A Conceptual Analysis of Equilibrium Bidding Strategy in a Combined Oligopoly and Oligopsony Wholesale Electricity Market

Iman Taheri Emami, Ehsan Samani, *Student Member, IEEE,* Hossein Askarian Abyaneh, *Senior Member, IEEE,* Hamed Mohsenian-Rad, *Fellow, IEEE,* and Alireza Bakhshai, *Fellow, IEEE*

***Abstract*—This paper proposes a semi-analytical method to obtain the equilibrium bidding strategies for generation and demand units in a *combined* oligopoly and oligopsony wholesale electricity market. Such market structure is the outcome of the increasing deployment of demand response programs that facilitate *active participation of demand-side players* in the price-setting process. In this analysis, the concept of supply function equilibrium (SFE) is used to investigate the oligopolistic competition among generation units. The SFE model is extended and the demand function equilibrium (DFE) is obtained to study the oligopsonistic competition among demand units. The economic behavior of a market participant is formulated as a bi-level programming (BLP) problem. The *imperfect competition* among generation units, as well as among demand units, are modeled as a non-cooperative game. Next, a direct method is developed to calculate all candidate equilibriums of the market, and the locational marginal prices (LMPs) in terms of the bidding strategies of the market participants. The BLP problem is solved by obtaining the coordinated Pareto-dominant Nash equilibrium of the market participants' non-cooperative games. Finally, the proposed analysis is examined in case studies. Accordingly, we report *insightful observations* with respect to the impact of the changes in the new market structure, at firm-level and market-level, such as in terms of mitigating market power of generation units, the market clearing prices and quantities, surplus for generation units and demand units, and potential impact on market efficiency.**

***Index Terms*—Bi-level programming, combined oligopoly and oligopsony, market equilibrium, non-cooperative game theory, equilibrium bidding strategy, wholesale electricity market.**

NOMENCLATURE

b	Index for bus, b=1,...,N_b.
i	Index for demand, i=1,...,N_d.
j	Index for generation unit, j=1,...,N_g.
l	Index for transmission line, l=1,...,N_l.
m	Index for strategic player, m=1,...,M.
a_j, b_j, c_j	Coefficients of generation cost function.
α_i, β_i	Coefficients of benefit function.
k_j	Bidding strategy of generation unit j.
t_m	Type of market player m, $t_m \epsilon \mathbf{t}$.
κ_i	Bidding strategy of demand unit i.
p_m	Belief of player m about the opponents' types.
C_j	Cost function of generation unit j (\$/h).
Π_j	Bidding price of generation unit j (\$/MWh).
B_i	Benefit function of demand unit i (\$/h).
MC_j	Marginal cost of generation unit j (\$/MWh).
MR_i	Marginal benefit of demand unit i (\$/MWh).
Θ_i	Bidding price of demand unit i (\$/MWh).
P_{gj}	Generation level of generation unit j (MW).
P_{di}	Consumption level of demand unit i (MW).
$P_{gj,min}$	Minimum generation of generation unit j.
$P_{gj,max}$	Maximum generation of generation unit j.
$P_{di,min}$	Minimum consumption of demand unit i.
$P_{di,max}$	Maximum consumption of demand unit i.
f_{lmax}	Power flow limit on line l (MW).
$\mathbf{A}_{eq}$	N_b-dimensional column vector of ones.
$\mathbf{P}_g$	Bus generation vector; P_{gb} is its element.
$\mathbf{P}_d$	Bus consumption vector; P_{db} is its element.
$\mathbf{T}_m$	Types of player m; t_m is its element.
$\mathbf{T}_{-m}$	Types of player m's opponents; $\mathbf{t}_{-m} \epsilon \mathbf{T}_{-m}$.
$\mathbf{T}$	Space for all players' type; $\mathbf{T} = \mathbf{T}_1 \times ... \times \mathbf{T}_M$.
$\mathbf{K}_m$	Strategy space of player m.
$\mathbf{k}$	Strategy profile of generation units.
$\mathbf{t}$	An element of type space of market players.
$\mathbf{t}_{-m}$	Type of market player m's opponents, $\mathbf{t}_{-m} \epsilon \mathbf{t}$.
$\boldsymbol{\kappa}$	Strategy profile of demand units.

I. Taheri Emami and H. Askarian Abyaneh are with the Department of Electrical Engineering, Amirkabir University of Technology, Tehran, Iran.

E. Samani and H. Mohsenian-Rad are with the Department of Electrical and Computer Engineering, University of California, Riverside, CA, USA.

A. Bakhshai is with the Department of Electrical Engineering and Computer Engineering, Queen's University, Kingston, ON K7L 3N6, Canada.

The corresponding author is H. Askarian Abyaneh, e-mail: askarian@aut.ac.ir.

I. INTRODUCTION

LACK of demand-side participation in the wholesale electricity markets has traditionally given a dominant position to the generation units in their relationship to the consumers [1]. However, the circumstances are gradually changing in recent years due to the advent of smart grid technologies and the increasing penetration of demand response. Demand response programs allow the more active participation of demand-side players in the price-setting process [2]. Due to transmission congestion, neither the supply side nor the demand side of the emerging wholesale electricity market is not perfectly competitive, but it has a *combined* oligopoly and oligopsony structure [3]-[4]. The oligopolistic and oligopsonistic nature of the emerging wholesale electricity market enables *both* generation and demand units to profitably manipulate the market outcome through exercising market power, or in other words, through strategic bidding [5]. Accordingly, it is important to understand the strategic bidding behavior of market participants; where both oligopolists and oligopsonists seek to choose their bidding strategy so as to maximize their profits. Addressing this open problem is the focus of this paper.

A. Literature Review

The problem of identifying the optimal bidding strategy has been extensively studied in the electricity market literature. Traditionally, the focus has been on the oligopolistic behavior of generation units while consumers are considered as inelastic demands. Since the oligopolistic competition of generation units eventually leads to economic equilibrium, these studies often used multifarious equilibrium-based methods to determine the optimal bidding strategy [6]. The oligopolistic competition of generation units can be modeled as a non-cooperative game with complete information [7]-[10] or incomplete information [11]-[14]. The optimal bidding strategy of generation units is examined in the presence of transmission constraints in [7]-[9], and with consideration of contingency constraints in [10]. The optimal bidding strategy problem has been scrutinized from viewpoint of risk-neutral generation units in [11]-[13], and risk-averse generation units in [14].

Under the smart grid environment, the demand-side players can actively participate in the wholesale electricity market. In this context, the demand-side player can be modeled as elastic demand, or strategic demand. The papers [15]-[19] have studied the oligopolistic behavior of generation units in the presence of elastic demands. The effects of consumers' price elasticity on the equilibrium of electricity markets have been investigated in [15]-[17]; while the beneficial impact of demand shifting in mitigation of generation units' market power has been examined in [18]-[19].

However, the above studies inherently ignore the ability of the demand-side players to act strategically or exercise market power. While this has been acceptable in the past in the traditional setting of the wholesale electricity markets, it is no longer accurate due to the increasing deployment of demand response programs and accordingly the increasing ability of the demand entities to act strategically when they participate in the wholesale electricity markets. A few papers have recently studied the bidding strategy problem from a strategic demand's perspective [20]-[27]. The optimal bidding strategy of a large consumer is calculated in [20]-[25]. The cooperative bidding strategies of buyers who collectively act as a super-player are determined in [26]. The studies in [20]-[26] are concerned with market scenarios where there is *one* dominant demand, who is a price-maker, and many small demands, who are price-takers. This approach cannot describe the strategic behaviors of market participants in an emerging environment where *multiple* demand-side players may act as price-makers. To carry out a more realistic analysis, in [27], a supply function equilibrium (SFE) model is proposed that considers multiple strategic players in both supply and demand sides. Note that, the SFE model was originally developed to study bidding behaviors of producers in the oligopoly wholesale electricity markets [28]. Accordingly, the SFE model needs to be extended to deal with the strategic bidding of market participants in a combined oligopoly and oligopsony wholesale electricity market.

B. Summary of Contributions

The contributions in this paper are summarized as follows:

- This paper investigates the optimal bidding strategy problem in the context of a combined oligopoly and oligopsony wholesale electricity market, i.e., when we *simultaneously* have an oligopoly on the supply side and an oligopsony on the demand side. The problem is formulated as a bi-level program (BLP). The profit of the market participant is maximized in the upper level (UL) while the market clearing problem is embedded in the lower level (LL). To determine the optimal solution of the BLP of a market participant at market equilibrium, the strategic bidding of all market participants need to be optimized jointly. A SFE model is employed to analyze the oligopolistic bidding behaviors of generation units. The demand function equilibrium (DFE) model is developed to cope with the oligopsonistic competition of demand units. The strategic interactions among generation units, as well as among demand units, are modeled as a non-cooperative game. Then, these two games are coordinated to reach a single equilibrium point that simultaneously solves the BLPs of all market participants.
- A semi-analytical two-phase solution method is proposed to solve the equilibrium problem. First, parametric local solutions of the LL problem are determined in terms of market participants' bidding strategies. Then, by substituting parametric local solutions into the equilibrium problem, it is transformed into a series of tractable equilibrium sub-problems with strict inequality constraints (EPSICs). The Pareto-dominant equilibrium represents a global solution for the original equilibrium problem.
- We then assess the impact of the paradigm shift in the electricity market structure from oligopoly-perfect competitive to oligopoly-oligopsony on the bidding behavior of market players and the market efficiency. Some of the *insightful observations* include: ***First***, the oligopoly-oligopsony duality mitigates the market power of generation units and decreases the bid price of demand units. ***Second***, this paradigm shift leads to a decrease in both equilibrium price and quantity. ***Third***, the dual figure of oligopolistic-oligopsonistic market tends to increase the consumer surplus (CS) and decrease the producer surplus (PS). ***Fourth***, the imperfect competition in both sides of the market has a detrimental impact on the market efficiency. The above observations can help market players to comprehend market behavior under a combined oligopoly and oligopsony structure. Moreover, they may help ISOs to better set forth their market policies in the future.

II. Electricity Market Model

In this section, the clearing process of an hour-ahead double-sided wholesale electricity market is formulated.

A. Generation Units and Demand Units

Suppose that, the generation cost of generation unit j can be represented by a quadratic function as follows:

$$C_j(P_{gj}) = \frac{1}{2}a_j {P_{gj}}^2 + b_j P_{gj} + c_j. \tag{1}$$

The marginal cost of generation unit j is:

$$MC_j(P_{gj}) = a_j P_{gj} + b_j. \tag{2}$$

The bid function of generation unit j is as follows:

$$\Pi_j = k_j MC_j(P_{gj}), \tag{3}$$

where k_j is equal to 1 for price takers. Additionally, the SFE model is extended and the DFE model is developed to cope with the oligopsonistic bidding behaviors of demand units. The benefit functions of demand units have to satisfy the following three properties [29]:

Property 1: The marginal benefit is non-increasing,

$$\frac{\partial^2 B_i}{\partial {P_{di}}^2} \leq 0. \tag{4}$$

Property 2: The marginal benefit is non-negative. That is,

$$\frac{\partial B_i}{\partial P_{di}} \geq 0. \tag{5}$$

Property 3: For zero consumption of power, the benefit function is equal to zero.

Suppose that, the benefit function of demand unit i is:

$$B_i(P_{di}) = -\frac{1}{2}\alpha_i {P_{di}}^2 + \beta_i P_{di}. \tag{6}$$

The marginal benefit of demand unit i is:

$$MR_i(P_{di}) = -\alpha_i P_{di} + \beta_i. \tag{7}$$

In the DFE model, the bid function of i can be constructed by parameterizing its marginal benefit. There are four possible types of parameterization:

1) $\alpha - parameterization$: the demand unit prepares its demand function by adjusting the slope of its marginal benefit while holding the intercept constant.
2) $\beta - parameterization$: the demand unit prepares its demand function by adjusting the intercept of its marginal benefit while holding the slope constant.
3) $\kappa - parameterization$: the demand unit prepares its demand function by multiplying its marginal benefit by a non-negative constant, say κ_i.
4) $(\alpha - \beta) - parameterization$: the demand unit prepares its demand function by adjusting both the slope and intercept of its marginal benefit.

Here, $\kappa - parameterization$ method is used to model the strategic bidding behaviors of demand units. Accordingly, the bid function of demand unit i is as follows:

$$\Theta_i = \kappa_i MR_i(P_{di}), \tag{8}$$

where κ_i is equal to 1 in a perfect competitive market. In $\kappa - parameterization$ method, demand unit i can exercise market power either by a physical withholding strategy or by a low bid-price strategy. These strategies are illustrated in Fig. 1. As can be seen in Fig. 1(a), unit i manipulates the market by reducing its demand from the competitive level. In Fig. 1(a), π^{comp}-π^{imp} and $d^{comp}(\pi^{imp})$-d^{imp} represent the value of price distortion and the value of withheld capacity by unit i, respectively. On the other hand, as can be seen in Fig. 1(b), unit i distorts the market price by bidding a lower price than the competitive level. In Fig 1(b), π^{comp}-π^{imp} and $\pi^{comp}(d^{imp})$-π^{imp} represent the value of price distortion and the value of economic withholding by unit i.

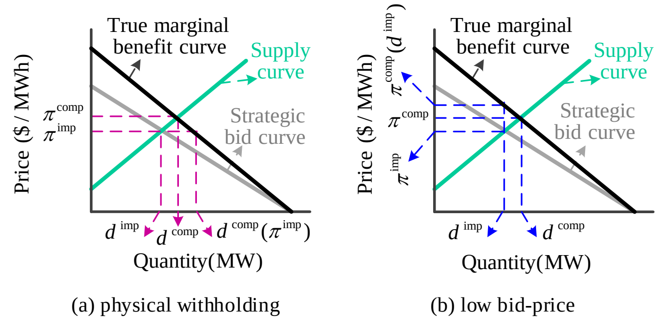


Fig. 1. Illustration of market manipulation strategies.

B. Market Clearing Mechanism

In the double-sided electricity market, generation and demand units submit their bids to the ISO which in turn, the ISO clears the market based on a merit order mechanism so that the social welfare (SW) is maximized [30]. The market clearing problem can be formulated as following bid-based transmission constrained economic dispatch (TCED):

$$\min_{P_{gj}, P_{di}} \quad \sum_{j=1}^{N_g} \Pi_j P_{gj} - \sum_{i=1}^{N_d} \Theta_i P_{di} \tag{9a}$$

$$\text{s.t.} \quad \sum_{i=1}^{N_d} P_{di} - \sum_{j=1}^{N_g} P_{gj} = 0 \tag{9b}$$

$$P_{di,min} \leq P_{di} \leq P_{di,max}; \; \forall i \tag{9c}$$

$$P_{gj,min} \leq P_{gj} \leq P_{gj,max}; \; \forall j \tag{9d}$$

$$\sum_{b=1}^{N_b} T_{lb}(P_{gb} - P_{db}) \geq -f_{lmax}; \; \forall l \tag{9e}$$

$$\sum_{b=1}^{N_b} T_{lb}(P_{gb} - P_{db}) \leq f_{lmax}; \; \forall l. \tag{9f}$$

The objective function (9a) minimizes the negative of the quasi-social welfare (QSW) of the market [31]. Equation (9b) reflects the power balance constraint. Constraints (9c)-(9f) represent generation capacity limits of generation units, consumption limits of demand units, and flow constraints of transmission lines, respectively. Moreover, T_{lb} as the shift factor, represents the sensitivity of the flow on line l to a change in the nodal injection at bus b and the withdrawal of equal power at the reference bus [32]. It is worth mentioning that the above model in (1)-(9) is more or less similar to the existing ISO electricity markets, such as in California ISO energy market [33].

III. Problem Statement

In a combined oligopoly and oligopsony market structure, market participants seek to choose the optimal bidding strategy to gain maximum profit. Here, the optimal bidding strategy problem of a given market participant is investigated within the paradigm of market economic equilibrium. In the following, game theory is employed to find equilibrium bidding strategy in the presence of complete and incomplete information.

A. Equilibrium Bidding Strategy in the Presence of complete Information

The profit of generation unit j can be defined by [34]:

$$u_j(P_{gj}, LMP_j) = P_{gj}LMP_j - C_j(P_{gj}). \quad (10)$$

Also, the profit of demand unit i is:

$$u_i(P_{di}, LMP_i) = B_i(P_{di}) - P_{di}LMP_i. \quad (11)$$

Accordingly, the optimal bidding strategy problem of generation unit j can be modeled as the following BLP problem:

$$\begin{aligned} &\max_{k_j} \quad u_j(P_{gj}, LMP_j) \\ &\text{s.t. bid-based TCED problem, i.e. (9).} \end{aligned} \quad (12)$$

The optimal bidding strategy problem of demand unit i is:

$$\begin{aligned} &\max_{\kappa_i} \quad u_i(P_{di}, LMP_i) \\ &\text{s.t. bid-based TCED problem, i.e. (9).} \end{aligned} \quad (13)$$

Regarding (12)-(13), the LL of the BLP problem of market participant m is parameterized by decisions of all market participants. Note that, the opponents of m try to maximize their profits. Since the conflicts of interest exist among market participants, m needs to maximize its profit at market equilibrium, which results in an equilibrium problem. In this regard, the oligopolistic competition of generation units, as well as oligopsonistic competition of demand units, is considered as a non-cooperative game with complete information. The equilibrium of generation units' oligopolistic competition and demand units' oligopsonistic competition can be determined by calculating Nash equilibrium (NE) of these two non-cooperative games. To achieve this purpose, the BLP problem of generation and demand units should be maximized simultaneously which are linked through the LL problem. The LL problem coordinates non-cooperative games of generation and demand units to reach a single equilibrium point. However, the coordinated NE bidding strategy of market participant m can be determined by solving the following equilibrium problem:

$$\begin{aligned} &\begin{cases} \max\limits_{k_1} \quad u_1(P_{g1}, LMP_1) \\ \vdots \\ \max\limits_{\kappa_M} \quad u_M(P_{dM}, LMP_M) \end{cases} \\ &\text{s.t. bid-based TCED problem, i.e. (9).} \end{aligned} \quad (14)$$

The global solution of (14) represents the NE bidding strategy of market participant m.

B. Equilibrium Bidding Strategy in the Presence of Incomplete Information

In the previous subsection, we assumed that each player has the full knowledge of its rivals' profit functions. In reality, market players lack such information about their opponents. Therefore, in this subsection, we expand our model and consider the oligopolistic competition of generation units, as well as the oligopsonistic competition of demand units, also as a non-cooperative game with *incomplete information*. An M-player Bayesian game can be denoted in the normal-form representation by G={$\mathbf{K}_1$,...,$\mathbf{K}_M$; $\mathbf{T}_1$,...,$\mathbf{T}_M$; p_1,...,p_M; u_1,...,u_M}, where we need to address two subjects:

1) Constructing the Types of Market Players: The types of generation unit j, $\mathbf{T}_j$, are defined based on its cost function. The cost function of unit j varies over time according to the change in the fuel prices. For the sake of simplicity, let us assume the fuel price is the only factor that change the cost function of unit j. Thus, the types of unit j can be constructed by estimating the cost coefficients of unit j over time. For estimating the cost functions of generation units based on observed bid data, readers are referred to [35]. Analogously, the types of demand unit i, $\mathbf{T}_i$, can be defined based on its benefit function.

2) Constructing the Beliefs of Market Players: The belief of player m, $p_m(\mathbf{t}_{-m}|t_m)$, describes the uncertainty of player m about the types of rival players, $\mathbf{t}_{-m}$, when player m has type t_m. Under Harsanyi's model, the posterior belief of player m about the types of other players can be extracted from a common prior using Bayes' rule as:

$$p_m(\mathbf{t}_{-m}|t_m) = \frac{p(\mathbf{t}_{-m}, t_m)}{\sum\limits_{\mathbf{T}_{-m}} p(\mathbf{t}_{-m}, t_m)}, \quad (15)$$

where $p(\mathbf{t}_{-m}, t_m)$ represents the joint probability distribution of t_m and $\mathbf{t}_{-m}$. In other words, $p(\mathbf{t}_{-m}, t_m)$ signifies the probability that player m is type t_m, and its opponents are types $\mathbf{t}_{-m}$. Player m infers $p(\mathbf{t}_{-m}, t_m)$ from the publicly available data as $p(\mathbf{t}_{-m}, t_m) = p(t_1)...p(t_m)...p(t_M)$, where $p(t_m)$ indicates the prior probability distribution of t_m. Bayesian games assume that the beliefs of players about the types of other players are mutually consistent, in the sense that they are derived from a common prior. Under common prior assumption (CPA), each player knows the beliefs of opponents about its type. It is worth mentioning that the private information of market players in a Bayesian game are included in the description of the type. The analysis in this paper is restricted to Bayesian games with a common prior. Abandoning the CPA, Harsanyi's model faces the issue of infinite hierarchy of beliefs. For representing Bayesian games without a common prior, readers are referred to [36-37].

The solution of an M-player Bayesian game is the Bayesian Nash equilibrium (BNE). Under Harsanyi's model, the BNE simultaneously maximizes the expected profit of all market participants, where the expected profit of generation unit m is [38]:

$$eu_m = \sum_{\mathbf{T}_m}\sum_{\mathbf{T}_{-m}} u_m(P_{gm}(\mathbf{t}), LMP_m(\mathbf{t}))p_m(\mathbf{t}_{-m}|t_m), \quad (16)$$

and the expected profit of demand unit m is:

$$eu_m = \sum_{\mathbf{T}_m}\sum_{\mathbf{T}_{-m}} u_m(P_{dm}(\mathbf{t}), LMP_m(\mathbf{t}))p_m(\mathbf{t}_{-m}|t_m), \quad (17)$$

and $P_{gm}(\mathbf{t})$, $P_{dm}(\mathbf{t})$, and $LMP_m(\mathbf{t})$ represent generation level of unit m, consumption level of unit m, and LMP at bus m when the type is $\mathbf{t}$.

IV. Solution Technique

In this section, a semi-analytical two-phase solution method is devised to solve the equilibrium problem. The idea is based on the fact that the equilibrium problem can be transformed into a series of EPSIC sub-problems, each sub-problem corresponds to a possible solution of the TCED problem. In the first phase, we determine all possible solutions of the TCED problem. In this regard, the dimension of the parametric TCED problem is reduced firstly by identifying and eliminating the redundant transmission constraints. Then, the candidate solutions of the reduced parametric TCED problem are determined in terms of bidding strategies of market participants. In the second phase, by substituting parametric local solutions in the LL, the equilibrium problem transformed into a series of EPSICs, which are now formulated in terms of the bidding strategies of market participants. Note that, the optimal solution of each EPSIC represents a local NE for the original equilibrium problem. These EPSICs are solved and Pareto dominant NE is designated as a global solution for the original problem. The detailed explanations about the proposed solution method have been provided in the following.

A. Parametric Solutions of the Bid-based TCED

1) Identifying the Redundant Transmission Constraints: Constraints of the bid-based TCED program are considered as the following linear system S:

$$\begin{cases} \text{Constraints} \quad (9b),\ (9c),\ (9d) \\ \mathbf{A}_{ineq,T}[\mathbf{P}_g - \mathbf{P}_d] \leq \mathbf{B}_{ineq,T}, \end{cases} \tag{18}$$

where $\mathbf{A}_{ineq,T}$ is a $2N_l \times N$ matrix. Moreover, $\mathbf{B}_{ineq,T}$ is a $2N_l$-dimensional column vector. The following transmission constraint:

$$\mathbf{A}^{\{n\}}_{ineq,T}[\mathbf{P}_g - \mathbf{P}_d] \leq \mathbf{B}^{\{n\}}_{ineq,T}, \tag{19}$$

is defined as redundant to the system S if and only if there is no vector $\mathbf{P}=\begin{bmatrix}\mathbf{P}_g{}^T & \mathbf{P}_d{}^T\end{bmatrix}^T$ such that [39]:

$$\begin{cases} \text{Constraints} \quad (9b),\ (9c),\ (9d) \\ \mathbf{A}^{-\{n\}}_{ineq,T}[\mathbf{P}_g - \mathbf{P}_d] \leq \mathbf{B}^{-\{n\}}_{ineq,T} \\ \mathbf{A}^{\{n\}}_{ineq,T}[\mathbf{P}_g - \mathbf{P}_d] \geq \mathbf{B}^{\{n\}}_{ineq,T}, \end{cases} \tag{20}$$

where $\mathbf{A}^{\{n\}}_{ineq,T}$ and $\mathbf{B}^{\{n\}}_{ineq,T}$ are the n^{th} row of $\mathbf{A}_{ineq,T}$ and $\mathbf{B}_{ineq,T}$. Also, matrices $\mathbf{A}^{-\{n\}}_{ineq,T}$ and $\mathbf{B}^{-\{n\}}_{ineq,T}$ are obtained by removing the n^{th} row of $\mathbf{A}_{ineq,T}$ and $\mathbf{B}_{ineq,T}$. Consider the following linear programming (LP) problem:

$$\begin{aligned} \max_{\mathbf{P}} \quad & z_n = \mathbf{A}^{\{n\}}_{ineq,T}[\mathbf{P}_g - \mathbf{P}_d] \\ \text{s.t.} \quad & S^{-\{n\}}, \end{aligned} \tag{21}$$

where $S^{-\{n\}}$ is obtained by removing the n^{th} constraint from the system S. From Theorem 2 in [39], the n^{th} constraint in (20) is redundant if and only if:

$$z^*_n < \mathbf{B}^{\{n\}}_{ineq,T}, \tag{22}$$

where z^*_n is the optimal solution of the LP problem in (21). Accordingly, the redundant transmission constraints of the TCED can be identified by solving a series of LPs. For alternative transmission constraints elimination methods, readers are referred to [40-41].

2) Implementing the Active Set Strategy: If the bidding strategy of market participants treated as parameters, the reduced bid-based TCED can be formulated as the following parametric quadratic programming (PQP) problem:

$$\min_{\mathbf{P}_g,\mathbf{P}_d} \quad \frac{1}{2}\mathbf{P}_g^{\mathrm{T}}\mathbf{H}_g\mathbf{P}_g + \mathbf{b}_g^{\mathrm{T}}\mathbf{P}_g + \frac{1}{2}\mathbf{P}_d^{\mathrm{T}}\mathbf{H}_d\mathbf{P}_d - \mathbf{b}_d^{\mathrm{T}}\mathbf{P}_d \tag{23a}$$

$$\text{s.t.} \quad \mathbf{A}_{eq}^T[\mathbf{P}_d - \mathbf{P}_g] = 0 \ : \lambda, \tag{23b}$$

$$\mathbf{A}_{red}[\mathbf{P}_g - \mathbf{P}_d] \leq \mathbf{B}_{red} \ : \boldsymbol{\mu}_{red}, \tag{23c}$$

where

$\mathbf{H}_g = diag(2k_b a_b);\ \mathbf{H}_d = diag(2\kappa_b \alpha_b);\ \forall b,$

where $\mathbf{b}_g$ and $\mathbf{b}_d$ are N_b-dimensional column vectors with $k_b b_b$ and $\kappa_b \beta_b$ as an element, respectively. The inequality constraints of reduced bid-based TCED are generalized in (23c). The active set strategy is employed to solve the PQP problem. This method splits the PQP problem into PQP sub-problems with equality constraints (PQPEC), each sub-problem associated with a particular combination of active constraints [42]. The optimal solution of a PQPEC represents a local solution for the PQP problem. By repeating this procedure for the remained combinations of active constraints, all parameterized local solutions of the PQP problem can be determined.

Consider the following PQPEC corresponding to a given active set AS:

$$\min_{\mathbf{P}} \quad \boldsymbol{\Gamma} = \frac{1}{2}\mathbf{P}^T\boldsymbol{\phi}_{11}\mathbf{P} - \mathbf{P}^T\boldsymbol{\phi}_{12} \tag{24a}$$

$$\text{s.t.} \quad \boldsymbol{\phi}_{21}\mathbf{P} = \boldsymbol{\phi}_{22} \ : \boldsymbol{\mu}_{eq}, \tag{24b}$$

where

$\boldsymbol{\phi}_{11} = \begin{bmatrix}\mathbf{H}_g & \mathbf{0} \\ \mathbf{0} & \mathbf{H}_d\end{bmatrix}, \boldsymbol{\phi}_{12} = \begin{bmatrix}-\mathbf{b}_g \\ \mathbf{b}_d\end{bmatrix}, \boldsymbol{\phi}_{21} = \begin{bmatrix}-\mathbf{A}_{eq}^T & \mathbf{A}_{eq}^T \\ \mathbf{A}_{AS} & -\mathbf{A}_{AS}\end{bmatrix}, \boldsymbol{\phi}_{22} = \begin{bmatrix}0 \\ \mathbf{B}_{AS}\end{bmatrix},$

where $\mathbf{A}_{AS}$ and $\mathbf{B}_{AS}$ are obtained by eliminating the rows of $\mathbf{A}_{red}$ and $\mathbf{B}_{red}$ which are associated with inactive constraints AS, respectively. The KKT optimality conditions for the PQPEC problem are:

$$\begin{bmatrix}\boldsymbol{\phi}_{11} & \boldsymbol{\phi}_{21}^T \\ \boldsymbol{\phi}_{21} & \mathbf{0}\end{bmatrix}\begin{bmatrix}\mathbf{P}^* \\ \boldsymbol{\mu}^*_{eq}\end{bmatrix} = \begin{bmatrix}\boldsymbol{\phi}_{12} \\ \boldsymbol{\phi}_{22}\end{bmatrix}. \tag{25}$$

Definition 1: The coefficient matrix and the matrix $\boldsymbol{\Psi}^T\boldsymbol{\phi}_{11}\boldsymbol{\Psi}$ are called the KKT matrix and the reduced Hessian. Moreover, $\boldsymbol{\Psi}$ is a matrix that form a basis for the null space of $\boldsymbol{\phi}_{11}$.

Theorem 1: Assume that the KKT matrix has full row rank. Then, the parametric system of the KKT conditions in (25) has a unique solution.

Proof: The parametric system of KKT conditions has a unique solution if the KKT matrix is non-singular. The KKT matrix is non-singular if the reduced Hessian is positive definite. The reduced Hessian is positive definite if and only if the value of $\boldsymbol{\Psi}^T\boldsymbol{\phi}_{11}\boldsymbol{\Psi}$ is positive for every non-zero column vector of $\boldsymbol{\Psi}$. For every real vector $\boldsymbol{\Psi}$ we have:

$$\boldsymbol{\Psi}^T\boldsymbol{\phi}_{11}\boldsymbol{\Psi} = \sum_j 2a_j k_j \Psi_j^2 + \sum_i 2\alpha_i \kappa_i \Psi_i^2, \tag{26}$$

which is always positive. Consequently, the parametric system of the KKT conditions in (25) has a unique solution.

Therefore, according to Theorem 1, it can be concluded that $\mathbf{P}^*$ is the unique global solution of the PQPEC in (24). It should be noted that $\mathbf{P}^*$ and $\boldsymbol{\mu}_{eq}^*$ are the explicit functions of the generation and demand units' bidding strategies. The candidate solution of the PQPEC is a feasible solution to the original PQP problem in (23) if satisfies the following strict inequality constraints: $\mathbf{A}_{IAS}[\mathbf{P}_g^* - \mathbf{P}_d^*] - \mathbf{B}_{IAS} < \mathbf{0}$ and $\boldsymbol{\mu}_{AS}^* > \mathbf{0}$, where $\mathbf{A}_{IAS}$ and $\mathbf{B}_{IAS}$ are obtained by eliminating the rows of $\mathbf{A}_{red}$ and $\mathbf{B}_{red}$ which are associated with active constraints AS, respectively. Moreover, $\boldsymbol{\mu}_{AS}$ is the vector of Lagrangian multipliers for active inequality constraints AS. Additionally, for a given active set AS, LMP at bus b can be determined in terms of bidding strategy of market players, as follows [30]:

$$LMP_b = \lambda^* - \sum_{l=1}^{N_l} \mu_l^* T_{lb} + \sum_{l=N_l+1}^{2N_l} \mu_l^* T_{lb}. \tag{27}$$

where μ_l^* is an element of $\boldsymbol{\mu}_{AS}^*$.

B. Solving the Equilibrium Problem in the Presence of Complete Information

The equilibrium problem in (14) can be transformed into a series of EPSICs, each EPSIC corresponds to a possible solution of the TCED problem. By substituting a given candidate solution of the parametric TCED ($\mathbf{P}^*$, $\boldsymbol{\mu}_{eq}^*$) in the equilibrium problem, we have the following EPSIC:

$$\begin{cases} \max\limits_{k_1} \quad u_1(P_{g1}, LMP_1) \\ \vdots \\ \max\limits_{\kappa_M} \quad u_M(P_{dM}, LMP_{dM}) \end{cases} \tag{28}$$
$$\text{s.t.} \quad \begin{array}{c} \mathbf{A}_{IAS}[\mathbf{P}_g^* - \mathbf{P}_d^*] - \mathbf{B}_{IAS} < \mathbf{0} \\ \boldsymbol{\mu}_{AS}^* > \mathbf{0}. \end{array}$$

In (28), the profit of market players and strict inequality constraints are the explicit function of market participants' bidding strategies. By differentiating the profit functions with respect to the bidding strategies of market participants, we get a system of nonlinear algebraic equations with strict inequalities. The system of nonlinear algebraic equations can be solved by the trust region Newton method. The detail of the trust region Newton method is given in [43]. It should be noted that the obtained solution is feasible if satisfies strict inequality constraints. However, the optimal solution of each EPSIC represents a coordinated local NE for the original equilibrium problem. To find all coordinated local NEs, the optimal solution of other EPSICs are determined. The candidate NEs are compared and the Pareto optimal NE is designated as a global solution for the original equilibrium problem [44]. It should be noted that the game may have a Pareto-optimal pure-strategy NE, several non-dominated pure-strategy NEs, or none at all.

C. Solving the Equilibrium Problem in the Presence of Incomplete Information

The proposed semi-analytical method in the previous subsections is employed to solve the equilibrium bidding strategy problem in the presence of incomplete information. In this regard, the coordinated BNE of incomplete-information non-cooperative games of generation and demand units are determined for all possible combinations of the active constraints of LL problem. For a given active set AS, the coordinated BNE solves the following problem:

$$\begin{cases} \max\limits_{k_1} \quad eu_1(P_{g1}, LMP_1) \\ \vdots \\ \max\limits_{\kappa_M} \quad eu_M(P_{dM}, LMP_M) \end{cases} \tag{29}$$
$$\text{s.t.} \quad \begin{array}{c} \mathbf{A}_{IAS}[\mathbf{P}_g^* - \mathbf{P}_d^*] - \mathbf{B}_{IAS} < \mathbf{0} \\ \boldsymbol{\mu}_{AS}^* > \mathbf{0}. \end{array}$$

where $\mathbf{P}_g^*$, $\mathbf{P}_d^*$, and $\boldsymbol{\mu}_{AS}^*$ are the functions of $\mathbf{k}(\mathbf{t})$, and $\boldsymbol{\kappa}(\mathbf{t})$. By differentiating the expected profit functions with respect to the bidding strategies of market participants, we get a system of nonlinear algebraic equations with strict inequalities that can be solved by the trust region Newton method. After determining the BNE corresponding to all combinations of active constraints, they are compared and Pareto-dominant BNE is designated as a global solution for the bidding strategy problem of each market player. The game may have a Pareto-optimal pure-strategy BNE, several non-dominated pure-strategy BNEs, or none at all.

D. Discussion on Computational Complexity

The proposed semi-analytical solution method in this study solves the equilibrium bidding strategy problem corresponding to each possible combination of active constraints in the LL sub-problem. Given the nature and the purpose of this study, the analysis in this paper does not need to be done in real-time; therefore, computational complexity is not a major concern. Nevertheless, it is worth to briefly discuss the subject of computational complexity in this section. The computational time in this type of analysis grows exponentially when there exists a large number of binding constraints in the reduced bid-based TCED problem. However, there are some ways to overcome the time complexity. In particular, the analysis in this paper can be extended in two ways in order address computational complexity.

First, one can consider the bounded rationality of the strategic players. In reality, the rationality of market players is limited by the tractability of the bidding strategy problem. Hence, the optimal bidding strategy of a market participant can be found by determining the local equilibrium corresponding to some combinations of active constraints. Even though the obtained equilibrium bidding strategy is less credible than its global counterpart, this alternative less-complex solution still has meaning as it can be adequate for the satisfaction of the market player due to limitations on its rationality.

Second, one can also parallelize the solution process. Determining all possible solutions of the parametric bid-based TCED problem is the most time-consuming process of the proposed methodology. This process can be performed in parallel; which can significantly help with reducing the computational time.

V. Analytical Case: IEEE 9-Bus System

The proposed semi-analytical approach is applied to the IEEE 9-bus test system in Fig. 2. The cost coefficients of generation units are provided in Table I. The inverse demand function of a price-taker demand unit i is considered as $\Theta_i = -0.04P_{di} + 20$, for $i \in \{1, 2, 3\}$. Consumption limits of each demand unit are 20 (MW) and 85 (MW).

A. Case 1: Elastic Demand Units

To provide intuition about the strategic bidding behavior of generation units in the presence of elastic demand units, the following two scenarios are taken into account in Case 1:

- **Case 1.A**: Transmission lines have large enough capacity.
- **Case 1.B**: The similar input data as Case 1.A, but the transmission constraints are contemplated.

In Case 1.A, generation units compete against one another to serve demand units. The market outcome is given in Table II. The proposed semi-analytical approach allows us to shed light on the origins and characteristics of generation units' bidding behavior. To illustrate how market forces work, the parameterized equilibrium of market is given in following:

$$P^*_{g1} = \frac{-10(3400k_1k_2+4900k_1k_3-251756k_2k_3+62475k_1k_2k_3)}{1496k_1k_2+2156k_1k_3+1666k_2k_3+27489k_1k_2k_3}$$

$$P^*_{g2} = \frac{-40(264k_1k_2-82295k_1k_3+294k_2k_3+4851k_1k_2k_3)}{1496k_1k_2+2156k_1k_3+1666k_2k_3+27489k_1k_2k_3}$$

$$P^*_{g3} = \frac{-40(220k_1k_3-57214k_1k_2+170k_2k_3+2805k_1k_2k_3)}{1496k_1k_2+2156k_1k_3+1666k_2k_3+27489k_1k_2k_3}$$

$$P^*_{di} = \frac{10(74800k_1k_2+107800k_1k_3+83300k_2k_3-31033k_1k_2k_3)}{1496k_1k_2+2156k_1k_3+1666k_2k_3+27489k_1k_2k_3}$$

$$\lambda^* = \frac{562190k_1k_2k_3}{1496k_1k_2+2156k_1k_3+1666k_2k_3+27489k_1k_2k_3}. \tag{30}$$

According to λ^*, each generation unit can increase the equilibrium price by increasing its bidding strategy. The positive correlation of equilibrium price and bidding strategy of generation units represents the force of self-interest. On the other hand, regarding P^*_g, the equilibrium quantity of each generation unit and its bidding strategy is negatively correlated. In other words, the competition force is reflected in the equilibrium quantity of each generation unit. According to the aforementioned descriptions, two opposing, but complementary forces of self-interest and competition act as Adam Smith's invisible hand and shape the bidding behavior of each strategic player and consequently the market equilibrium.

TABLE I
Cost Coefficients of Generation Units

Unit #	a	b	$P_{g,min}$	$P_{g,max}$
1	0.22	5	20	100
2	0.17	1.2	20	120
3	0.245	1	30	90

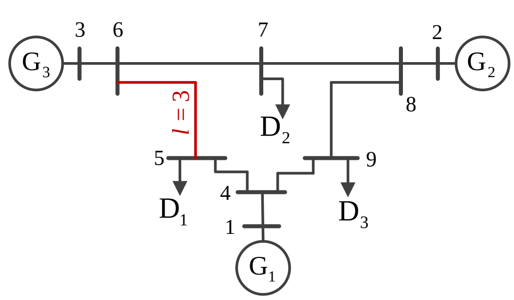


Fig. 2. The IEEE 9-bus test system.

TABLE II
Market outcome pertaining to Case 1.A

Unit	k	P_g	LMP	Profit
G_1	1.038	52.9	17.27	340.8
G_2	1.065	88.3	17.27	756.1
G_3	1.045	63.4	17.27	539.6
Unit	κ	P_d	LMP	Profit
D_1	1	68.2	17.27	93.2
D_2	1	68.2	17.27	93.2
D_3	1	68.2	17.27	93.2

To study the strategic interactions in an congested network, Case 1.B is designed. After solving 18 LPs, constraint of line 3 is merely recognized as critical ones. Table III and Table IV report market outcome and line flows associated with Case 1.B, respectively. To show beyond the shadow price of line 3's constraint, μ^*_3 is decomposed in the following:

$$\mu^*_3 = \frac{\gamma_{\mu a}k_1k_2+\gamma_{\mu b}k_1k_3-\gamma_{\mu c}k_2k_3+\gamma_{\mu d}k_1k_2k_3}{10^5(\gamma_1k_1+\gamma_2k_2+\gamma_3k_3+\gamma_4k_1k_2+\gamma_5k_1k_3+\gamma_6k_2k_3+\gamma_7k_1k_2k_3)}, \tag{31}$$

where γ_1=6569.1648, γ_2=29817, γ_3=14826, γ_4=35764, γ_5=195860, γ_6=121630, γ_7=1453000, $\gamma_{\mu a}$=351426816, $\gamma_{\mu b}$=220741632, $\gamma_{\mu c}$=143744028, $\gamma_{\mu d}$=186250397. Regarding Table III and (31), the value of μ_3 at candidate NE is 2.76. Note that (31) shows how generation units can create congestion in line 3 through strategic bidding. Congestion occurs in line 3 when the numerator of (31) is greater than zero.To show how bidding strategies of generation units influence nodal prices, LMP at bus 5 is decomposed as follows:

$$LMP_5 = \frac{100(\gamma_{5a}k_1k_2+\gamma_{5b}k_1k_3-\gamma_{5c}k_2k_3+\gamma_{5d}k_1k_2k_3)}{\gamma_1k_1+\gamma_2k_2+\gamma_3k_3+\gamma_4k_1k_2+\gamma_5k_1k_3+\gamma_6k_2k_3+\gamma_7k_1k_2k_3}, \tag{32}$$

where γ_{5a}=89754, γ_{5b}=37305, γ_{5c}=66122, γ_{5d}=278940. The proposed decomposition illustrates how generation units are able to exercise market power through strategic bidding. Moreover, it can be used by generation unit j to evaluate the sensitivity of LMP_j to its bidding strategy.

TABLE III
Market outcome pertaining to Case 1.B

Unit	k	P_g	LMP	Profit
G_1	1.058	54.2	17.91	376.3
G_2	1.105	83	16.92	719.5
G_3	1.206	50.8	16.22	457.3
Unit	κ	P_d	LMP	Profit
D_1	1	42.8	18.29	36.8
D_2	1	84.4	16.62	142.9
D_3	1	60.8	17.57	73.2

TABLE IV
Line flows pertaining to Case 1.B

Line #	1	2	3	4	5	6	7	8	9
Limit (MW)	60	35	20	70	45	65	95	70	80
Flow (MW)	-54.2	-22.8	20	-50.8	-30.8	53.6	83	-29.4	31.4

B. Case 2: Strategic Demand Units

To study the effects of market structure on the strategic bidding behavior of players and market efficiency, the following three scenarios are defined in Case 2:

- **Case 2.A**: Transmission lines have large enough capacity.
- **Case 2.B**: The similar input data as Case 2.A, but the transmission constraints are contemplated.
- **Case 2.C**: The similar input data as Case 2.A, but each player has incomplete information about opponents.

In Case 2.A, 3 generation units compete against one another to serve 3 strategic demand units. The market result for Case 2.A is summarized in Table V. Note that the oligopsonistic competition of demand units in Case 2.A is a symmetric game. The parameterized equilibrium of the market in Case 2.A is given in (33). Eq. (33) describes the structure of conflict of interests among market participants. First, generation units (demand units) are willing to boost (depress) equilibrium price through exercising market power. Regarding λ^*, strategic demand units play an active role in the price setting. Second, each generation unit (demand unit) is willing to increase its generation level (consumption level) by bidding lower (higher) price. Third, bidding low (high) price by a given generation unit (demand unit) has a negative consequence on the generation level (consumption level) of rival players. The above-mentioned forces shape the bidding behavior of each market participant. Accordingly, each generation unit (demand unit) has to choose between selling (buying) more power and increasing (decreasing) the equilibrium price considering the bidding behavior of rival players.

Case 2.B scrutinizes the bidding behavior of transmission-constrained market players under a combined oligopoly and oligopsony structure. The market outcome pertaining to Case 2.B is summarized in Table VI. To investigate the optimal bidding strategy problem from the viewpoints of transmission-constrained market players, we provide a visual description for the bidding behavior of demand unit 1. In this regard, the best response function of demand unit 1 to the NE bidding strategies of its opponents is illustrated in Fig. 3. As can be seen, for a given active set, the best response function of demand unit 1 is convex. Accordingly, this unit can maximize its profit by setting its bidding strategy at 0.878. It should be noted that the bidding behavior of rival players forms the best response function of market players. For example, consider the profit function of demand unit 1. Fig. 4 exhibits the profit function of this unit when bidding strategies of demand units 2, and 3 are varied between 0.8 and 1. This figure points out that the profit of demand unit 1 increases as the bid price of demand unit 3 (demand unit 2) falls (rises) and decreases as the bid price of demand unit 3 (demand unit 2) rises (falls).

TABLE V
MARKET OUTCOME PERTAINING TO CASE 2.A

Unit	k	P_g	LMP	Profit
G_1	1.035	49.5	16.45	297.2
G_2	1.062	84.1	16.45	682
G_3	1.043	60.3	16.45	486.2
Unit	κ	P_d	LMP	Profit
D_1	0.945	64.6	16.45	146.2
D_2	0.945	64.6	16.45	146.2
D_3	0.945	64.6	16.45	146.2

TABLE VI
MARKET OUTCOME PERTAINING TO CASE 2.B

Unit	k	P_g	LMP	Profit
G_1	1.05	46.8	16.07	276.4
G_2	1.085	70.9	14.38	507.2
G_3	1.164	42.2	13.2	296.7
Unit	κ	P_d	LMP	Profit
D_1	0.878	24.6	16.7	69.2
D_2	0.776	52.6	13.89	266
D_3	0.928	82.7	15.48	237.6

To analyze the impacts of uncertainty, the bidding behavior of market players is investigated in the presence of incomplete information in Case 2.C. In this case, demand units and generation unit 1 have type 1. The cost coefficients of generation units in type 1 are summarized in Table I. For type 2, the cost coefficients of each generation unit are 1.05 times those for type 1. The benefit coefficients of demand units in type 1 are α=0.04, and β=20. Additionally, it is supposed that all market players except generation unit 1 believe that rival players have type 1. Generation unit 1 is not sure about the types of generation units 2 and 3. Generation unit 1's belief about the types of other generation units is $p_1(\mathbf{t}_{-1}|t_1)$=0.25, where $\mathbf{t}_{-1}\epsilon\{(1,1),(1,2),(2,1),(2,2)\}$. Table VII reports market outcome pertaining to Case 2.C. In the following, we take a closer look at bidding behavior of generation unit 1 and demand unit 1 in cases 2.A and 2.C. Compared to Case 2.A, generation unit 1 will choose to bid a higher price in Case 2.C. To explain this, note that, in Case 2.A the optimal bidding strategy of generation unit 1 is determined when the type of generation units is (1,1,1). In Case 2.C, optimal bidding strategy of generation unit 1 is determined considering 4 different combinations of types (1,1,1), (1,2,1), (1,1,2), and (1,2,2). In types (1,2,1), (1,1,2), and (1,2,2), the opponents of generation unit 1 have a higher cost functions in comparison with type (1,1,1). Accordingly, generation unit 1 would increase its bid price and expect higher profit compared to Case 2.A. Moreover, Table VII shows that the bidding strategy of demand unit 1 is the same as that in Table V, so that the expected profit of demand unit 1 is the same as that in Case 2.A. The reason is that demand unit 1 solves the bidding strategy problem in the belief that the type of other players is the same as that in Case 2.A.

To analyze the impact of the paradigm shift in the market structure at firm-level, bidding behaviors of market players in cases 1.B and 2.B (cases with a congested network) are compared. Regarding Table III and Table VI, generation units 1, 2, and 3 slightly reduced their bidding strategies in Case 2.B in comparison with Case 1.B. On the contrary, demand units 1, 2, and 3 choose to exercise market power through

TABLE VII
MARKET OUTCOME PERTAINING TO CASE 2.C

Unit	G_1	G_2	G_3	D_i
Bidding strategy	1.063	1.055	1.038	0.945
Expected profit	317.3	653.4	467.1	146.2

$$
\begin{aligned}
P_{g1}^{*} &= \frac{-5(20825k_1k_2k_3(\kappa_1\kappa_2+\kappa_1\kappa_3+\kappa_2\kappa_3)+\kappa_1\kappa_2\kappa_3(3400k_1k_2+4900k_1k_3-251756k_2k_3))}{9163k_1k_2k_3\kappa_1\kappa_3+1496k_1k_2\kappa_1\kappa_2\kappa_3+2156k_1k_3\kappa_1\kappa_2\kappa_3+1666k_2k_3\kappa_1\kappa_2\kappa_3}\\
P_{g2}^{*} &= \frac{-20(21617k_1k_2k_3(\kappa_1\kappa_2+\kappa_1\kappa_3+\kappa_2\kappa_3)+\kappa_1\kappa_2\kappa_3(264k_1k_2-82295k_1k_3+294k_2k_3))}{9163k_1k_2k_3\kappa_1\kappa_3+1496k_1k_2\kappa_1\kappa_2\kappa_3+2156k_1k_3\kappa_1\kappa_2\kappa_3+1666k_2k_3\kappa_1\kappa_2\kappa_3}\\
P_{g3}^{*} &= \frac{-20(935k_1k_2k_3(\kappa_1\kappa_2+\kappa_1\kappa_3+\kappa_2\kappa_3)+\kappa_1\kappa_2\kappa_3(-57214k_1k_2+220k_1k_3+170k_2k_3))}{9163k_1k_2k_3\kappa_1\kappa_3+1496k_1k_2\kappa_1\kappa_2\kappa_3+2156k_1k_3\kappa_1\kappa_2\kappa_3+1666k_2k_3\kappa_1\kappa_2\kappa_3}\\
P_{di}^{*} &= \frac{5(k_1k_2k_3(458150\kappa_i\sum_{i\neq i}\kappa_i-947333\prod_{i\neq i}\kappa_i)-\kappa_1\kappa_2\kappa_3(74800k_1k_2+107800k_1k_3+83300k_2k_3))}{9163k_1k_2k_3\kappa_1\kappa_3+1496k_1k_2\kappa_1\kappa_2\kappa_3+2156k_1k_3\kappa_1\kappa_2\kappa_3+1666k_2k_3\kappa_1\kappa_2\kappa_3}\\
\lambda^{*} &= \frac{562193.2k_1k_2k_3\kappa_1\kappa_2\kappa_3}{9163k_1k_2k_3\kappa_1\kappa_3+1496k_1k_2\kappa_1\kappa_2\kappa_3+2156k_1k_3\kappa_1\kappa_2\kappa_3+1666k_2k_3\kappa_1\kappa_2\kappa_3}.
\end{aligned}
\tag{33}
$$

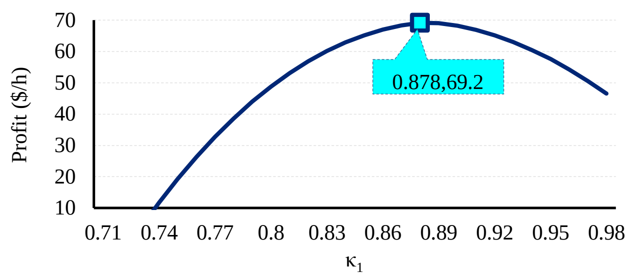


Fig. 3. Best response function of demand unit 1.

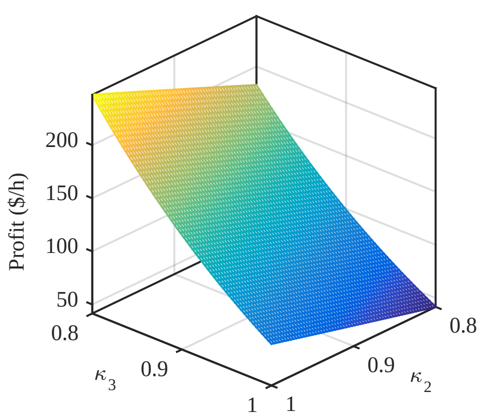


Fig. 4. The impact of bidding strategies of demand units 2 and 3 on the profit of demand unit 1.

bidding lower prices in Case 2.B. For example, compared to Case 1.B, demand unit 1 reduces its bidding factor by 12% in Case 2.B. The consequence is that the awarded consumption of D_1 and the LMP at bus 5 are significantly decreased from 42.8 (MW) and 17.27 (\$/MWh) in Case 1.B to 24.6 (MW) and 16.45 (\$/MWh) in Case 2.B. Accordingly, the profit of demand unit 1 is increased from 36.7 (\$/h) in Case 1.B to 69.2 (\$/h) in Case 2.B. Comparison among bidding behavior of demand unit 3 in cases 1.B and 2.B results in some interesting observations. Even though demand unit 3 bid lower price in Case 2.B in comparison with Case 1.B (by reducing its bidding factor from 1 to 0.928), surprisingly the awarded consumption of demand unit 3 is increased. To explain this, note that, the awarded consumption of demand unit 3 depends on the bidding behavior of rival players, i.e. demand units 1 and 2. The impact of bidding strategies of demand units 1 and 2 on the consumption level of demand unit 3 is illustrated in Fig. 5, where Plane 1 and Plane 2 represent the awarded consumption of D_3 in cases 2.B and 1.B, respectively. Regarding Fig. 5, the two planes intersect on a line that can be characterized by the equation $\kappa_2 + 1.667\kappa_1 - 2.44 = 0$. If ($\kappa_1$,$\kappa_2$) satisfies $\kappa_2 + 1.667\kappa_1 - 2.44 < 0$, then the awarded consumption of D_3 in Case 2.B is bigger than that in Case 1.B.

C. Insightful Observations

Comparison among the above case studies can result in some interesting and insightful observations. In particular, by comparing Table II and Table V, we can report **Observations 1 and 2**, as listed in the last bullet item in Section I.B. To explain this, note that, the oligopoly-oligopsony duality *mitigates* the market power of generation units and *decreases* the bid price of demand units. Hence, the equilibrium occurs at a *lower* demand. Besides, the energy price is reduced from 17.27 (\$/MWh) in Case 1.A to 16.45 (\$/MWh) in Case 2.A; which leads to transfer welfare from generation units to demand units. Next, consider the level of CS, PS, QSW, and total demand (TD) in cases 1.A and 2.A that are shown in Fig. 6. Based on the results in this figure, we can report **Observations 3 and 4**, as we listed in the last bullet item in Section I.B. In particular, notice that the strategic bidding by demand units *increases* CS, whereas it reduces both PS and SW. To explain why the strategic bidding behavior of demand units has such impacts on the market, the integrated inverse supply function (IISF) of generation units and the integrated inverse demand function (IIDF) of demand units pertaining to cases 1.A and 2.A are illustrated in Fig. 7. The slope and intercept of IISF are 0.0724 and 2.453 in Case 1.A and 0.0722 and 2.446 in Case 2.A. It is worth mentioning that the slope and intercept of IISF are nearly the same in cases 1.A and 2.A, whereas the slope and intercept of IIDF are changed from -0.0133 and 20 in Case 1.A to -0.0126 and 18.9 in Case 2.A, respectively.

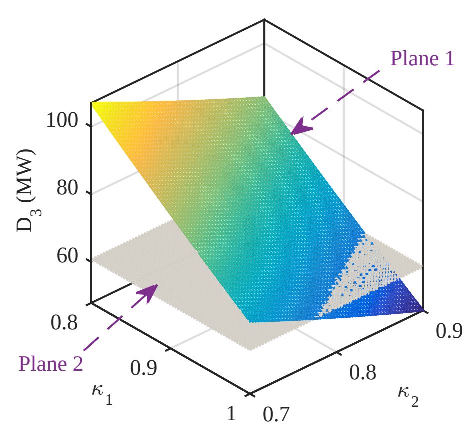
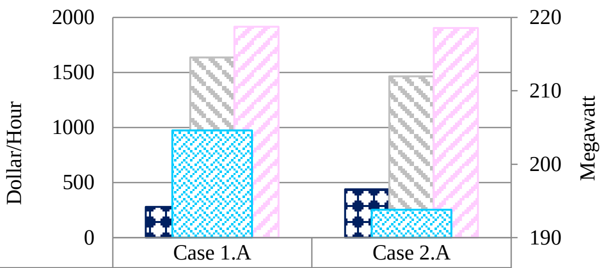


Fig. 5. The impact of bidding strategies of demand units 1 and 2 on the awarded consumption of demand unit 3.



	Case 1.A	Case 2.A
CS ($/h)	279.1	438.5
PS ($/h)	1637	1464
QSW ($/h)	1916.1	1902.5
TD (MW)	204.6	193.8

Fig. 6. The impact of consumers' strategic bidding on the market outcome pertaining to uncongested cases 1.A and 2.A.

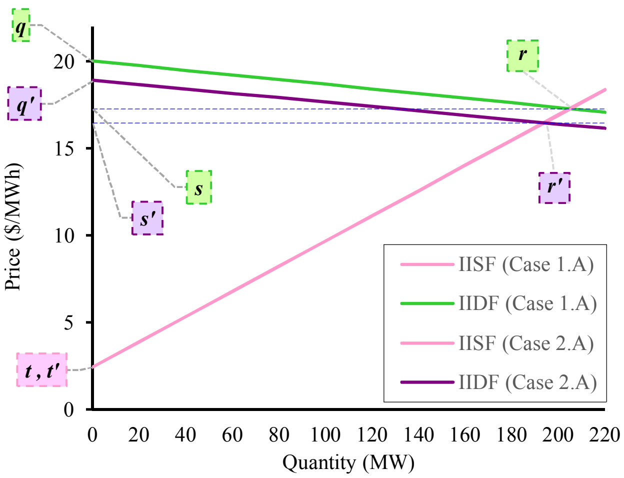


Fig. 7. The impact of strategic bidding by demand units on market equilibrium.

Regarding Fig. 7, a decrease in the intercept of IIDF results in shifting the equilibrium quantity and price from (r,s) in Case 1.A to (r',s') in Case 2.A. Consequently, as can be seen in Fig. 7, the strategic bidding behavior of demand units has detrimental effects on CS, PS, and SW. CS, PS, and SW associated with Case 1.A (Case 2.A) are equal to areas rsq ($r's'q'$), rst ($r's't'$), and rqt ($r'q't'$), respectively. According to Table III and Table VI, strategic bidding behaviors of demand units have the same effect on market outcome in the presence of congestion. As presented in Fig. 8, strategic bidding by demand units in Case 2.B decreases the LMPs at all buses in comparison with Case 1.B. Referring to Fig. 8, demand units benefit from lower LMPs. The level of CS, PS, QSW, and TD in cases 1.B and 2.B are calculated and displayed in Fig. 9. According to Fig. 9, strategic bidding by consumers reduces TD and worsens both PS and QSW whereas enhances CS.

VI. Numerical Case Study: IEEE 30-Bus System

To show the credibility of the proposed solution method, several analyzes are carried out on the IEEE 30-bus test system in Fig. 10. The information on cost coefficients of generation units is reported in [10]. The inverse demand function of a price-taker consumer is considered as $\Theta_i = -0.04P_{di} + 20$.

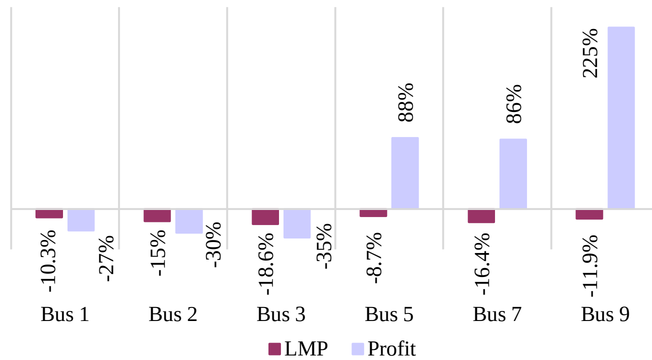
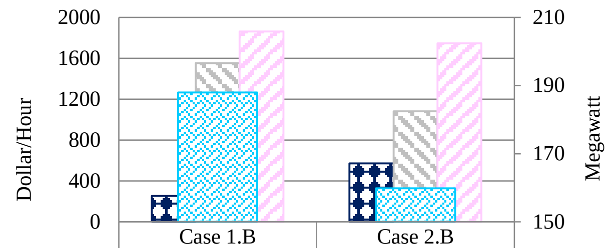


Fig. 8. The impact of consumers' strategic bidding on the percentage change of LMPs and players' profit pertaining to congested cases 1.B and 2.B.



	Case 1.B	Case 2.B
CS ($/h)	252.9	572.8
PS ($/h)	1553	1080.3
QSW ($/h)	1860.7	1746.1
TD (MW)	188	159.9

Fig. 9. The impact of consumers' strategic bidding on the market outcome pertaining to congested cases 1.B and 2.B.

A. Market Analysis

To study the market interactions, three scenarios are taken into account:

- **Case 3.A**: Generation units 1 to 4 are strategic players, and all other market participants are non-strategic players. Also, transmission lines have large enough capacity.
- **Case 3.B**: Similar input data as Case 3.A is utilized, but demand units are strategic players.
- **Case 3.C**: Similar input data as Case 3.B is utilized, but the transmission constraints are considered.

The market outcome that correspond to cases 3.A and 3.B are summarized in Table VIII and Table IX, respectively. Also, the consumption of each demand unit is equal to 102.3 (MW) and 95.2 (MW) in cases 3.A and 3.B, respectively. After solving 82 LPs, transmission constraints of lines 2 and 33 are recognized critical in Case 3.C. The upper flow limits of lines 2 and 33 are 90 MW and 100 MW, respectively. The NE which is related to the combination with two active constraints is merely feasible. The simulation results pertaining to supply and demand sides of Case 3.C are reported in Table X and Table XI.

B. Insightful Observations

Similar to the analysis in Section V.C, we can verify the same insightful observations based on the numerical case studies in this section. In particular, the comparison of Table IX and Table X confirms **Observations 1 and 2**. Regarding

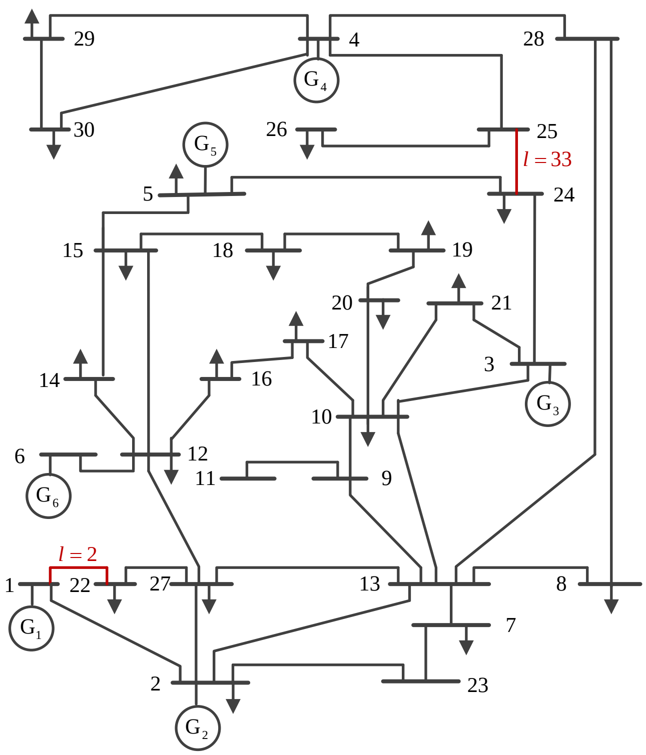


Fig. 10. The IEEE 30-bus test system.

Table IX and Table X, the oligopoly-oligopsony duality *mitigates* the market power of generation units and *decreases* the equilibrium price and quantity. To investigate the effects of transmission line constraints on the market power of different players in an oligopoly-oligopsony structure, LMPs in cases 3.B and 3.C are compared in Fig. 11. Besides, Fig. 12 depicts the impacts of transmission constraints on the profit of market players. Moreover, Fig. 13 reports CS, PS, QSW, and TD pertaining to cases 3.A, 3.B, and 3.C. Making a comparison between CS, PS, and QSW in cases 3.A and 3.B we can similarly confirm **Observations 3 and 4**.

C. Discussion on the Time Complexity

The computational challenges over the elements of the game (number of strategic players and the types) and the system scale (number of buses and transmission lines) are investigated in this section. First, we analyze the impact of the number of strategic players and the system scale on the computational requirements of the proposed algorithm. The process of determining the NE bidding strategies involves three important steps:

TABLE VIII
MARKET OUTCOME PERTAINING TO CASE 3.A

Unit	k	P_g	LMP	Profit
G_1	1.034	344.5	15.91	2414.7
G_2	1.04	386.9	15.91	2858.5
G_3	1.012	117.8	15.91	889
G_4	1.078	689.9	15.91	4763.9
G_5	1	258.2	15.91	1664.4
G_6	1	258.2	15.91	1664.4

TABLE IX
MARKET OUTCOME PERTAINING TO CASE 3.B

Unit	k	P_g	LMP	Profit
G_1	1.031	314.18	15.06	2129.5
G_2	1.038	364.5	15.06	2526.5
G_3	1.01	111.29	15.06	790.6
G_4	1.075	645.9	15.06	4147.8
G_5	1	241.2	15.06	1454.4
G_6	1	241.2	15.06	1454.4

TABLE X
MARKET OUTCOME PERTAINING TO SUPPLY-SIDE OF CASE 3.C

Unit	k	P_g	LMP	Profit
G_1	1.11	283.5	14.81	2024.3
G_2	1.14	323.9	14.92	2429.8
G_3	1.21	86.8	14.33	686.1
G_4	1.35	561.5	17.03	5108.5
G_5	1	242.1	14.18	1250.9
G_6	1	256	14.61	1348.6

TABLE XI
MARKET OUTCOME PERTAINING TO DEMAND-SIDE OF CASE 3.C

Unit	κ	P_d	LMP	Profit
D_2	0.88	76.1	14.92	270.9
D_5	0.87	92.4	14.18	366.6
D_7	0.92	91.5	15.03	287.2
D_8	0.95	102.8	15.09	292.9
D_{10}	0.91	101.7	14.5	352.8
D_{12}	0.86	75.2	14.61	292
D_{14}	0.86	77.5	14.53	303.6
D_{15}	0.84	69.3	14.47	287.1
D_{16}	0.89	90.9	14.56	328.9
D_{17}	0.89	92.2	14.52	335.5
D_{18}	0.92	106.4	14.48	361
D_{19}	0.92	106.4	14.48	360.2
D_{20}	0.92	106.4	14.48	359.8
D_{21}	0.93	113.8	14.37	381.9
D_{22}	0.94	98.1	15.11	287
D_{24}	0.85	94.2	13.8	406.8
D_{26}	0.97	37.2	17.95	48.4
D_{27}	0.85	57.7	15.04	219.6
D_{29}	0.97	61.1	17.03	106.7
D_{30}	0.97	61.1	17.03	106.7

- Step a) Calculating the solutions of the KKT optimality conditions in (25).
- Step b) Constructing the EPSIC problem in (28).
- Step c) Solving the EPSIC problem in (28).

Hence, the impact of the number of strategic players and the system scale on the computational time of the process can be examined in terms of the execution times of the above-mentioned steps. In this regard, the proposed method in this paper was tested on IEEE 9-bus, IEEE 30-bus, IEEE 57-bus, and IEEE 118-bus test systems. All tests are executed on a MATLAB platform, with a PC consisting of a 2-GHz processor and 8 GB of RAM. The execution times of different steps are reported in Table XII. Regarding Table XII, the computation time of Step a increases rapidly with the number

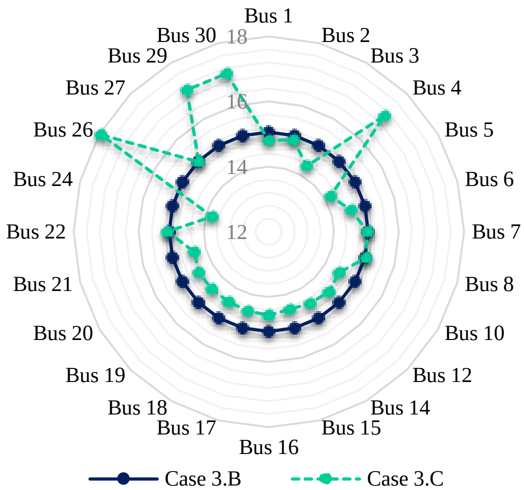


Fig. 11. The impact of congestion on the nodal prices.

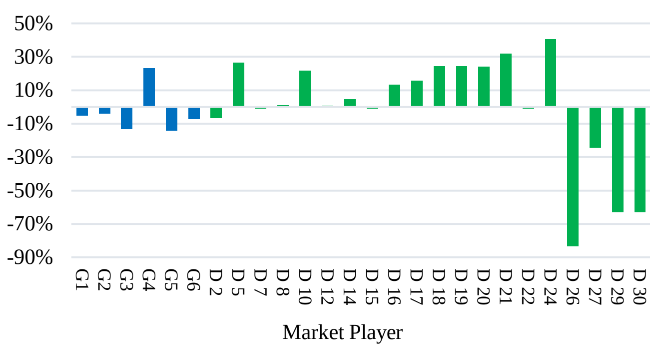
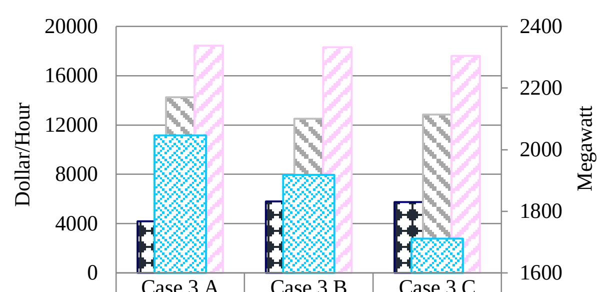


Fig. 12. The impact of congestion on the profit of market players.

of strategic players and the system scale, while the increase in the computation time of Step b and Step c is almost negligible. Due to the computational complexity of Step a, implementing the proposed solution method on a large-scale system with a substantial number of strategic players requires a significant amount of time.

In a Bayesian game, the computational requirements for constructing the BNE problem in (29) will scale linearly with the number of types. Accordingly, the proposed solution method would be intractable for large-scale systems with a substantial number of types.



	Case 3.A	Case 3.B	Case 3.C
CS ($/h)	4186.1	5796.7	5755.6
PS ($/h)	14258.9	12503.3	12848.2
QSW ($/h)	18445	18300	17593.2
TD (MW)	2046	1918.4	1711.9

Fig. 13. Market outcomes for cases 3.A, 3.B, and 3.C.

TABLE XII
THE EXECUTION TIMES OF THE IMPORTANT STEPS

Test case	M	Execution time of (sec) Step a	Step b	Step c
IEEE 9-bus	3	0.63	0.042	0.0
IEEE 9-bus	6	0.84	0.054	0.4
IEEE 30-bus	4	0.76	0.047	0.1
IEEE 30-bus	24	8.82	3.447	19.8
IEEE 57-bus	7	80.66	0.099	0.7
IEEE 57-bus	15	153.33	1.035	6.4
IEEE 57-bus	30	436.47	5.844	32.8
IEEE 118-bus	10	658.11	0.32	2.2
IEEE 118-bus	20	1327.8	2.2	12.9
IEEE 118-bus	40	3442.7	11.246	61.9

VII. Conclusions

This paper provides a semi-analytical approach to study the emerging wholesale electricity markets under a smart grid environment where demand units play an active role in the wholesale electricity market. Because of transmission congestion, neither the supply side nor the demand side of such wholesale electricity market is perfectly competitive. Therefore, the bidding strategy problem has to be studied in the context of a combined oligopoly and oligopsony electricity market. In this regard, we separately modeled the imperfect competition among generation units, as well as among demand units. The analysis was done as an equilibrium problem, and a two-phase semi-analytical approach we proposed to find the coordinated market equilibrium. In the first phase, parametric local solutions of the bid-based TCED problem were determined. In the second phase, the equilibrium problem was transformed into a series of equilibrium sub-problems with strict inequality constraints, each sub-problem associated with a parametric local solution of the TCED. The candidate NEs were accordingly determined by solving these sub-problems. The optimal bidding strategy of each market participant was obtained by ascertaining the Pareto-dominant NE. The proposed model has been tested on the IEEE 9-bus test system and the IEEE 30-bus test system. The analytical case studies and the test results showed that paradigm shift in the market structure from oligopoly-perfect competitive to oligopoly-oligopsony has a major impact on optimal bidding strategies of market participants and market equilibrium. At the firm-level, the oligopoly-oligopsony duality mitigated the potential to exercise market power by generation units; it also decreased the bid price of demand units. At the market-level, it declined energy price and PS while improved CS. Moreover, it had a detrimental effect on market efficiency and led to lower QSW in comparison with the oligopoly-perfect competitive structure. Additionally, it was observed that transmission line congestion does not have similar impact on the optimal bidding strategy and bidding profit of all market participants in a perfectly imperfect competitive environment; but it does reduce the QSW of the market as a whole. The results in this paper can help both generation and demand units to comprehend market behavior under a combined oligopoly and oligopsony structure. Moreover, they are insightful to

ISOs to better set forth their market policies in the future.

The proposed framework in this paper can be extended in two directions. First, the model can be extended to consider the strategic bidding of market participants in a multi-period market clearing structure. Second, the solution method can be extended in two ways to reduce the computational complexity. First extension would be to consider the bounded rationality of strategic players. Second extension would be to parallelize the solution process.

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Iman Taheri Emami received the B.Sc. degree in electrical engineering from Noshirvani Institute of Technology, Babol, Iran, in 2008 and the M.Sc. degree from Shahid Bahonar University (SBU), Kerman, Iran, in 2013. He is currently pursuing the Ph.D. degree at the Electrical Engineering Department of Amirkabir University of Technology, Tehran, Iran. His research is on economics, data analysis, and optimization of power systems and smart grids.

Ehsan Samani (S'16) received the B.Sc. and M.Sc. degrees in electrical engineering from the University of Tehran, Tehran, Iran, in 2016 and 2018, respectively. He is working toward a Ph.D. degree in electrical engineering at the University of California, Riverside, CA, USA. His research interests include analyzing energy storage systems, renewable energies, and virtual bidding in wholesale electricity markets by using data analysis, mathematical modeling, optimization methods, and machine learning techniques. Ehsan has put his knowledge into practice by working at Tesla, Inc. and ICF International, Inc. as an energy markets engineering intern.

Hossein Askarian Abyaneh was born in Abyaneh, Iran. He received the Ph.D. degree in electrical power system engineering from the University of Manchester Institute of Science and Technology, Manchester, U.K., in 1988. Currently, he is a Professor with the Department of Electrical Engineering, Amirkabir University of Technology (AUT), Tehran, Iran. His research interests are relay protection, power quality, restructuring, and deregulation in power systems.

Hamed Mohsenian-Rad (M'09-SM'14-F'20) received the Ph.D. degree in electrical and computer engineering from the University of British Columbia, Vancouver, BC, Canada, in 2008. He is currently a Professor of electrical engineering and a Bourns Family Faculty Fellow at the University of California, Riverside, CA, USA. His research is on monitoring, data analysis, and optimization of power systems and smart grids. He is the author of the textbook *Smart Grid Sensors: Principles and Applications* by Cambridge University Press - 2022. He was the recipient of the National Science Foundation (NSF) CAREER Award, the Best Paper Award from the IEEE Power & Energy Society General Meeting, and the Best Paper Award from the IEEE Conference on Smart Grid Communications. He has been the PI on ten million dollars research grants in the area of smart grid. He has served as Editor for the IEEE TRANSACTIONS ON POWER SYSTEMS, IEEE TRANSACTIONS ON SMART GRID and the IEEE POWER ENGINEERING LETTERS.

Alireza Bakhshai is a Professor in the Department of Electrical and Computer Engineering at Queen's University. Dr. Bakhshai received his BSc and MSc from the Isfahan University of Technology, Iran, in 1984 and 1986, respectively, and his PhD from Concordia University in 1997. He was a Postdoctoral fellow at Concordia University during 1997-8. From 1986 to 1993, and from 1998 to 2004, he served on the faculty of Electrical and Computer Engineering at Isfahan University of Technology.