

## Continuous measurement of entangled qubits

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We have developed Bayesian formalism to describe the process of continuous measurement of entangled qubits. We start with the case of two qubits and then generalize it to an arbitrary number of qubits.

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The problem of quantum measurement of a qubit (two-level system) received renewed attention recently in relation to its importance for quantum computing. The case of sufficiently fast (instantaneous) measurement can be readily described by “orthodox” collapse postulate [1], and this is the case assumed at present by all quantum algorithms. However, in practice, especially for solid-state qubits, the act of measurement is not instantaneous. Because of typically low coupling between a solid-state qubit and a detector, it takes a considerable time before the qubit state is completely destroyed by the act of measurement. Correspondingly, because of fundamentally unavoidable noise of the detector, the information about the state of the measured qubit is available not immediately, but only after some time sufficient to get an acceptably large signal-to-noise ratio. It is important that the time scale of measurement (and collapse) process may be comparable to the time scale of “free” qubit evolution (e.g., due to Rabi oscillations) or to the duration of the detector on-off operation sequence. (For example, if the detector is switched off when signal-to noise ratio is still on the order of unity, the measurement is only partially completed.)

So, for practical needs we should be able to describe the measurement of a solid-state qubit as a continuous process. The formalism suitable to describe a continuous measurement of an *ensemble* of qubits has been developed two decades ago [2] (for its use in quantum computing problems see, e.g., Ref. [3]). In contrast, the formalism describing the process of measurement of a *single* qubit have been presented only recently [4–6] and is still in the stage of active development. (In fact, it can be considered as a direct continuation of the well-developed field of selective or conditional quantum measurements — see, e.g., Refs. [7–12] and references in Ref. [5]). This formalism is called Bayesian (because of the essential role of the Bayes formula [13]) and combines advantages of the “orthodox” approach [1] (the ability to treat single quantum systems) and the Leggett’s approach [2] (the ability to treat continuous measurement).

The Bayesian approach has been applied so far only to the continuous measurement of a single qubit. [4–6,14–16] In this paper we apply it to derive the equations describing continuous measurement of entangled qubits.

Let us consider first the case of two entangled qubits, one of which is continuously measured by a detector (Fig. 1). As a main example, we consider qubits made of double quantum dots while the detector is a quantum point contact [realizations based on single-electron transistors and superconducting quantum interference devices (SQUIDs) are also possible — see Ref. [5]]. Let us denote four basis vectors characterizing the state of two qubits as  $|1\rangle \equiv |\uparrow\uparrow\rangle$ ,

$|2\rangle \equiv |\uparrow\downarrow\rangle$ ,  $|3\rangle \equiv |\downarrow\uparrow\rangle$ , and  $|4\rangle \equiv |\downarrow\downarrow\rangle$ . (The basis for the first qubit is determined by its interaction with the detector, while for the second qubit, the basis is arbitrary.) The qubits can interact with each other as well as be noninteracting (the entanglement can be a result of previous interaction). The free evolution of qubits is described by the Schrödinger equation  $d\Psi/dt = (-i/\hbar)H_{qb}\Psi$ , where  $H_{qb}$  is the Hamiltonian of qubits only ( $H_{qb}$  accounts for energy asymmetries and “tunnelings” within qubits and for interaction between qubits, but does not include interaction with the detector). Correspondingly, in the case without detector the density-matrix  $\rho$  of a double-qubit system evolves as  $d\rho/dt = (-i/\hbar)[H_{qb}, \rho]$ .

The detector output is characterized by two dc currents,  $I_\uparrow$  and  $I_\downarrow$ , corresponding to two states of the first qubit, and the frequency-independent spectral density  $S$  of the detector noise. As usual [5] we assume weakly responding (linear) detector,  $|\Delta I| \ll I_0$ , where  $\Delta I \equiv I_\uparrow - I_\downarrow$  and  $I_0 \equiv (I_\uparrow + I_\downarrow)/2$ , to neglect individual electrons passing through the detector and consider the detector current  $I(t)$  as a continuous function of time. For the same purpose we assume that the time scale  $e/I_0$  (where  $e$  is the electron charge) is much shorter than other time scales in the problem (due to collapse, dephasing, and free evolution of qubits).

Let us start with the simplest case when qubits are “frozen,” [17]  $H_{qb} = 0$  (so all the evolution is due to the measurement only), the initial state of qubits is pure, and the detector is ideal (for example, quantum point contact at low temperature is an ideal detector [5], as well as single-electron transistor well inside the cotunneling range [18]). We can always represent the initial pure state as  $\Psi = \alpha|\uparrow\rangle \otimes (a_\uparrow|\uparrow\rangle + b_\uparrow|\downarrow\rangle) + \beta|\downarrow\rangle \otimes (a_\downarrow|\uparrow\rangle + b_\downarrow|\downarrow\rangle)$ , where the states of the second qubit are normalized,  $|a_\uparrow|^2 + |b_\uparrow|^2 = |a_\downarrow|^2 + |b_\downarrow|^2 = 1$ , and consequently  $|\alpha|^2 + |\beta|^2 = 1$ . Since the detector is not coupled to the second qubit, the evolution due to measurement affects only the factors  $\alpha$  and  $\beta$ , which can be calcu-

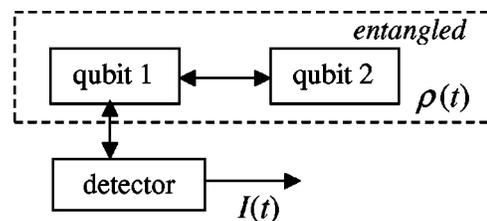


FIG. 1. Schematic of two entangled qubits, one of which is continuously measured by a detector. The noisy detector output  $I(t)$  is used to monitor the evolution of the double-qubit density-matrix  $\rho(t)$ .

lated using the single-qubit Bayesian result [5] (the overall wave function phase is of course not important).

$$\frac{\alpha(\tau)}{\alpha(0)} = \left[ \frac{P_{\uparrow}(\tau)}{|\alpha(0)|^2 P_{\uparrow}(\tau) + |\beta(0)|^2 P_{\downarrow}(\tau)} \right]^{1/2}, \quad (1)$$

$$\frac{\beta(\tau)}{\beta(0)} = \left[ \frac{P_{\downarrow}(\tau)}{|\alpha(0)|^2 P_{\uparrow}(\tau) + |\beta(0)|^2 P_{\downarrow}(\tau)} \right]^{1/2}, \quad (2)$$

where  $P_{\uparrow}(\tau)$  and  $P_{\downarrow}(\tau)$  characterize the conditional probabilities (for the first qubit in  $|\uparrow\rangle$  and  $|\downarrow\rangle$  states) of getting a particular realization of the detector output [19]  $I(t)$ ,

$$P_{\uparrow}(\tau) = (2\pi D)^{-1/2} \exp[-(\bar{I}(\tau) - I_{\uparrow})^2/2D], \quad (3)$$

$$P_{\downarrow}(\tau) = (2\pi D)^{-1/2} \exp[-(\bar{I}(\tau) - I_{\downarrow})^2/2D], \quad (4)$$

$$\bar{I}(\tau) \equiv \tau^{-1} \int_0^{\tau} I(t) dt, \quad D \equiv S/2\tau. \quad (5)$$

In the language of double-qubit density matrix the evolution described by Eqs. (1) and (2) can be rewritten as

$$\frac{\rho_{11}(\tau)}{\rho_{11}(0)} = \frac{\rho_{22}(\tau)}{\rho_{22}(0)} = \frac{\rho_{12}(\tau)}{\rho_{12}(0)} = \frac{P_{\uparrow}(\tau)}{\rho_{\uparrow}P_{\uparrow}(\tau) + \rho_{\downarrow}P_{\downarrow}(\tau)}, \quad (6)$$

$$\frac{\rho_{33}(\tau)}{\rho_{33}(0)} = \frac{\rho_{44}(\tau)}{\rho_{44}(0)} = \frac{\rho_{34}(\tau)}{\rho_{34}(0)} = \frac{P_{\downarrow}(\tau)}{\rho_{\uparrow}P_{\uparrow}(\tau) + \rho_{\downarrow}P_{\downarrow}(\tau)}, \quad (7)$$

$$\begin{aligned} \frac{\rho_{13}(\tau)}{\rho_{13}(0)} &= \frac{\rho_{14}(\tau)}{\rho_{14}(0)} = \frac{\rho_{23}(\tau)}{\rho_{23}(0)} = \frac{\rho_{24}(\tau)}{\rho_{24}(0)} \\ &= \frac{[P_{\uparrow}(\tau)P_{\downarrow}(\tau)]^{1/2}}{\rho_{\uparrow}P_{\uparrow}(\tau) + \rho_{\downarrow}P_{\downarrow}(\tau)}, \end{aligned} \quad (8)$$

where  $\rho_{\uparrow} \equiv \rho_{11}(0) + \rho_{22}(0)$  and  $\rho_{\downarrow} \equiv \rho_{33}(0) + \rho_{44}(0)$  correspond to initial probabilities to find the first qubit in  $|\uparrow\rangle$  and  $|\downarrow\rangle$  states.

If the initial state  $\rho(0)$  is not pure, its evolution can be calculated in the following way. Let us represent  $\rho(0)$  as

$$\rho(0) = \sum_s p_s(0) \rho_s(0), \quad (9)$$

where  $p_s$  is the classical probability of a pure state  $|s\rangle$ ,  $\rho_s$  is its density matrix, and the sum is over a necessary number of pure states. (Of course, such representation is not unique in general.) To calculate  $\rho(\tau)$  we can apply the ‘‘double Bayes’’ procedure: classical Bayes theorem to obtain probabilities  $p_s(\tau)$ ,

$$p_s(\tau) = \frac{p_s(0)[\rho_{s,\uparrow}P_{\uparrow}(\tau) + \rho_{s,\downarrow}P_{\downarrow}(\tau)]}{\sum_r p_r(0)[\rho_{r,\uparrow}P_{\uparrow}(\tau) + \rho_{r,\downarrow}P_{\downarrow}(\tau)]}, \quad (10)$$

and the quantum Bayesian result [Eqs. (6)–(8)] to calculate each  $\rho_s(\tau)$ . It is easy to show that the resulting evolution of  $\rho(\tau) = \sum_s p_s(\tau) \rho_s(\tau)$  satisfies Eqs. (6)–(8), which therefore are valid for arbitrary mixed states as well. Notice that Eq.

(8) has an obvious interpretation as the conservation of the ‘‘degree of purity’’ similar to the one-qubit case [5].

Besides the derivation of Eqs. (6)–(8) using one-qubit Bayesian result (as above), they can also be obtained (in the case of pure states) directly using the ‘‘quantum Bayes theorem’’ [20], which says that the classical Bayes formula [13] is applicable not only to the probabilities described by the diagonal matrix elements (that is obvious because of the correspondence principle), but also applicable to the wave function amplitudes. Besides that, Eqs. (6)–(8) can be easily derived ‘‘microscopically’’ in the case of a low-transparency quantum point contact at zero temperature. In this case, solving the Schrödinger equation for the qubits coupled to the detector (for the model see Refs. [21] and [5]) one can obtain the following Bloch equations [21] for the density matrix  $\tilde{\rho}_{ij}^n$  which contains index  $n$  corresponding to the number of electrons passed through the detector:

$$d\tilde{\rho}_{11}^n/dt = -(I_{\uparrow}/e)\tilde{\rho}_{11}^n + (I_{\uparrow}/e)\tilde{\rho}_{11}^{n-1}, \quad (11)$$

$$d\tilde{\rho}_{33}^n/dt = -(I_{\downarrow}/e)\tilde{\rho}_{33}^n + (I_{\downarrow}/e)\tilde{\rho}_{33}^{n-1}, \quad (12)$$

$$d\tilde{\rho}_{12}^n/dt = -(I_{\uparrow}/e)\tilde{\rho}_{12}^n + (I_{\uparrow}/e)\tilde{\rho}_{12}^{n-1}, \quad (13)$$

$$d\tilde{\rho}_{13}^n/dt = -(I_0/e)\tilde{\rho}_{13}^n + (\sqrt{I_{\uparrow}I_{\downarrow}}/e)\tilde{\rho}_{13}^{n-1}, \quad (14)$$

$$d\tilde{\rho}_{14}^n/dt = -(I_0/e)\tilde{\rho}_{14}^n + (\sqrt{I_{\uparrow}I_{\downarrow}}/e)\tilde{\rho}_{14}^{n-1}. \quad (15)$$

The equations for other components of  $\tilde{\rho}_{ij}^n$  are similar and can be obtained by the substitutions:  $\tilde{\rho}_{11}^n \rightarrow \tilde{\rho}_{22}^n$ ,  $\tilde{\rho}_{33}^n \rightarrow \tilde{\rho}_{44}^n$ ,  $\tilde{\rho}_{33}^n \rightarrow \tilde{\rho}_{34}^n$ ,  $\tilde{\rho}_{13}^n \rightarrow \tilde{\rho}_{23}^n$ , and  $\tilde{\rho}_{13}^n \rightarrow \tilde{\rho}_{24}^n$ . Solving these equations and collapsing the number  $n$  at time  $\tau$  (measuring the charge passed through the detector and obtaining, for example, charge  $me$ ),

$$\tilde{\rho}_{ij}^n(\tau+0) = \delta_{nm} \rho_{ij}(\tau+0), \quad (16)$$

$$\rho_{ij}(\tau+0) = \frac{\tilde{\rho}_{ij}^m(\tau-0)}{\sum_k \tilde{\rho}_{kk}^m(\tau-0)}, \quad (17)$$

one reproduces Eqs. (6)–(8).

Now let us take into account finite detector ideality (efficiency),  $\eta \leq 1$ , where in the one-qubit case [5]  $\eta \equiv (\Delta I)^2/4S\Gamma$  is the ratio of the ‘‘information acquisition rate’’ [22]  $(\Delta I)^2/4S$  and the ensemble dephasing rate  $\Gamma$ . Similar to the derivation of Ref. [4], let us consider first the case of a detector with neglected output (which is equivalent to ‘‘pure environment’’). Then, averaging Eqs. (6)–(8) over the probability distribution  $\rho_{\uparrow}P_{\uparrow}(\tau) + \rho_{\downarrow}P_{\downarrow}(\tau)$  of  $\bar{I}(\tau)$  [see Eqs. (3)–(5)], we get the following: the right-hand side of Eqs. (6) and (7) becomes unity (which means that  $\rho_{11}$ ,  $\rho_{22}$ ,  $\rho_{12}$ ,  $\rho_{33}$ ,  $\rho_{44}$ , and  $\rho_{34}$  do not change on average), while the right-hand side of Eq. (8) is replaced by  $\exp[-(\Delta I)^2/4S]$  (which means that  $\rho_{13}$ ,  $\rho_{14}$ ,  $\rho_{23}$ , and  $\rho_{24}$  decay on average with the rate  $(\Delta I)^2/4S$ ). Similar to the one-qubit case, we can regard a nonideal detector as two detectors ‘‘in parallel’’ [23], neglecting the output of the second detector. In this

way, we obtain the following result for a nonideal detector: Eqs. (6) and (7) remain valid, while Eq. (8) should be replaced by

$$\begin{aligned} \frac{\rho_{13}(\tau)}{\rho_{13}(0)} &= \frac{\rho_{14}(\tau)}{\rho_{14}(0)} = \frac{\rho_{23}(\tau)}{\rho_{23}(0)} = \frac{\rho_{24}(\tau)}{\rho_{24}(0)} \\ &= \frac{[P_{\uparrow}(\tau)P_{\downarrow}(\tau)]^{1/2}}{\rho_{\uparrow}P_{\uparrow}(\tau) + \rho_{\downarrow}P_{\downarrow}(\tau)} \exp(-\gamma\tau), \end{aligned} \quad (18)$$

where  $\gamma = (\eta^{-1} - 1)(\Delta I)^2/4S$ .

Notice that since the second qubit is not coupled to the detector, the state of the second qubit changes only due to its entanglement with the first qubit. In particular, in the case of no initial entanglement [when  $\rho(0)$  can be represented as a direct product], the state remains disentangled, the second qubit density matrix does not change, and Eqs. (6)–(8) reduce to the Bayesian result for the first qubit.

If the qubits are not frozen,  $H_{qb} \neq 0$ , the evolution due to  $H_{qb}$  should be added to the evolution due to measurement. (Simple adding of contributions from two evolutions is possible because the measurement process in the Bayesian formalism is assumed to be Markovian that requires, for example, the “attempt” frequency of tunneling electrons in the detector to be much larger than the typical frequency of qubit system evolution.) In differential form [we use Stratonovich representation [5], so we take the usual derivatives of Eqs. (6), (7), and (18)] we get the following Bayesian equations:

$$\dot{\rho}_{11} = \frac{-i}{\hbar} [H_{qb}, \rho]_{11} + \rho_{11}(\rho_{33} + \rho_{44}) \frac{2\Delta I}{S} (I(t) - I_0), \quad (19)$$

$$\dot{\rho}_{33} = \frac{-i}{\hbar} [H_{qb}, \rho]_{33} - \rho_{33}(\rho_{11} + \rho_{22}) \frac{2\Delta I}{S} (I(t) - I_0), \quad (20)$$

$$\dot{\rho}_{12} = \frac{-i}{\hbar} [H_{qb}, \rho]_{12} + \rho_{12}(\rho_{33} + \rho_{44}) \frac{2\Delta I}{S} (I(t) - I_0), \quad (21)$$

$$\dot{\rho}_{34} = \frac{-i}{\hbar} [H_{qb}, \rho]_{34} - \rho_{34}(\rho_{11} + \rho_{22}) \frac{2\Delta I}{S} (I(t) - I_0), \quad (22)$$

$$\begin{aligned} \dot{\rho}_{13} &= \frac{-i}{\hbar} [H_{qb}, \rho]_{13} - \rho_{13}(\rho_{11} + \rho_{22} - \rho_{33} - \rho_{44}) \frac{\Delta I}{S} \\ &\quad \times (I(t) - I_0) - \gamma\rho_{13}, \end{aligned} \quad (23)$$

$$\begin{aligned} \dot{\rho}_{14} &= \frac{-i}{\hbar} [H_{qb}, \rho]_{14} - \rho_{14}(\rho_{11} + \rho_{22} - \rho_{33} - \rho_{44}) \frac{\Delta I}{S} \\ &\quad \times (I(t) - I_0) - \gamma\rho_{14}. \end{aligned} \quad (24)$$

The equations for remaining components can be obtained from Eq. (19) by substitution  $\{11\} \rightarrow \{22\}$ , from Eq. (20) by substitution  $\{33\} \rightarrow \{44\}$ , and from Eq. (23) by substitutions  $\{13\} \rightarrow \{23\}$  and  $\{13\} \rightarrow \{24\}$ .

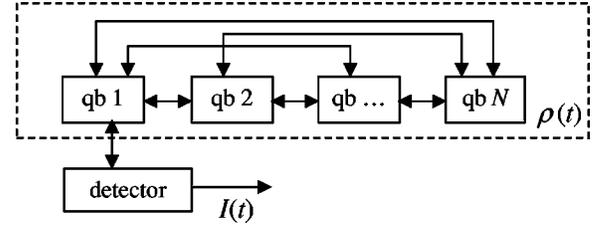


FIG. 2. Schematic of  $N$  entangled qubits, one of which is continuously measured by a detector. Bayesian formalism allows us to monitor  $N$ -qubit density-matrix  $\rho(t)$ , using detector output  $I(t)$ .

These equations allow us to monitor the evolution of the double-qubit density matrix if we know the initial state  $\rho(0)$  (for example, we have prepared qubits ourselves) and we know the detector output  $I(t)$  from an experiment. [To emphasize the noisy nature of  $I(t)$  we show this time dependence in Eqs. (19)–(24) explicitly, while the time dependence of the density matrix  $\rho$  is not shown explicitly.] To simulate the measurement process numerically, we need (similar to Ref. [5]) to complement these equations by the formula

$$I(t) - I_0 = \frac{\Delta I}{2} (\rho_{11} + \rho_{22} - \rho_{33} - \rho_{44}) + \xi(t), \quad (25)$$

where  $\xi(t)$  is a zero-correlated (“white”) random process with zero average and the same spectral density as the detector noise,  $S_{\xi} = S$ . [Equation (25) is derived from the probability distribution  $\rho_{\uparrow}P_{\uparrow}(\tau) + \rho_{\downarrow}P_{\downarrow}(\tau)$  for the average current  $\bar{I}(\tau)$  at sufficiently small  $\tau$ , so that evolution due to  $H_{qb}$  can be neglected.]

The conventional (ensemble-averaged) evolution equations can be obtained from Eqs. (19)–(24) by averaging over  $\xi(t)$  (it is easier to do using Itô representation – see similar procedure in Ref. [5]). In the resulting equations the terms proportional to  $\Delta I$  disappear, while  $\gamma$  is replaced by  $\gamma + (\Delta I)^2/4S$ . This result can be also obtained directly, since ensemble averaging is equivalent to the use of completely nonideal detector:  $\Delta I = 0$ ,  $\eta = 0$ .

The generalization of Eqs. (19)–(25) to the case of arbitrary number  $N$  of entangled qubits, one of which is being continuously measured (Fig. 2), is pretty obvious. If both basis vectors  $i$  and  $j$  (from the set of  $2^N$  basis vectors) correspond to the state  $|\uparrow\rangle$  of the measured qubit, then the evolution of the matrix element  $\rho_{ij}$  is given by the equation

$$\dot{\rho}_{ij} = \frac{-i}{\hbar} [H_{qb}, \rho]_{ij} + \rho_{ij} \rho_{\downarrow} \frac{2\Delta I}{S} (I(t) - I_0). \quad (26)$$

If both  $i$  and  $j$  correspond to the state  $|\downarrow\rangle$  of the measured qubit, then

$$\dot{\rho}_{ij} = \frac{-i}{\hbar} [H_{qb}, \rho]_{ij} - \rho_{ij} \rho_{\uparrow} \frac{2\Delta I}{S} (I(t) - I_0). \quad (27)$$

Finally, if  $i$  corresponds to the state  $|\uparrow\rangle$  while  $j$  corresponds to the state  $|\downarrow\rangle$ , then

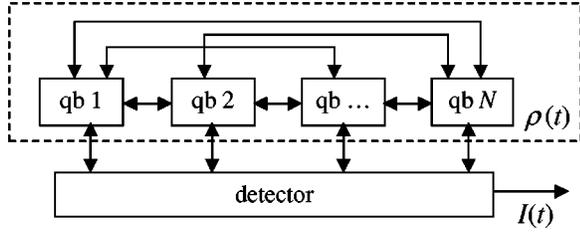


FIG. 3.  $N$  entangled qubits, continuously measured by a detector, coupled to all qubits.

$$\dot{\rho}_{ij} = \frac{-i}{\hbar} [H_{qb}, \rho]_{ij} - \rho_{ij} (\rho_{\uparrow} - \rho_{\downarrow}) \frac{\Delta I}{S} [I(t) - I_0] - \gamma \rho_{ij}. \quad (28)$$

In these equations  $H_{qb}$  is again the Hamiltonian of qubits (without detector) while  $\rho_{\uparrow}(t)$  and  $\rho_{\downarrow}(t)$  (now time dependent) are the sums of the diagonal matrix elements of  $\rho(t)$ , corresponding to the states  $|\uparrow\rangle$  and  $|\downarrow\rangle$  of the measured qubit. Equation (25) should be generalized as

$$I(t) - I_0 = \frac{\Delta I}{2} [\rho_{\uparrow}(t) - \rho_{\downarrow}(t)] + \xi(t). \quad (29)$$

Now let us generalize the formalism to the case when the detector is coupled to all qubits (Fig. 3). Classically, in this case there are up to  $2^N$  different dc current levels  $I_i$ , corresponding to various combinations of qubit states. Some of these levels can coincide, for example, if the detector is not coupled to some qubits or if some qubits are coupled to the detector equally strong. Applying the quantum Bayes theorem in the case of frozen qubits,  $H_{qb} = 0$ , and taking into account finite ideality  $\eta$  of the detector, we obtain the following equations:

$$\frac{\rho_{ij}(\tau)}{\rho_{ij}(0)} = \frac{\sqrt{P_i(\tau)P_j(\tau)}}{\sum_k \rho_{kk}(0)P_k(\tau)} \exp(-\gamma_{ij}\tau), \quad (30)$$

$$P_i(\tau) = (2\pi D)^{-1/2} \exp[-(\bar{I}(\tau) - I_i)^2/2D], \quad (31)$$

$$\gamma_{ij} = (\eta^{-1} - 1)(I_i - I_j)^2/4S, \quad (32)$$

where  $\bar{I}(\tau)$  and  $D$  are defined by Eq. (5), and the sum in Eq. (30) is over all  $2^N$  basis vectors  $k$  (the basis is defined by the interaction between the detector and each qubit). Correspondingly, the probability distribution of  $\bar{I}(\tau)$  is  $\sum_i \rho_{ii}(0)P_i(\tau)$ . Notice that the exponent due to nonideality in Eq. (30) disappears for diagonal matrix elements ( $i=j$ ) and also if the classical currents  $I_i$  and  $I_j$  for two different configurations coincide. This is because  $I_i = I_j$  means equal coupling of the detector to the states  $i$  and  $j$ , so the detector noise cannot destroy the coherence between these states.

Let us briefly discuss what will happen to Eqs. (30)–(32) if we relax the assumption of weak detector response,  $|I_i - I_j| \ll (I_i + I_j)/2$ . As an example, let us again consider a low-transparency quantum point contact at zero temperature. In the case of moderate or strong response, each electron passed through the detector brings significant information and, cor-

respondingly, changes significantly the density-matrix  $\rho$  of the qubits. Then the language of continuous detector current is not applicable anymore, and instead of considering average current  $\bar{I}(\tau)$  we should count the number  $n$  of electrons passed through the detector during time  $\tau$ . Equation (30) in this case does not change (except  $\gamma_{ij} = 0$  since the detector is ideal), while the Gaussian distribution in Eq. (31) should be replaced by the Poissonian distribution:  $P_i(\tau) = (n!)^{-1} (I_i \tau / e)^n \exp(-I_i \tau / e)$ . It is not easy to introduce nonideality for a detector with finite response. If, however, we define  $\eta$  in a way [6] similar to optical quantum efficiency as a probability to observe (not to miss) each electron tunneled through a detector (unfortunately, this definition is hardly justified in typical solid-state setups), then we can keep the exponential term in Eq. (30) and should replace Eq. (32) by  $\gamma_{ij} = (\eta^{-1} - 1)(\sqrt{I_i} - \sqrt{I_j})^2/2e$ .

Returning to the case of weak detector response and continuous current, differentiating Eq. (30) over time, and adding the free evolution due to  $H_{qb}$ , we finally obtain the following equation:

$$\begin{aligned} \dot{\rho}_{ij} = & \frac{-i}{\hbar} [H_{qb}, \rho]_{ij} + \rho_{ij} \frac{1}{S} \sum_k \rho_{kk} \left[ \left( I(t) - \frac{I_k + I_i}{2} \right) \right. \\ & \left. \times (I_i - I_k) + \left( I(t) - \frac{I_k + I_j}{2} \right) (I_j - I_k) \right] - \gamma_{ij} \rho_{ij}. \end{aligned} \quad (33)$$

Equation (25) in this case is replaced by

$$I(t) = \sum_i \rho_{ii}(t) I_i + \xi(t). \quad (34)$$

Our final generalization is to the case of several detectors, coupled to  $N$  qubits. Each detector has its own set of up to  $2^N$  classical current levels. It is important to notice that coupling of qubits to different detectors can define different sets of basis vectors. So, generalization of Eq. (33) requires us to sum the terms due to measurement over all detectors, choosing particular basis for each detector.

In conclusion, we have developed the Bayesian formalism describing continuous measurement of entangled solid-state qubits. The case of two qubits, one of which is measured by a detector is considered in detail and then generalized to an arbitrary case. For nonideal detectors we have assumed the absence of correlation between output and backaction noises, so the formalism applicable to nonideal detectors with such correlation [5] still has to be developed. The results of this paper [evolution Eqs. (19)–(24), (26)–(28), and (33)] can be experimentally tested. However, such experiments seem to be still a little beyond the reach of the present-day solid-state technology. They could be attempted after proposed Bayesian experiments with a single solid-state qubit, in particular, Bell-type experiment [15].

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- [17] The condition  $H_{qb}=0$  means symmetric qubits with no “tunnelings” (mixing of  $|\uparrow\rangle$  and  $|\downarrow\rangle$ ) and no interaction between qubits. Actually, the formalism remains almost the same if we allow for energy asymmetries and the interaction which does not include mixing. However, it is easier to include these effects later, together with the general form of  $H_{qb}$ .
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- [19] In the case of frozen qubits the probability of a particular output realization  $I(t)$  is completely determined by the average current  $\bar{I}(\tau)$  of this realization. This is typically not true, however, in the presence of free evolution,  $H_{qb}\neq 0$ .
- [20] C.W. Gardiner, *Quantum Noise* (Springer, Berlin, 1991), Chap. 2.2.
- [21] S.A. Gurvitz, *Phys. Rev. B* **56**, 15 215 (1997).
- [22] Actually, the average classical information acquisition rate (bit/s) is  $[(\Delta I)^2/S]\rho_{\uparrow}\rho_{\downarrow}(\rho_{\uparrow}-\rho_{\downarrow})\log_2(\rho_{\uparrow}/\rho_{\downarrow})$ .
- [23] In the case of nonideal detector we neglect the correlation between the output noise and the backaction noise. In the case of a significant correlation the formalism is more complicated—see Ref. [5].