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Simple quantum feedback of a solid-state qubit

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We propose an experiment on quantum feedback control of a solid-state qubit, which is within the reach of the present-day technology. Similar to an earlier proposal [R. Ruskov and A. N. Korotkov, Phys. Rev. B **66**, 041401(R) (2002)], the feedback loop is used to maintain coherent oscillations in a qubit for an arbitrarily long time; however, this is done in a significantly simpler way, which eases the bandwidth problem. The main idea is to use the quadrature components of the noisy detector current to monitor approximately the phase of qubit oscillations. The price for simplicity is a less-than-ideal operation: the fidelity is limited to about 95%. The feedback loop operation can be experimentally verified by the appearance of a positive in-phase component of the detector current relative to an external oscillating signal used for synchronization.

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The needs of quantum computing[1] are fueling the rapid progress in experiments with solid-state qubits. In particular, quantum coherent (Rabi) oscillations have been demonstrated using superconducting charge, flux, and phase qubits, as well as double-quantum-dot qubits.[2] Successful experiments with two superconducting qubits have also been performed.[3] One of the directions for the advanced qubit control is realization of the quantum feedback control of a solid-state qubit,[4] which can be used in a quantum computer for qubit initialization and is also an important demonstration by itself, clarifying the controversial issue of gradual collapse of a quantum state. (In optics quantum feedback control was proposed more than a decade ago and has been already demonstrated experimentally.[5])

For the analysis of a quantum feedback we have to take into account the process of continuous qubit collapse. Therefore, the conventional approach to continuous quantum measurement[6,7] is inapplicable, and it is necessary to use the recently developed Bayesian approach[8] or the equivalent (though technically much different) approach of quantum trajectories.[9] The possibility of a quantum feedback is based on the fact that measurement by an ideal solid-state detector (with a 100% quantum efficiency η) does not decohere a single qubit,[8] even though it decoheres an ensemble of qubits because each qubit evolves in a different way. The random evolution of a qubit in the process of measurement can be monitored using the noisy detector output, with the accuracy depending on η, so that for an ideal detector ($\eta=1$) even the monitoring of a qubit wave function is possible. An example of a theoretically ideal solid-state detector is[8] the quantum point contact (η comparable to 1 has been demonstrated experimentally[10]). The single-electron transistor is significantly nonideal[7,8,11] ($\eta \ll 1$) in the semiclassical "orthodox" mode of operation; however, it can reach ideality in some modes based on cotunneling or Cooper pair tunneling.[12]

Monitoring of the quantum state in real time can naturally be used for continuous feedback control of a quantum system. In the proposal of Ref. 4 the quantum feedback is used to maintain quantum coherent oscillations in a qubit for an arbitrarily long time. This is done by measuring the noisy current $I(t)$ in a weakly coupled detector and using the quantum Bayesian equations[8] to translate information contained in $I(t)$ into the evolution of qubit density matrix $\rho(t)$. After that, $\rho(t)$ is compared with the desired quantum state $\rho_d(t)$, and the calculated difference is used to control the qubit Hamiltonian in order to decrease the difference. Notice that the measurement backaction necessarily shifts the phase of coherent oscillations in a random way; however, the information contained in $I(t)$ is sufficient to monitor this change and therefore restore the desired phase.

An important difficulty in such an experiment is the necessity to solve the Bayesian equations in real time. Moreover, the bandwidth of the line delivering $I(t)$ to the circuit solving the Bayesian equations, should be significantly wider than the frequency Ω of coherent oscillations. Unfortunately, these conditions are unrealistic for the present-day experiments with solid-state qubits.

In this paper we propose and analyze a much simpler way (Fig. 1) of processing the information carried by the detector current $I(t)$. The idea is to use the fact that besides noise, $I(t)$ contains an oscillating contribution due to coherent oscillations in the measured qubit. Therefore, if we apply $I(t)$ to a simple tank circuit (which is in resonance with Ω), then the phase of the tank circuit oscillations will depend on the phase of the qubit oscillations. Instead of using the tank circuit, a theoretically almost equivalent procedure is to mix $I(t)$ with the signal from a local oscillator (Fig. 1) in order to determine two quadrature amplitudes of $I(t)$ at the frequency Ω, which will carry information on the phase of coherent

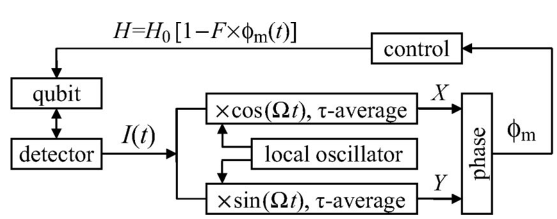

FIG. 1. Schematic of the proposed quantum feedback loop. Two quadrature components of the detector current $I(t)$ are used to monitor approximately the phase difference between qubit coherent oscillations and a local oscillator, which is then used to control the qubit parameter H (feedback strength is characterized by parameter F). The phase can also be monitored using a tank circuit. Positive average in-phase quadrature $\langle X \rangle$ is an experimental indication of quantum feedback operation.

oscillations. Since the diffusion of the oscillation phase is a slow process (assuming weak coupling to the detector and environment), the further circuitry can be relatively slow, limited by the qubit dephasing rate, but not limited by the much higher frequency Ω. The simplicity of the information processing and alleviation of the bandwidth problem are the main advantages of this proposal in comparison with Ref. 4. (The bandwidth of the line between the detector and the mixer should still be much larger than Ω; however, it is not a problem for the on-chip mixing.) The experiment can be realized using either superconducting or GaAs technology.[2,3,10]

The idea of this proposal partially stems from the fact that in the absence of feedback, the qubit oscillations lead to a noticeable peak in the spectral density $S_I(\omega)$ of the detector current at $\omega \approx \Omega$, with the peak-to-pedestal ratio up to four times[13] (somewhat similar experiments have been reported recently[14]). Since 4 is not a big number, one would expect quite inaccurate phase information carried by the current quadratures and therefore poor operation of the feedback. Surprisingly, the quantum feedback operates much better than expected.

Let us consider a "charge" qubit (either double quantum dot or single Cooper pair box) with Hamiltonian $\mathcal{H}_{qb}=(\varepsilon/2)(c_2^\dagger c_2 - c_1^\dagger c_1)+H(c_1^\dagger c_2 + c_2^\dagger c_1)$, where $c_{1,2}^\dagger$ and $c_{1,2}$ are the creation and annihilation operators in the basis of "localized" (charge) states, ε is their energy asymmetry, and the tunneling $H=H_0+H_{fb}(t)$ can be controlled by the feedback loop (H_{fb}). We assume the standard coupling[8,13,15] between the charge qubit and the detector (quantum point contact or single-electron transistor). Instead of writing Hamiltonian explicitly, we will characterize the measurement by two levels of the average detector current, I_1 and I_2, corresponding to the two charge states, by the detector output noise S_I, and by the qubit ensemble dephasing rate Γ due to detector backaction and environment. Assuming a sufficiently large detector voltage and quasicontinuous detector current $I(t)$, we describe the qubit evolution by the Bayesian equations[8] (in Stratonovich form),

$$\dot{\rho}_{11} = -2H \,\mathrm{Im}\, \rho_{12} + 2\rho_{11}\rho_{22}[I(t)-I_0]\Delta I/S_I, \tag{1}$$

$$\dot{\rho}_{12} = i\varepsilon\rho_{12} + iH(\rho_{11}-\rho_{22}) - \gamma\rho_{12} - (\rho_{11}-\rho_{22})\rho_{12}[I(t)-I_0]\Delta I/S_I, \tag{2}$$

where $\hbar=1, \Delta I = I_1 - I_2, I_0=(I_1+I_2)/2$, and $\gamma=\Gamma-(\Delta I)^2/4S_I$. The decoherence rate $\gamma=\gamma_d+\gamma_e$ of the single qubit is due to the detector nonideality, $\gamma_d=(\eta^{-1}-1)(\Delta I)^2/4S_I$, and the additional coupling with environment (γ_e). The current $I(t)=I_0+(\rho_{11}-\rho_{22})\Delta I/2+\xi(t)$ has the noise component $\xi(t)$ with the flat (white) spectral density S_I. [Averaging over $\xi(t)$ would lead to the standard master (Bloch) equation[6,16] with the ensemble dephasing rate Γ.] Notice that in the case $\varepsilon=0$ (which is assumed unless mentioned otherwise), we can disregard the evolution of $\mathrm{Re}\,\rho_{12}$ (it becomes zero at $t \gg \Gamma^{-1}$), so only two degrees of freedom are left, which may be parametrized as $\rho_{11}-\rho_{22}=P\cos(\Omega t+\phi)$ and $2\,\mathrm{Im}\,\rho_{12}=P\sin(\Omega t+\phi)$, where the feedback-maintained frequency Ω (see below) is assumed to be equal (unless stated otherwise) to the bare frequency $\Omega_0=(4H_0^2+\varepsilon^2)^{1/2}$ of coherent oscillations. Moreover, in the ideal case $\gamma=0$ the state eventually becomes pure,[8] so that $P=1$ and the evolution can be described by only one parameter $\phi(t)$.

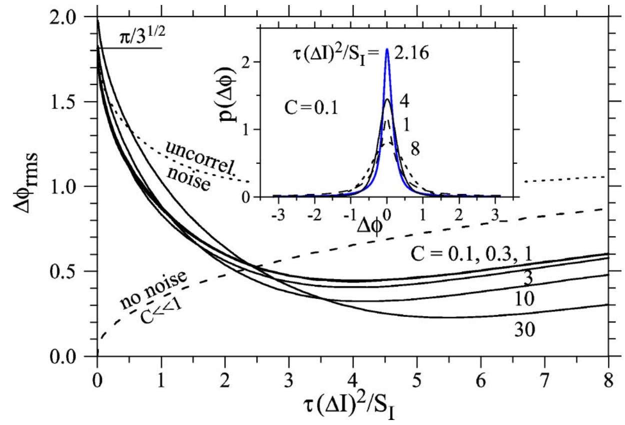

FIG. 2. Dependence of monitoring inaccuracy $\Delta\phi_{rms}$ on averaging time τ without feedback for several values of coupling C. The dashed and dotted lines are for classical signals (see text). Inset: Distribution of $\Delta\phi$ for several τ at weak coupling.

We assume that two quadrature components of the detector current (Fig. 1) are determined as

$$X(t) = \int_{-\infty}^{t} [I(t')-I_0]\cos(\Omega t')e^{-(t-t')/\tau}dt', \tag{3}$$

$$Y(t) = \int_{-\infty}^{t} [I(t')-I_0]\sin(\Omega t')e^{-(t-t')/\tau}dt', \tag{4}$$

where Ω is the local oscillator frequency applied to the mixer and τ is the averaging (relaxation) time constant.[17] Similar formulas are also applicable to the case of a tank circuit with the resonant frequency Ω and quality factor $Q=\Omega\tau/2$. If the detector current would be a harmonic signal $I(t)=I_0+\frac{1}{2}P\Delta I\cos(\Omega t+\phi_0)$, then $\phi_0=-\arctan(\langle Y\rangle/\langle X\rangle)$, so it is natural to use

$$\phi_m(t) \equiv -\arctan(Y/X) \tag{5}$$

as a monitored estimate of the phase shift $\phi(t)$ between the coherent oscillations and the local oscillator ($\langle\cdots\rangle$ means averaging over time).

Let us assume $\gamma=0$ and analyze first how close is the estimate $\phi_m(t)$ to the actual phase $\phi(t)$ without feedback, in which case ϕ evolves in a diffusive manner due to the detector backaction. Figure 2 shows the rms phase difference $\Delta\phi_{rms}=\langle(\phi_m-\phi)^2\rangle^{1/2}$ (solid lines) as a function of τ for several values of the dimensionless qubit-detector coupling $C\equiv(\Delta I)^2/S_I H_0$, calculated numerically using Monte Carlo simulation of the measurement process.[8] At weak coupling, $C \lesssim 1$, the curves practically coincide, and the minimum $\Delta\phi_{rms}\approx 0.44$ is achieved at $\tau \approx 4S_I/(\Delta I)^2=1/\Gamma$, as expected, since Γ determines the phase diffusion:[8,9,13] $\langle[\phi(t)-\phi(0)]^2\rangle/t=\Gamma$. At a larger τ, ϕ_m includes too much irrelevant information from a distant past, while at a smaller τ the quadrature amplitudes suffer too much from noise. At

$\tau \to 0$ (as well as at $\tau \to \infty$) $\Delta\phi_{rms} \to \pi/\sqrt{3} \approx 1.81$, corresponding to the uniform distribution of $\Delta\phi = \phi_m - \phi$ within $\pm\pi$ interval (all phases are defined modulo 2π).

It is important to notice that the calculated $\Delta\phi_{rms}$ is significantly smaller than for a naive classical case, in which the noise $\xi(t)$ is not correlated with the diffusive evolution of ϕ. The dotted line in Fig. 2 shows the result for such a case at weak coupling [for this curve we assumed $I(t) - I_0 = \sqrt{2}(\Delta I/2)\cos(\Omega t + \phi) + \xi(t)$, diffusive evolution of ϕ with coefficient $\Gamma/2$, and no correlation between ξ and ϕ, which corresponds to correct spectrum[13]].Even more surprisingly, at $\tau > 2.5 S_I/(\Delta I)^2$ the inaccuracy $\Delta\phi_{rms}$ in the quantum case is smaller than for the classical noiseless case shown by the dashed line [we assumed $\xi(t) = 0$, while ϕ evolves as above], which means that the *noise improves the monitoring accuracy*. This quantum behavior can be understood from the phase evolution equation[4] which follows[18] from Eqs. (1) and (2),

$$\dot{\phi} = -[I(t) - I_0]\sin(\Omega t + \phi)(\Delta I/S_I) + \Omega_0 - \Omega. \tag{6}$$

A comparison with the equation for $\dot{\phi}_m$ (see below) shows that the quadrature component of the noise ξ, which shifts the observed phase ϕ_m, also shifts the actual phase ϕ in the same direction. In other words, when the noise looks like oscillations, it forces the real coherent oscillations to evolve closer to what is observed.

The inset in Fig. 2 shows the distribution of $\Delta\phi$ in the weak-coupling limit for several values of τ. The distributions are significantly non-Gaussian with the central part significantly narrower than $\Delta\phi_{rms}$. It is interesting that the value $\tau = 4S_I/(\Delta I)^2$ corresponding to the minimum $\Delta\phi_{rms}$, does not provide the highest peak of the $\Delta\phi$ distribution. Let us compare the monitored phase evolution $\dot{\phi}_m = -[I(t) - I_0]\sin(\Omega t + \phi_m)/(X^2 + Y^2)^{1/2}$, with Eq. (6). We would expect the best approximation of ϕ by ϕ_m when $\langle X^2 + Y^2\rangle = (S_I/\Delta I)^2$. Using definitions (3) and (4) and the current-current correlation function[13] $\langle I(0)I(t)\rangle = (S_I/2)\delta(t) + (\Delta I/2)^2\cos(\Omega t)\exp[-(\Delta I)^2 t/8S_I]$, we obtain $\langle X^2 + Y^2\rangle = S_I\tau[1/4 + 1/(1 + 8S_I/(\Delta I)^2\tau)]$ at $\Omega\tau \gg 1$ and $C \ll 1$, so the condition $\langle X^2 + Y^2\rangle = (S_I/\Delta I)^2$ is satisfied at $\tau(\Delta I)^2/S_I = (2/5)(\sqrt{41} - 1) \approx 2.16$. We checked numerically that this value indeed corresponds to the highest peak of $\Delta\phi$ distribution.

A reasonably small difference between ϕ and ϕ_m in the absence of feedback implies that we can expect decent operation of the quantum feedback loop in which the phase estimate ϕ_m is used for determining the feedback action. Similar to Ref. 4 we consider the feedback loop, which aims to suppress the fluctuations of the oscillation phase, so that the goal is $\phi(t) = 0$ (or as small as possible). It has been shown that this goal can be fully reached using the linear feedback rule $H_{fb}(t)/H_0 = -F\phi(t)$, which requires exact monitoring of ϕ; here we analyze the operation of the feedback loop with $H_{fb}/H_0 = -F\phi_m(t)$, where F is the dimensionless feedback factor (by definition $|\phi_m| \leq \pi$).

We characterize the performance of the feedback loop by the synchronization degree $D = \langle P(t)\cos\phi(t)\rangle = 2\mathcal{F}_q - 1$,

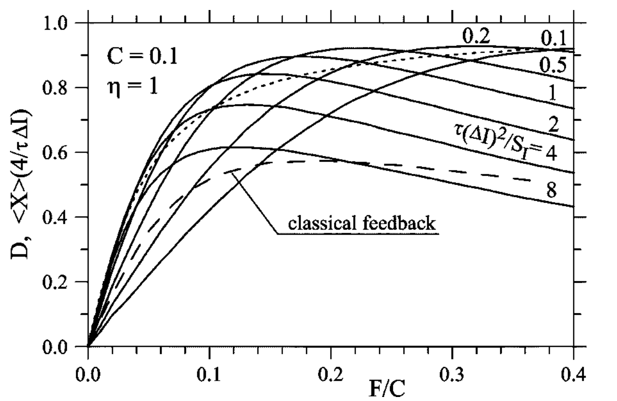

FIG. 3. Solid lines: Dependence of the synchronization degree D on the feedback factor F in ideal case ($\gamma = 0$) for several τ. Experimentally, D can be measured via average in-phase current quadrature $\langle X\rangle$. The dashed line is for classical feedback, the dotted line is for quantum feedback of Ref. 4.

where $\mathcal{F}_q = \langle \mathrm{Tr}\, \rho(t)\rho_d(t)\rangle$ is fidelity and ρ_d corresponds to the desired perfect coherent oscillations ($P_d = 1, \phi_d = 0$). Figure 3 shows (solid lines) the dependence of D on the feedback factor F for several time constants τ in the case of weak coupling $C = 0.1$ and $\gamma = 0$ (we normalize F by C, so the results practically do not depend on C for $C \lesssim 1$) (Ref. 19). One can see that each curve has a maximum, so that the "oversteering" effect at a larger F makes the feedback performance worse. Somewhat unexpectedly, $\tau = 1/\Gamma = 4S_I/(\Delta I)^2$ is no longer an optimum, and the smaller time constants are actually better. It can be shown that the feedback loop can operate even at $\tau \ll \Omega^{-1} \ll \Gamma^{-1}$; however, we are not interested in this wide-bandwidth regime. Limiting ourselves to $\tau \sim S_I/(\Delta I)^2$, we see that the maximum achievable synchronization degree D_{max} is about 90% (that corresponds to the fidelity $\mathcal{F}_q$ of about 95%). It is impossible to reach 100%, because the monitored simple phase estimate ϕ_m is different from the actual ϕ; however, the fidelity is still surprisingly high for such a simple feedback loop. It is interesting to note that a very crude estimate of D_{max} as $\cos(\Delta\phi_{rms})$ using $\min(\Delta\phi_{rms}) \simeq 0.44$ from the analysis without feedback, works quite well, $\cos(0.44) = 0.90$ (though for different τ). The dashed line in Fig. 3 shows the feedback performance for the classical signal corresponding to the dotted line in Fig. 2, assuming $\tau(\Delta I)^2/S_I = 1$. As expected, the performance is much worse than for the quantum feedback. The dotted line in Fig. 3 shows the operation of the quantum feedback of Ref. 4 based on the exact monitoring of ϕ; while the dotted line can go below solid lines, at large F/C it approaches unity as $D \approx \exp(-C/32F)$.

An important question is how the operation of the quantum feedback loop can be verified experimentally. One of the easiest ways is to check that the average value $\langle X\rangle$ of the in-phase quadrature component $X(t)$ becomes positive, while in the absence of feedback positive and negative values of X are obviously equally probable. Notice that *any* Hamiltonian control of a qubit that is not based on the information obtained from the detector (i.e., feedback control) leads to[20] $\langle X\rangle = 0$. It is easy to show that

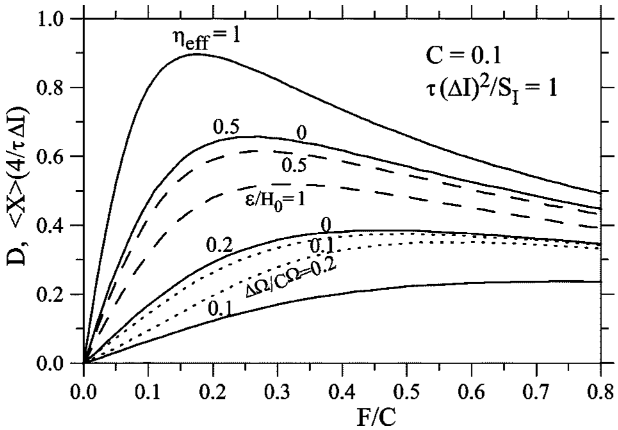

FIG. 4. Solid lines: Synchronization degree D (and in-phase current quadrature $\langle X\rangle$) as functions of F for several values of the detection efficiency η_{eff}. The dashed and dotted lines illustrate the effects of the energy mismatch ($\varepsilon \neq 0$) and the frequency mismatch ($\Omega \neq \Omega_0$).

$\langle X\rangle = [D + \langle P\cos(2\Omega t+\phi)\rangle]\tau\Delta I/4$, and since the second term in brackets vanishes at weak coupling (and $\varepsilon=0$), therefore $\langle X\rangle$ is directly related to D. The numerical results for $\langle X\rangle/(\tau\Delta I/4)$ practically coincide with the curves for D in Fig. 3 (within the thickness of the line).

The ideal case $\gamma=0$ is obviously not realizable in the experiment because of the detector nonideality ($\eta<1$) and presence of the extra environment ($\gamma_e>0$). Both effects can be taken into account simultaneously introducing effective efficiency of quantum detection $\eta_{eff}=[\eta^{-1}+4\gamma_e S_I/(\Delta I)^2]^{-1}$. Figure 4 shows (solid lines) the feedback performance for several values of η_{eff} assuming $\tau(\Delta I)^2/S_I=1$. One can see that $\eta_{eff}\sim 0.1$ is still sufficient for a noticeable operation of the quantum feedback loop. Note that D_{max} is limited by the state purity factor, $D_{max}<P$, which is (Ref. 18) $P\approx\sqrt{2\eta_{eff}}$ at $\eta_{eff}\ll 1$ and $C/\eta \ll 1$ ($D_{max}=P$ can be reached by the feedback of Ref. 4 but not by the feedback studied here).

Finally, let us discuss how accurately the conditions $\Omega=\Omega_0$ and $\varepsilon=0$ should be satisfied in the experiment. If Ω is different from Ω_0, then without feedback the phase ϕ grows linearly in time [Eq. (6)]. However, if the feedback loop operation is faster than $|\Delta\Omega|=|\Omega-\Omega_0|$, the linear growth of ϕ is stopped by adjusting the qubit frequency Ω_0 to match the desired frequency Ω. The dotted lines in Fig. 4 show the feedback operation for $\eta_{eff}=0.2$ and two values of $\Delta\Omega$, confirming that the operation is still satisfactory at $|\Delta\Omega|\ll C\Omega\sim\Gamma\sim\tau^{-1}$. Notice that the frequency mismatch leads to nonzero $\langle\phi_m\rangle$ and therefore can be noticed and corrected. Energy mismatch ($\varepsilon\neq 0$) also worsens the performance of the feedback loop; however, the dashed lines in Fig. 4 ($\eta_{eff}=0.5$) show that a relatively large mismatch ($\varepsilon \lesssim H_0$) can be tolerated.

In conclusion, we have proposed and analyzed the quantum feedback loop for a solid-state qubit, based on monitoring the phase of coherent oscillations via quadrature components of the current in a weakly coupled detector.

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[17] Notice that our procedure is significantly different from the homodyne detection for quantum feedback in optics, since in our case the mixing with local oscillator is after the measurement, not before it.

[18] At finite η (assuming $\varepsilon=0$ and $\Omega=\Omega_0$) the phase equation is $\dot\phi = -[I(t)-I_0]\sin(\Omega t+\phi)(\Delta I/S_I)/P-(\gamma/2)\sin(2\Omega t+2\phi)$. At $C/\eta \ll 1$ the last term can be neglected and the purity factor $P \equiv [(\rho_{11}-\rho_{22})^2+(2\,\mathrm{Im}\,\rho_{12})^2]^{1/2}$ can be approximated using an assumption (not quite accurate) of non-fluctuating P, that leads to $P^2\approx 1+1/2\eta-\sqrt{(1+1/2\eta)^2-2}$. In particular, $P\approx\sqrt{2\eta}$ at $\eta\ll 1$.

[19] Notice that $H_{fb}/H_0 \ll 1$ because $F/C\lesssim 1$ and $C\ll 1$, so the feedback requires only a weak change of the tunnel barrier between the qubit states.

[20] Some procedures, which include a periodic dissipation phase (nonunitary operation), can be used to achieve nonzero $\langle X\rangle$, though much less efficiently than by the quantum feedback.