Supplemental Material for "Catch-Disperse-Release Readout for Superconducting Qubits"

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Evolution of Wigner function and probability distributions

In this supplemental material, we present the evolution of the Wigner function for resonator field corresponding to initial states $|00\rangle$ and $\overline{|10\rangle}$. Here the eigenstate $\overline{|10\rangle} = \cos\theta_1|00\rangle - \sin\theta_1|01\rangle$ with $\tan\theta_1 = 2g(\Delta_0^2 + 4g^2)^{-1/2}$, where Δ_0 is the initial qubit-resonator detuning and g is the qubit-resonator field coupling. Note that for large Δ_0, the eigenstate $\overline{|10\rangle} \approx |10\rangle$ (in $|nm\rangle$, n represents qubit state and m represents the Fock state). We also present the evolution of the probability distributions of the measurement result x_φ at different times, and the waveform used for qubit frequency and for the microwave pulse. In the results presented up to page 14, we assumed a two-level model of the qubit.

We compute the Wigner function using the formula

$$W(\alpha) = Tr\left[\mathrm{D}(-\alpha)\rho\mathrm{D}(\alpha)e^{i\pi a^\dagger a}\right],$$

where $D(\alpha) = e^{-|\alpha|^2}e^{\alpha a^\dagger}e^{-\alpha^* a}$ is the usual displacement operator, ρ is the density operator for the resonator field, and $a^\dagger$ and a are creation and annihilation operators for the resonator field.

The qubit frequency and the microwave pulse used in all the results shown in the following slides have the form

$$\omega_q(t) = \omega_0 + \frac{(\Delta_0 - \Delta)}{2}\left[\mathrm{Erf}\left(\frac{t - t_\mathrm{q}}{\sqrt{2}\sigma_\mathrm{q}}\right) - \mathrm{Erf}\left(\frac{t - t_\mathrm{qe}}{\sqrt{2}\sigma_\mathrm{qe}}\right)\right]$$

$$B(t) = \frac{B_0}{2}\left[\mathrm{Erf}\left(\frac{t - t_\mathrm{B}}{\sqrt{2}\sigma_\mathrm{B}}\right) - \mathrm{Erf}\left(\frac{t - t_\mathrm{B} - \tau_\mathrm{B}}{\sqrt{2}\sigma_\mathrm{B}}\right)\right]$$

The parameters used for the plots presented in all the slides are:

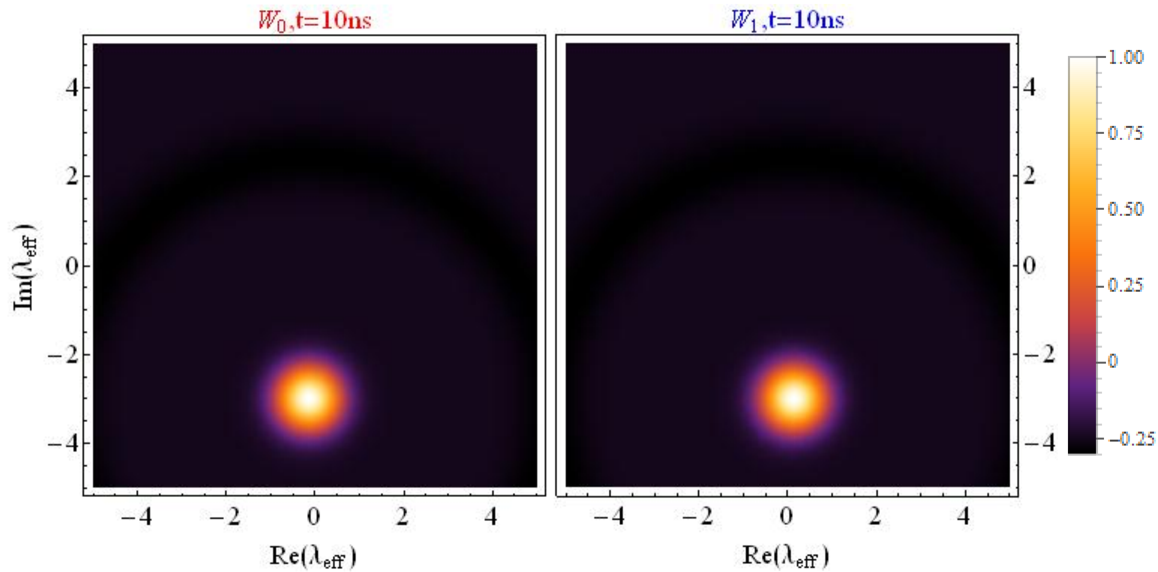
$$\frac{\omega_0}{2\pi} = 6\mathrm{GHz}, \frac{\Delta_0}{2\pi} = 1\mathrm{GHz}, \frac{\Delta}{2\pi} = 80\mathrm{MHz}, \frac{g}{2\pi} = 30\mathrm{MHz}$$

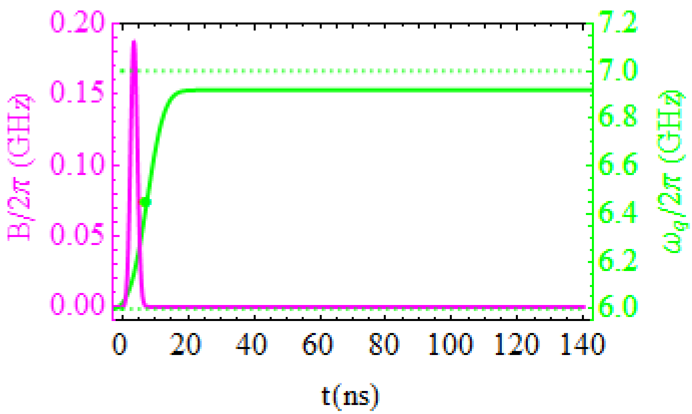
$$\sigma_\mathrm{q} = 4\mathrm{ns},\ \sigma_\mathrm{qe} = 1\mathrm{ns},\ t_\mathrm{q} = 10\mathrm{ns},\ t_{q\mathrm{e}} = 160\mathrm{ns}$$

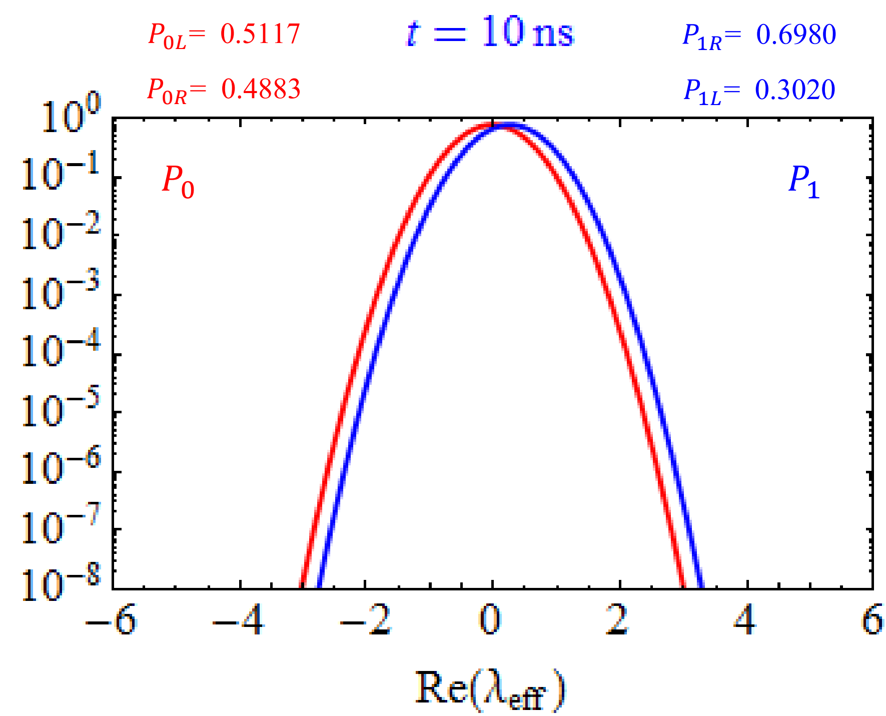
$$\frac{B_0}{2\pi} = 0.4974\mathrm{GHz}\ (\sim 9\ \text{photons}), \sigma_B = 1\mathrm{ns}, t_\mathrm{B} = 3\mathrm{ns}, \tau_\mathrm{B} = 1\mathrm{ns}$$

W_0— Wigner function for resonator state corresponding to an initial state $|00\rangle$

W_1— Wigner function for resonator state corresponding to an initial state $\overline{|10\rangle}$





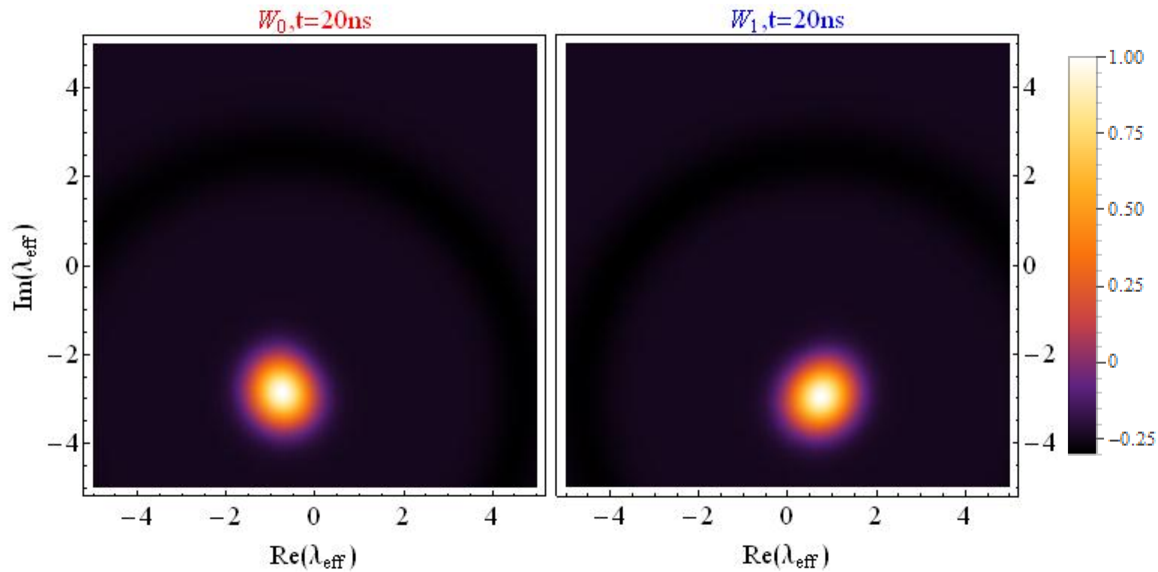


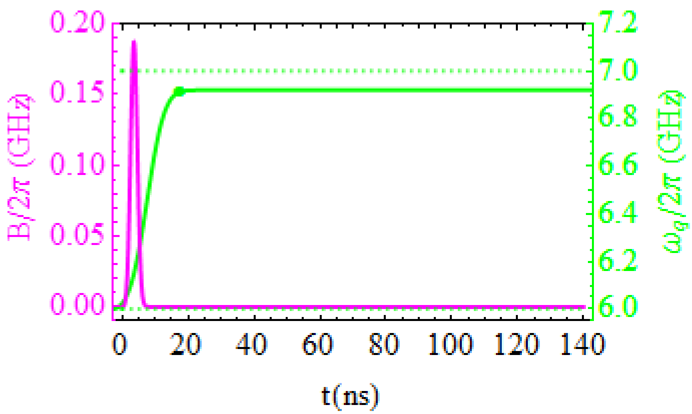
Initial state:$|00\rangle$ (qubit in state $|0\rangle$)

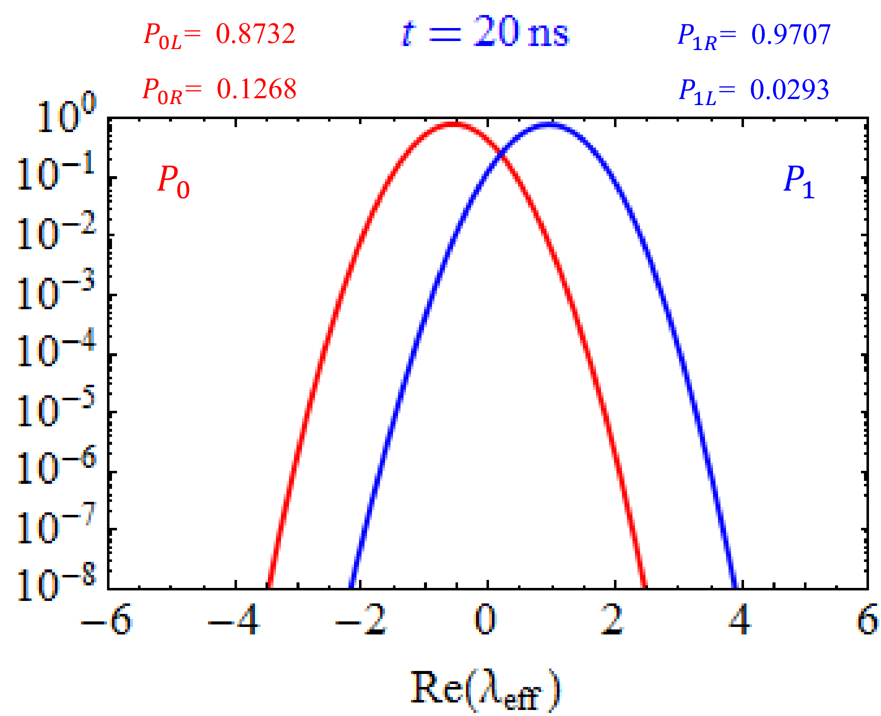
P_{0L} — probability for qubit to be in state $|0\rangle$
P_{0R} — probability for qubit to be in state $|1\rangle$

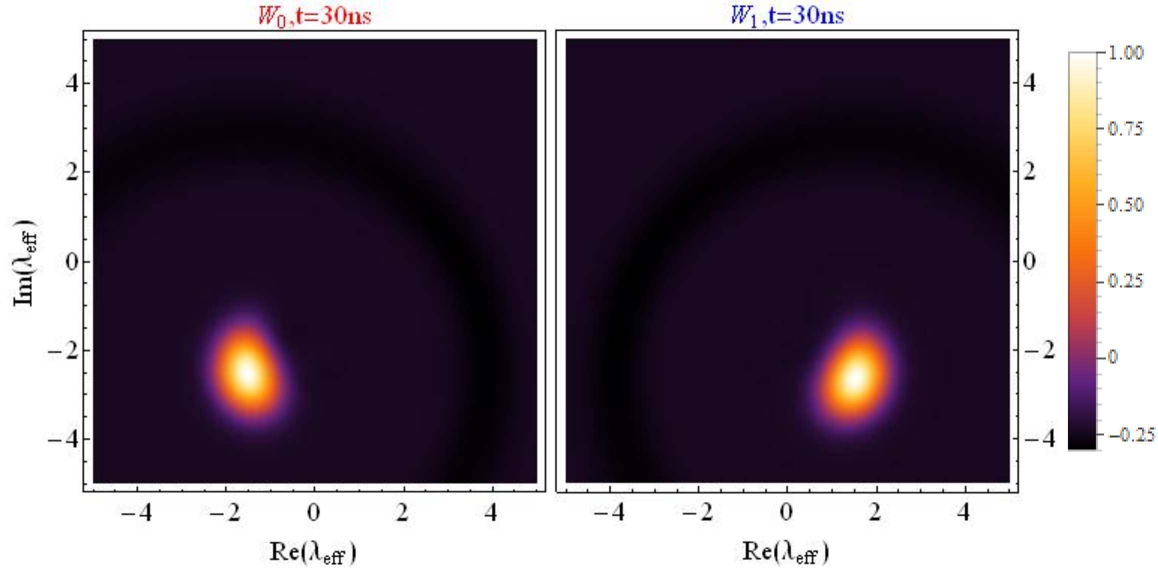
Initial state: $\overline{|10\rangle}$ (qubit in state $|1\rangle$)

P_{1R} — probability for qubit to be in state $|1\rangle$
P_{1L} — probability for qubit to be in state $|0\rangle$

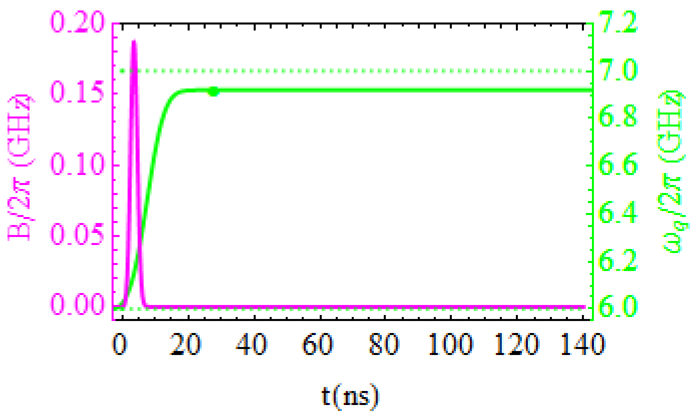

W_0,t=20ns
W_1,t=20ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25


B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)

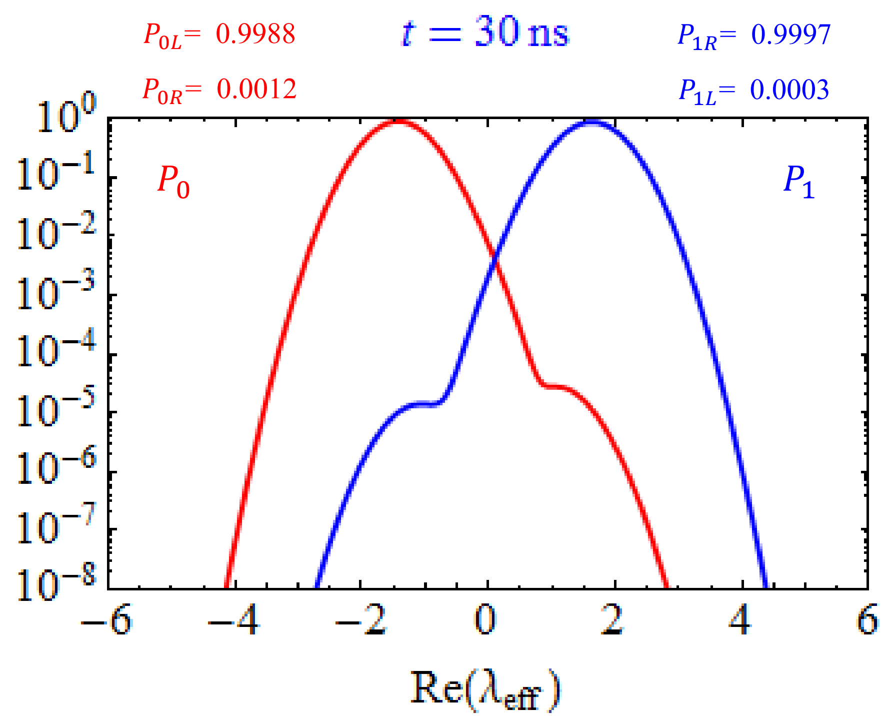

P_{0L}= 0.8732
P_{0R}= 0.1268
$t = 20$ ns
P_{1R}= 0.9707
P_{1L}= 0.0293
P_0
P_1
Re(λ_{eff})



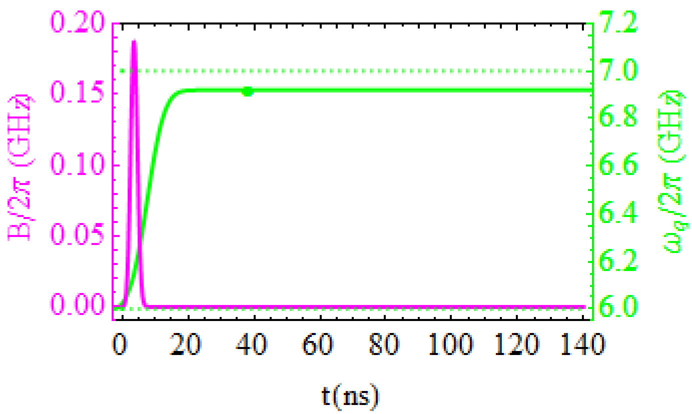
W_0,t=30ns
W_1,t=30ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})



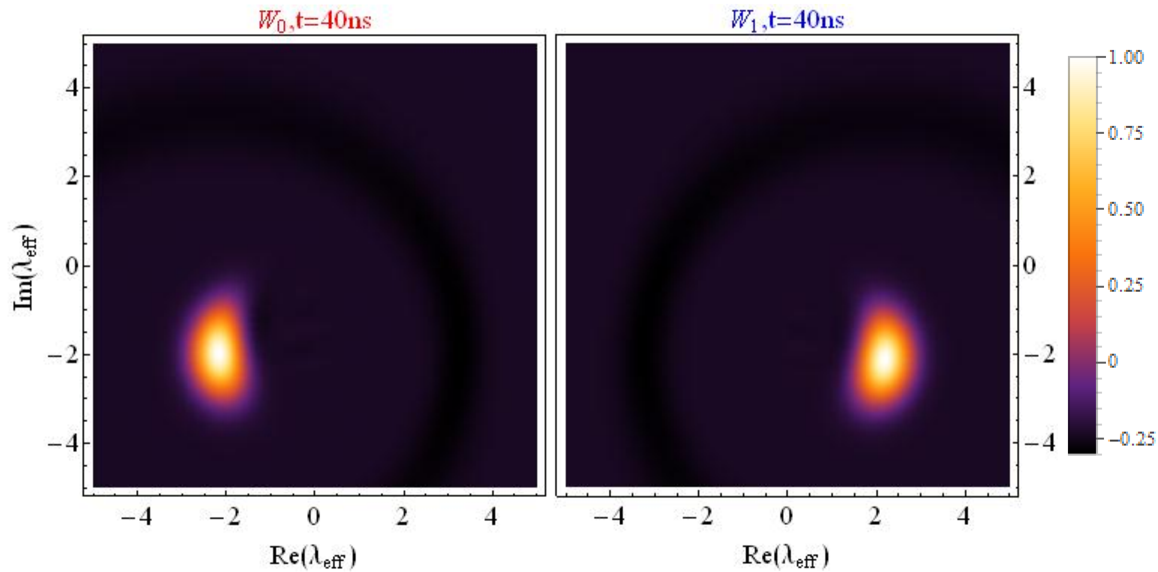
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)



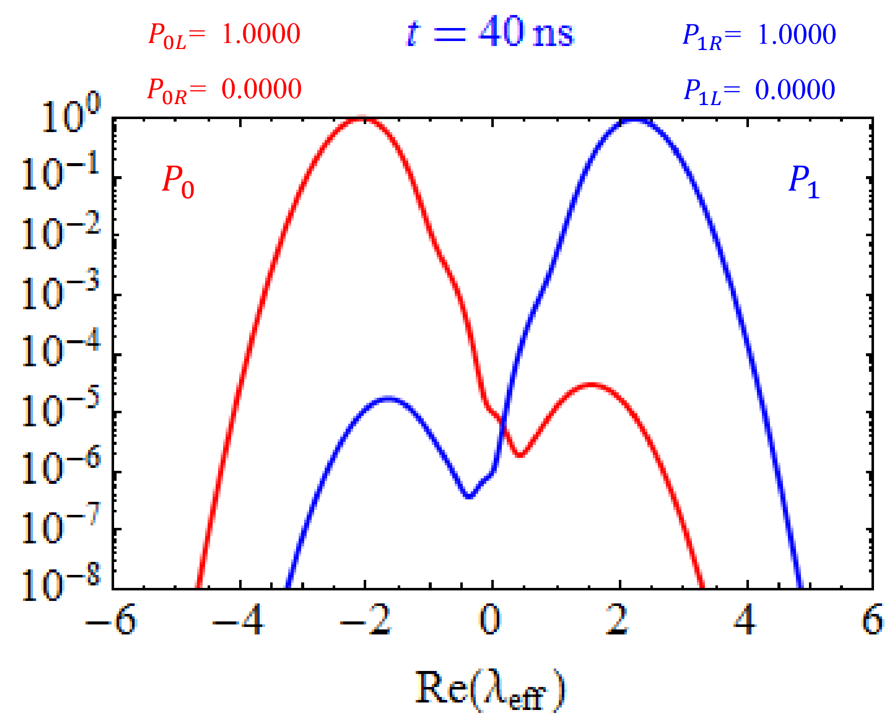
P_{0L}= 0.9988
P_{0R}= 0.0012
$t = 30$ ns
P_{1R}= 0.9997
P_{1L}= 0.0003
P_0
P_1
Re(λ_{eff})



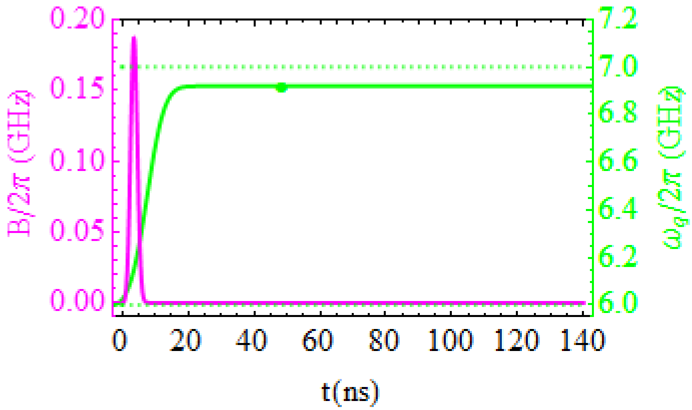
0.20
0.15
0.10
0.05
0.00
B/2π (GHz)
7.2
7.0
6.8
6.6
6.4
6.2
6.0
ωq/2π (GHz)
0 20 40 60 80 100 120 140
t(ns)



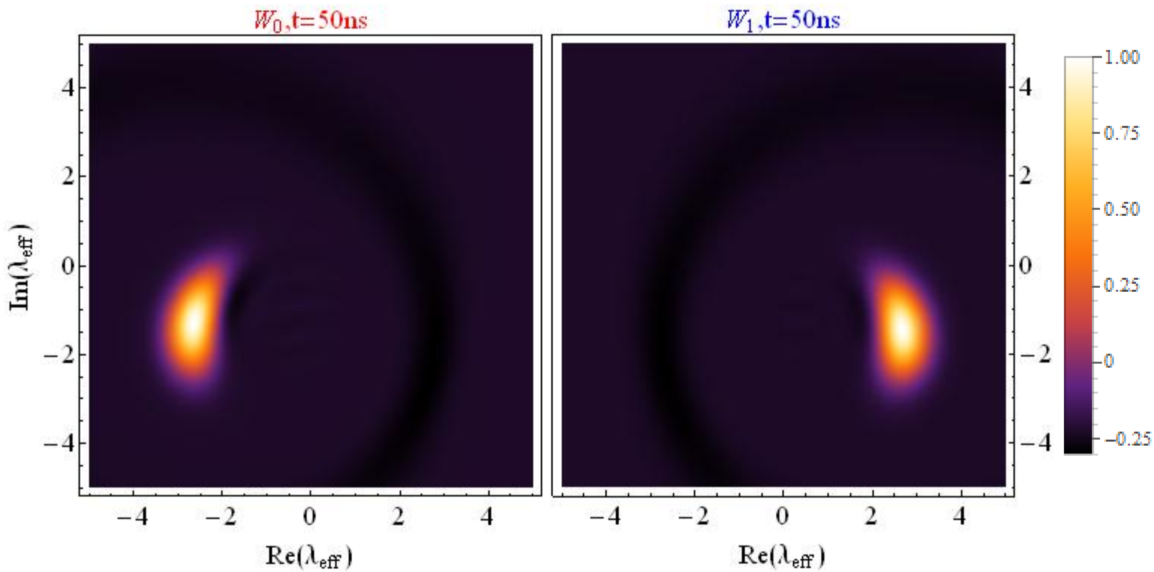
W0,t=40ns
W1,t=40ns
Im(λeff)
Re(λeff)
Re(λeff)
1.00
0.75
0.50
0.25
0
−0.25



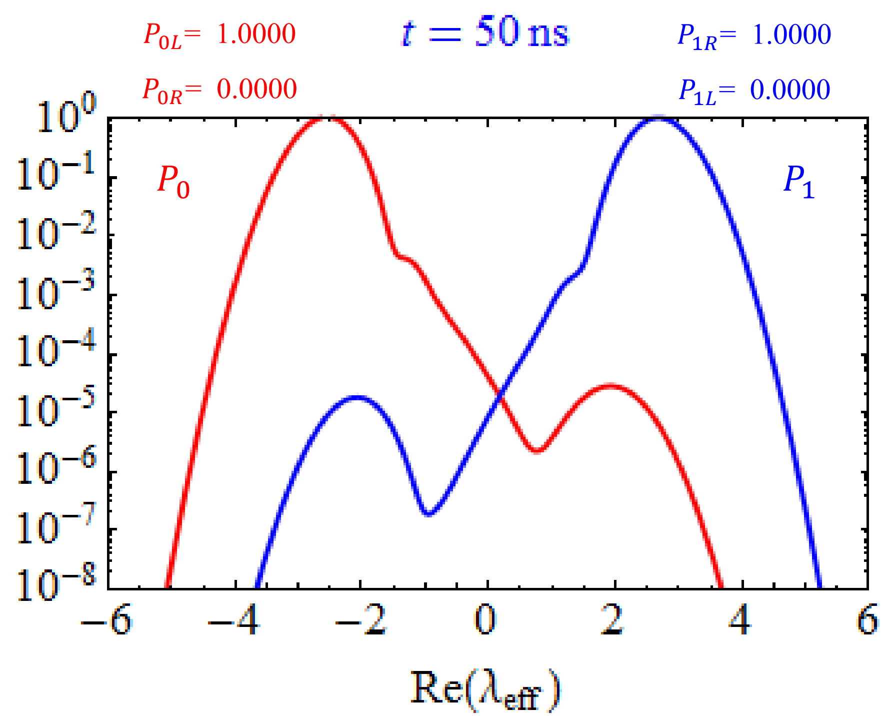
P0L= 1.0000
P0R= 0.0000
t = 40 ns
P1R= 1.0000
P1L= 0.0000
P0
P1
Re(λeff)



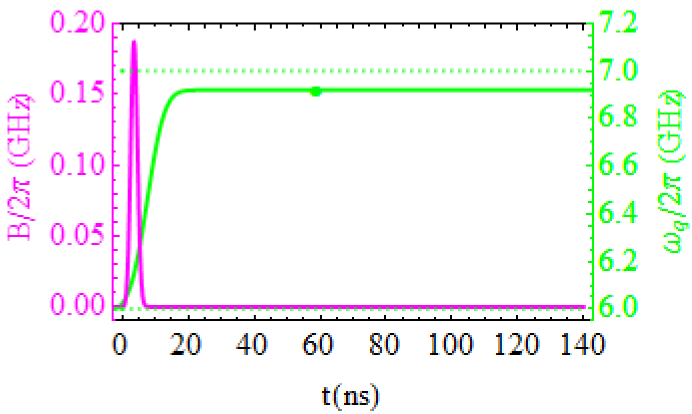
0.20
0.15
0.10
0.05
0.00
B/2π (GHz)
7.2
7.0
6.8
6.6
6.4
6.2
6.0
ω_q/2π (GHz)
0
20
40
60
80
100
120
140
t(ns)



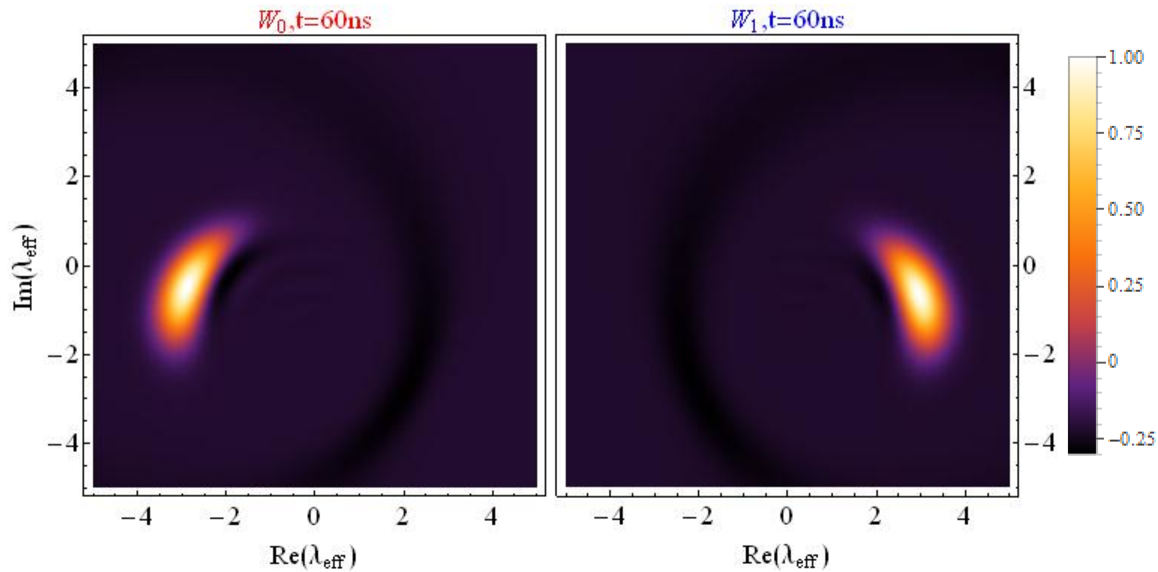
W_0,t=50ns
W_1,t=50ns
Im(λ_eff)
Re(λ_eff)
Re(λ_eff)
1.00
0.75
0.50
0.25
0
−0.25



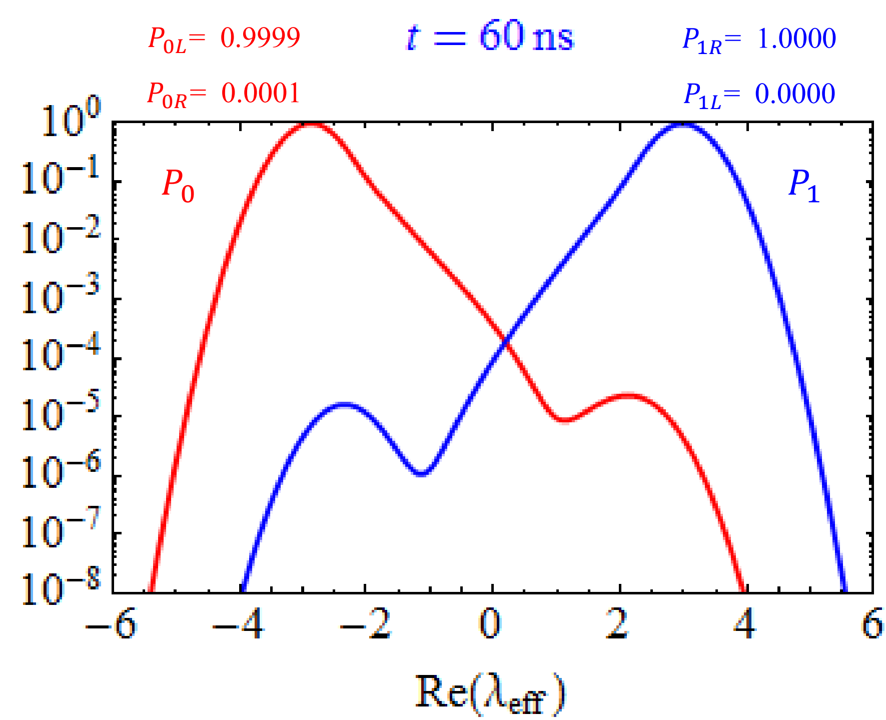
P_0L= 1.0000
P_0R= 0.0000
t = 50 ns
P_1R= 1.0000
P_1L= 0.0000
P_0
P_1
Re(λ_eff)



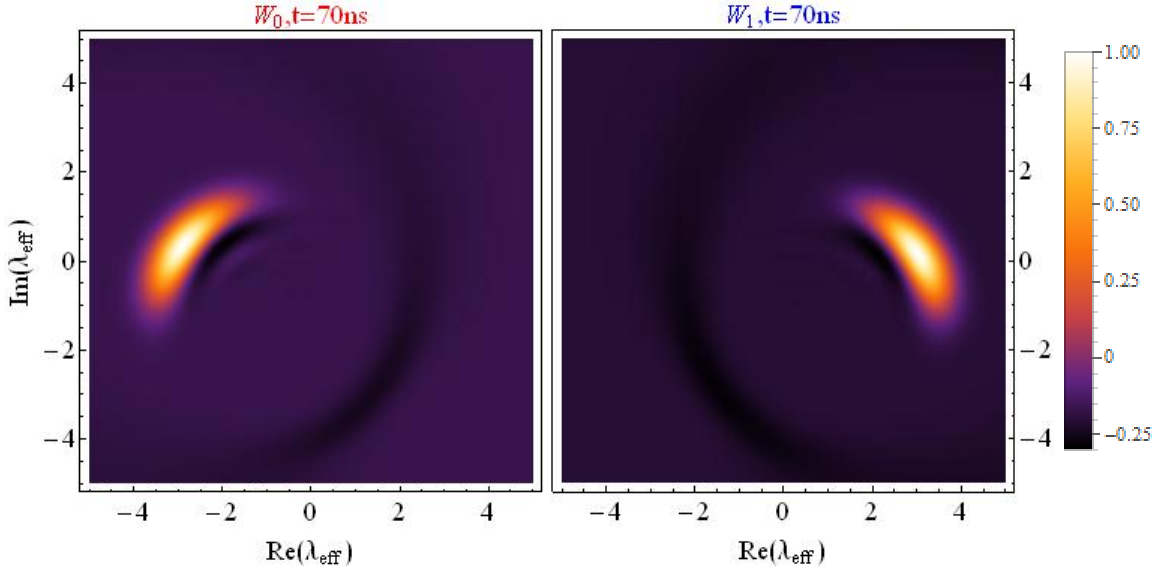
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)



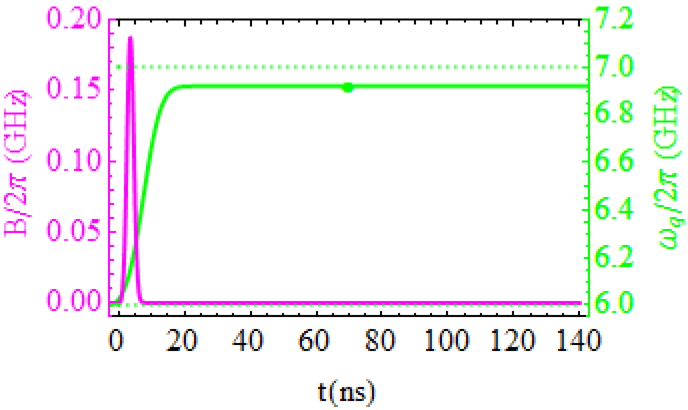
W_0,t=60ns
W_1,t=60ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})



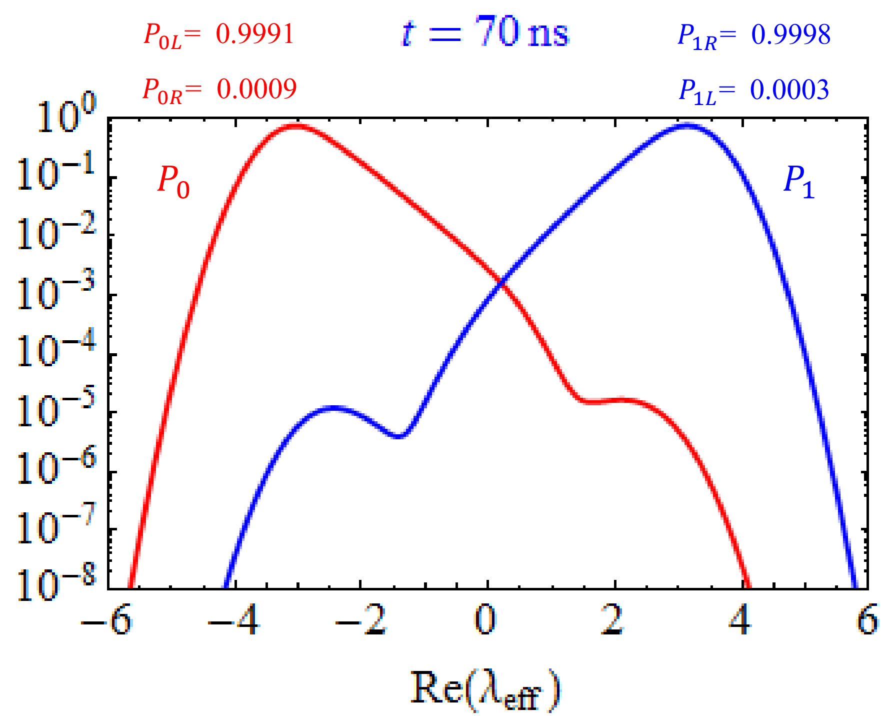
P_{0L}= 0.9999
t = 60 ns
P_{1R}= 1.0000
P_{0R}= 0.0001
P_{1L}= 0.0000
P_0
P_1
Re(λ_{eff})



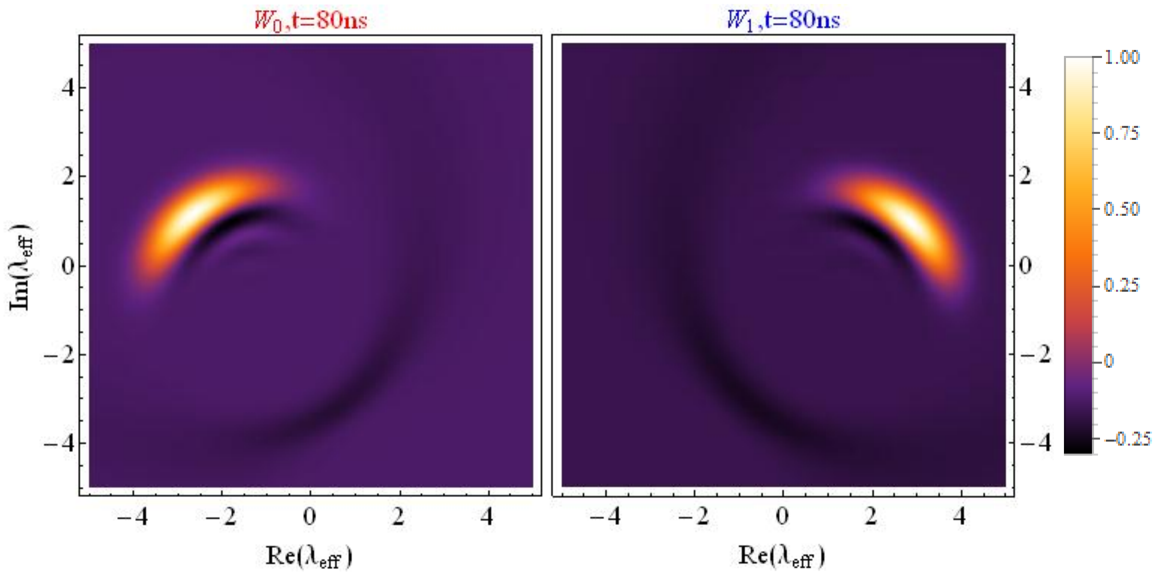
W_0,t=70ns
W_1,t=70ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25



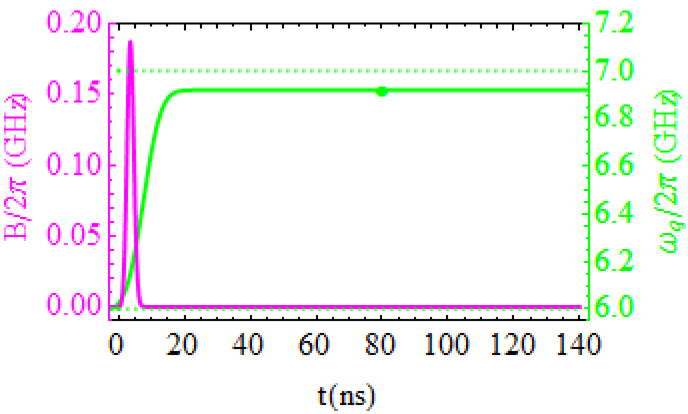
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)



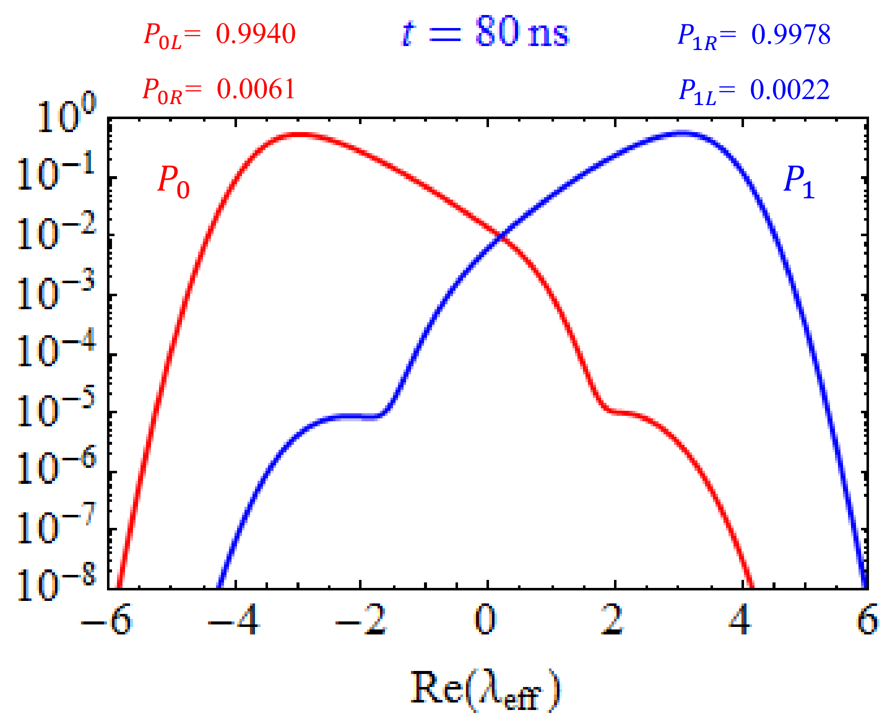
P_{0L}= 0.9991
$t = 70$ ns
P_{1R}= 0.9998
P_{0R}= 0.0009
P_{1L}= 0.0003
P_0
P_1
Re(λ_{eff})



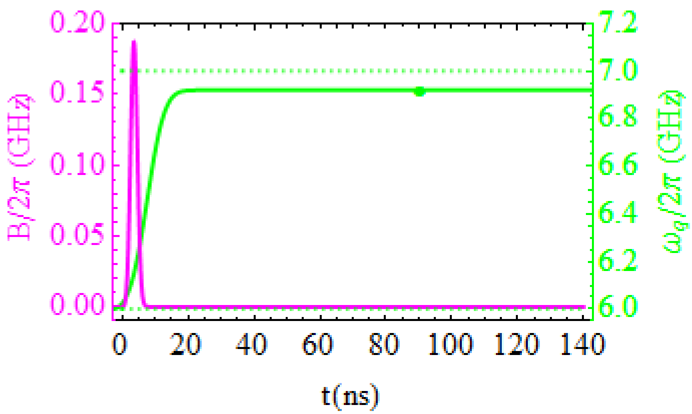
W_0,t=80ns
W_1,t=80ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25

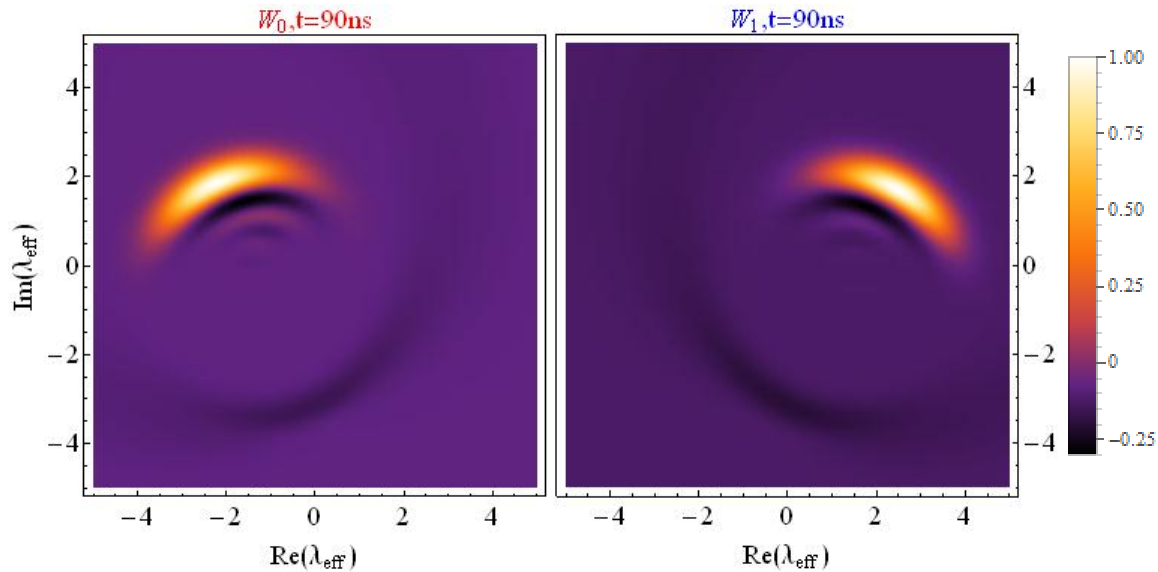


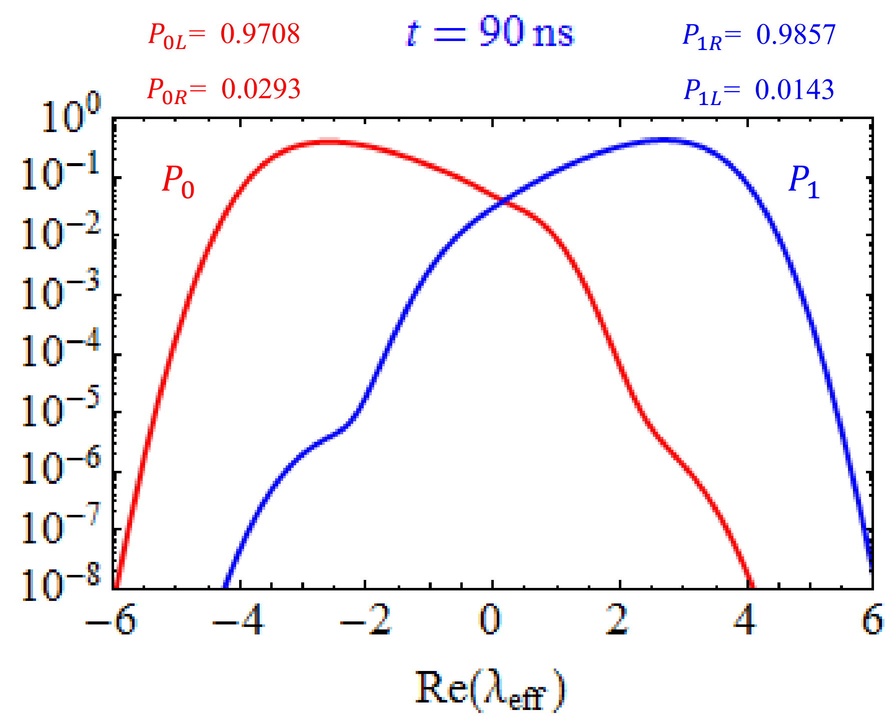
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)

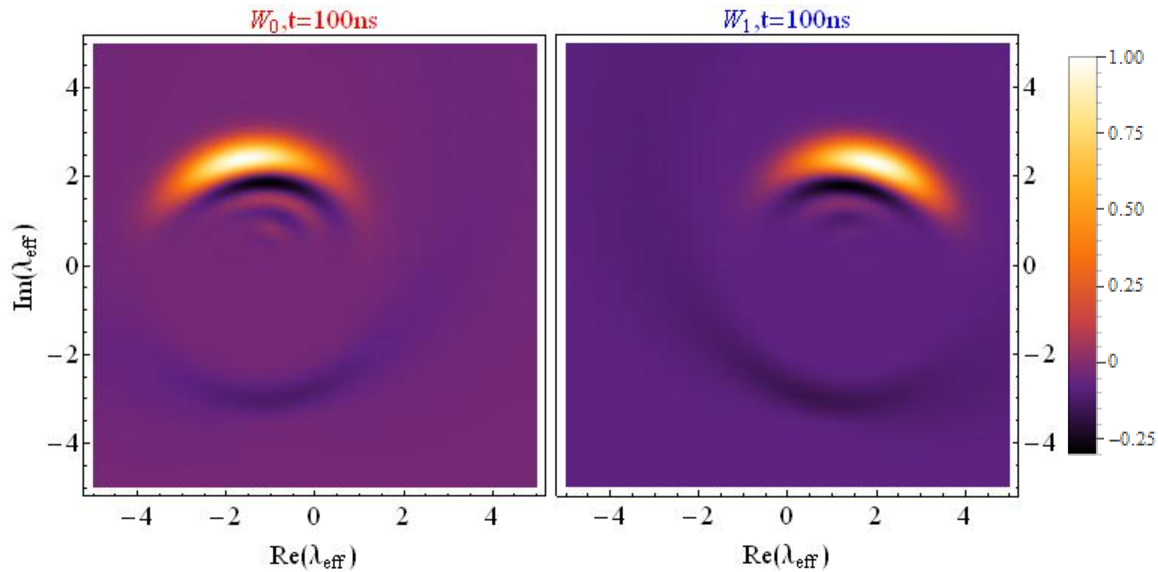


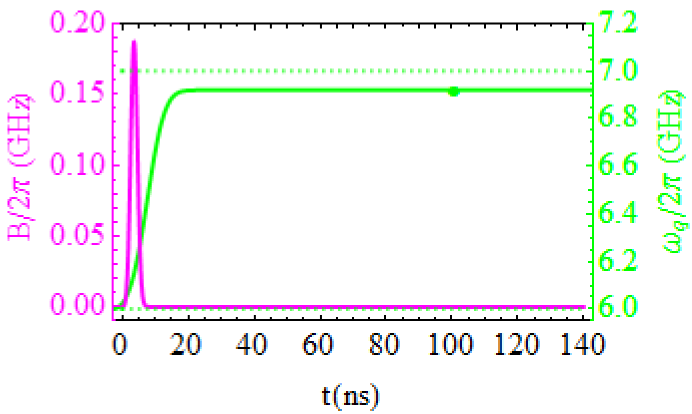
P_{0L}= 0.9940
t = 80 ns
P_{1R}= 0.9978
P_{0R}= 0.0061
P_{1L}= 0.0022
P_0
P_1
Re(λ_{eff})

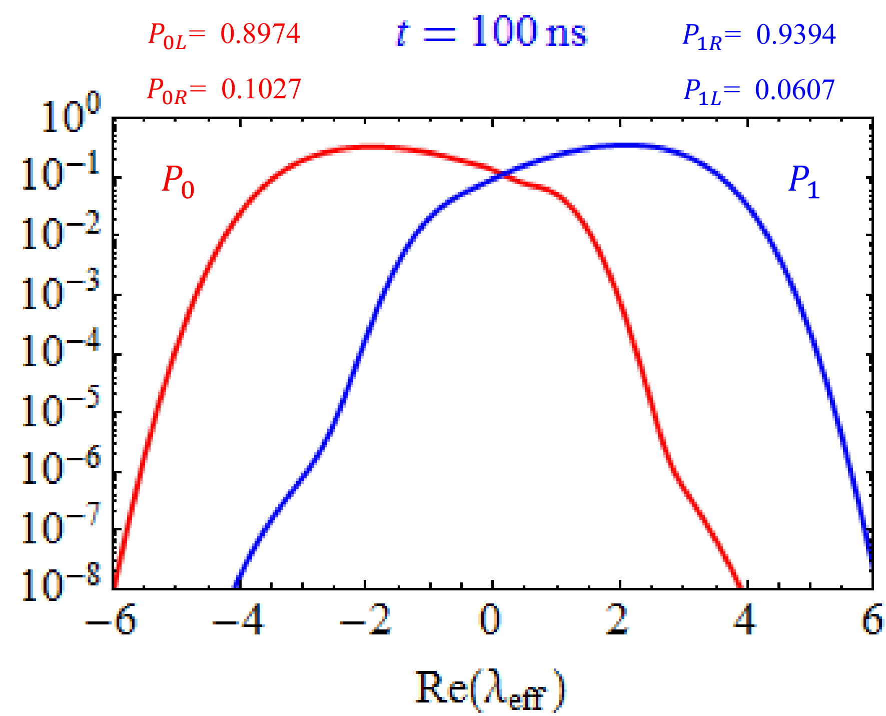

B/2π (GHz)
ω_q/2π (GHz)
t(ns)
0.20
0.15
0.10
0.05
0.00
7.2
7.0
6.8
6.6
6.4
6.2
6.0
0
20
40
60
80
100
120
140

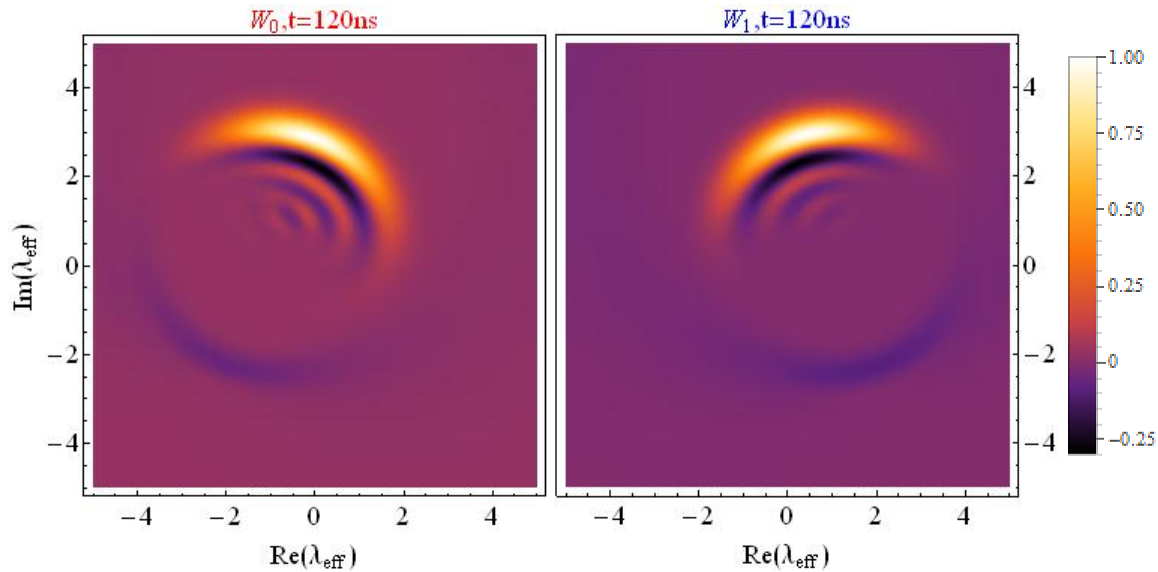

W_0,t=90ns
W_1,t=90ns
Im(λ_eff)
Re(λ_eff)
Re(λ_eff)
1.00
0.75
0.50
0.25
0
−0.25

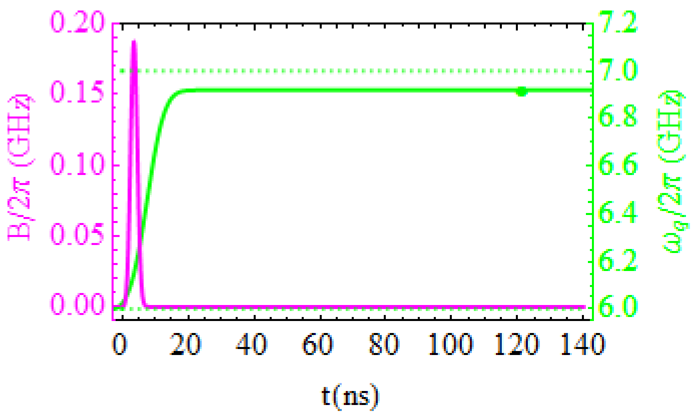

P_0L= 0.9708
P_0R= 0.0293
t = 90 ns
P_1R= 0.9857
P_1L= 0.0143
P_0
P_1
Re(λ_eff)


W_0,t=100ns
W_1,t=100ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25


B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)

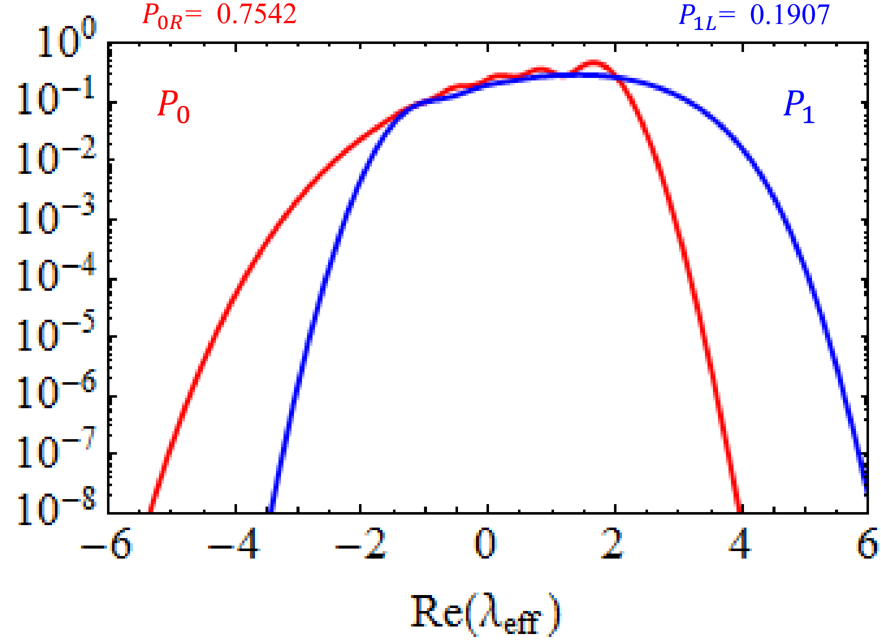

P_{0L}= 0.8974
P_{0R}= 0.1027
$t = 100$ ns
P_{1R}= 0.9394
P_{1L}= 0.0607
P_0
P_1
Re(λ_{eff})

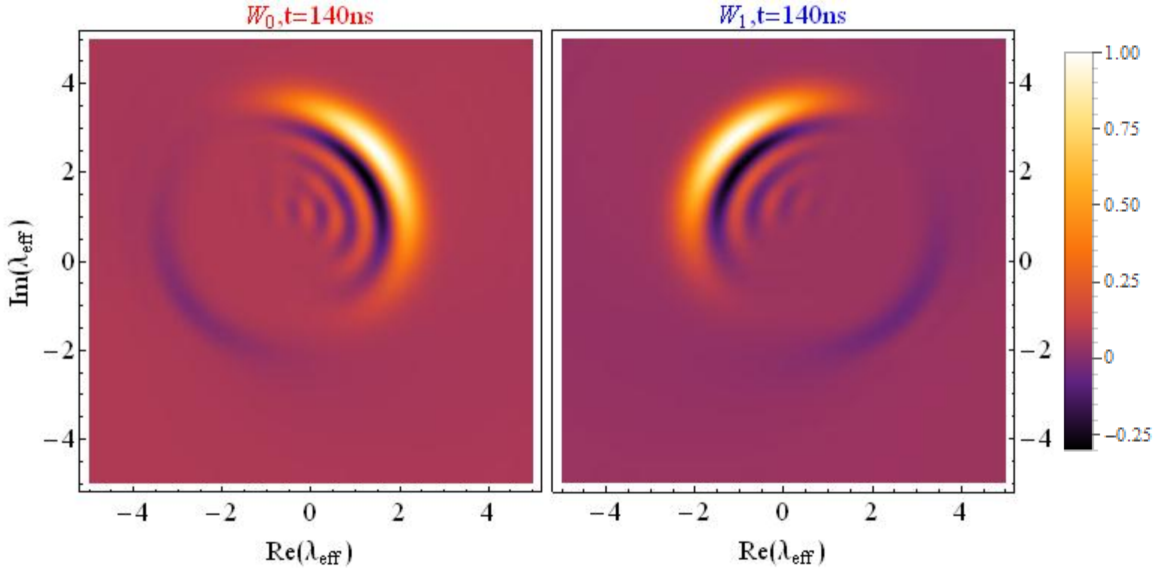


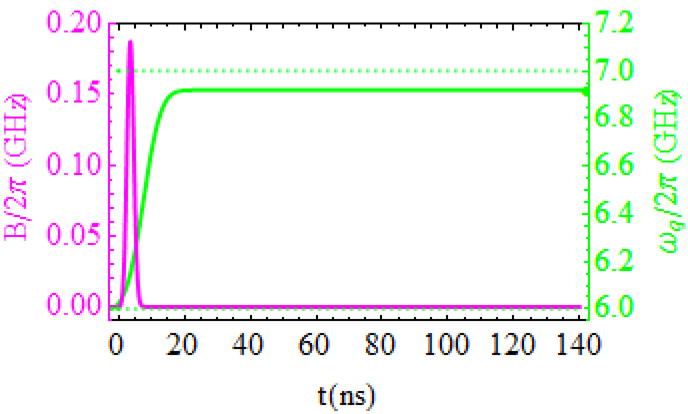


P_{0L}= 0.2459 $t = 120$ ns P_{1R}= 0.8094

P_{0R}= 0.7542 P_{1L}= 0.1907







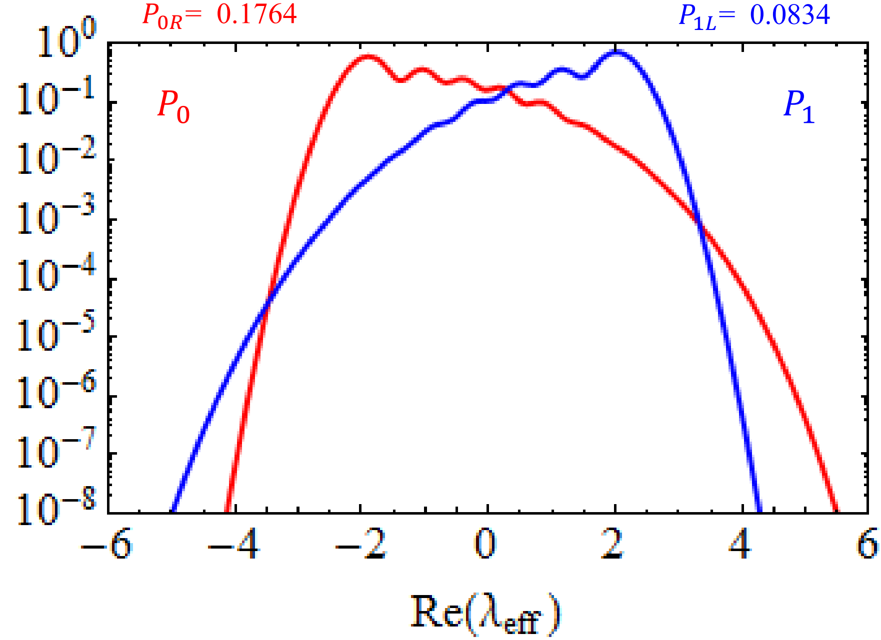
P_{0L}= 0.8237

P_{0R}= 0.1764

t = 140 ns

P_{1R}= 0.9167

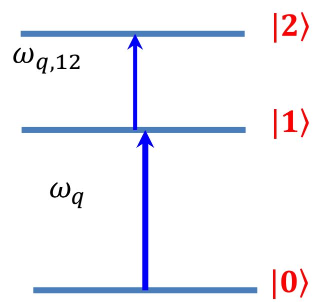
P_{1L}= 0.0834



Effect of level $|2\rangle$ on the evolution of the Wigner function of the resonator field

In the previous slides, we assumed the two-level model of the qubit. Real superconducting qubits, however, are only slightly anharmonic oscillators, thus the effect of the next excited level $|2\rangle$ is often important as shown the following slides.

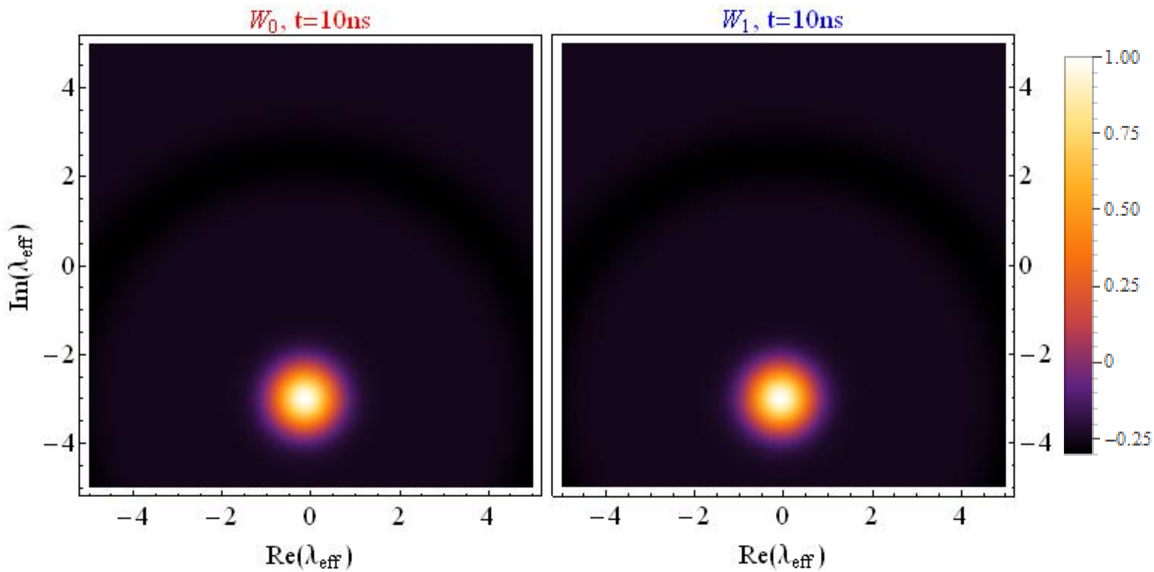
The qubit anharmonicity is given by $\mathcal{A} = \omega_q - \omega_{q,12}$. For phase and transmon superconducting qubits the anharmonicity is about 3%. Here we assume $\mathcal{A}/2\pi = 200$ MHz in all results shown in the following slides. All other parameters are the same as in page 2.

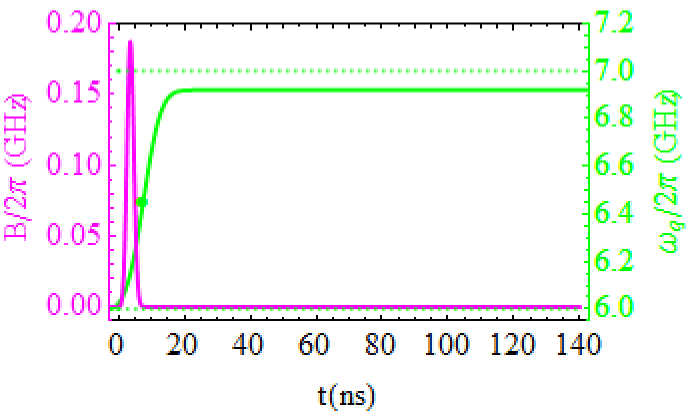


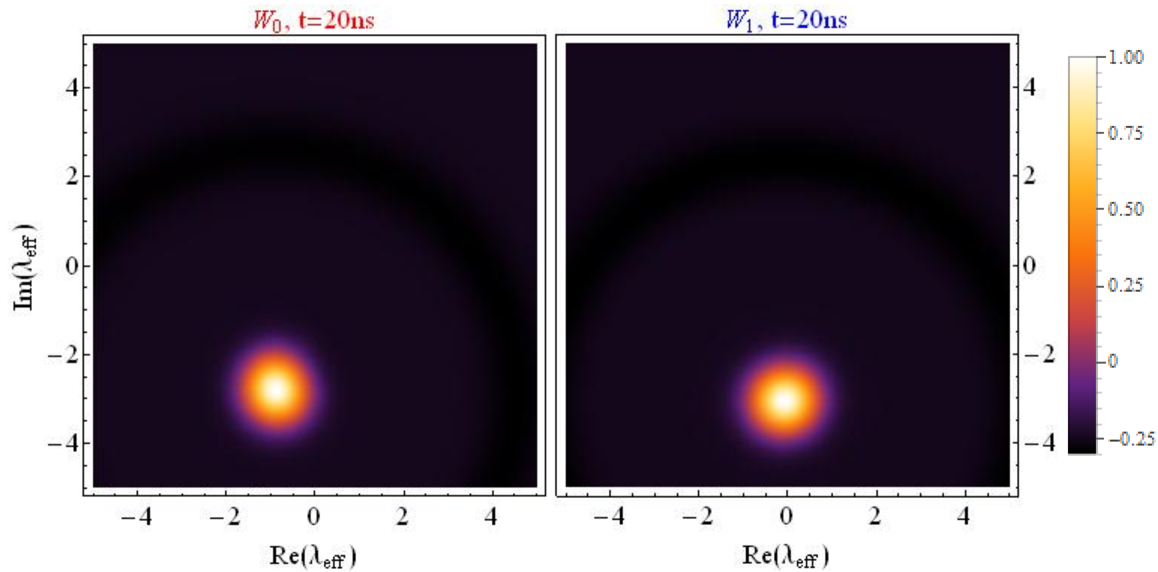
Three-level model of the qubit

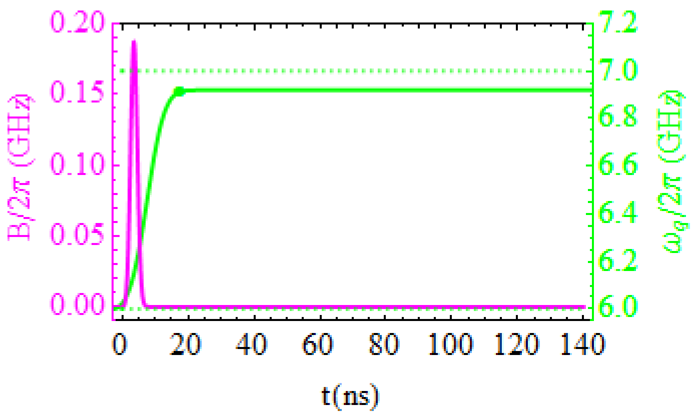
W_0— Wigner function for resonator state corresponding to an initial state $|00\rangle$

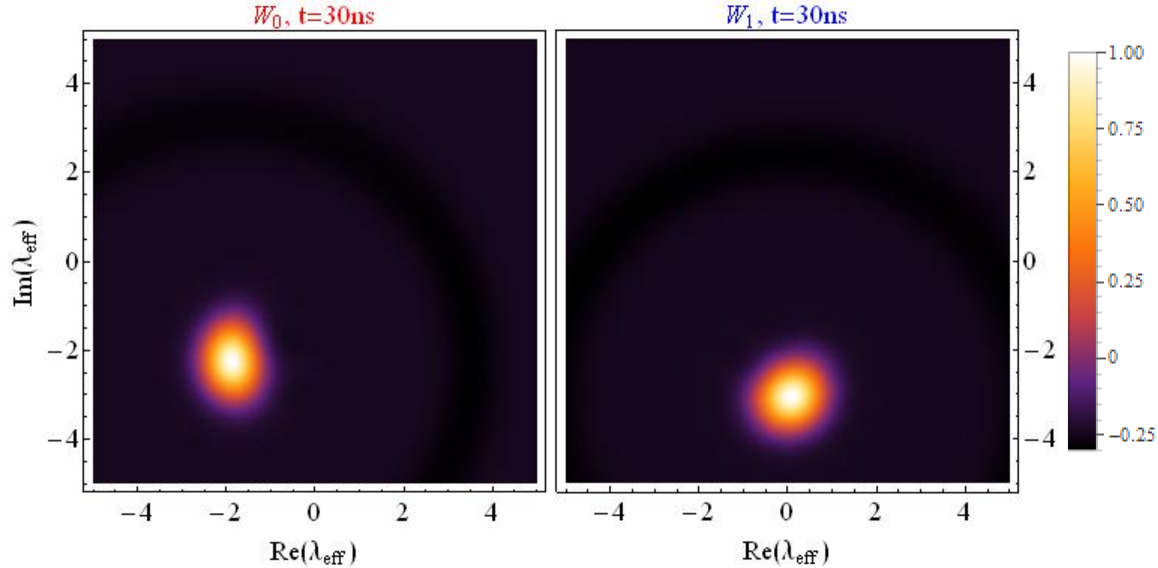
W_1—Wigner function for resonator state corresponding to an initial state $\overline{|10\rangle}$



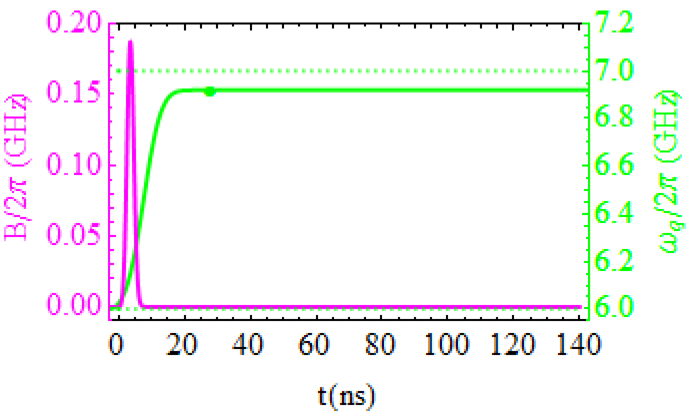



W_0, t=20ns
W_1, t=20ns
$\mathrm{Im}(\lambda_{\mathrm{eff}})$
$\mathrm{Re}(\lambda_{\mathrm{eff}})$
$\mathrm{Re}(\lambda_{\mathrm{eff}})$
1.00
0.75
0.50
0.25
0
−0.25

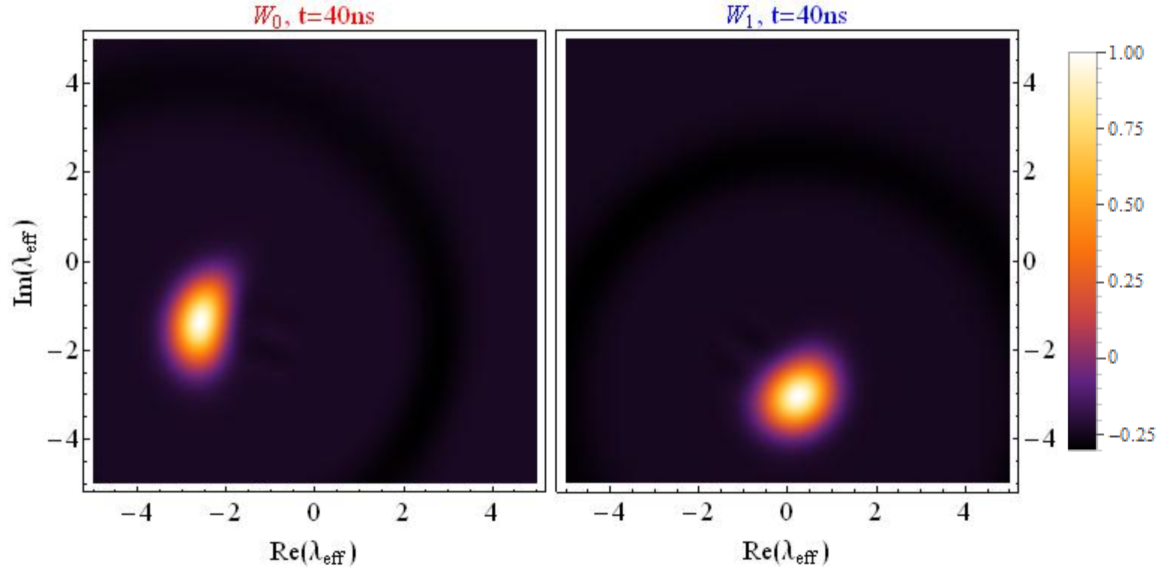

$B/2\pi$ (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)

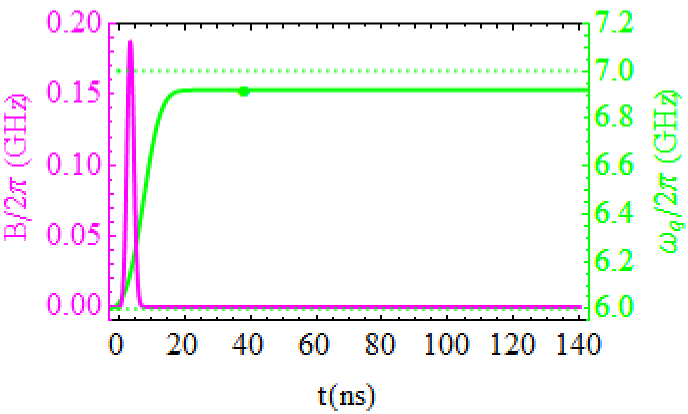


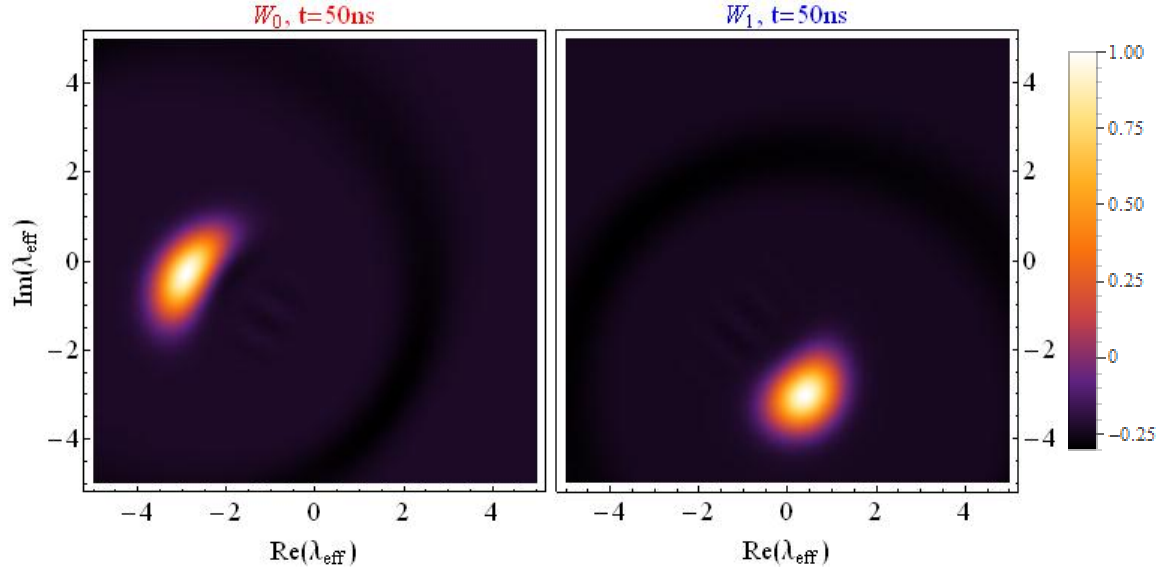
W_0, t=30ns
W_1, t=30ns
Im($\lambda_{\rm eff}$)
Re($\lambda_{\rm eff}$)
Re($\lambda_{\rm eff}$)
1.00
0.75
0.50
0.25
0
−0.25
4
2
0
−2
−4

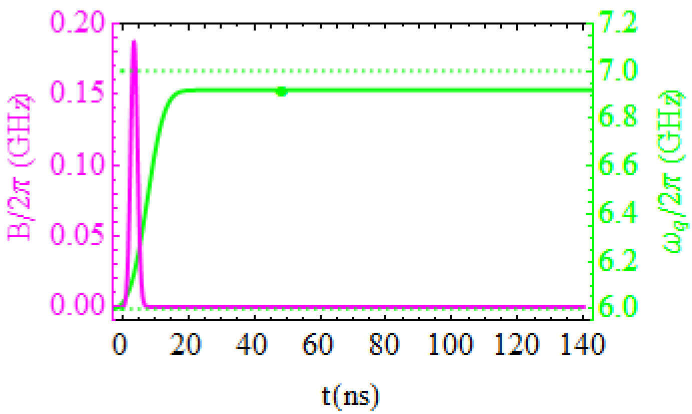


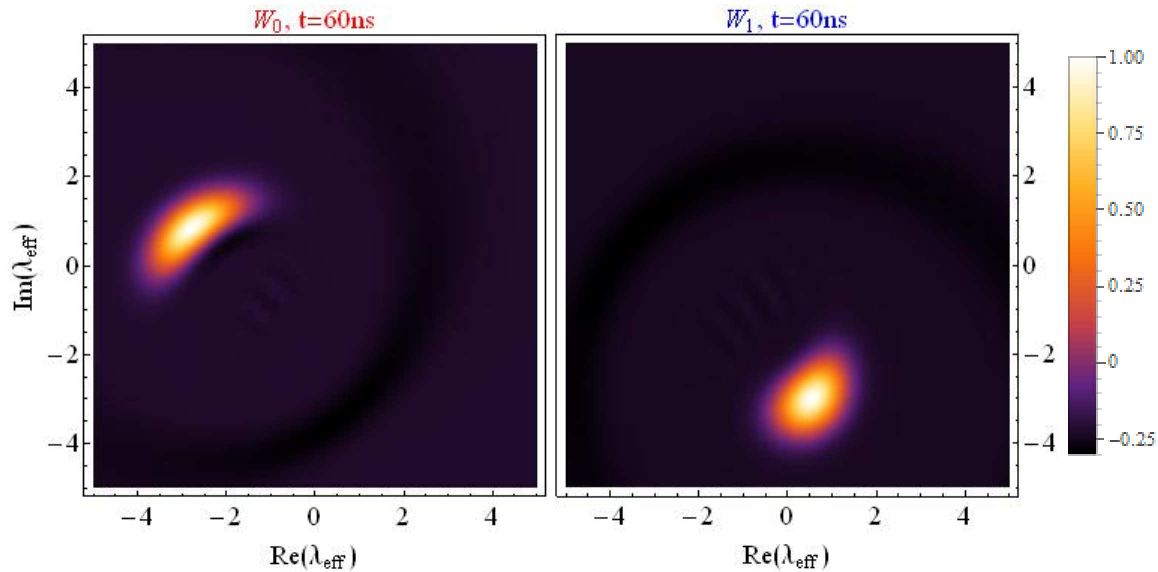
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)
0.20
0.15
0.10
0.05
0.00
7.2
7.0
6.8
6.6
6.4
6.2
6.0
0
20
40
60
80
100
120
140


W_0, t=40ns
W_1, t=40ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
4
2
0
−2
−4
1.00
0.75
0.50
0.25
0
−0.25

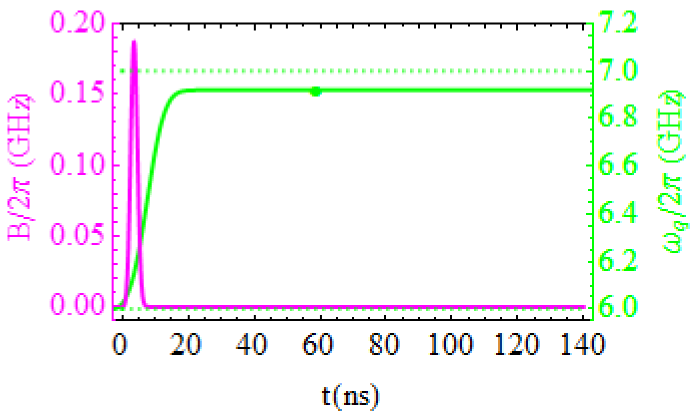

B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)
0.20
0.15
0.10
0.05
0.00
7.2
7.0
6.8
6.6
6.4
6.2
6.0
0 20 40 60 80 100 120 140


W_0, t=50ns
W_1, t=50ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
4
2
0
−2
−4
−4
−2
0
2
4
1.00
0.75
0.50
0.25
0
−0.25

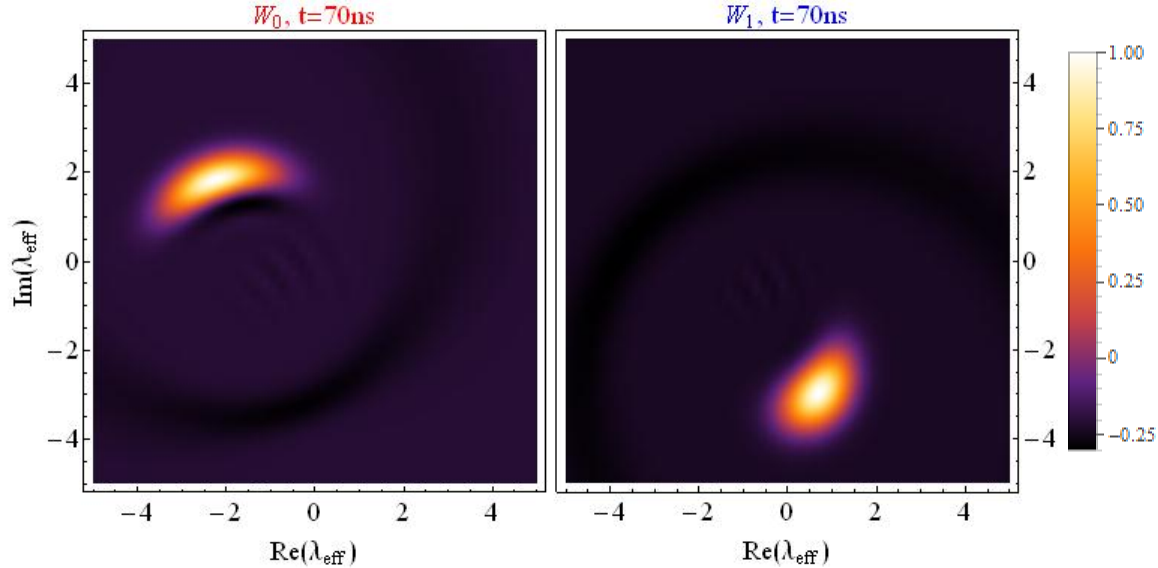

B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)
0.20
0.15
0.10
0.05
0.00
7.2
7.0
6.8
6.6
6.4
6.2
6.0
0
20
40
60
80
100
120
140



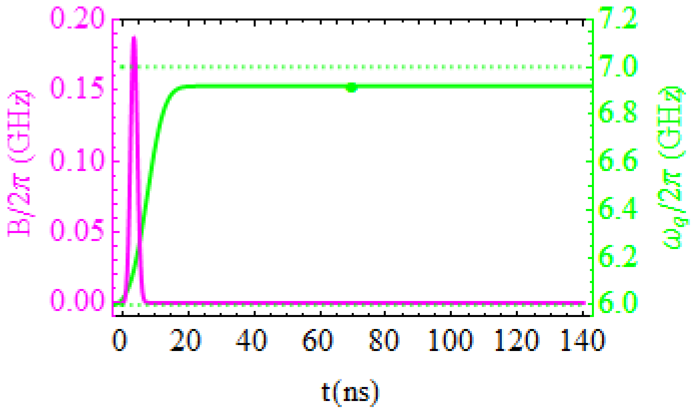
W_0, t=60ns
W_1, t=60ns
Im(λ_eff)
Re(λ_eff)
Re(λ_eff)
1.00
0.75
0.50
0.25
0
−0.25



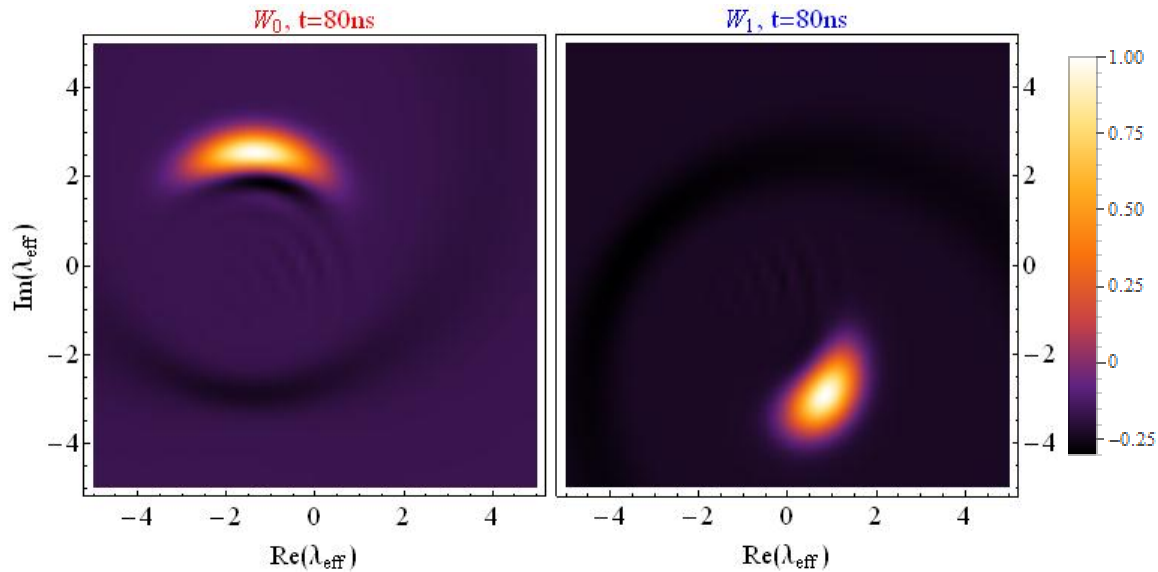
B/2π (GHz)
ω_q/2π (GHz)
t(ns)



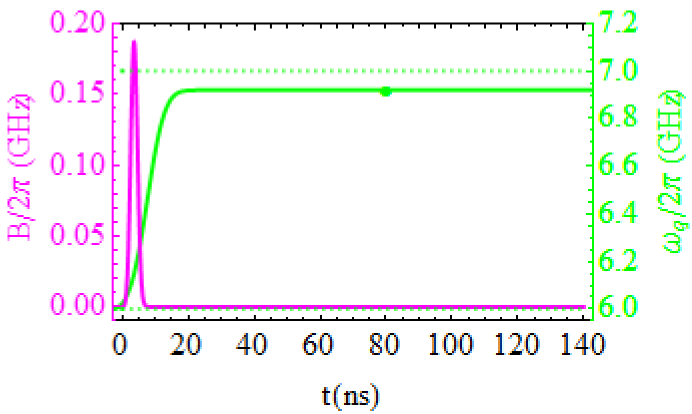
W_0, t=70ns
W_1, t=70ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25



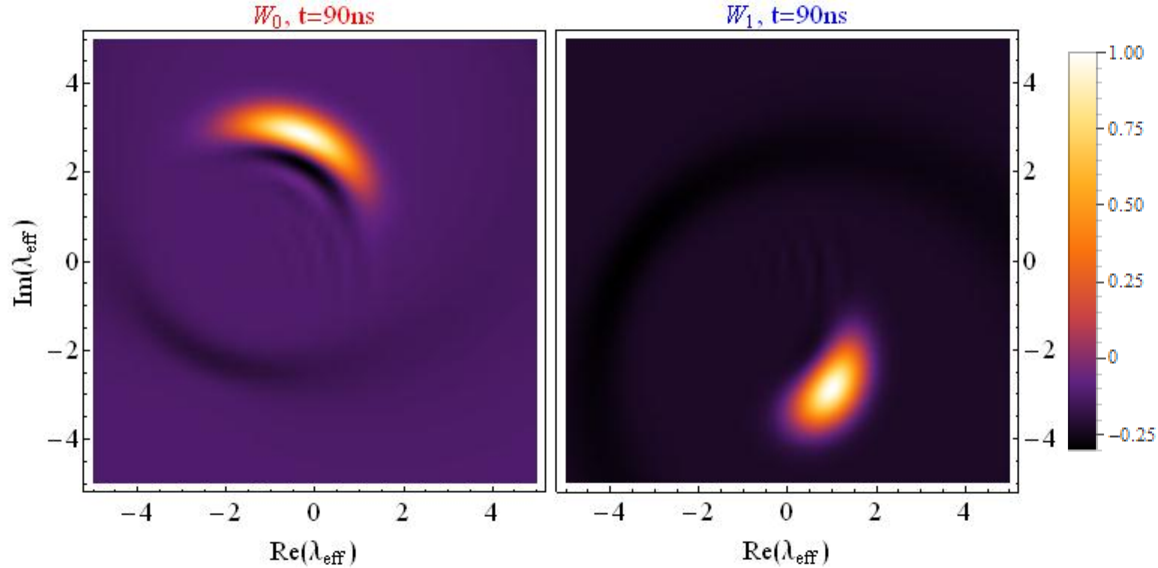
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)



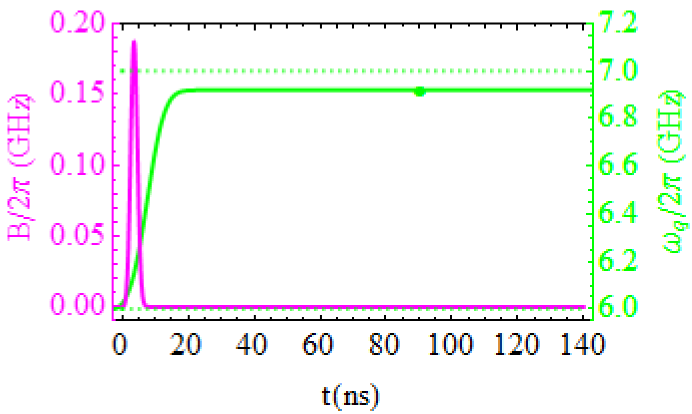
W_0, t=80ns
W_1, t=80ns
$\mathrm{Im}(\lambda_{\mathrm{eff}})$
$\mathrm{Re}(\lambda_{\mathrm{eff}})$
$\mathrm{Re}(\lambda_{\mathrm{eff}})$



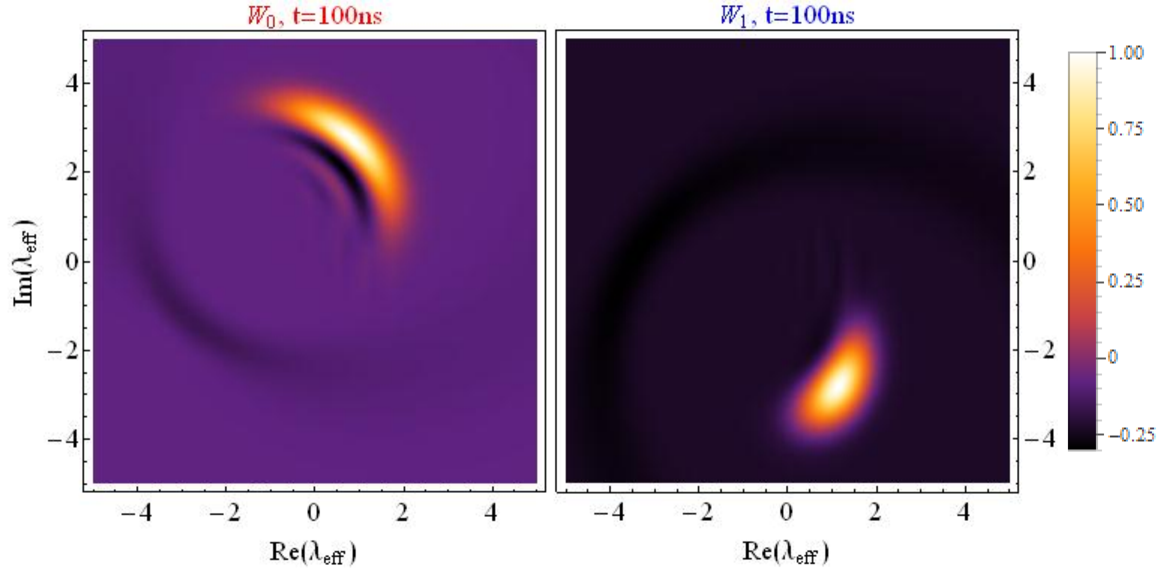
$B/2\pi$ (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)

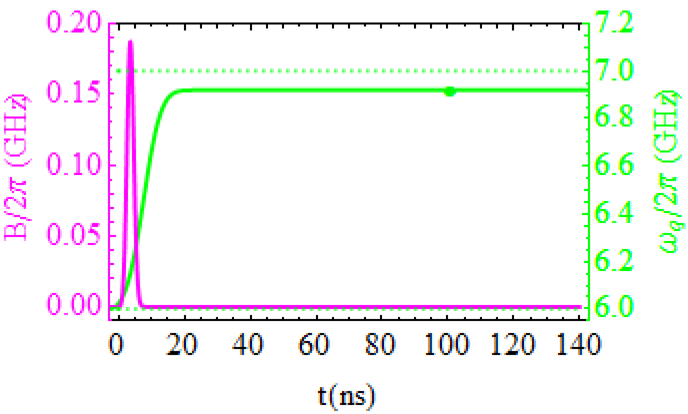


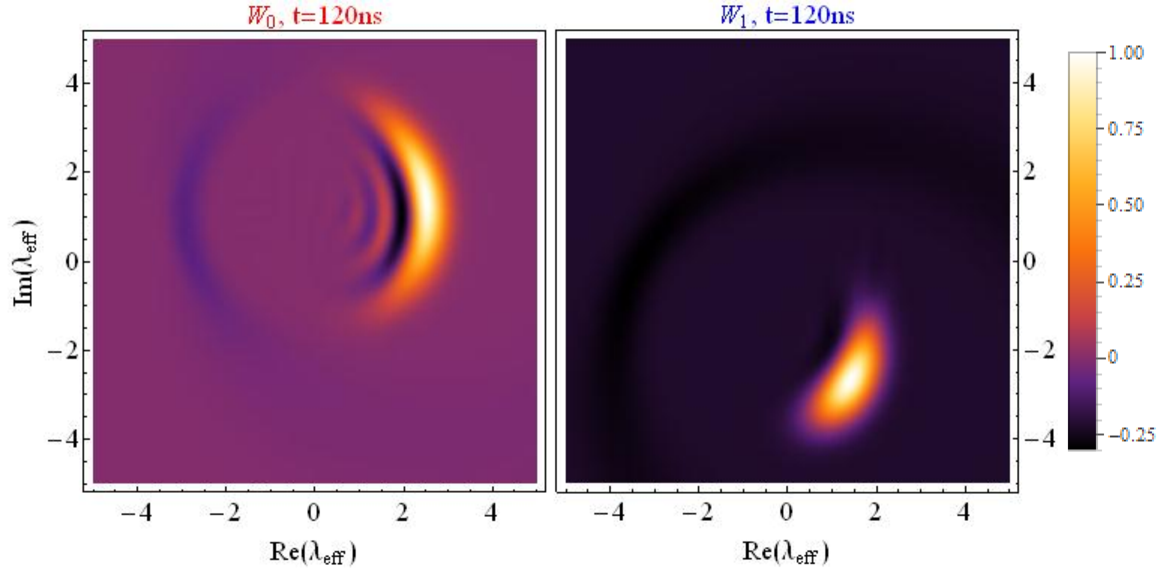
W_0, t=90ns
W_1, t=90ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})

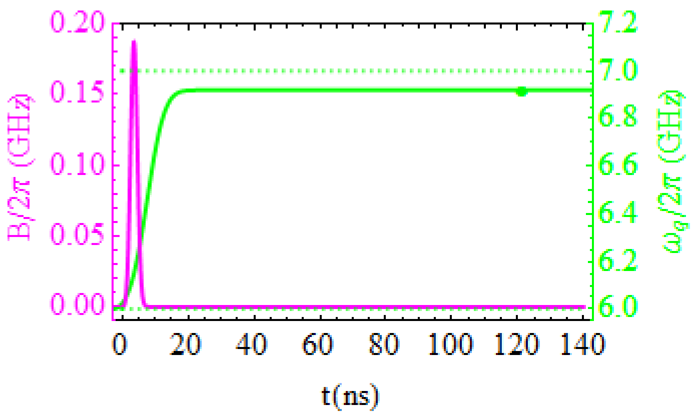


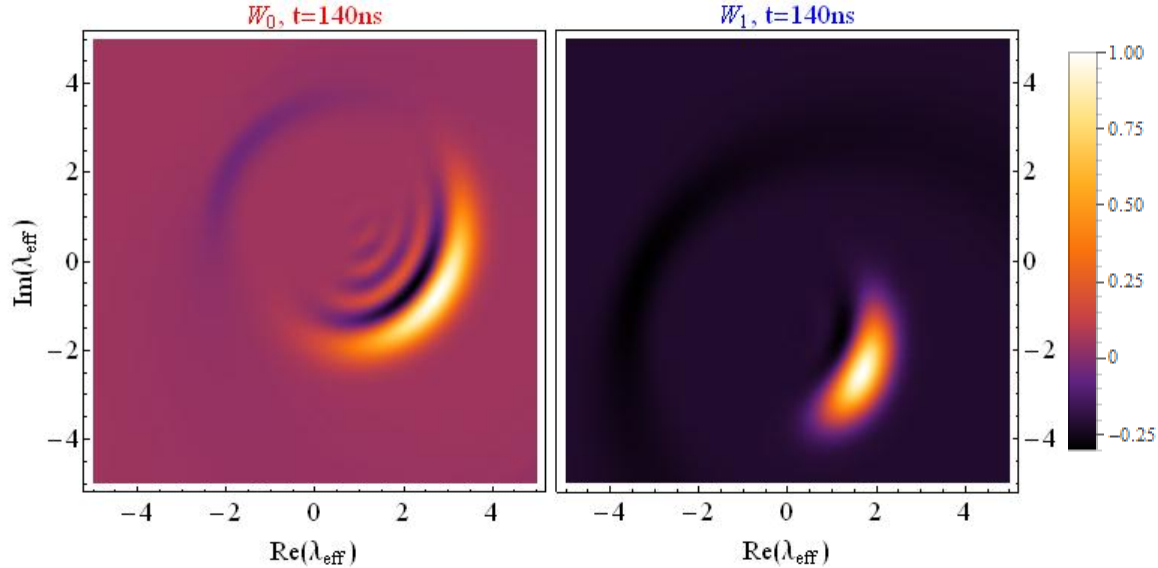
B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)


W_0, t=100ns
W_1, t=100ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25

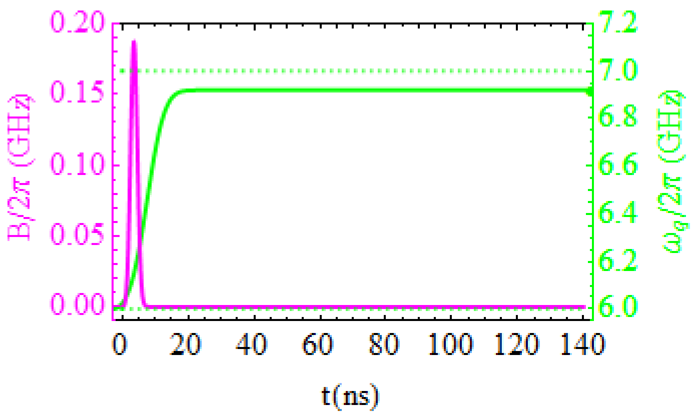

B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)


W_0, t=120ns
W_1, t=120ns
$\mathrm{Im}(\lambda_{\mathrm{eff}})$
$\mathrm{Re}(\lambda_{\mathrm{eff}})$
$\mathrm{Re}(\lambda_{\mathrm{eff}})$
1.00
0.75
0.50
0.25
0
−0.25


$B/2\pi$ (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)



W_0, t=140ns
W_1, t=140ns
Im(λ_{eff})
Re(λ_{eff})
Re(λ_{eff})
1.00
0.75
0.50
0.25
0
−0.25



B/2π (GHz)
$\omega_q/2\pi$ (GHz)
t(ns)