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Quantum feedback of a double-dot qubit

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Abstract

We discuss an experimental proposal on quantum feedback control of a double-dot qubit, which seems to be within the reach of the present-day technology. Similar to the earlier proposal, the feedback loop is used to maintain the coherent oscillations in the qubit for an arbitrary long time; however, this is done in a significantly simpler way. The main idea is to use the quadrature components of the noisy detector current to monitor approximately the phase of qubit oscillations.

The principle of feedback control is widely used in physics and engineering. However, continuous feedback control of quantum systems is a relatively new and not well studied subject. Quantum feedback in optics has been proposed theoretically a decade ago [1] and has been recently demonstrated experimentally [2]. For a solid-state system (qubit), the quantum feedback has been discussed for the first time only few years ago [3], and there are no experiments yet.

The possibility of a quantum feedback is based on the fact that measurement by an ideal solid-state detector (with 100% quantum efficiency η) does not decohere a single qubit [4,5], even though it decoheres an ensemble of qubits because each qubit evolves in a different way. An example of theoretically ideal detector is [4] the quantum point contact (η comparable to 1 has been demonstrated experimentally [6]).

The random evolution of a qubit in the process of measurement can be monitored using the noisy detector output [4,5], and this monitoring can naturally be used for continuous feedback control of the qubit. In the proposal of Ref. [3], the quantum feedback is used to maintain quantum coherent (Rabi) oscillations in a qubit for an arbitrary long time, synchronizing them with an external classical signal. This is done by measuring the noisy current $I(t)$ in a weakly coupled detector and using the quantum Bayesian equations [4] to translate information contained in $I(t)$ into the evolution of qubit density matrix $\rho(t)$. After that $\rho(t)$ is compared with the desired quantum state $\rho_d(t)$, and the calculated difference is used to control the qubit Hamiltonian in order to decrease the difference.

An important difficulty in such experiment is a necessity to solve the Bayesian equations in real time. Moreover, the bandwidth of the line delivering $I(t)$ to the circuit solving the Bayesian equations, should be significantly wider than the Rabi frequency Ω. Unfortunately, these conditions are unrealistic for the present-day experiments with solid-state qubits.

In this paper, we discuss (see also Ref. [7]) a much simpler way (Fig. 1) of processing the information carried by the detector current $I(t)$. The idea is to use the fact that besides noise, $I(t)$ contains an oscillating contribution due to Rabi oscillations in the measured qubit. Therefore, if we apply $I(t)$ to a simple tank circuit (which is in resonance with Ω), then the phase of the tank circuit oscillations will depend on the phase of Rabi oscillations. Instead of using the tank circuit, almost equivalent theoretically procedure is to mix $I(t)$ with the signal from a local oscillator (Fig. 1) in order to determine two quadrature amplitudes of $I(t)$ at frequency Ω, which will carry information on the phase of Rabi oscillations. Since diffusion of the Rabi phase is a slow process (assuming weak coupling to the detector and environment), the further circuitry can be relatively slow, limited by the qubit dephasing rate, but not limited by much higher Rabi frequency. The simplicity of the information processing and alleviation of the bandwidth problem are the main advantages of this proposal in comparison with Ref. [3].

Let us consider a 'charge' qubit made of double quantum dot [8] occupied by a single electron, described by

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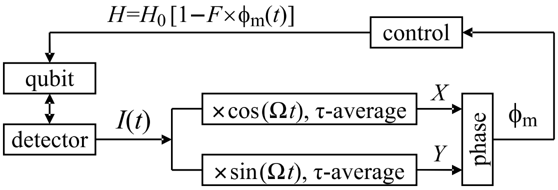

Fig. 1. Schematic of the proposed quantum feedback loop.

Hamiltonian $\mathcal{H}_{qb}=(\varepsilon/2)(c_2^\dagger c_2-c_1^\dagger c_1)+H(c_1^\dagger c_2+c_2^\dagger c_1)$, where $c_{1,2}^\dagger$ and $c_{1,2}$ are the creation and annihilation operators in the basis of 'localized' (charge) states, ε is their energy asymmetry, and the tunneling between dots $H=H_0+H_{fb}(t)$ can be controlled by the feedback loop (H_{fb}). The qubit state is measured by the quantum point contact [6], which barrier height depends on the qubit charge. Instead of writing Hamiltonians explicitly [9,4], we will characterize the measurement by two levels of the average detector current, I_1 and I_2, corresponding to the two charge states, by the detector output noise S_I, and by the qubit ensemble dephasing rate Γ due to detector back-action and environment. Assuming sufficiently large detector voltage and quasicontinuous detector current $I(t)$, we describe the qubit evolution by the Bayesian equations [4]

$$\dot\rho_{11}=-2H\,\mathrm{Im}\,\rho_{12}+2\rho_{11}\rho_{22}[I(t)-I_0]\Delta I/S_I, \tag{1}$$

$$\dot\rho_{12}=\mathrm{i}\varepsilon\rho_{12}+\mathrm{i}H(\rho_{11}-\rho_{22})-\gamma\rho_{12}-(\rho_{11}-\rho_{22})\rho_{12}[I(t)-I_0]\Delta I/S_I, \tag{2}$$

where $\hbar=1$, $\Delta I=\mathrm{I}_1-I_2$, $I_0=(I_1+I_2)/2$, and $\gamma=\Gamma-(\Delta I)^2/4S_I$. The decoherence rate $\gamma=\gamma_d+\gamma_e$ of the single qubit is due to detector nonideality, $\gamma_d=(\eta^{-1}-1)(\Delta I)^2/4S_I$, and due to additional coupling with environment (γ_e). The current $I(t)=I_0+(\rho_{11}-\rho_{22})\Delta I/2+\xi(t)$ has the noise component $\xi(t)$ with the flat spectral density S_I. [Averaging over $\xi(t)$ would lead to the standard master equation with ensemble dephasing Γ.] Notice that in the case $\varepsilon=0$ (which is assumed unless mentioned otherwise), we can disregard the evolution of Re ρ_{12} (it becomes zero at $t\gg\Gamma^{-1}$), so only two degrees of freedom are left, which may be parameterized as $\rho_{11}-\rho_{22}=P\cos(\Omega t+\phi)$ and $2\,\mathrm{Im}\,\rho_{12}=P\sin(\Omega t+\phi)$, where the feedback-maintained frequency Ω (see below) is assumed to be equal (unless stated otherwise) to the bare Rabi frequency $\Omega_0=(4H_0^2+\varepsilon^2)^{1/2}$.

We assume that two quadrature components of the detector current (Fig. 1) are determined as

$$X(t)=\int_{-\infty}^{t}[I(t')-I_0]\cos(\Omega t')\mathrm{e}^{-(t-t')/\tau}\,\mathrm{d}t', \tag{3}$$

$$Y(t)=\int_{-\infty}^{t}[I(t')-I_0]\sin(\Omega t')\mathrm{e}^{-(t-t')/\tau}\,\mathrm{d}t', \tag{4}$$

where Ω is the local oscillator frequency applied to the mixer, and τ is the averaging time constant. Similar formulas are also applicable to the case of a tank circuit with the resonant frequency Ω and quality factor $Q=\Omega\tau/2$. If the detector current would be a harmonic signal $I(t)=I_0+P(\Delta I/2)\cos(\Omega t+\phi_0)$, then $\phi_0=-\mathrm{arc\,tan}(\langle Y\rangle/\langle X\rangle)$, so it is natural to use

$$\phi_m(t)\equiv-\arctan(Y/X) \tag{5}$$

as a monitored estimate of the phase shift $\phi(t)$ between the Rabi oscillations and the local oscillator ($\langle\ldots\rangle$ means averaging over time). Similar to Ref. [3] we consider the feedback loop, which aim is to suppress the fluctuations of the Rabi phase, so that the goal is $\phi(t)=0$ (or as small as possible). The linear feedback rule is assumed: $H_{fb}/H_0=-F\phi_m(t)$, where F is the dimensionless feedback factor (by definition $|\phi_m|\le\pi$).

We characterize the performance of the feedback loop by the synchronization degree $D=\langle P(t)\cos\phi(t)\rangle=2\langle\mathrm{Tr}\,\rho(t)\rho_d(t)\rangle-1$, where ρ_d corresponds to the desired perfect Rabi oscillations ($P_d=1$, $\phi_d=0$). The feedback operation is simulated numerically using Monte Carlo algorithm [4]. Fig. 2 shows the dependence of D on the feedback factor F for several time constants τ in the case of weak coupling $\mathcal{C}=0.1$ and $\gamma=0$ (we normalize F by $\mathcal{C}$, so the results practically do not depend on $\mathcal{C}$ for $\mathcal{C}\lesssim1$). Limiting ourselves to $\tau\sim S_I/(\Delta I)^2$ (excluding wide-bandwidth regime), we see that the maximum achievable synchronization degree D_{max} is about 90%. It is impossible to reach 100% because the monitored simple phase estimate ϕ_m is significantly different from the actual ϕ; however, the fidelity is still surprisingly large for such a simple feedback loop.

An important question is how the operation of the quantum feedback loop can be verified experimentally. One of the easiest ways is to check that the average value $\langle X\rangle$ of the in-phase quadrature component $X(t)$ becomes positive, while in absence of feedback ($F=0$) positive and negative values of X are obviously equally probable. Notice that *any* Hamiltonian control of a qubit which is not based on

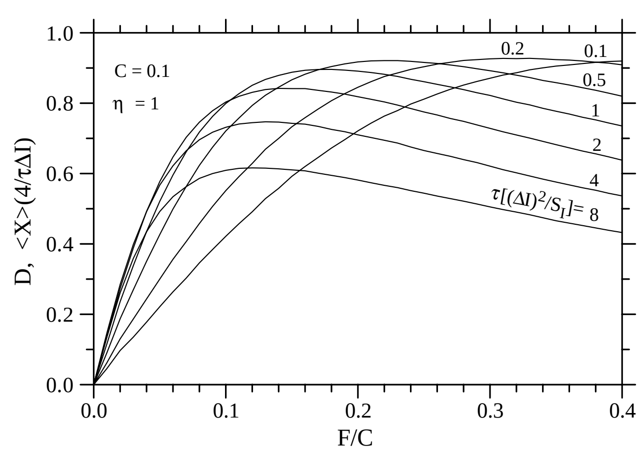

Fig. 2. Dependence of the synchronization degree D on the feedback factor F in ideal case ($\gamma=0$) for several τ. Experimentally D can be measured via average in-phase current quadrature $\langle X\rangle$.

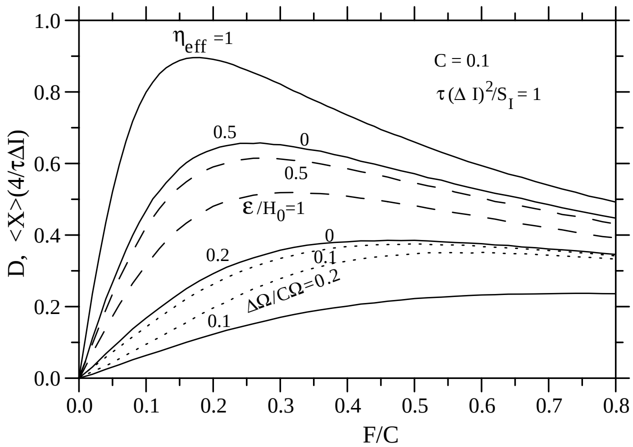

Fig. 3. Solid lines: synchronization degree D (and in-phase current quadrature $\langle X\rangle$) as functions of F for several values of the detection efficiency $\eta_{\rm eff}$. Dashed and dotted lines illustrate the effects of the energy mismatch ($\varepsilon \neq 0$) and the frequency mismatch ($\Omega \neq \Omega_0$).

the information obtained from the detector (i.e. feedback control) cannot provide nonzero $\langle X\rangle$. It is easy to show that $\langle X\rangle = [D + \langle P\cos(2\Omega t + \phi)\rangle]\tau\Delta I/4$, and since the second term in brackets vanishes at weak coupling (and $\varepsilon = 0$), therefore, $\langle X\rangle$ is directly related to D. The numerical results for $\langle X\rangle/(\tau\Delta I/4)$ practically coincide with the curves for D in Fig. 2 (within the thickness of the line).

The ideal case $\gamma = 0$ is obviously not realizable in an experiment because of finite nonideality of a detector ($\eta < 1$) and presence of an extra environment ($\gamma_e > 0$). Both effects can be taken into account simultaneously introducing effective efficiency of quantum detection $\eta_{\rm eff} = [\eta^{-1} + 4\gamma_e S_I/(\Delta I)^2]^{-1}$. Fig. 3 shows (solid lines) the feedback performance for several values of $\eta_{\rm eff}$ assuming $\tau(\Delta I)^2/S_I = 1$. One can see that $\eta_{\rm eff} \sim 0.1$ is still a sufficient value for a noticeable operation of the quantum feedback loop.

Finally, let us discuss how accurately the conditions $\Omega = \Omega_0$ and $\varepsilon = 0$ should be satisfied in an experiment. Dotted lines in Fig. 3 show the feedback operation for $\eta_{\rm eff} = 0.2$ and two values of $\Delta\Omega$, confirming still good operation at $|\Delta\Omega| = |\Omega - \Omega_0| \ll C\Omega \sim \Gamma \sim \tau^{-1}$. Energy mismatch ($\varepsilon \neq 0$) also worsens the performance of the feedback loop; however, the dashed lines in Fig. 3 ($\sim \eta_{\rm eff} = 0.5$) show that a relatively large mismatch ($\varepsilon \lesssim H_0$) can be tolerated.

In conclusion, we have shown that a quite simple feedback loop based on the monitoring of quadrature components of the detector current, can maintain coherent oscillations in a double-dot qubit with a surprisingly good fidelity. The work was supported by NSA and ARDA under ARO grant W911NF-04-1-0204.

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