

Coherence of a Josephson phase qubit under partial-collapse measurement

Leonid P. Pryadko¹ and Alexander N. Korotkov²

¹*Department of Physics & Astronomy, University of California, Riverside, California 92521, USA*

²*Department of Electrical Engineering, University of California, Riverside, California 92521, USA*

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We discuss quantum evolution of a decaying state in relation to a recent experiment of Katz *et al.* Based on exact analytical and numerical solutions of a simple model, we identify a regime where the qubit retains coherence over a finite time interval independently of the rates of the three competing decoherence processes. In this regime, the quantum decay process can be continuously monitored via a “weak” measurement without affecting the qubit coherence.

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An impressive recent progress in implementing simple quantum systems related to quantum computing¹ permits one to address experimentally the long-standing controversies on quantum measurement.^{2,3} In particular, it is becoming possible to study directly what happens “inside” the quantum-state collapse during a continuous (weak, partial, incomplete, etc.) measurement. While continuous measurement (CM) of an ensemble of quantum systems can be described just as an ensemble decoherence,^{3,4} a much more subtle and interesting topic is CM of a single quantum system.

This topic is well developed in quantum optics,⁵ with the most advanced experiments, including a demonstration of a continuous quantum feedback.⁶ While the formalisms and terminology used by different groups in relation to the continuous (partial, etc.) collapse are quite diverse, the most widely known theoretical approaches are the so-called positive-operator-valued measure¹ (POVM) and “quantum trajectory.”⁵ In condensed matter physics a similar approach has been introduced as the “quantum Bayesian” formalism.⁷

The first direct condensed matter experiment on partial collapse has been realized recently.⁸ (A somewhat similar experiment was proposed in optics⁹ but never realized.) The experimental setup of Ref. 8 was based on the Josephson phase qubit¹⁰ [Fig. 1(a)], described by an asymmetric double-well potential profile. The two lowest levels (with energies E_0 and E_1) in the shallow “left” well were used as qubit states $|0\rangle$ and $|1\rangle$ [Fig. 1(b)]. The levels in the deep “right” well were significantly broadened, essentially creating a continuum of states. With some (over)simplification, the experiment can be presented in the following way. The qubit was prepared in a superposition state $\psi(0) = \alpha_0(0)|0\rangle + \alpha_1(0)|1\rangle$, and then the barrier was lowered for a time t to allow a partial tunneling from the state $|1\rangle$ into the right well ($\Gamma t \sim 1$, where Γ is the tunneling rate). Selecting only the cases when the tunneling had *not* happened (“null result”), the qubit state was subsequently examined by quantum-state tomography. Experimental results⁸ were consistent¹¹ with the simple formula

$$\psi(t) = \frac{\alpha_0(0)e^{-iE_0 t}|0\rangle + \alpha_1(0)e^{-iE_1 t}e^{-\Gamma t/2}|1\rangle}{\sqrt{|\alpha_0(0)|^2 + |\alpha_1(0)|^2 e^{-\Gamma t}}}, \quad (1)$$

which follows from the quantum Bayes rule^{7,12} for partial measurement of the qubit. Notice that for $\Gamma t \gg 1$ this formula

describes the “orthodox” projective collapse onto state $|0\rangle$ [this regime is usually used for phase qubit measurement¹⁰ by sensing the tunneling into the right well with a nearby superconducting quantum interference device (SQUID)], while for $\Gamma t \sim 1$ the collapse is only partial. Therefore, the experiment⁸ has shown that after the partial collapse the qubit remains almost perfectly pure, while its evolution is information related; in particular, the amplitude of state $|0\rangle$ gradually grows without “physical” interaction.

The purpose of this work is to understand why and how well a metastable qubit may retain coherence despite decohering processes in its environment. This is important for understanding of the partial-collapse measurement in Ref. 8, but even more so for future experiments on continuous monitoring for qubit decay where measurement-induced decoherence would be inherent. We focus on a simplified model with the level structure as in Fig. 1(c), where the (qubit) states in the left well experience no direct decoherence, whereas those in the right well and the tunneling Hamiltonian are subject to decoherence. As we discuss, in this case the qubit remains pure as long as the tunneling out of the left well is an irreversible process. We use analytical and numerical techniques to illustrate situations where such an irreversibility is due to the choice of system parameters (e.g., for a nearly continuous spectrum in the right well) or where it happens dynamically due to the evolution properties in the right well.

We write the system Hamiltonian in the block form,

$$H = \begin{pmatrix} H_L & T \\ T^\dagger & H_R \end{pmatrix}. \quad (2)$$

Here H_L is the two-level Hamiltonian in the left well, $H_L = \text{diag}\{E_0, E_1\}$, H_R is the Hamiltonian in the right well, H_R

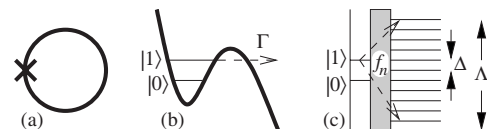


FIG. 1. (a) Schematic of the phase qubit: superconducting loop interrupted by a Josephson junction. (b) Potential profile and level structure of the phase qubit. Upper level $|1\rangle$ decays with the rate Γ . (c) Idealized level structure used here. Left-well levels with energies E_0 and E_1 form a qubit. The state $|1\rangle$ can decay into the right well where the average level spacing is Δ and the bandwidth is Λ . The tunneling amplitude to the state $|n\rangle$ is f_n .

$=\sum_n E_n |n\rangle\langle n|$, and T is the corresponding tunneling Hamiltonian (here and below \sum_n denotes summation over the right-well states only). Unless mentioned otherwise, we assume that only the transitions from the upper qubit state are allowed, $T_{1n} \equiv f_n \neq 0$, while the state $|0\rangle$ is fully disconnected, $T_{0n}=0$.

Let us start with the simplest case of *no decoherence* during the tunneling time interval t , followed by an ideal orthodox quantum measurement which distinguishes left and right wells (the technical realization of such a measurement is discussed below). Then the state of the system can be described by a wave function $\psi=(\psi_L, \psi_R)^t$, which starts as $\psi_L(0)=\alpha_0(0)|0\rangle+\alpha_1(0)|1\rangle$ and $\psi_R(0)=0$, evolves according to the Schrödinger equation $\dot{\psi}=-iH\psi$ before the measurement, and finally undergoes orthodox projective collapse at time t . In particular, the left-well component ψ_L is either zeroed if the escape is detected, or is rescaled to become the new wave function of the system^{2,13} if the measurement finds no escape: $(\psi_L, \psi_R) \rightarrow (\psi_L/\|\psi_L\|, 0)$. In this case it is trivial to see that the qubit remains fully coherent in the null-result scenario of no escape that is interesting for us.

Before the measurement, the left-well components evolve as $\alpha_0(t)=e^{-iE_0 t}\alpha_0(0)$ and

$$\alpha_1(t) = \frac{i\alpha_1(0)}{2\pi} \int_{-\infty}^{\infty} \frac{d\epsilon e^{-i\epsilon t}}{\epsilon - E_1 - \sum_n |f_n|^2 / (\epsilon - E_n + i0)}, \quad (3)$$

so after the null-result measurement the qubit state, up to a phase, becomes $\psi=A[e^{-iE_0 t}\alpha_0(0)|0\rangle+\alpha_1(t)|1\rangle]$, where the normalization $A=[|\alpha_0(0)|^2+|\alpha_1(t)|^2]^{-1/2}$. Generically, $A>1$. This corresponds to an increase of the component α_0 , even though the state $|0\rangle$ is fully disconnected. Notice that this result coincides with Eq. (1), up to a trivial substitution.

The integration in Eq. (3) can be formally done as a sum over residues ϵ_n , the exact eigenvalues of the Hamiltonian (2). Qualitatively, we can characterize the spectrum of the right well by the average energy spacing Δ , average tunneling amplitude f (the rms of $|f_n|$ at energies near E_1), and the total energy bandwidth $\Lambda \gg \Delta, f$ [see Fig. 1(c)]. Then at time $t \leq \Lambda^{-1}$ the contributions of different residues add nearly in phase, and $|\alpha_1|^2$ changes quadratically in t . At $t \gg \Delta^{-1}$ the resonant processes of return from the right well become important; the form of $\alpha_1(t)$ differs qualitatively depending on the number of strongly coupled levels ($\sim f/\Delta$) and their exact position. In the intermediate range, $\Lambda^{-1} \ll t \ll \Delta^{-1}$, the level discreteness is unimportant and the residue summation can be approximated by an integration. We obtain

$$\alpha_1(t) = Z\alpha_1(0)e^{-i(E_1+\delta E_1)t}e^{-\Gamma t/2}, \quad (4)$$

where the decay rate is $\Gamma=2\pi\tilde{D}(\epsilon)$, while the energy shift δE_1 and the prefactor Z are given by the integrals

$$\delta E_1 = \mathcal{P} \int \frac{dE \tilde{D}(E)}{\epsilon - E}, \quad \frac{1}{Z} \equiv 1 + \int \frac{dE \tilde{D}(E)}{(\epsilon - E + i0)^2},$$

all evaluated at $\epsilon=E_1+\delta E_1-i\Gamma/2$. Here we introduced the smoothed tunneling density of states (TDOS) $\tilde{D}(E)$ instead

of $D(E)=\sum_n |f_n|^2 \delta(E-E_n)$; unlike in Ref. 14 we assume $\tilde{D}(E)$ to have no discontinuities or singularities.

Replacing \tilde{D} by f^2/Δ , the decay rate can be written as $\Gamma=2\pi f^2/\Delta$, and the evolution is well exponential if $\Delta \ll f \ll \sqrt{\Delta\Lambda}$. In this case (which is essentially tunneling into continuum) we obtain the simple formula (1) with small corrections δE_1 and Z .

Now let us add *decoherence* processes into the picture. We will consider only decoherence in the right well and between the wells, excluding explicit left-well decoherence, which has a trivial effect.

A simple model describing decoherence of right-well levels can be introduced by adding imaginary parts to their energies, $E_n \rightarrow E_n - i\tilde{\gamma}_n/2$. Physically, this corresponds to processes of energy relaxation to levels that do not interact with $|1\rangle$. Then the wave function formalism [Eq. (3)] is still valid, so the qubit remains pure after the null-result measurement, while the conditions for the exponential decay are now more relaxed since the TDOS is naturally smoothed. Despite simplicity, this model is well applicable to experiment.⁸

In a more complete model, we consider the dissipative dynamics of the system (2) within the master equation in the Lindblad form,¹⁵

$$\dot{\rho} = -i[H, \rho] + \sum_i (\gamma_i/2)([\Lambda_i \rho, \Lambda_i^\dagger] + [\Lambda_i, \rho \Lambda_i^\dagger]), \quad (5)$$

where ρ is the density matrix (DM), Λ_i are the decoherence operators, and γ_i are the corresponding decoherence rates. We will specifically consider the cases of phase noise between the wells, $\Lambda_0=\sum_n |n\rangle\langle n|$ (cf. Ref. 16), as well as the incoherent transitions up and down the ladder of levels in the right well, $\Lambda_1=\Lambda_2^\dagger=\sum_n |n\rangle\langle n+1|$ (cf. Ref. 17).

We have found an exact real-time analytical solution of Eq. (5) in the case of uniformly coupled equidistant states in the right well with infinite bandwidth: $f_n=f$, $E_n=n\Delta$, $\Lambda \rightarrow \infty$. The solution is constructed using the momentum representation in the right well, $|\phi\rangle \equiv (2\pi)^{-1/2} \sum_n |n\rangle e^{in\phi}$. Then the Hamiltonian H_R becomes a differential operator, $H_R\psi(\phi)=-i\Delta\partial_\phi\psi(\phi)$, the tunneling operator picks $\phi=0$ since $Tf\psi(\phi)|\phi\rangle d\phi=|1\rangle\sqrt{2\pi}f\psi(0)$, and the decoherence operators are diagonal: $\Lambda_0=1_R$, $\Lambda_1=e^{-i\phi}$, $\Lambda_2=e^{i\phi}$. Then, e.g., the off-diagonal component ρ_{10} of the qubit DM can be found from the equations

$$\dot{\rho}_{10} = i(E_0 - E_1)\rho_{10} - igb_0(0), \quad (6)$$

$$\dot{b}_0(\phi) = (iE_0 - \gamma/2)b_0(\phi) - \Delta\partial_\phi b_0(\phi) - i\rho_{10g}\delta(\phi), \quad (7)$$

where $\gamma \equiv \gamma_0 + \gamma_1 + \gamma_2$ is the net dephasing rate, $g = \sqrt{2\pi}f$, and $b_0(\phi)$ encodes the off-diagonal components of the DM between the level $|0\rangle$ and the right-well levels. Equation (7) is the first-order quasilinear partial differential equation (PDE); it can be integrated in quadratures for any form of $\rho_{10} \equiv \rho_{10}(t)$. The solution describes the chiral propagation of the decaying amplitude b_0 around the circle from $\phi=0$. As a result, before a full turn is completed, for $t < 2\pi/\Delta$, the amplitude does not return back to the left well. For this time interval, the only effect on the component ρ_{01} is a relaxation

with the rate $\Gamma/2$, independent of the dephasing γ :

$$\rho_{10}(t) = \rho_{10}(0)e^{i(E_0-E_1)t}e^{-\Gamma t/2}, \quad \Gamma = 2\pi f^2/\Delta. \quad (8)$$

Similarly, the evolution of the qubit DM component ρ_{11} is coupled with the right-well DM components $\rho_R(\phi, \phi')$ and components $b_1(\phi)$ involving level $|1\rangle$ and right-well levels. Solving in turn the PDEs for $\rho_R(\phi, \phi')$ and $b_1(\phi)$, we have obtained the self-consistent equation for ρ_{11} . Again, over the same time interval, there is no return tunneling from the right well, and the evolution of ρ_{11} is not affected by decoherence described by operators $\Lambda_{0,1,2}$, so that $\rho_{11}(t) = \rho_{11}(0)e^{-\Gamma t}$, $t < 2\pi/\Delta$. Evolution of ρ_{00} is trivial: $\rho_{00}(t) = \rho_{00}(0)$.

With probability $\rho_{00}(t) + \rho_{11}(t)$, an ideal projective measurement at time t will show that the system has not decayed to the right well. In this case the density matrix needs to be changed (collapsed) as^{2,13}

$$\begin{pmatrix} \rho_L & (b_0, b_1)^\dagger \\ (b_0, b_1) & \rho_R \end{pmatrix} \rightarrow \begin{pmatrix} \rho_L/\text{Tr}(\rho_L) & 0 \\ 0 & 0 \end{pmatrix}. \quad (9)$$

It is easy to see that, if the system originated in a pure state, $\rho_L(0) = \psi_L(0)\psi_L^\dagger(0)$, the density matrix after such a measurement at time $t < 2\pi/\Delta$ also describes a pure state, which exactly corresponds to the formula (1). We emphasize that the absence of return tunneling from the right well is both necessary and sufficient for the qubit purity in our model.

The obtained analytical solution of the ideal case can be now used as a starting point of the perturbation theory for situations more realistic experimentally. In particular, a weak *nonlinearity in the right-well spectrum*, e.g., $E_n = \Delta(n + \beta n^2)$, $\beta \ll 1$, in the phase representation corresponds to a dispersive term $\delta H_R = -\beta \Delta \partial^2 / \partial \phi^2$. The analog of Eq. (7) would then include not only propagation but also dispersion of the wave packet, and the qubit decoherence due to reverse tunneling processes may start earlier. The effect is exponentially small for $2\pi - t\Delta \gg \delta\phi$, where $\delta\phi \approx 2(t\beta\Delta)^{1/2}$ is the rms width of the packet. A similar dispersive effect results from *random level spacing* in the right well, or due to *phase noise* in the right well. With many levels in the right well effectively coupled, we expect these effects to be weak at sufficiently early times, as long as the corresponding phase broadening $\delta\phi$ is small, $\delta\phi \ll 1$.

A different sort of perturbation results if the tunneling amplitudes f_n are not equal to each other, or if the number of states in the right well is finite. In this case the transformed tunneling Hamiltonian would not correspond to a δ function in phase space but instead would couple to a finite range of phases, $|\phi| \lesssim \delta\phi \sim \Delta/\Lambda$. As a result, some reverse tunneling from the right well back to the qubit may start early. However, the associated decoherence is not expected to be significant as long as $\delta\phi \ll 2\pi - t\Delta$. Additionally, the decay in the model (5) would be exponential only at $t \gg \Lambda^{-1}$, which leads to an additional prefactor as in Eq. (4). We also note that the inelastic escape to levels decoupled from $|1\rangle$ also reduces the return tunneling probability and extends the time interval of qubit coherence.

We illustrate these arguments by a numerical simulations of Eq. (5) in Fig. 2: open symbols represent the purity $P(t) = [(\rho_{11} - \rho_{00})^2 + 4|\rho_{01}|^2]^{1/2} / (\rho_{00} + \rho_{11})$, while closed symbols

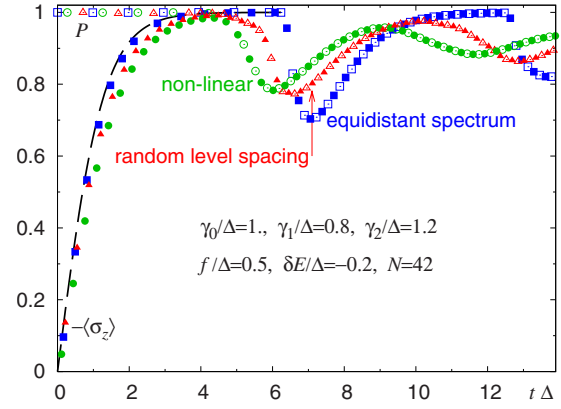


FIG. 2. (Color online) Simulation results for the evolution projected onto the left well beginning with the state with $\langle\sigma_x\rangle=1$; parameters as shown. Open symbols represent the purity $P(t)$ which starts at $P(0)=1$, closed symbols represent the longitudinal qubit polarization $-\langle\sigma_z\rangle$; the two curves merge near $t\Delta=4$. Blue squares: equidistant spectrum in the right well, $E_n = \delta E + n\Delta$; numerical results are almost on top of the analytical result with $N \rightarrow \infty$ (dashes). The broad minima at $t \geq 2\pi/\Delta$ and $t \geq 4\pi/\Delta$ correspond to arrival of incoherent echoes from the right well. Green circles: data with added spectrum nonlinearity, $\beta=0.02$ (see text). Red triangles: random level spacing, $E_{n+1} - E_n = (0.5 + \xi_n)\Delta$, where ξ_n are uniformly distributed random numbers, $0 \leq \xi_n < 1$.

represent the diagonal component of qubit polarization, $-\langle\sigma_z\rangle \equiv (\rho_{00} - \rho_{11}) / (\rho_{00} + \rho_{11})$. Even with a not very large parameter $\Lambda/\Delta=40$ (which corresponds to a total of $N=42$ energy levels), the results of numerical simulation agree almost perfectly with the analytical result shown by dotted lines. In agreement with our arguments, neither weak spectrum nonlinearity (circles) nor randomized level spacing (triangles) reduces the qubit coherence for sufficiently early evolution time.

Let us briefly discuss what happens with Eq. (1) when the state $|0\rangle$ can also tunnel into the right well with the rate Γ_0 . The tunneling would then produce a matrix element h_{10} between the levels $|1\rangle$ and $|0\rangle$. In the simplest case of no return processes, such a matrix element would be purely imaginary (anti-Hermitian), with the magnitude¹⁸ $|h_{10}| \leq (\Gamma_0\Gamma)^{1/2}/2$, where the equality is achieved when the tunneling matrix elements from the two states are all proportional to each other, $T_{0n} = \zeta T_{1n}$. The corresponding dynamics is not trivial, but the level mixing is small if $|h_{10}| \ll |E_1 - E_0|$. Even in this case there may be a significant effect on level decay rate. However, if $\Gamma_0 + \Gamma_1 \ll |E_1 - E_0|$, then Eq. (1) can be simply replaced by the result expected from the quantum Bayes rule:^{7,12,19} the term $\alpha_0(0)$ should be substituted with $\alpha_0(0)\exp(-\Gamma_0 t/2)$.

So far we have assumed an ideal orthodox measurement at time t , which distinguishes between the left and right wells, but does not affect the states in the left well. For a superconducting phase qubit such a measurement can be technically realized by biasing the measurement SQUID at time t to the point at which the SQUID switches to the finite-voltage state only for a right-well flux.¹⁰ Due to strong nonlinearity of such a detector, in the case of no switching the back action onto the left-well qubit states can be made prac-

tically negligible. It is important to mention that a linear detector would necessarily disturb the qubit states because for a phase qubit the “distance” between the wells is comparable to the “width” of the qubit well.

Strictly speaking, the results discussed in this work assume measurement only once at time t . However, there is a sense in which our results describe qubit evolution before the decay. Our qubit “ages” in the process of no decay [as in Eq. (1)], unlike the case of a radioactive atom which remains “as new” before the decay actually happens. For such an interpretation, we necessarily need to consider repeated (or continuous) measurements with time resolution better than Γ^{-1} , and it is important that presence or absence of extra measurements within the time interval t should not affect the non-decayed qubit state at time t . Clearly, this is not the case if orthodox measurements are repeated too frequently with time interval Δt shorter than the scale of the quantum Zeno effect, $\Delta t \lesssim \Lambda^{-1}$. However, in the regime of an exponential decay with $\Delta t \gg \Lambda^{-1}$ [e.g., as in Eq. (4) with $Z \approx 1$] an extra measurement has no effect: a composition of two evolutions (1) with durations t_1 and t_2 is the same as a similar evolution with duration $t_1 + t_2$. In actual experiment the measurement SQUID can monitor the decay continuously, and then Δt corresponds to the intrinsic time resolution of the detector. In this case the frequent partial collapses can be replaced by introduction of the interwell phase noise [Eq. (5)] with $\gamma_0 = 1/\Delta t$. Our results indicate that this does not lead to significant qubit dephasing even in the case of good time resolution

$\Delta t \ll \Gamma^{-1}$, as long as the conditions of exponential decay are well satisfied. We conclude that in the regime of exponential decay, Eq. (1) can really be interpreted as actual qubit evolution in time before decay.

In conclusion, we have analyzed the quantum dynamics of a model with two-well structure resembling the experiment of Ref. 8 with the (qubit) states in the left well nearly coherent, while those in the right well and the transition Hamiltonian are subject to decoherence. The analytical solution of the master equation (5) obtained for an infinitely wide right-well spectrum with equal level spacing Δ gives pure qubit subspace for $t < 2\pi/\Delta$, independent of the decoherence rates $\gamma_{0,1,2}$ [see Eq. (5)]. This property of coherence preservation over a finite time interval remains in effect in a perturbed system where the solvability conditions are only approximate. We have identified a regime where the quantum evolution during tunneling can be experimentally accessed via a repeated “stroboscopic” measurement or a continuous “weak” measurement. In this regime the qubit state will remain pure in spite of the phase noise associated with the measurement.

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