Entanglement of solid-state qubits by measurement

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We show that two identical solid-state qubits can be made fully entangled (starting from a completely mixed state) with probability 1/4 just by measuring them with a detector, equally coupled to the qubits. This happens in the case of repeated strong (projective) measurements as well as in a more realistic case of weak continuous measurement. In the latter case, the entangled state can be identified by a flat spectrum of the detector shot noise, while the nonentangled state (probability 3/4) leads to a spectral peak at the Rabi frequency with the maximum peak-to-pedestal ratio of 32/3.

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Prospective solid-state realizations of quantum computers may have significant advantages due to natural scalability, simple electrical control of parameters, and use of well developed technology. A number of theoretical proposals have been put forward¹ and interesting experimental results have been achieved, including demonstrations of charge qubits² using single-Cooper-pair boxes, flux qubits^{3,4} using superconducting loops interrupted by Josephson junctions, and combined charge-flux qubits⁵ with the quality factor as high as⁵ 25 000. Obviously, the next important experimental step is the demonstration of entangled solid-state qubits.

Entanglement of qubits can be produced using their direct interaction. In this paper, we discuss an alternative way, when two solid-state qubits are made entangled just by their simultaneous measurement with one detector, which thus provides an indirect coupling between qubits. A somewhat similar idea of entanglement via indirect dissipative coupling has been discussed earlier in quantum optics for the preparation of entangled atoms in an optical cavity by monitoring the cavity decay.⁶ Moreover, it has been shown that some entanglement can be produced just by coupling to a common environment.⁷ However, in this case the degree of entanglement is very small, while in our setup the full 100% entanglement of qubits can be achieved. The stability of the entangled state is due to equal coupling of the qubits with the detector, so that this state is essentially a decoherence-free subspace.⁸ Our procedure works with a probability less than unity, and in this respect it is somewhat similar to the operation of conditional quantum gates⁹ based on linear optical elements.

In contrast to qubits represented by photons, which are physically destroyed by the acts of measurement, solid-state qubits only change their state due to measurement, which allows somewhat more freedom in designing quantum operations. On the other hand, it is quite difficult to realize simple projective measurements of solid-state qubits because of typically weak coupling with detector. Therefore, instead of a simple abrupt collapse, we have to deal with dephasinglike processes in the case of ensemble measurements¹⁰ or with the continuous (weak) measurements^{11–14} in the case of single qubits.

The theory of nonaveraged ("selective" or "conditional") continuous measurement of single solid-state qubits has been under active development for the past four years (see recent

review, Ref. 15, and references therein) and exists in two almost identical variants: the so-called Bayesian formalism¹³ and a version of the quantum trajectory approach¹⁴ adapted to solid-state setups from quantum optics.¹² The main feature of the theory compared to the ensemble-averaged approach¹⁰ is the account of the noisy detector output that naturally bridges the concept of qubit dephasing due to measurement with the "orthodox" collapse postulate. It has been shown¹³ that a single solid-state qubit does not decohere (moreover, is gradually purified) in the process of measurement by a good (ideal) detector [for example, by a quantum point contact (QPC)], which leads to a number of experimental predictions.¹⁵ In particular, the theory shows¹⁶ that the qubit Rabi oscillations monitored by a weakly coupled detector can be evidenced by the peak in the detector current spectral density at the Rabi frequency; however, the peak height cannot be larger than four times the noise pedestal (this fact seems to have recent experimental confirmation¹⁷).

In this paper, we consider two identical qubits performing Rabi oscillations, which are continuously measured by an equally coupled detector. We have found that the system is gradually collapsed into one of the two regimes: either qubits become fully entangled (Bell state), which can be identified by a flat spectrum of the detector current, or the qubits' state falls into the orthogonal subspace that can be identified by the Rabi spectral peak, which for an ideal detector is 32/3 times higher than the noise pedestal. The probabilities of two scenarios are 1/4 and 3/4, respectively, so on average the peak-to-pedestal ratio is equal to 8, twice as large as for a single qubit.

Figure 1 shows possible realizations of our setup. In the first realization [Fig. 1(a)], each qubit is made of a double quantum dot¹⁸ (DQD), occupied by a single electron, while the detector is a QPC located in between DQD's. The second possible realization [Fig. 1(b)] is based on single-Cooperpair boxes as qubits,² which are measured by a single electron transistor (SET). Other possible realizations (not shown) can be based on flux qubits^{3,4} or combined charge-flux qubits.⁵

In the Hamiltonian of the system, $\mathcal{H} = \mathcal{H}_{QB} + \mathcal{H}_{DET} + \mathcal{H}_{INT}$, the first term describes two qubits alone, $\mathcal{H}_{QB} = (\varepsilon_a/2)(a_{\downarrow}^{\dagger}a_{\downarrow} - a_{\uparrow}^{\dagger}a_{\uparrow}) + H_a(a_{\uparrow}^{\dagger}a_{\downarrow} + a_{\downarrow}^{\dagger}a_{\uparrow}) + (\varepsilon_b/2)(b_{\downarrow}^{\dagger}b_{\downarrow} - b_{\uparrow}^{\dagger}b_{\uparrow}) + H_b(b_{\uparrow}^{\dagger}b_{\downarrow} + b_{\downarrow}^{\dagger}b_{\uparrow})$, where ε_a and ε_b are energy asymmetries, which are assumed to be zero, the amplitudes



FIG. 1. Schematic of two qubits measured by an equally coupled detector. (a) Realization based on double quantum dots measured by a quantum point contact, (b) realization based on single-Cooper-pair boxes (SCPB) measured by a single-electron transistor. Measurement can entangle qubits.

 H_a and H_b describe the tunneling within qubits (we consider $H_a \approx H_b$), and the direct interaction term $Ua_{\uparrow}^{\dagger}a_{\uparrow}b_{\uparrow}^{\dagger}b_{\uparrow}$ is neglected. The frequencies of free Rabi oscillations of qubits, $\Omega_a = (4H_a^2 + \varepsilon_a^2)^{1/2} = 2H_a$ and $\Omega_b = 2H_b$ (we use $\hbar = 1$) obviously coincide, $\Omega_a = \Omega_b = \Omega$ [$\Omega \equiv (\Omega_a + \Omega_b)/2$] if $H_a = H_b$. For simplicity, we limit ourselves by the case¹⁹ of DQD qubits, measured by a low-transparency QPC, so that the detector Hamiltonian is $\mathcal{H}_{DET} = \Sigma_l E_l c_l^{\dagger} c_l + \Sigma_r E_r c_r^{\dagger} c_r + \Sigma_{l,r} T(c_l^{\dagger} c_r + c_r^{\dagger} c_l)$ and the interaction term is $\mathcal{H}_{INT} = \Sigma_{l,r} \Delta T_a (a_{\uparrow}^{\dagger} a_{\uparrow} - a_{\downarrow}^{\dagger} a_{\downarrow}) (c_l^{\dagger} c_r + c_r^{\dagger} c_l) + \Sigma_{l,r} \Delta T_b (b_{\uparrow}^{\dagger} b_{\uparrow}) - b_{\downarrow}^{\dagger} b_{\downarrow}) (c_l^{\dagger} c_r + c_r^{\dagger} c_l)$; equal coupling implies $\Delta T_a = \Delta T_b$.

The four basis states of two qubits, $|1\rangle \equiv |\uparrow_a\uparrow_b\rangle$, $|2\rangle \equiv |\uparrow_a\downarrow_b\rangle$, $|3\rangle \equiv |\downarrow_a\uparrow_b\rangle$, $|4\rangle \equiv |\downarrow_a\downarrow_b\rangle$, correspond to four values of the average current through the detector: $I_{1,2,3,4} = 2\pi (T \pm \Delta T_a \pm \Delta T_b)^2 \rho_l \rho_r e^2 V$, where V is the QPC voltage and $\rho_{l(r)}$ are densities of states. The measurement process tends to collapse the two-qubit state into this "measurement" basis. However, in the case of equal coupling two currents coincide, $I_2 = I_3 \equiv I_{23}$, so the measurement cannot distinguish between states $|2\rangle$ and $|3\rangle$. Besides the measurement basis, it is convenient to introduce also the Bell basis: $|1\rangle^B \equiv (|\uparrow_a\downarrow_b\rangle - |\downarrow_a\uparrow_b\rangle)/\sqrt{2}$, $|2\rangle^B \equiv (|\uparrow_a\uparrow_b\rangle - |\downarrow_a\downarrow_b\rangle)/\sqrt{2}$. Note that $|1\rangle^B$ and $|2\rangle^B$ are eigenstates of \mathcal{H}_{QB} if $H_a = H_b$, while states $|3\rangle^B - i \sin(\Omega t + \phi)|4\rangle^B$.

Before considering continuous measurements, let us discuss a simpler case of a sequence of orthodox projective measurements which can be realized if the coupling with the detector is strong ($C \ge 1$, see below) and the detector voltage is applied during short-time intervals. Since states $|2\rangle$ and $|3\rangle$ are mutually indistinguishable, the two-qubit density matrix ρ is projected each time into one of the three subspaces, corresponding to states $|1\rangle$, $|23\rangle$, and $|4\rangle$ (we use notation $|23\rangle$ for the subspace spanned by $|2\rangle$ and $|3\rangle$). The projective measurements are separated by time intervals Δt of unitary evolution due to \mathcal{H}_{OB} .

Assume that the first measurement resulted in the current I_{23} ; then the state is projected into the $|23\rangle$ subspace, which is also a subspace $|13\rangle^B$ in the Bell basis. If the state would be exactly $|1\rangle^B$ (which does not evolve under \mathcal{H}_{QB}), then all subsequent measurements would give the same result I_{23} and state $|1\rangle^B$ would remain unchanged. However, if the two qubits would be in state $|3\rangle^B$, then the next measurement

would result in I_{23} only with probability $p = (\cos \Omega \Delta t)^2$, while the probabilities of results I_1 and I_4 would be (1 - p)/2 each. Therefore, if a long sequence of current measurements repeatedly gives the result I_{23} , the two-qubit density matrix ρ purifies and becomes close to the fully entangled state $|1\rangle^B$.

A simple analysis shows that after N successful measurements (all results are I_{23})

$$\rho_{11}^{B}(N) = \rho_{11}^{B}(0) / [\rho_{11}^{B}(0) + \rho_{33}^{B}(0)(\cos\Omega\Delta t)^{2(N-1)}], \quad (1)$$

where $\rho_{11}^B(0)$, $\rho_{33}^B(0)$, and $\rho_{11}^B(N)$ are the corresponding density-matrix elements in the Bell basis before and after the measurements, while the probability of a successful sequence is $P(N) = \rho_{11}^B(0) + \rho_{33}^B(0)(\cos \Omega \Delta t)^{2(N-1)}$. For large N, the difference from state $|1\rangle^B$ becomes exponentially small, while the probability of success is close to $\rho_{11}^B(0)$, which is equal to 1/4 for the fully mixed initial state $\rho_{ij,mix}$ $= \rho_{ij,mix}^B = \delta_{ij}/4$. The purification rate depends on Δt , and is the fastest when Δt is close to $(1/4 + k/2)2\pi/\Omega$ (k is an integer), which is a regime opposite to the quantum nondemolition measurements.²⁰

If some measurement in the sequence gives I_1 or I_4 , then ρ_{11}^B becomes zero. In this case, to obtain the Bell state $|1\rangle^B$, one has to apply some perturbation which mixes two subspaces (for example, a noise affecting ε_a and/or ε_b) and repeat the procedure. Thus, the probability $1 - (3/4)^M$ to obtain state $|1\rangle^B$ becomes arbitrary close to unity for a sufficiently large number M of attempts.

The procedure can obviously be used for the *preparation* of entangled states in a solid-state quantum computer, so it is important to discuss what happens if the conditions $H_a = H_b$ and $I_2 = I_3$ are not satisfied exactly. In the case of slightly different H_a and H_b , Eq. (1) changes insignificantly $[\cos \Omega \Delta t$ should be replaced with $\cos \Omega \Delta t/\cos(\Delta \Omega \Delta t/2)$, where $\Delta \Omega \equiv \Omega_a - \Omega_b]$, however, the probability of an *N*-long successful sequence becomes $P(N) = \rho_{11}^B(0) [\cos(\Delta \Omega \Delta t/2)]^{2(N-1)} + \rho_{33}^B(0) (\cos \Omega \Delta t)^{2(N-1)}$ and decreases to zero at $N \rightarrow \infty$. Estimating the average length of a successful sequence, $\overline{N} \sim [\sin(\Delta \Omega \Delta t/2)]^{-2}$, one can estimate a typical inaccuracy $1 - \rho_{11}^B \sim [\cos \Omega \Delta t/\cos(\Delta \Omega \Delta t/2)]^{2/[\sin(\Delta \Omega \Delta t/2)]^2}$, which is as small as $\sim \exp[-(2\Omega/\Delta \Omega)^2]$ if $\Delta t \ll \Omega^{-1}$ (quantum Zeno regime) and even smaller, $\sim (\cos \Omega \Delta t)^{(32/\pi^2)(\Omega/\Delta \Omega)^2}$, if Δt is close to $\pi/2\Omega$.

To analyze the effect of a small difference between I_2 and I_3 because of slightly different coupling, we use the standard theory¹¹ of weak quantum measurements and take into account the detector shot noise $S_i = 2eI_i$. We assume that during a short measurement interval δt currents I_1 and I_4 can be unambiguously identified, while currents I_2 and I_3 are almost indistinguishable: $\epsilon = (I_2 - I_3)^2 / 4D \ll 1$, where $D = S_{23}/2\delta t$ is the variance of the measured noisy current. Each successful measurement tends to shift the state towards either $|2\rangle$ or $|3\rangle$ and so decreases the amount of entangled state $|1\rangle^B$, that competes with the purification due to Eq. (1) and leads to an iterative formula $\rho_{I_1}^B(N+1) \simeq \rho_{I_1}^B(N) - \epsilon/4$



FIG. 2. Two numerical realizations of ρ_{11}^B evolution starting from the fully mixed state. The upper (solid) line illustrates the scenario of collapse into a fully entangled Bell state $|1\rangle^B$, while the lower (dotted) line shows a collapse into the orthogonal subspace. Two insets show the corresponding spectral densities $S_I(\omega)$ of the detector noise (solid/dashed lines are the numerical/analytical results).

+[1- $\rho_{11}^B(N)$](sin $\Omega\Delta t$)² when ρ_{11}^B is close to unity. Therefore, a typical inaccuracy is $1-\rho_{11}^B \simeq \epsilon/4(\sin \Omega\Delta t)^2$.

Now instead of instantaneous measurements let us consider a more realistic case of a continuous measurement, realized when the detector voltage is applied all the time. For the analysis we will use the Bayesian formalism¹³ assuming weakly responding linear detecting regime, $|\Delta I_{a,b}| \ll I_i$, $\Delta I_a \equiv I_1 - I_3 = I_2 - I_4$, $\Delta I_b \equiv I_1 - I_2 = I_3 - I_4$, and symmetric weak coupling, $C_a \approx C_b \leq 1$, $C_{a,b} \equiv (\Delta I_{a,b})^2 / S_0 H_{a,b}$, where the frequency-independent detector noise spectral density S_0 does not depend significantly on the qubit's state.

The evolution of the two-qubit density matrix ρ can be described by the equation²¹ (in the Itô representation)

$$\frac{d}{dt}\rho_{ij} = \left[I(t) - \sum_{k} \rho_{kk}I_{k}\right] \left(I_{i} + I_{j} - 2\sum_{k} \rho_{kk}I_{k}\right) \frac{\rho_{ij}}{S_{0}} - \left[(I_{i} - I_{j})^{2}/4S_{0} + \gamma_{ij}\right]\rho_{ij} - i\left[\mathcal{H}_{QB}, \rho\right]_{ij}, \quad (2)$$

where the extra dephasing rate $\gamma_{ij} = (\eta^{-1} - 1)(I_i - I_j)^2/4S_0$ depends on detector ideality η ($0 \le \eta \le 1$) and vanishes for the QPC as a detector¹³ ($\eta = 1$); however, this term is important, for example, for the SET. To simulate individual realizations of the random measurement process, the noisy detector current I(t) can be calculated as $I(t) = \xi(t)$ $+ \Sigma_k \rho_{kk} I_k$, where $\xi(t)$ is a white noise with spectral density S_0 . Notice that averaging over noise $\xi(t)$ eliminates the first term in Eq. (2) and leads to the standard master equation.

We have performed extensive Monte Carlo simulations and found the following (Fig. 2). In the symmetric case, $H_a = H_b$, $C_a = C_b = C$ [$C \equiv (C_a + C_b)/2$, we used C from 1/4 to 1], any initial state either evolves into the fully entangled Bell state $|1\rangle^B$ ($\rho_{11}^B \rightarrow 1$) or ends up in the orthogonal subspace ($\rho_{11}^B \rightarrow 0$) performing oscillations²² within this subspace so that the "signal" $z \equiv \rho_{11} - \rho_{44} = 2 \operatorname{Re} \rho_{24}^B$ (which affects the detector current) oscillates with frequency Ω and amplitude fluctuating between 0 and 1. Both states correspond to the same²³ average detector current [since I_{23} PHYSICAL REVIEW B 67, 241305(R) (2003)

= $(I_1+I_4)/2$]; however, the spectral density of the detector current is different. In state $|1\rangle^B$ it is flat and equal to S_0 (since z=0), while in the oscillating state it exhibits a peak²³ at frequency Ω (lower inset in Fig. 2) with the peak height (32/3) ηS_0 , confirming the analytical result discussed below.

The fact of collapsing eventually either into state $|1\rangle^{B}$ or into the orthogonal subspace can be understood using an analogy with the sequential measurement case, and is because neither unitary evolution due to \mathcal{H}_{QB} nor nonunitary evolution due to measurement mixes two subspaces [see Eq. (2)]. The probability of two scenarios are obviously equal to $\rho_{11}^{B}(0)$ and $1-\rho_{11}^{B}(0)$, since the ensemble-averaged value $\langle \rho_{11}^{B}(t) \rangle$ does not change with time (as follows from the master equation).

To find analytically the spectral density of the detector current for the oscillating state, we have used two methods¹⁶ leading to the same result. The first one is based on the master equation and the collapse ansatz. Using the classical equation $I(t) = z\Delta I + \xi(t)$, we calculate the current correlation function $K_I(\tau) = \langle I(0)I(\tau) \rangle$ as $K_I(\tau > 0)$ $= (\Delta I)^2 K_{\tau}(\tau)$, while $K_{\tau}(\tau)$ is calculated in the following way. At time $\tau = 0$, the two-qubit state is collapsed into one of the three basis states of the subspace: $|1\rangle$ (corresponding to z=1), $|4\rangle$ (z=-1), or $|3\rangle^B$ (z=0). The probabilities of these collapses are 1/3 each, since for $\rho_{11}^B = 0$ the stationary solution of the master equation is $\rho_{11} = \rho_{44} = \rho_{33}^B = 1/3$ (this state is diagonal in the basis $\{|1\rangle, |4\rangle, |3\rangle^B\}$ as well as in the Bell basis and has zero entanglement²⁴). In each of the three cases, the value of z at time τ is obtained from the solution of the master equation [averaged Eq. (2)] for two relevant components:

$$dz/d\tau = -\Omega y, \quad dy/d\tau = \Omega z - \Gamma y,$$
 (3)

where $y \equiv 2 \operatorname{Im} \rho_{23}^B$ and $\Gamma = \eta^{-1} (\Delta I)^2 / 4S_0$. So $z(\tau) = \pm G(\tau)$, $G(\tau) \equiv \exp(-\Gamma \tau/2) [\cos \tilde{\Omega} \tau + (\Gamma/2\tilde{\Omega}) \sin \tilde{\Omega} \tau]$ [here $\tilde{\Omega} \equiv (\Omega^2 - \Gamma^2 / 4)^{1/2}$] in the two first cases, while $z(\tau) = 0$ in the third one. Summing the three contributions to $\langle z(0)z(\tau) \rangle$ with probability weights 1/3 each, we obtain $K_{\hat{z}} = (2/3)G(\tau)$ and the current spectral density

$$S_{I}(\omega) = S_{0} + \frac{8}{3} \frac{\Omega^{2}(\Delta I)^{2} \Gamma}{(\omega^{2} - \Omega^{2})^{2} + \Gamma^{2} \omega^{2}}.$$
 (4)

In the case $\Gamma \ll \Omega$, the spectral peak at the Rabi frequency Ω corresponds to the *Q*-factor of $8 \eta/C$ (as for one qubit¹⁶) and has the peak height equal to $(32/3) \eta S_0$.

The second method of $S_I(\omega)$ calculation is based on the Bayesian equation (2) assuming $\eta = 1$ and random evolution of a pure state²² with $z=A(t)\cos[\Omega t+\Phi(t)]$ [then y $=A(t)\sin[\Omega t+\Phi(t)]$]. In this method¹⁶ the correlation between noise $\xi(0)$ and evolution of the density matrix at a later time should be taken into account, so $K_I(\tau>0)$ $=(\Delta I)^2 K_z(\tau) + \Delta I K_{\xi z}(\tau)$, while correlation functions $K_z(\tau)$ and $K_{\xi z}(\tau)$ should be calculated by averaging of a long individual realization over time. We have proved that the result for $K_I(\tau)$ calculated by this method coincides with the result of the previous method for arbitrary coupling C; however, the formalism is much simpler for weak coupling, $\mathcal{C} \ll 1$. In this case, the stochastic differential equations for A(t) and $\Phi(t)$ can be averaged over oscillations with frequency Ω and the correlation functions can be calculated analytically: $K_{z}(\tau) = (5/12)G(\tau)$ and $K_{\xi_z}(\tau > 0)$ $=G(\tau)\Delta I/4$. This gives us a natural partition of the relative spectral peak height 32/3 into two contributions: "classical" part 20/3 comes from oscillations of the signal z, while the "quantum" contribution equal to 4 is due to the partial collapse of ρ correlated with the detector noise. Comparing this partition with the partition 4=2+2 for a one-qubit measurement,¹⁶ we observe that the classical part grows faster than the quantum part when the number of qubits is increased.

Numerical simulations show that if the two Rabi frequencies Ω_a and Ω_b are slightly different, or a small difference between C_a and C_b is due to asymmetry of the coupling (different ΔI_a and ΔI_b), then the two-qubit density matrix ρ makes rare abrupt jumps between a state very close to $|1\rangle^{B}$ and the oscillating state. To find the switching rates analytically, we have used the master equation starting from the entangled initial condition $\rho_{11}^B = 1$ and calculated the linear term in $\rho_{11}^B(t)$ dependence at $t \ge \Gamma^{-1}$ [but when $\rho_{11}^B(t)$ is still close to unity]. In this way we have obtained the rate $\Gamma_{B\to O} = (\Delta \Omega)^2 / 2\Gamma$ of switching from the Bell state to the oscillating state due to slightly different Rabi frequencies, and the rate $\Gamma_{B\to O} = (\Delta C/C)^2 \Gamma/8$ when $\Omega_a = \Omega_b$, but couplings ΔI_a and ΔI_b are slightly different. To find the rate of the reverse switching, note that the stationary master equation has the solution $\rho_{ij,st}^B = \rho_{ij,st} = \delta_{ij}/4$, therefore the sys-

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tem should spend on average one-fourth of the time in the state $|1\rangle^{B}$, and so $\Gamma_{O\rightarrow B} = \Gamma_{B\rightarrow O}/3$. The numerical histograms of switching time distributions confirm these formulas. Taking into account rare switching events, the average spectral density of the detector current is given by Eq. (4) multiplied by 3/4, so the spectral peak height is equal to $8 \eta S_{0}$.

Finally, we have studied the effect of environmental dephasing, modeling it with two small dephasing rates γ_a and γ_b acting separately onto two qubits. This leads to a slightly mixed ρ even for an ideal detector and to switching events with $\Gamma_{B\to O} = 3\Gamma_{O\to B} = (\gamma_a + \gamma_b)/2$. Note that a controllable weak external noise can be used in a simple feedback protocol to restore the entangled state after an undesirable switching to the oscillating state.

In conclusion, we have found that the continuous measurement of two identical solid-state qubits by the equally coupled detector leads to either a full spontaneous entanglement of qubits (Bell state) or to a collapse into the orthogonal oscillating state. Slight asymmetry of the two-qubit configuration as well as environmental dephasing leads to switching between two regimes. It is important to mention that for an experimental observation of the phenomenon, the quantum ideality η of the detector should not necessarily be close to unity; it should only be large enough to distinguish the Rabi spectral peak with the peak-to-pedestal ratio of $32 \eta/3$.

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