) only at $T \lesssim 0.15e^2/C_\Sigma$, while at $T = 0.3e^2/C_\Sigma$ the modulation is already negligible, $(I_{\text{max}} - I_{\text{min}})/I_{\text{max}} \approx 5\%$. Notice that the maximum relative modulation is achieved at small voltages and does not depend on ratios $C_1/C_2$ and $R_1/R_2$.

*NISIN* transistor with $\Delta(0) = 0.5e^2/C_\Sigma$ shows considerable modulation crudely up to $T \approx 0.2e^2/C_\Sigma$, while *SISIS* transistors with $\Delta(0) = 0.5e^2/C_\Sigma$ and $\Delta(0) = 2.0e^2/C_\Sigma$ operate well almost up to the critical temperature $T_c$ ($T_c/(e^2/C_\Sigma) = 0.28$ and 1.14, respectively). The case $\Delta(0) = 0.5e^2/C_\Sigma$ corresponds to the typical present-day experimental situation with aluminum junctions and $C_\Sigma \approx 0.4$ F (see, e.g. Ref. [4]). Comparison of Fig. 1C and D shows that the increase of $\Delta(0)$ provides further improvement of the transistor performance at high temperatures. Using Fig. 1D one can predict the operation of the niobium-based SET with $C_\Sigma \approx 0.2$ F (current state-of-the-art for aluminum junctions) at temperatures up to 7 K.

Superconductivity improves the SET performance at relatively high temperatures because, in contrast to the Coulomb blockade, the superconducting energy gap is not smeared by the finite temperature. In the normal metal case the $I-V$ curve has a cusp at the Coulomb blockade threshold

$$V_i = \min_{i,n}[V_{i,n} \mid V_{i,n} > 0], \quad \text{where} \quad V_{i,n} = \frac{e}{C_i} \left( \frac{1}{2} + (-1)^i \left( n + \frac{Q_0}{e} \right) \right),$$

(1)

and this cusp is rounded within the voltage interval proportional to the temperature. In *SISIS* case the jump of the $I-V$ curve at $V_i$, which is shifted due to the energy gap,

$$V_i = \min_{i,n}[V_{i,n} + 2\Delta(T)C_\Sigma/eC_1 \mid V_i > 4\Delta(T)],$$

(2)

remains sharp even at $T \sim \Delta(T)$, and the subthreshold current increase is only proportional to $\exp(-T/\Delta(T))$. This explains why *SISIS* transistor shows considerable dependence on $Q_0$ for the temperatures almost up to $T_c$ even if $T \gtrsim e^2/C_\Sigma$. In *NISIN* case the $I-V$ curve in the vicinity of

$$V_i = \min_{i,n}[V_{i,n} + \Delta(T)C_\Sigma/eC_1 \mid V_i > 2\Delta(T)]$$

(3)

is rounded by the finite temperature, that makes *NISIN* transistor worse than *SISIS* transistor, however, it is still better than usual *NININ* transistor.

Now let us consider the noise-limited sensitivity of the SET. The minimum detectable charge for the given bandwidth $\Delta f$ is

$$\delta Q_0 = (S_f \Delta f)^{1/2}/(\partial I/\partial Q_0)$$

(4)

where the spectral density $S_f$ of the current noise is taken in the low frequency limit. The ultimate low-temperature ($T \ll e^2/C_\Sigma$) sensitivity in the *NININ* case is [7, 8]

$$\min \delta Q_0 \simeq 2.7C_\Sigma(R_{\text{min}}T_{\Delta f})^{1/2}, \quad R_{\text{min}} = \min(R_1, R_2).$$

(5)

This result can be somewhat improved in the *NISIN* SET (with the same resistances) operating near the threshold $V_i$ of quasiparticle tunneling. At low temperatures, $T \ll \min(e^2/C_\Sigma, \Delta(T))$, and for $V$ close to nondegenerate $V_i$, we can use approximation

$$S_f \simeq 2eI, \quad I \simeq I_{0,i}(V - V_i)C_1C_2/C(C_\Sigma),$$

(6)

where

$$I_{0,i}(v) = (1/eR_i)[T \Delta(T)/2]^{1/2} \int_0^{\infty} dy/\sqrt{y}[1 + \exp(y + (\Delta - ev)/T)]^{-1}$$

(7)
The minimum detectable charge $\delta Q_0$, the current $I$, and the ratio $S_0/2eI$ as functions of the bias voltage $V$ for SISIS SET. Dashed lines show $\delta Q_0$ for NININ and NISIN SETs. The best sensitivity is achieved in SISIS case.

is the ‘seed’ $I$–$V$ curve of $i$th junction. Then the ultimate sensitivity is given by equation

$$\min \delta Q_0 = C_\Sigma(2e\Delta f)^{1/2} \min \sqrt{\frac{I_0(v)}{(dI_0/dv)}},$$

and finally we get the result

$$\min \delta Q_0 \simeq 2.6C_\Sigma(R_{\min}T\Delta f)^{1/2}\frac{[T/(\Delta(T))]^{1/4}}{[\Delta(T)/T]^{1/4}}$$

which is better than NININ sensitivity when $T < \Delta(T)$. The main reason for the improvement is the increase [3–5] of the transfer coefficient $\partial I/\partial Q_0 \simeq V(1/C_\Sigma)(\partial I/\partial V)$, because the differential resistance $R_d$ of the ‘seed’ $I$–$V$ curve near the onset of quasiparticle tunneling is less than $R$. Notice that the ‘orthodox’ theory used here is valid only if $R_d \gg R_0$ because the cotunneling processes [17, 5] impose the lower bound for $(\partial I/\partial V)^{-1}$ on the order of $R_0$ [18]. For relatively high temperatures the ratio of minimum $\delta Q_0$ in NISIS and NININ cases is larger than $[\Delta(T)/T]^{1/4}$ (e.g., compare the dashed lines in Fig. 2) because NININ sensitivity starts to deviate up from the low-temperature approximation at smaller $T$ than NISIS sensitivity.

The improvement of the ultimate sensitivity is more significant in SISIS SET. For pure BCS model the ‘orthodox’ theory gives infinite derivative $\partial I/\partial Q_0$ at $V = V_0$ even for finite temperature leading to $\delta Q_0 \to 0$. Hence, the ‘orthodox’ ultimate sensitivity depends on the imperfection of the current jump which is described in our model by the energy gap spread $w_0$ ($w_0 \ll \min\{\Delta(T), e^2/C_\Sigma\}$).

Figure 2 shows $\delta Q_0$ together with current $I$ and ratio $S_0/2eI$, as functions of the voltage for the symmetric SISIS SET with parameters $\Delta(0) = 0.5e^2/C_\Sigma, w_0 = 0.05\Delta(0), T = 0.1e^2/C_\Sigma$, and $Q_0 = 0.25e$ (numerical calculations are done using the method described in Refs [7] and [8]). Dashed lines show $\delta Q_0$ for similar NININ and NISIN SETs. One can see that the sensitivity of SISIS SET is much better than for NININ and NISIN cases within a relatively narrow voltage range which corresponds to the jump of current.

In contrast to NININ and NISIN cases, the approximation $S_0 \simeq 2eI$ is not accurate in the vicinity of $V_0$ for SISIS SET even at low temperatures (see Fig. 2) because the relatively large tunneling rate in the junction determining $V_0$ is comparable to the tunneling rate in the other junction. This approximation is valid only if $T \ll \Delta(T) \ll e^2/C_\Sigma$, and would lead to inaccuracy typically about 10% for the analytical calculation.
of \( \min \delta Q_0 \) if \( T \ll \Delta(T) \sim e^2/C_\Sigma \). Nevertheless, it can be used as a crude estimate. Using eqn (8) and smoothed by \( w_0 \) low-temperature \( (T \ll \Delta(T)) \) ‘seed’ \( I-V \) curve for SIS junction [15] we get

\[
\min \delta Q_0 \simeq 1.8C_\Sigma \left(R_{\text{min}}\Delta f\frac{w_0^2}{\Delta(T)}\right)^{1/2}.
\]

Notice that the numerical factor depends on the particular model describing the shape of the current jump. Comparing eqn (10) with the result for \( NININ \) SET, we see that the temperature \( T \) is replaced in \( SISIS \) case by \( w_0^2/\Delta(T) \). Hence, the ultimate sensitivity is better in \( SISIS \) SET (resistances are the same) with sufficiently narrow width of the current jump, \( w_0 < (T\Delta(T))^{1/2} \).

In the case of very sharp ‘seed’ \( I-V \) curve, \( w_0 \ll \Delta(T)R_0/R \), the slope of the jump of the SET \( I-V \) curve is determined by cotunneling [17] and it cannot be sharper than crudely \( R_0^{-1} \) [18]. Then \( \min \delta Q_0 \) is on the order of \( C_\Sigma\Delta f\Delta(T)R_0^2/R \)\(^{1/2} \) (we assume \( \Delta(T) \gg e^2/C_\Sigma \), \( R_1 = R_2 \)), and the ultimate sensitivity is better than for \( NININ \) SET if \( T \gtrsim \Delta(T)(R_0/R)^2 \).

In conclusion, the superconductivity of electrodes can considerably improve the performance of the single electron transistor as an electrometer at relatively large temperatures, if the superconducting energy gap is comparable or larger than \( e^2/C_\Sigma \).

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