Demand Side Management for Wind Power Integration in Microgrid Using Dynamic Potential Game Theory

Chenye Wu, *Student Member, IEEE,* Hamed Mohsenian-Rad, *Member, IEEE,*
Jianwei Huang, *Member, IEEE,* and Yuexuan Wang, *Member, IEEE*

Abstract—We propose a novel demand side management method to tackle the intermittency in wind power generation. Our focus is on an isolated microgrid with one wind turbine, one fast-responding conventional generator, and several users. Users act as independent decision makers in shaping their own load profiles. Using dynamic potential game theory, we analyze and coordinate the interactions among users to efficiently utilize the available renewable and conventional energy resources to minimize the total energy cost in the system. We further model the inter-temporal variations of the available wind power as a Markov chain based on field data. Using techniques from dynamic potential game theory, we first derive closed-form expressions for the best responses for the users that participate in demand side management. Then, we investigate the efficiency of the constructed game model at the equilibrium. Finally, the system performance is assessed using computer simulation. In particular, our proposed scheme saves 38% generation cost compared with the case without demand side management.

Index Terms—Smart Grid, Wind Power Integration, Markov Chain, Dynamic Potential Game Theory, Nash Equilibrium.

I. INTRODUCTION

Renewable energy sources, in particular wind power, are becoming significant power generation technologies around the world [1]. However, the intermittency and inherent stochastic nature of wind power becomes the major obstacle for reaching a large market penetration. One possible solution for this issue is to use fast-responding generators (such as natural gas units which are usually expensive and have high carbon footprints) to compensate the fluctuations of the wind turbines output. Alternatively, we can implement advanced demand side management (DSM) programs that adjust the controllable load to match the available power generated. Our focus in this paper is on this latter case, in particular, to study how to balance supply and demand in an isolated microgrid [2], which is an important concept for renewable energy integration. By studying the microgrid, which is a subsystem of the grid, we can better understand how to make use of current technology to achieve the highest renewable energy penetration.

C. Wu and Y. Wang are with the Institute for Interdisciplinary Information Sciences, Tsinghua University, Beijing, China. e-mails: wucy05@mails.thu.edu.cn, amywang@tsinghua.edu.cn. H. Mohsenian-Rad is with the Department of Electrical and Computer Engineering, Texas Tech University, Lubbock, TX, USA, e-mail: hamed.mohsenian-rad@ttu.edu. J. Huang is with the Department of Information Engineering, The Chinese University of Hong Kong, Shatin, Hong Kong. e-mail: jwhuang@ie.cuhk.edu.hk.

A. Related Work

The literature on integrating wind power into smart grid emerged only recently. In [3], Neely *et al.* used Lyapunov theory to obtain a *centralized* optimal queueing system for allocating renewable energy to delay tolerant consumers. In [4], Shimizu *et al.* designed a *centralized* load control system for integration of photovoltaic and wind power by utilizing electric cars' batteries. In [5], Liu designed a wait-and-see load dispatch for a hybrid system with thermal generators and wind turbines. In [6], He *et al.* proposed a multiple timescale dispatch for smart grid with integrated wind power. Different from the previous works, our focus is on applying game theory [7] to design a *decentralized* demand side management system, where users in an isolated microgrid are modeled as independent decision makers. The users share the cost of power generation, while each is interested in managing his own load to minimize his own energy expenses.

Game theory has already been applied to power networks and demand side management. In [8]–[10], Mohsenian-Rad *et al.* used game theory to address demand response management via price predication and optimal energy consumption scheduling. In [11], Wu *et al.* proposed a *static game* model to engage electric cars' batteries and a backup battery bank to integrate wind power into smart grid. In [12], Mohamed applied game theory to explore various management issues in a microgrid, such as finding the optimal operating strategy of various generators to minimize the operating costs together with the emission cost and level for a microgrid. In [13], Alibhai *et al.* used techniques from auction theory to coordinate the distributed energy resources to meet users' demands. Unlike [12] and [13], our approach addresses demand side management with focus on understanding the interactions among users (not sources), who share conventional and renewable generators. Different from [8]–[11] that adopted static game models, here we apply dynamic game models to take into account the decision dependencies over multiple periods of time.

B. Our Contributions

In this paper, we propose a decentralized demand side management program based on game theory solutions to integrate wind power and to minimize the total energy cost in an isolated microgrid. The contributions in this paper include:

- *Dynamic Potential Game Formulation:* Our game model captures the self interests of end users who participate in wind power integration over a substantial period of time.
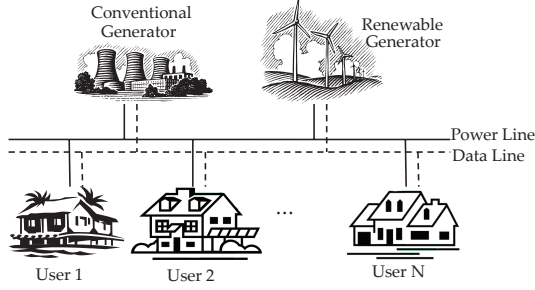
Fig. 1. An isolated microgrid with conventional and renewable generators.

- *Equilibrium Analysis:* We obtain the closed-form expressions to characterize the users' best strategies at the Nash equilibria of the formulated game model.
- *Simulation Studies*: Using field data for wind power generation, we run computer simulations to assess the performance for the proposed demand side management method. The results show that the generation cost is reduced by 38% compared to the benchmark method.

The rest of this paper is organized as follows. The system model is explained in Section II. The equilibrium analysis for single-user and multi-user cases are presented in Section III and Section IV, respectively. Simulation results are presented in Section V. The paper is concluded in Section VI.

II. SYSTEM MODEL

Consider an isolated microgrid as shown in Fig. 1, where a set $\mathcal{N} = \{1, \ldots, N\}$ of users share the energy generated by two types of generators. The first type is a conventional fast-responding fuel generator such as a gas or coal unit, and the other type is a wind turbine. We are interested in demand side management during time period $[1, H]$. The overall period is divided into H time slots, and the granularity depends on the frequency of measuring wind speed. For example, if measurements are made available every one hour, for the day-ahead planning, we have $H = 24$.

For each user $n \in \mathcal{N}$, let x_n^h denote user n's load during time slot h. Without loss of generality, we assume that each user has exactly one appliance with controllable/shiftable load, which needs to be satisfied within the the interval $[1, H]$. In particular, for each user $n \in \mathcal{N}$, the start and end time slots of the valid scheduling for the shiftable load are denoted by α_n and β_n, that is, a valid scheduling should make sure that the load is satisfied during this period. For example, for a dishwasher after lunch, we can set $\alpha_n = 1:00$ PM and $\beta_n = 5:00$ PM, such that the dishes are washed between lunch and dinner. Examples for such appliances also include washer, dryer, and electric vehicles [8]. In this regard, it is required that

$$\sum_{h=\alpha_n}^{\beta_n} x_n^h = E_n, \quad \forall n \in \mathcal{N}, \tag{1}$$

where E_n denotes the total energy consumption needed to finish the operation of user n's appliance. The energy profile

x_n^h outside the time frame $[\alpha_n, \beta_n]$ should be zero. That is,

$$\begin{cases} x_n^h \geq 0, & \alpha_n \leq h \leq \beta_n, \\ x_n^h = 0, & \text{otherwise}, \end{cases} \quad \forall n \in \mathcal{N}. \tag{2}$$

At a time slot h, the total load in the system is

$$l^h = \sum_{n \in \mathcal{N}} x_n^h. \tag{3}$$

Next, let v^h denote the power generation level of the wind turbine at time slot h. If the total load at each time slot matches the total renewable power generated at that time slot, then there will be no need to use the conventional generator. Otherwise, the conventional generator is used to compensate the mismatch between load and renewable power supply. At each time slot h, the total conventional power that needs to be generated is $l^h - v^h$. Assuming the conventional generator is a thermal generator, the generation cost at each time slot h can be approximated as a quadratic function as [14],

$$C(l^h - v^h) = k (l^h - v^h)^2. \tag{4}$$

For the renewable power generation, we assume that the generation cost is constant regardless of the exact amount of power generated [15]. That is, wind power generation only introduces a fixed so-called 'sunk cost' that does not affect decision making for demand side management. Thus, the model in (4) can be considered as the total cost of power generation in the microgrid to support the total load l^h at time slot h, as far as demand side management planning is concerned. From (4), any deviation (either positive or negative) of load l^h from renewable energy supply v^h is penalized. For the case when $l^h < v^h$, this is because the excessive power generation can degrade the power quality in the system with voltage frequency exceeding its nominal value [16]. Therefore, for the best operation of the microgrid with a minimum cost, the total load at each time slot should be kept as close to the total renewable power generated as possible.

A. Wind Power Prediction

The amount of power generated by a wind turbine follows a stochastic process due to the fluctuations in wind speed. A sample 10-day measurement of the hourly wind speed in West Texas is shown in Fig. 2(a). Given the wind speed, the generated wind power at each time instance is obtained based on the wind power versus wind speed curve in Fig. 2(b).

TABLE I
WIND POWER STATES.

State Index	Wind Power Range (kW)	Indicator of the State (kW)
State 1	0 - 30	13.02
State 2	30 - 60	42.37
State 3	60 - 90	73.88
State 4	90 - 120	104.13
State 5	120 - 150	133.91
State 6	150 - 180	165.77

To predict the wind power over time, we model it as a Markov chain [19]. Based on the measured data, we define
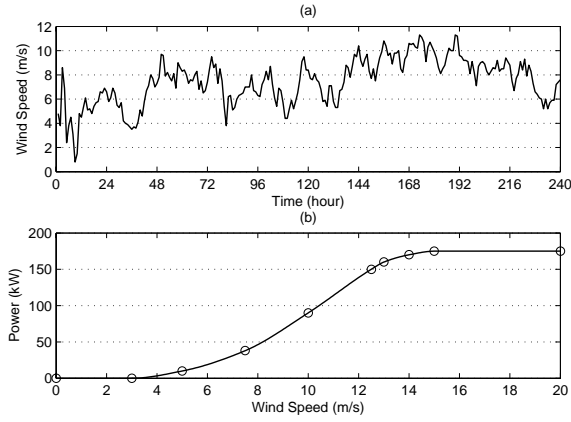
Fig. 2. An example trend for wind speed that we use for wind power prediction: (a) 10 days wind speed measurement in Crosbyton, TX in August 2009 [17]. (b) Power versus speed curve for a typical wind turbine. [18].

a Markov chain with six states. The first state indicates that the wind turbine output is between 0 to 30 kW. The rest of the states are defined similarly as in Table I. Based on hourly field data over a six-month period, we obtain the transition probabilities as in Table II. We can see that the transition probability matrix is sparse, as with a high probability the Markov chain will either stay at the current state or jump to an adjacent state. The transition probability matrix can help users to predict the available wind power at upcoming time slots for decision making for demand side management.

B. Cost Sharing

The total power generation cost in (4) is shared by users based on their portions of load. In this regard, each user n's electricity bill payment at each time slot h is calculated as

$$p_n(x_n^h, x_{-n}^h) = \frac{E_n}{\sum_{m \in \mathcal{N}} E_m} C\left(\sum_{n \in \mathcal{N}} x_n^h - v^h\right), \quad (5)$$

where x_{-n}^h denotes the load for all users other than user n. The cost sharing model in (5) is proportional to users' *total* load over *all* H time slots, not their instantaneous load at each time slot. Although the latter approach can more accurately incorporate fluctuations in users' load profiles, it makes our game-theoretic analysis more complicated. Therefore, in this paper, we only use (5) as the model for cost allocation.

In this setting, each user individually decides its energy consumption schedule at the beginning of each time slot. Such decision is made to maximize the user's own payoff

TABLE II
WIND POWER TRANSITION MATRIX.

	1	2	3	4	5	6
1	0.86	0.12	0.01	0.00	0.00	0.00
2	0.21	0.60	0.16	0.02	0.00	0.00
3	0.03	0.29	0.53	0.13	0.02	0.01
4	0.00	0.07	0.37	0.38	0.15	0.03
5	0.01	0.00	0.06	0.37	0.37	0.19
6	0.00	0.01	0.01	0.07	0.19	0.73

based on the prediction on wind power generation (using the Markov chain discussed in Section II-A). The actual electricity cost for each user at the end of each time slot, however, is calculated based on the true measured amount of wind power generated. For notional simplicity, we define $\mathbf{x_n}$ as the energy consumption profile vector for user n:

$$\mathbf{x_n} = [x_n^1, \cdots, x_n^H]. \quad (6)$$

The set of feasible energy consumption profile is defined as

$$\mathcal{X}_n = \left\{ \mathbf{x}_n \middle| \text{Constraints (1) and (2)} \right\}. \quad (7)$$

Each user aims to select $\mathbf{x}_n \in \mathcal{X}_n$ to minimize his own total payment $\sum_{h=1}^{H} p_n(x_n^h; x_{-n}^h)$ during the microgrid operation.

III. SINGLE USER ANALYSIS

To gain insights, we first consider a simple scenario with only one user. In this setting, the total cost of power generation is paid by that user only. We define user 1's payoff as

$$f_1(\mathbf{x_1}) = -\sum_{h=1}^{H} \mathbb{E}\{p_1(x_1^h)\} = -\sum_{h=1}^{H} \mathbb{E}\{k(x_1^h - v^h)^2\}. \quad (8)$$

As there is only one user, we do not study a game model here. However, it is still a challenging task for user 1 to select his energy consumption schedule $x_1^{\alpha_1}, \ldots, x_1^{\beta_1}$ to maximize his payoff. Recall that from (2), $x_1^h = 0$ for all $h < \alpha_1$ and all $h > \beta_1$. To maximize the payoff, user 1 can apply the *backward induction* technique. Starting from the last time slot, user 1 finds his optimal energy consumption schedule backward in time. For example, at the last time slot $h = \beta_1$, assuming that the optimal energy consumption schedules at the previous time slots (i.e., the sequence $x_1^{\alpha_1*}, \ldots, x_1^{\beta_1-1*}$) are known, the optimal energy consumption schedule $x_1^{\beta_1*}$ is obtained as

$$x_1^{\beta_1*} = E_1 - \sum_{i=1}^{\beta_1-1} x_1^{i*}. \quad (9)$$

Using backward induction, the following theorem provides the optimal energy consumption schedules in all time slots.

Theorem 1: Assume that E_1 is large enough such that

$$E_1 \geq \sum_{i=h+1}^{\beta_1} \mathbb{E}\{v^i - v^h | v^{h-1}\}, \quad \forall \alpha_1 \leq h \leq \beta_1. \quad (10)$$

User 1's payoff is maximized when we select:

$$x_1^{h*} = \frac{E_1 - \sum_{i=\alpha_1}^{h-1} x_1^{i*} - \sum_{i=h+1}^{\beta_1} \mathbb{E}\{v^i - v^h | v^{h-1}\}}{\beta_1 - h + 1}, \quad \alpha_1 \leq h \leq \beta_1. \quad (11)$$

The proof of Theorem 1 is given in Appendix A. At the beginning of time slot $h = \alpha_1$, user 1 calculates *the whole*

sequence of optimal energy consumption schedules as in (11); however, he only implements his schedule at that time slot:

$$x_1^{\alpha_1*} = \frac{E_1 - \sum_{i=2}^{\beta_1} \mathbb{E}\{v^i - v^1 | v^{\alpha_1 - 1}\}}{\beta_1 - \alpha_1 + 1}. \tag{12}$$

As time goes by, user 1 shall update his decisions, again using backward induction in (11), based on the *updated measurements* on the status of available wind power during the past time slots. Next, we study the case where the energy sources in the microgrid are shared by multiple users.

IV. MULTIPLE USERS ANALYSIS

For the general case when the microgrid includes several users, the payoff function for user $n \in \mathcal{N}$ is

$$f_n(\mathbf{x_n}, \mathbf{x_{-n}}) = \frac{-E_n}{\sum_{n \in \mathcal{N}} E_n} \sum_{h=1}^{H} \mathbb{E}\left\{ k \left(\sum_{n \in \mathcal{N}} x_n^h - v^h \right)^2 \right\}. \tag{13}$$

Each user's payoff function not only depends on his own strategy, but also depends on other users' strategies. This leads to a game theoretical model between users as below.

Energy Consumption Scheduling Game for Micro-grid:

- **Players**: Set $\mathcal{N}$ of all users.
- **Strategies**: For each user n, vector $\mathbf{x_n} \in \mathcal{X}_n$ as in (7).
- **Payoffs**: For each user n, $f_n(\mathbf{x}_n, \mathbf{x}_{-n})$ as in (13).

Next, we show the properties of the above *ECS-MG* game.

A. Potential Game

For a game with a set of players $\mathcal{N}$, feasible set of actions $\mathcal{X}_n$, and payoff functions $f_n(\mathbf{x}_n, \mathbf{x}_{-n})$, a function $\Phi : \mathcal{X}_n \to \mathbb{R}$ is called an *ordinal potential function* [20], if for each n and each $\mathbf{x_{-n}} = (\mathbf{x}_m \in \mathcal{X}_m, \ \forall m \in \mathcal{N} \backslash \{n\})$, the following holds:

$$\begin{aligned} f_n(\mathbf{y}, \mathbf{x}_{-n}) - f_n(\mathbf{z}, \mathbf{x}_{-n}) > 0 \Leftrightarrow \\ \Phi(\mathbf{y}, \mathbf{x}_{-n}) - \Phi(\mathbf{z}, \mathbf{x}_{-n}) > 0, \ \forall \mathbf{y}, \mathbf{z} \in \mathcal{X}_n. \end{aligned} \tag{14}$$

If a game has an ordinal potential function, it is called an ordinal potential game. It is easy to verify that

$$\Phi = \sum_{h=1}^{H} \mathbb{E}\left\{ k \left(\sum_{n \in \mathcal{N}} x_n^h - v^h \right)^2 \right\} \tag{15}$$

is an ordinal potential function for the *ECS-MG* game, *i.e.,* the *ECS-MG* game is an ordinal potential game. We first analyze a simple two-user two-slot case, and then characterize users' best responses and Nash equilibria for the general case.

B. The Two-User Two-Slot Scenario

Consider the case when $N = 2$ and $H = 2$. Also assume that $\alpha_1 = \alpha_2 = 1$ and $\beta_1 = \beta_2 = 2$. In this case, although the *ECS-MG* game has two stages, decision making can be reduced to only one stage since:

$$x_1^2 = E_1 - x_1^1 \quad \text{and} \quad x_2^2 = E_2 - x_2^1. \tag{16}$$

Thus, we can obtain user n's (n=1,2) best response as follows:

$$\begin{aligned} x_n^{1*}(x_{-n}^1) &= \operatorname*{argmax}_{x_n^1 \in \mathcal{X}_n} \ f_n(\mathbf{x_n}, \mathbf{x}_{-n}) \\ &= \frac{E_1 + E_2 + \mathbb{E}\{v^1 | v^0\} - \mathbb{E}\{v^2 | v^0\}}{2} - x_{-n}^1. \end{aligned}$$

Note that, given all other users' strategies $\mathbf{x_{-n}}$, there is a single best strategy for user n (n=1,2). Next, we will characterize the Nash equilibria based on the best strategies. We can prove the following theorem, which can directly obtained from [20].

Theorem 2: If the potential function Φ in (15) has a maximum value at $\mathbf{x_n^*} \in \mathcal{X}_n$, then $\mathbf{x_n^*}$ is a pure strategy Nash equilibrium for the *ECS-MG* game.

For the two-user two-slot case, we can verify that the Jacobian matrix $\boldsymbol{A}$ for Φ is negative definite as follows:

$$\boldsymbol{A} = \begin{pmatrix} \dfrac{\partial^2 \Phi}{\partial^2 x_1^1} & \dfrac{\partial^2 \Phi}{\partial x_1^1 \partial x_2^1} \\ \dfrac{\partial^2 \Phi}{\partial x_2^1 \partial x_1^1} & \dfrac{\partial^2 \Phi}{\partial^2 x_2^1} \end{pmatrix} = -4k \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}. \tag{17}$$

If $k > 0$, then $\boldsymbol{A}$ is negative definite and Φ is strictly diagonal and has a unique global maximum value. Thus, any pair (x_1^{1*}, x_2^{1*}) that satisfies the following equality

$$l^{1*} = x_1^{1*} + x_2^{1*} = \frac{E_1 + E_2 + \mathbb{E}\{v^1 | v^0\} - \mathbb{E}\{v^2 | v^0\}}{2} \tag{18}$$

is a Nash equilibrium for *ECS-MG* game. Note that, from (18) and (1) we also automatically have:

$$\begin{aligned} l^{2*} = x_1^{2*} + x_2^{2*} &= E_1 + E_2 - l^{1*} \\ &= \frac{E_1 + E_2 - \mathbb{E}\{v^1 | v^0\} + \mathbb{E}\{v^2 | v^0\}}{2}. \end{aligned} \tag{19}$$

Clearly, there are infinite pairs (x_1^{1*}, x_2^{1*}) that satisfy (18), indicating that the *ECS-MG* game has infinite number of Nash equilibria in this case. However, all these equilibria result in equal total energy costs. That is because only the total load, not the way distributed among users, determines the energy cost. As one option, users can choose their energy consumption proportionally, such that for each user n, we have

$$x_n^h = \frac{E_n}{\sum_{m \in \mathcal{N}} E_m} l^h, \ 1 \leq h \leq H, \tag{20}$$

which can easily be implemented in practice.

C. Generalization

We are now ready to investigate the *ECS-MG* game in a general case with an arbitrary number of users $N \geq 1$ and an arbitrary number of stages $H \geq 1$. Same as the two-user two-stage case, a Nash equilibrium is characterized only based on the total load, not the exact distribution of the load among users. Again, we will use the backward induction technique to obtained closed-form solutions for the Nash equilibria. To simply the analysis in this paper, we assume that:

Assumption 3: For each time slot h, there exists *at least one* end user n in the microgrid such that $\alpha_n \leq h \leq \beta_n$.

Using backward induction, we can show the following.

Theorem 4: If Assumption 3 holds and assume $E_1, \cdots, E_N$ are large enough such that (21) is always non-negative, then for the general case with $N \geq 1$ and $H \geq 1$, the Nash equilibrium for each sub game of the *ECS-MG* game at time slot $1 \leq h \leq H$, is constructed as

$$\sum_{n \in \mathcal{N}} x_n^{h*} = \frac{\sum_{n \in \mathcal{N}} \left(E_n - \sum_{i=1}^{h-1} x_n^{i*} \right) - \sum_{i=h+1}^{H} \mathbb{E}\{v^i - v^h | v^{h-1}\}}{H - h + 1}, \tag{21}$$

where

$$x_n^{*h} = 0, \quad \text{if } h < \alpha_n \text{ or } h > \beta_n. \tag{22}$$

The proof of Theorem 4 is similar to that of Theorem 1 and is omitted. Note that, although Theorem 4 provides the Nash equilibrium for *the whole rest* of the game as in (21), users only implement their energy consumption schedules only for the first time slot. As time goes by, the Nash equilibrium will be updated, using a similar backward induction approach, based on the update measurements on available wind power.

V. SIMULATION RESULTS

Consider a one-day (24 hours) energy consumption scheduling problem with $N = 50$ users. Simulation results on cost of conventional generation are shown in Fig. 3. Here, we compare three scenarios. First, the case when no demand side management program is implemented. In this case, each user n randomly schedule their energy consumption within interval $[\alpha_n, \beta_n]$. Second, the case where demand side management is done via the *ECS-MG* game in a decentralized fashion. In this case, the wind power prediction is done using the Markov Chain in Section II-A. Finally, the case where demand side management is done via the *ECS-MG* game in a decentralized fashion but with perfect information of the wind power generation in all future time slots. This last case provides an upper bound on the system performance, indicating the minimum cost of conventional power generation that can be reached. We can see that by implementing the proposed demand side management program, the cost of power generation reduces by 38% at the Nash equilibrium of the *ECS-MG*, compared to the case where no demand side management is implemented. Improving the wind power prediction accuracy can further reduce the total power generation cost by a maximum of 21%.

The results on the users' individual electricity bill payments are shown in Fig. 4. For the ease of presentation, the bill amount are shown for 10 users only. The results are similar for the rest of the users. Here, we can see that all users benefit from participating in the proposed demand side management program. Therefore, demand side management can not only help the whole system, but also help each individual user.

VI. CONCLUSIONS

In this paper, we consider an isolated microgrid consisting of N end users, who obtain energy from a renewable power generator and a backup conventional power plant. These N end users share the total energy cost of the conventional
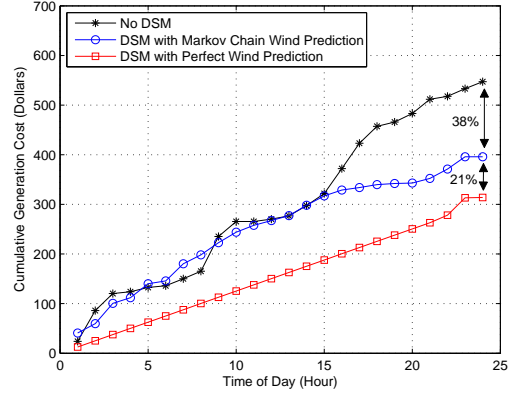

Fig. 3. The cost of conventional generation over 24 hours.
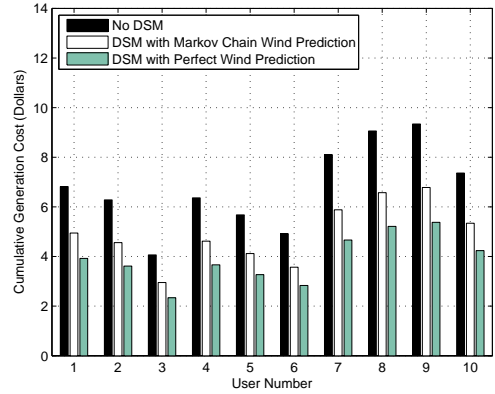

Fig. 4. The individual daily electricity bill for 10 users.

power plant. We design a cost allocation mechanism, which effectively minimize the total energy cost. We also study the Nash equilibrium of the dynamic potential game constructed among users by applying backward induction.

We plan to extend the results in several directions. For example, we would like to modify the game model to include uncertainty on other users' behaviors. This will lead to a more realistic formulation of *game with incomplete information*.

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APPENDIX

A. Proof for Theorem 1

Without loss of generality, we assume that $\alpha_1 = 1$ and $\beta_1 = H$. As stated in Theorem 1, we also assume that E_1 is large enough such that (11) is always non-negative. We now prove the result by induction.

Step 1: Consider the case of $H = 1$. From (1), the only feasible energy consumption schedule is $x_1^1 = E_1$. It is easy to verify that (11) leads to the same solution in this case.

Step 2: Assume that (11) holds for $H = 1, \cdots, M$, where $M \geq 1$. Next we will show that (11) also holds for $H = M+1$. We can divide the $M + 1$ time slots into two periods: the first slot and the rest M slots. For the optimal decisions during slots $\{2, \cdots, M + 1\}$, we can obtain the optimal schedule by using the optimal schedule for an M-slot decision problem (by replacing E_1 with $E_1 - x_1^1$ and each time slot i with $i + 1$).

More precisely, when $H = M$, we have

$$x_1^{'h} = \frac{E_1' - \sum_{i=1}^{h-1} x_1^{'i} - \sum_{i=h+1}^{M} \mathbb{E}\{v^{'i} - v^{'h}|v^{'h-1}\}}{M - h + 1}. \quad (23)$$

For $H = M + 1$, the last M-slot decision problem, we have

$$
\begin{aligned}
x_1^h = x_1^{'h-1} &= \frac{E_1' - \sum_{i=1}^{h-2} x_1^{'i} - \sum_{i=h}^{M} \mathbb{E}\{v^{'i} - v^{'h-1}|v^{'h-2}\}}{M - (h-1) + 1} \\
&= \frac{E_1 - x_1^1 - \sum_{i=1}^{h-2} x_1^{i+1} - \sum_{i=h+1}^{M+1} \mathbb{E}\{v^i - v^h|v^{h-1}\}}{(M+1) - h + 1} \\
&= \frac{E_1 - \sum_{i=1}^{h-1} x_1^i - \sum_{i=h+1}^{M+1} \mathbb{E}\{v^i - v^h|v^{h-1}\}}{(M+1) - h + 1}. \quad (24)
\end{aligned}
$$

Note that, the second line is because we have $x_1^h = x_1^{'h-1}$, $2 \leq h \leq M + 1$ and $v^h = v^{'h-1}$, $1 \leq h \leq M + 1$. For $h = 1$, let the first order derivative of (8) with respect to x_1^1 be zero so that we can obtain the optimal value of x_1^1. Note that using backward induction, $\forall\, h \geq 2$, x_1^h is a function of x_1^1. Thus, we have:

$$x_1^1 - \mathbb{E}\{v^1|v^0\} - \sum_{h=1}^{M+1} \left(x_1^h - \mathbb{E}\{v^h|v^0\}\right)\frac{\partial x_1^h}{\partial x_1^1} = 0. \quad (25)$$

To obtain the closed-form solution of x_1^1, we first need to show the following lemma.

Lemma 5: For $h = 2, \cdots, M + 1$,

$$\frac{\partial x_1^h}{\partial x_1^1} = -\frac{1}{M}. \quad (26)$$

Proof of Lemma 5: We prove this lemma also by induction.

Step 2.1: Since

$$x_1^2 = \frac{E_1 - x_1^1 - \sum_{i=3}^{M+1} \mathbb{E}\{v^i - v^2|v^1\}}{M}, \quad (27)$$

obviously, $\partial x_1^2/\partial x_1^1 = -1/M$.

Step 2.2: Assume for $h = 2, \cdots, m$, (26) holds. Then, for $h = m + 1$, we have

$$
\begin{aligned}
\frac{\partial x_1^{m+1}}{\partial x_1^1} &= -\frac{1}{M + 1 - m}\left(\sum_{i=1}^{m} \frac{\partial x_1^i}{\partial x_1^1}\right) \\
&= -\frac{1}{M + 1 - m}\left(1 - \frac{m-1}{M}\right) = -\frac{1}{M}. \quad (28)
\end{aligned}
$$

Note that in the second line $\frac{\partial x_1^1}{\partial x_1^1} = 1$. Thus, we complete the proof for Lemma 5. ■

Back to Step 2 of the proof for Theorem 1, based on Lemma 5 and the fact that $\sum_{h=2}^{M+1} x_1^h = E_1 - x_1^1$, we can obtain

$$x_1^1 = \frac{1}{M + 1}\left(E_1 - \sum_{i=1}^{M+1} \mathbb{E}\{v^i - v^h|v^0\}\right). \quad (29)$$

Thus, we complete the proof. ■