

# Topic 1: Basics of Power Systems

ECE 5332: Communications and Control for Smart

Spring 2012



# Power Systems

- The Four Main Elements in Power Systems:
  - Power Production / Generation
  - Power Transmission
  - Power Distribution
  - Power Consumption / Load
- Of course, we also need monitoring and control systems.

# Power Systems

- Power Production:
  - Different Types:
    - Traditional
    - Renewable
  - Capacity, Cost, Carbon Emission
  - Step-up Transformers



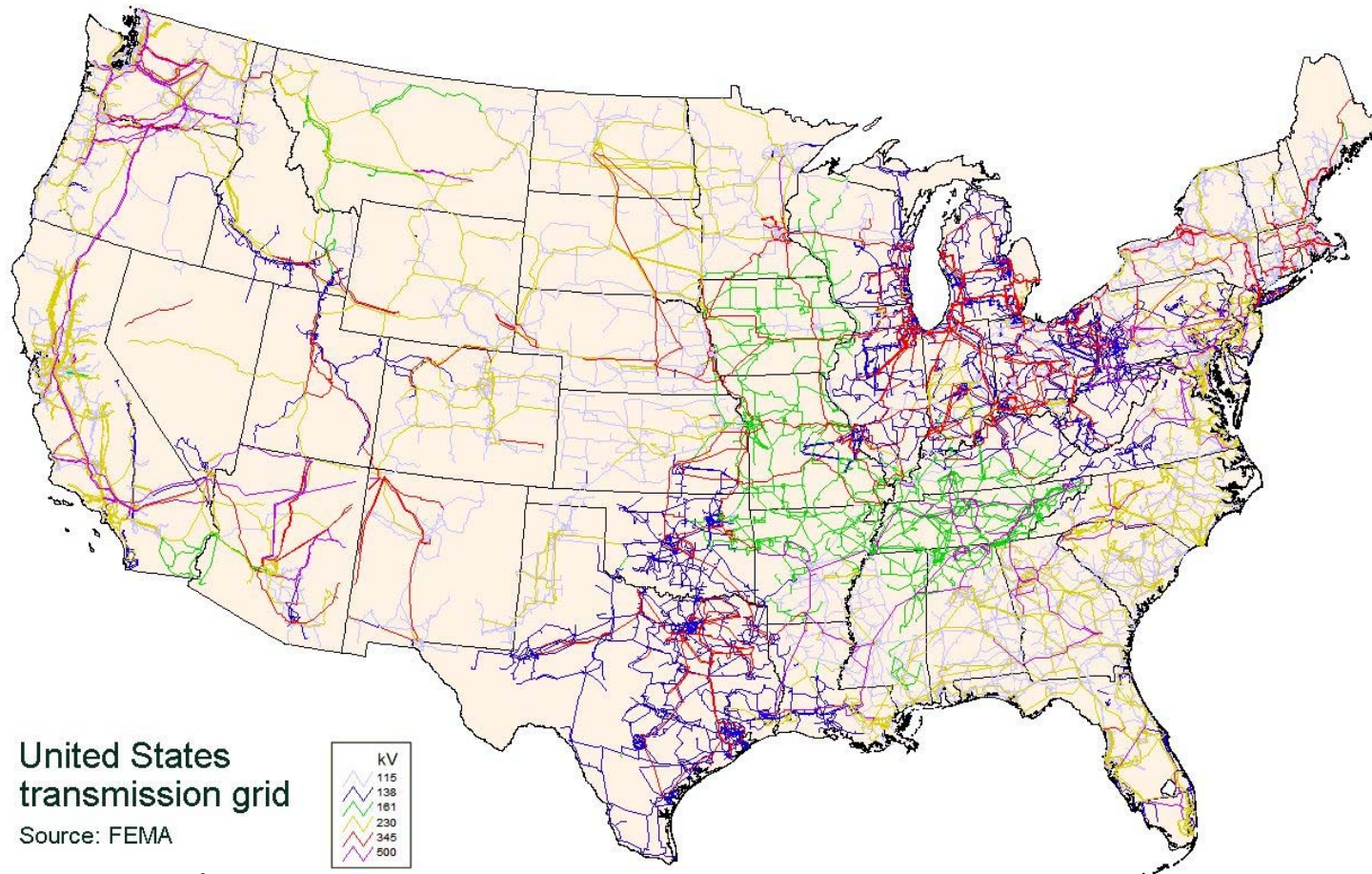
# Power Systems

- Power Transmission:
  - High Voltage (HV) Transmission Lines
  - Several Hundred Miles
  - Switching Stations
    - Transformers
    - Circuit Breakers



# Power Systems

- The Power Transmission Grid in the United States:



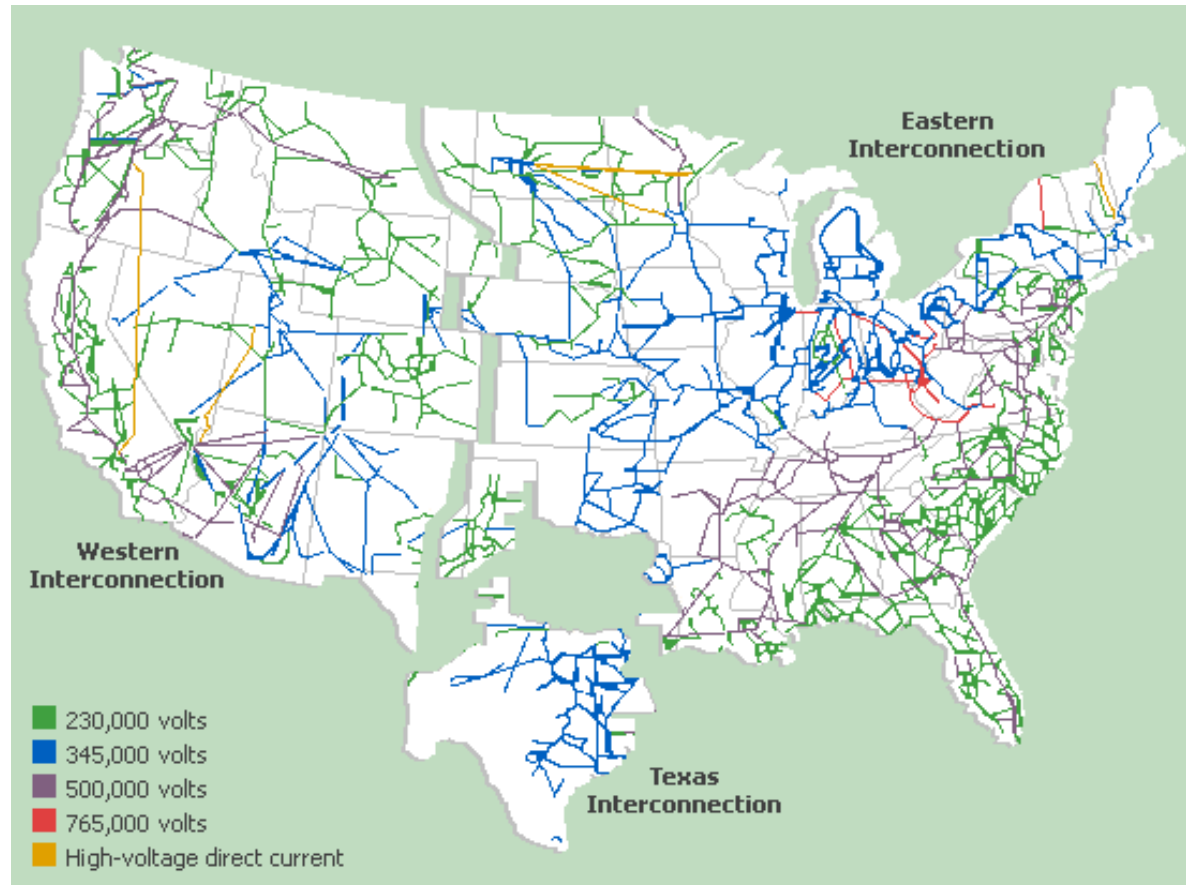
United States  
transmission grid

Source: FEMA

[www.geni.org](http://www.geni.org)

# Power Systems

- Major Inter-connections in the United States:



[www.geni.org](http://www.geni.org)

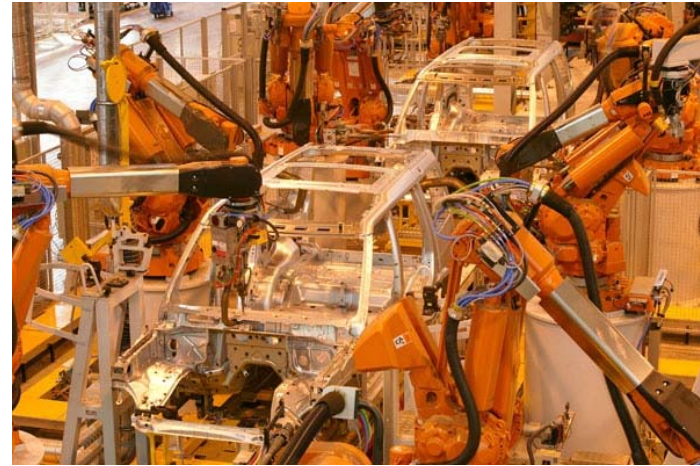
# Power Systems

- Power Distribution:
  - Medium Voltage (MV) Transmission Lines ( $< 50$  kV)
  - Power Deliver to Load Locations
  - Interface with Consumers / Metering
  - Distribution Sub-stations
    - Step-Down Transformers
    - Distribution Transformers



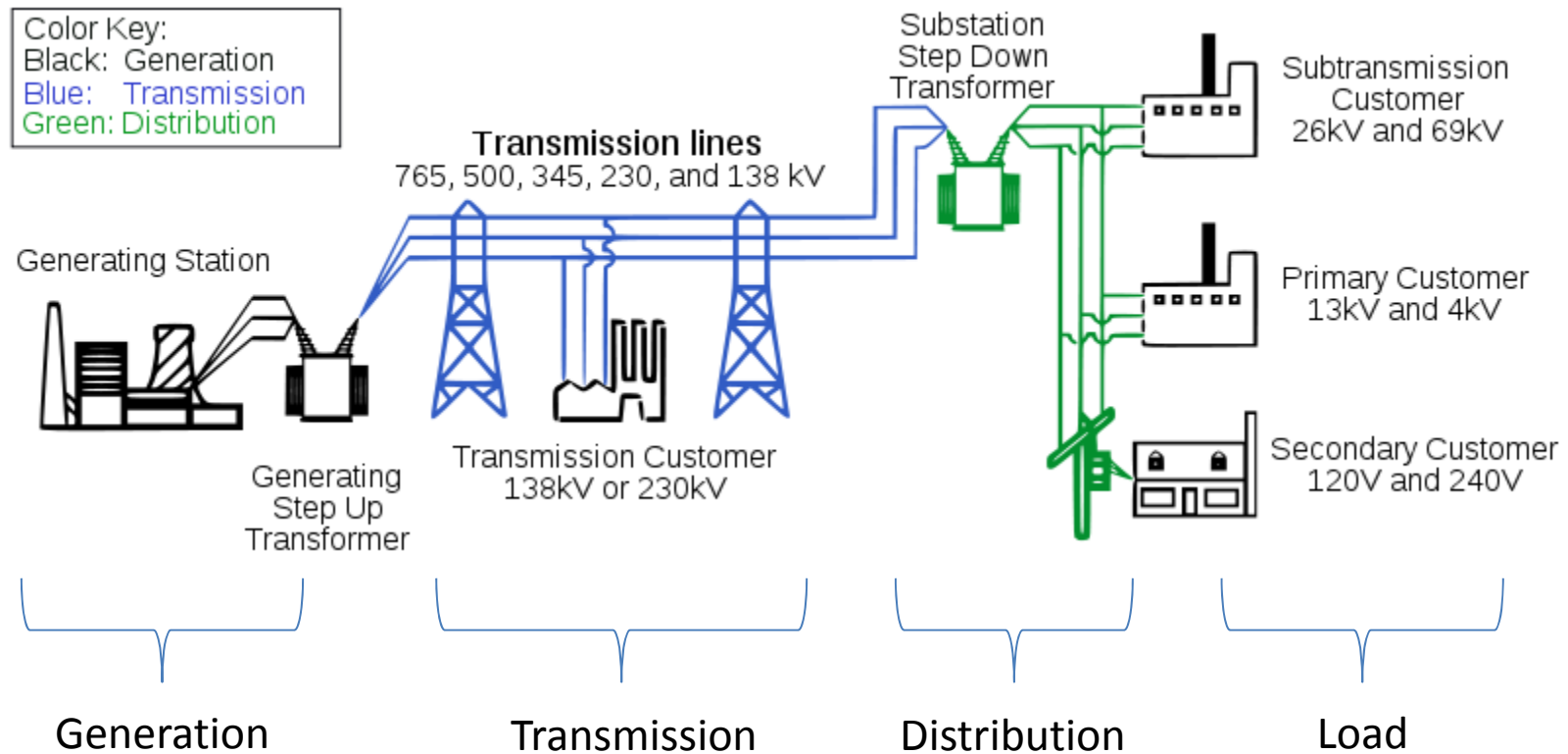
# Power Systems

- Power Consumption:
  - Industrial
  - Commercial
  - Residential
  
- Demand Response
  - Controllable Load
  - Non-Controllable





# Power Systems



# Power Systems

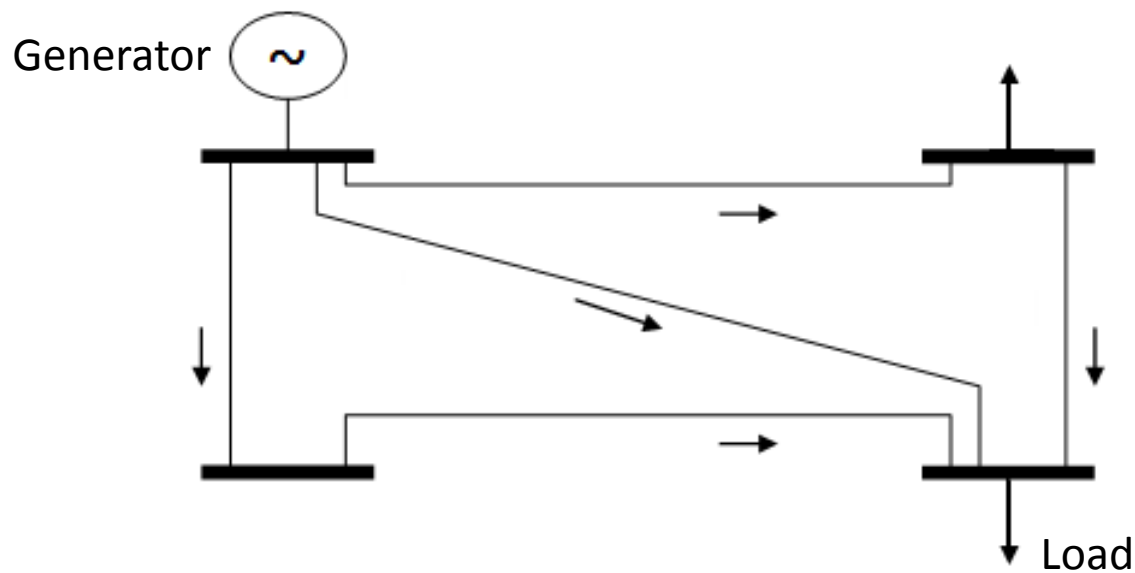
- Power System Control:
  - Data Collection: Sensors, PMUs, etc.
  - Decision Making: Controllers
  - Actuators: Circuit Breakers, etc.



# Power Grid Graph Representation

**Nodes:** Buses

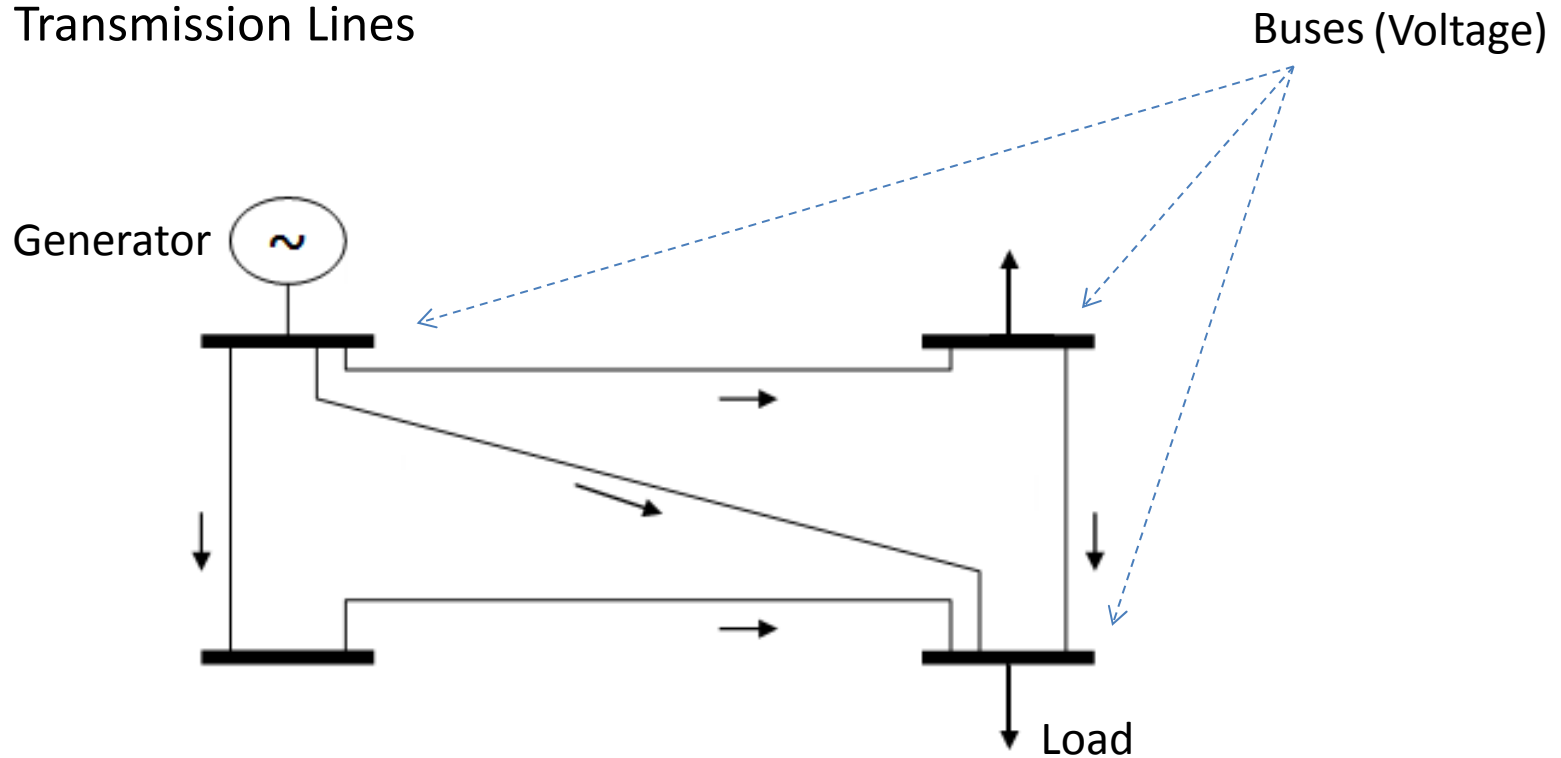
**Links:** Transmission Lines



# Power Grid Graph Representation

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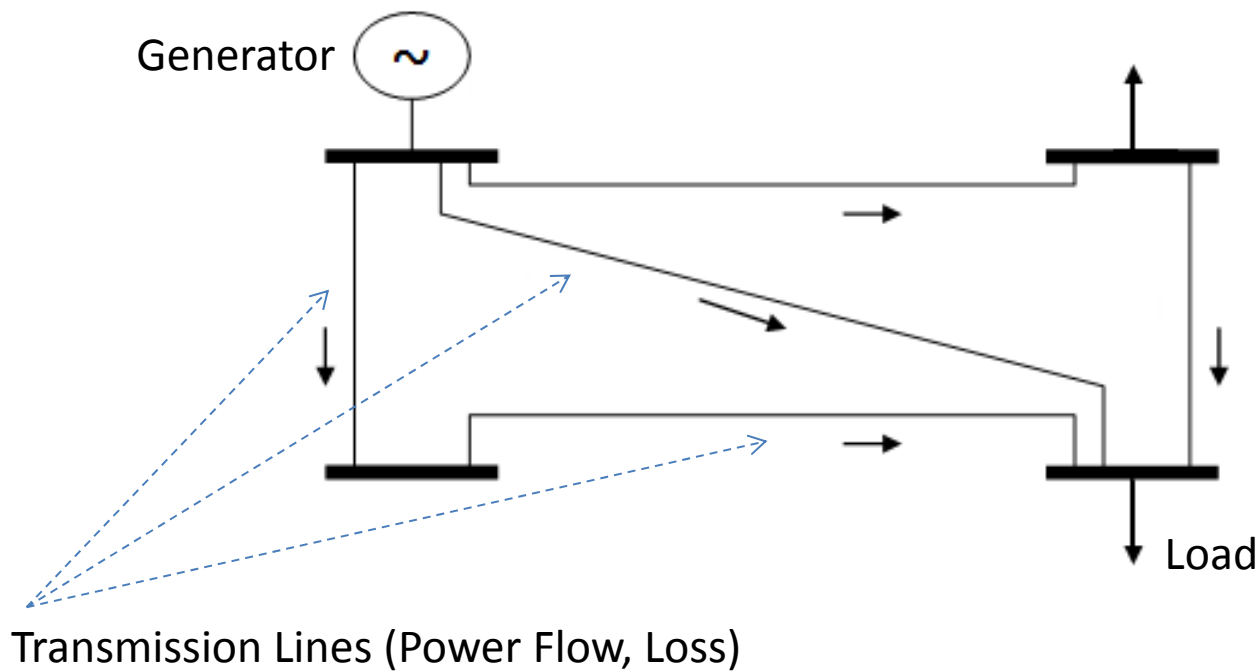
**Links:** Transmission Lines



# Power Grid Graph Representation

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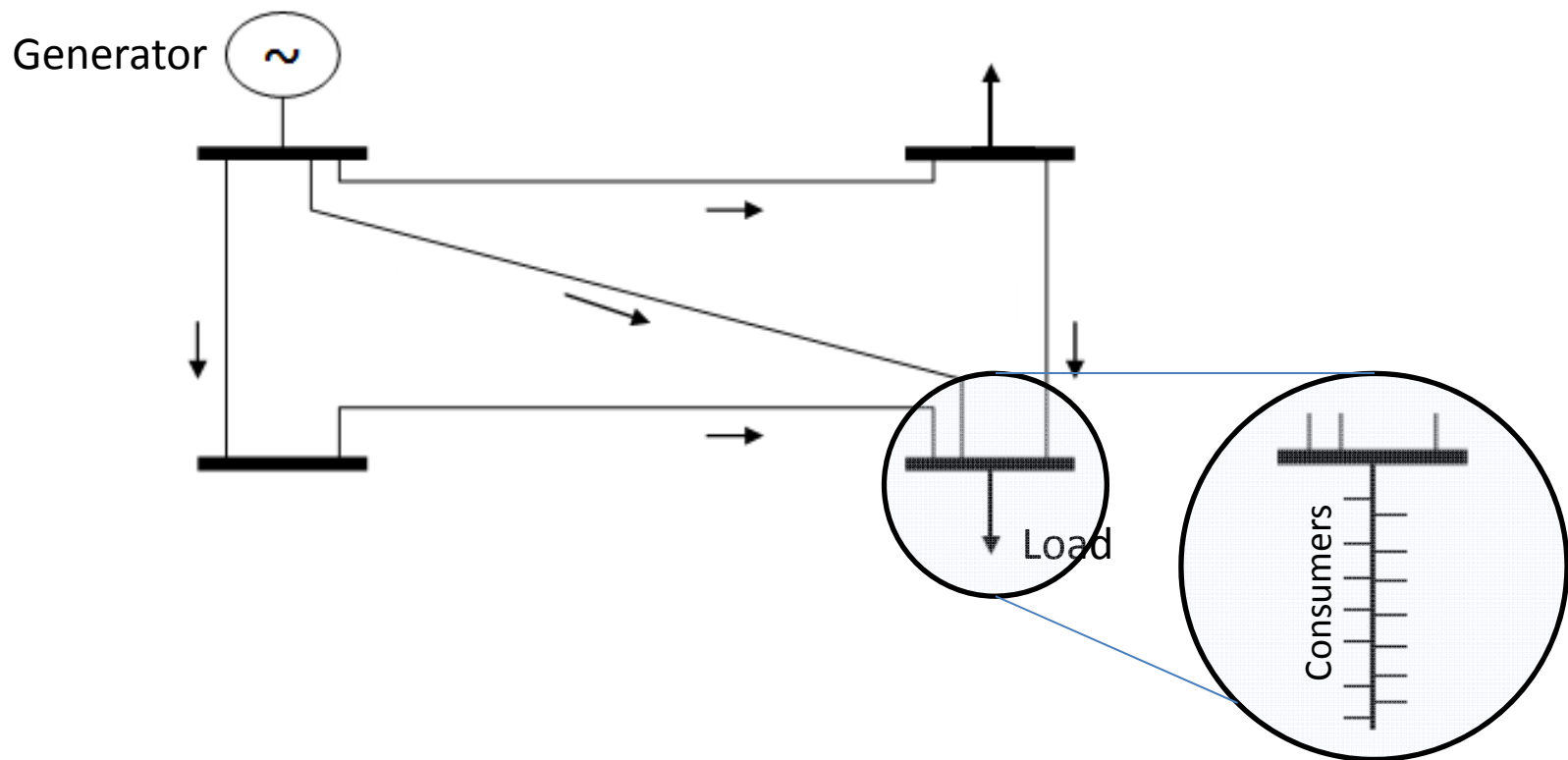
**Links:** Transmission Lines



# Power Grid Graph Representation

**Nodes:** Buses

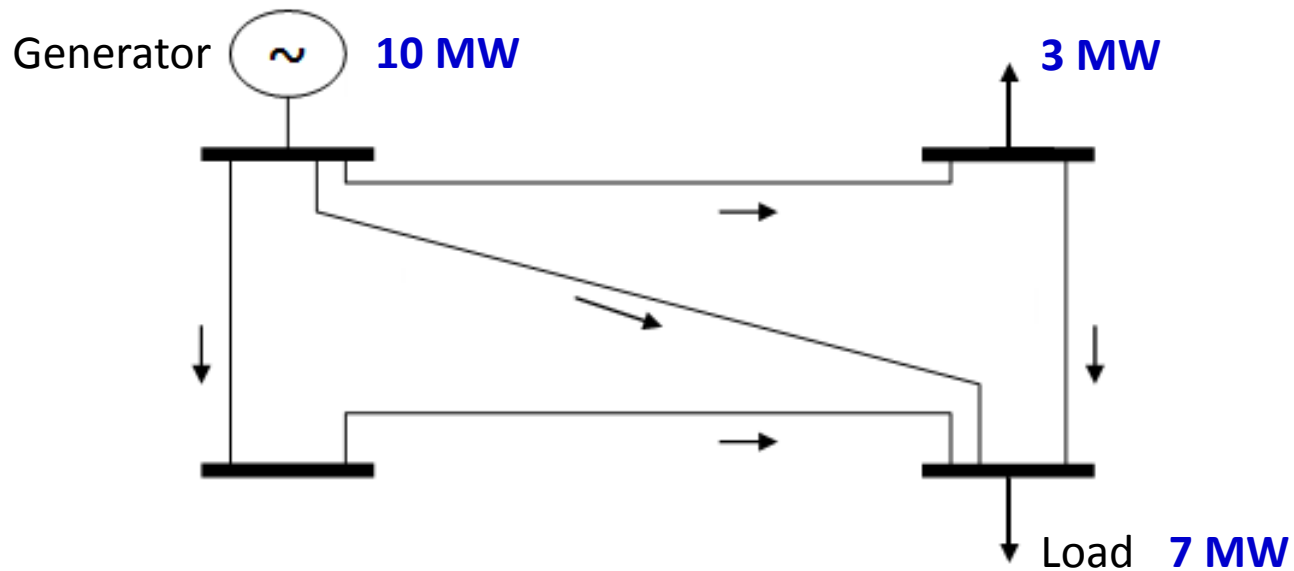
**Links:** Transmission Lines



# Power Grid Graph Representation

**Nodes:** Buses

**Links:** Transmission Lines



# Transmission Line Admittance

- Admittance  $y$  is defined as the inverse of impedance  $z$ :
  - $z = r + j x$  (r: Resistance, x: Reactance)
  - $y = g + j b$  (g: Conductance, b: Susceptance)
  - $y = 1 / z$
  - Parameter  $g$  is usually positive
  - Parameter  $b$ :
    - Positive: Capacitor
    - Negative: Inductor



# Transmission Line Admittance

- For the transmission line connecting bus  $i$  to bus  $k$ :
  - Admittance:  $y_{ik}$
  - Example:

$$y_{ik} = 1 - j 4 \quad (\text{per unit})$$

- Note that,  $y_{ii}$  is denoted by  $y_i$  and indicates:
  - Susceptance for any shunt element (capacitor) to ground at bus  $i$ .

# Y-Bus Matrix

- We define:

- $Y_{\text{bus}} = [ Y_{ij} ]$  where

- Diagonal Elements: 
$$Y_{ii} = y_i + \sum_{k=1, k \neq i}^N y_{ik}$$

- Off-diagonal Elements: 
$$Y_{ij} = -y_{ij}$$

- Note that  $Y_{\text{bus}}$  matrix depends on the power grid topology and the admittance of all transmission lines.
- $N$  is the number of busses in the grid.

# Y-Bus Matrix

- Example: For a grid with 4-buses, we have:

$$Y_{bus} = \begin{bmatrix} y_1 + y_{12} + y_{13} + y_{14} & -y_{12} & -y_{13} & -y_{14} \\ -y_{21} & y_2 + y_{21} + y_{23} + y_{24} & -y_{23} & -y_{24} \\ -y_{31} & -y_{32} & y_3 + y_{31} + y_{32} + y_{34} & -y_{34} \\ -y_{41} & -y_{42} & -y_{43} & y_4 + y_{41} + y_{42} + y_{43} \end{bmatrix}$$

- After separating the real and imaginary parts:

$$Y_{bus} = G + jB$$

# Bus Voltage

- Let  $V_i$  denote the voltage at bus  $i$ :
- Note that,  $V_i$  is a phasor, with **magnitude** and **angle**.

$$V_i = |V_i| \angle \theta_i$$

- In most operating scenarios we have:

$$|V_i| \approx |V_j| \quad \theta_i \neq \theta_j$$

# Power Flow Equations

- Let  $S_i$  denote the **power injection** at bus  $i$ :

$$S_i = P_i + j Q_i$$

Active Power      Reactive Power

- Generation Bus:  $P_i > 0$
- Load Bus:  $P_i < 0$       (negative power injection)

# Power Flow Equations

- Using Kirchhoff laws, **AC Power Flow Equations** become:

$$P_k = \sum_{j=1}^N |V_k| |V_j| (G_{kj} \cos(\theta_k - \theta_j) + B_{kj} \sin(\theta_k - \theta_j))$$

$$Q_k = \sum_{j=1}^N |V_k| |V_j| (G_{kj} \sin(\theta_k - \theta_j) - B_{kj} \cos(\theta_k - \theta_j))$$

- Do we know all notations here?
- If we know enough variables, we can obtain the rest of variables by solving a **system of nonlinear equations**.

# Power Flow Equations

- The AC Power Flow Equations are complicated to solve.
- Next, we try to simplify the equations in three steps.
- **Step 1:** For most networks,  $G \ll B$ . Thus, we set  $G = 0$ :



$$P_k = \sum_{j=1}^N |V_k| |V_j| (B_{kj} \sin(\theta_k - \theta_j))$$
$$Q_k = \sum_{j=1}^N |V_k| |V_j| (-B_{kj} \cos(\theta_k - \theta_j))$$

# Power Flow Equations

- **Step 2:** For most neighboring buses:  $|\theta_i - \theta_j| \leq 10^\circ$  to  $15^\circ$ .

- As a result, we have: 
$$\begin{cases} \sin(\theta_k - \theta_j) \approx \theta_k - \theta_j \\ \cos(\theta_k - \theta_j) \approx 1 \end{cases}$$



$$P_k = \sum_{j=1}^N |V_k| |V_j| (B_{kj} (\theta_k - \theta_j))$$


$$Q_k = \sum_{j=1}^N |V_k| |V_j| (-B_{kj})$$



# Power Flow Equations

- **Step 3:** In per-unit,  $|V_i|$  is very close to 1.0 (0.95 to 1.05).

- As a result, we have:  $|V_i||V_j| \approx 1$ .


$$P_k = \sum_{j=1}^N B_{kj} (\theta_k - \theta_j)$$
$$Q_k = \sum_{j=1}^N (-B_{kj}) = -B_{kk} - \sum_{\substack{j=1, \\ j \neq k}}^N B_{kj} = -b_k$$

- $P_k$  has a linear model and  $Q_k$  is almost fixed.

# Power Flow Equations

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- As a result, we have:  $|V_i||V_j| \approx 1$ .

## DC Power Flow Equations

$$P_k = \sum_{j=1}^N B_{kj} (\theta_k - \theta_j)$$
$$Q_k = \sum_{j=1}^N (-B_{kj}) = -B_{kk} - \sum_{\substack{j=1, \\ j \neq k}}^N B_{kj} = -b_k$$

- $P_k$  has a linear model and  $Q_k$  is almost fixed.

# Power Flow Equations

- Given the **power injection values** at all buses, we can use

$$P_k = \sum_{j=1}^N B_{kj} (\theta_k - \theta_j)$$

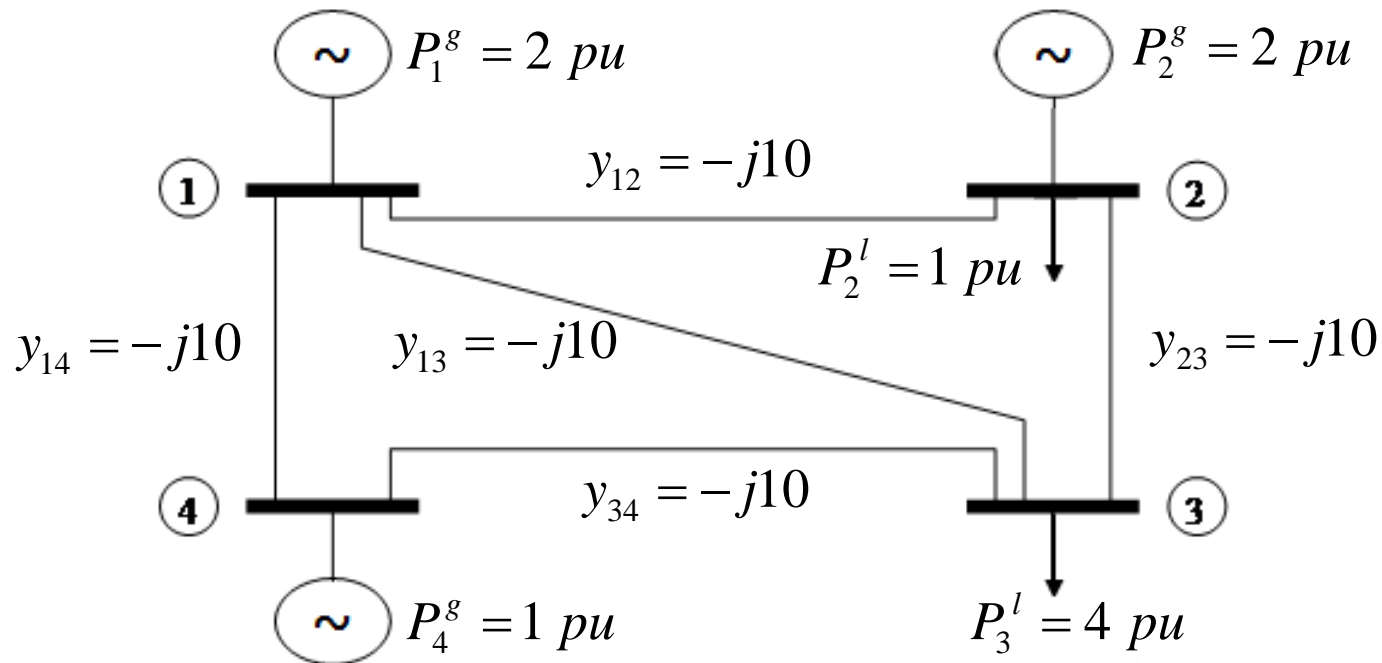
to obtain the **voltage angles** at all buses.

- Let  $P_{ij}$  denote the **power flow** from bus  $i$  to bus  $j$ , we have:

$$P_{ij} = B_{ij} (\theta_i - \theta_j)$$

# Power Flow Equations

- Example: Obtain power flow values in the following grid:



# Power Flow Equations

- First, we obtain the Y-bus matrix:

$$Y_{bus} = j \begin{bmatrix} b_1 + b_{12} + b_{13} + b_{14} & -b_{12} & -b_{13} & -b_{14} \\ -b_{21} & b_2 + b_{21} + b_{23} + b_{24} & -b_{23} & -b_{24} \\ -b_{31} & -b_{32} & b_3 + b_{31} + b_{32} + b_{34} & -b_{34} \\ -b_{41} & -b_{42} & -b_{43} & b_4 + b_{41} + b_{42} + b_{43} \end{bmatrix}$$

$$= j \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix} = j B = j \begin{bmatrix} B_{11} & B_{12} & B_{13} & B_{14} \\ B_{21} & B_{22} & B_{23} & B_{24} \\ B_{31} & B_{32} & B_{33} & B_{34} \\ B_{41} & B_{42} & B_{43} & B_{44} \end{bmatrix}$$

# Power Flow Equations

- Next, we write the (active) power flow equations:

$$P_1 = (B_{12} + B_{13} + B_{14})\theta_1 - B_{12}\theta_2 - B_{13}\theta_3 - B_{14}\theta_4$$

$$P_2 = -B_{21}\theta_1 + (B_{21} + B_{23} + B_{24})\theta_2 - B_{23}\theta_3 - B_{24}\theta_4$$

$$P_3 = -B_{31}\theta_1 - B_{32}\theta_2 + (B_{31} + B_{32} + B_{34})\theta_3 - B_{34}\theta_4$$

$$P_4 = -B_{41}\theta_1 - B_{42}\theta_2 - B_{43}\theta_3 + (B_{41} + B_{42} + B_{43})\theta_4$$

- This can be written as:

$$\begin{bmatrix} P_1 \\ P_2 \\ P_3 \\ P_4 \end{bmatrix} = \begin{bmatrix} B_{12} + B_{13} + B_{14} & -B_{12} & -B_{13} & -B_{14} \\ -B_{21} & B_{21} + B_{23} + B_{24} & -B_{23} & -B_{24} \\ -B_{31} & -B_{32} & B_{31} + B_{32} + B_{34} & B_{34} \\ -B_{41} & -B_{42} & -B_{43} & B_{41} + B_{42} + B_{43} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

# Power Flow Equations

- From the last two slides, we finally obtain:

$$\begin{bmatrix} \phantom{\theta_1} \\ \phantom{\theta_2} \\ \phantom{\theta_3} \\ \phantom{\theta_4} \end{bmatrix} = \begin{bmatrix} \phantom{\theta_1} \\ \phantom{\theta_2} \\ \phantom{\theta_3} \\ \phantom{\theta_4} \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

- Therefore, the voltage angles are obtained as:

$$\begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} \phantom{\theta_1} \\ \phantom{\theta_2} \\ \phantom{\theta_3} \\ \phantom{\theta_4} \end{bmatrix}^{-1} \begin{bmatrix} \phantom{\theta_1} \\ \phantom{\theta_2} \\ \phantom{\theta_3} \\ \phantom{\theta_4} \end{bmatrix}$$

# Power Flow Equations

- However, the last matrix in the previous slide is **singular!**
- Therefore, we cannot take the inverse.
- The system of equations would have infinite solutions.
- The problem is that the **four angles are not independent.**
- What matters is the **angular/phase difference.**
- We choose one bus (e.g., bus 1) as **reference bus:  $\theta_1 = 0$**  .



# Power Flow Equations

- We should also remove the corresponding rows/columns:

$$\begin{bmatrix} 2 \\ 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 30 & -10 & -10 & -10 \\ -10 & 20 & -10 & 0 \\ -10 & -10 & 30 & -10 \\ -10 & 0 & -10 & 20 \end{bmatrix} \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} \quad \longrightarrow \quad \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix} \begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}$$

- The angular differences (with respect to  $\theta_1$ ):

$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ -4 \\ 1 \end{bmatrix} = \begin{bmatrix} -0.025 \\ -0.15 \\ -0.025 \end{bmatrix} \quad \longrightarrow \quad \begin{aligned} \theta_2 - \theta_1 &= -0.025 \\ \theta_3 - \theta_1 &= -0.15 \\ \theta_4 - \theta_1 &= -0.025 \end{aligned}$$

# Power Flow Equations

- Finally, the power flow values are calculated as:

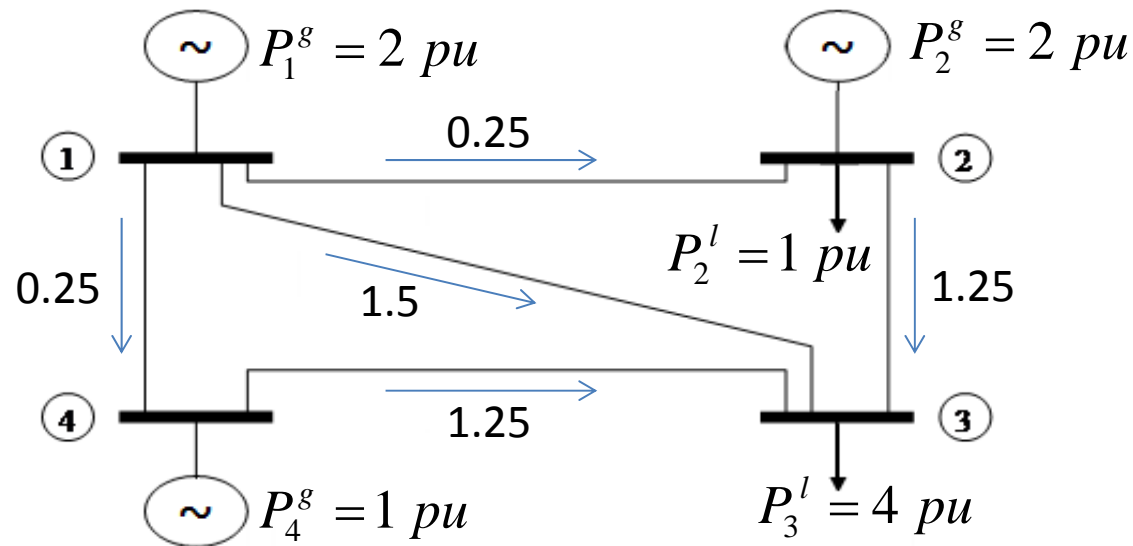
$$P_{12} = B_{12}(\theta_1 - \theta_2) = 10(0 - -0.025) = 0.25$$

$$P_{13} = B_{13}(\theta_1 - \theta_3) = 10(0 - -0.15) = 1.5$$

$$P_{14} = B_{14}(\theta_1 - \theta_4) = 10(0 - -0.025) = 0.25$$

$$P_{23} = B_{23}(\theta_2 - \theta_3) = 10(-0.025 - -0.15) = 1.25$$

$$P_{34} = B_{34}(\theta_3 - \theta_4) = 10(-0.15 - -0.025) = -1.25$$



# Power Flow Equations

- What if the generator connected to bus 1 is **renewable**?
- What if the **capacity** of transmission link (1,3) is 1 pu?
- What if we can apply **demand response** to load bus 3?
- What if one of the transmission lines **fails**?

# Economic Dispatch Problem

- In the example we discussed earlier, we had:

$$\text{Power Supply} = \text{Power Load}$$

- In particular, we had:

$$P_1^g + P_2^g + P_4^g = P_2^l + P_3^l$$

- However, generation levels  $P_1^g$ ,  $P_2^g$ , and  $P_4^g$  assumed given.
- **Q:** What if the generators have **different generation costs?**

# Economic Dispatch Problem

- For thermal power plants, generation cost is quadratic:

$$\text{Generation Cost} = C(P) = a_1 + a_2 \times P + a_3 \times P^2$$

- Example: a grid with three power plants:

$$C_1(P_1) = 561 + 7.92 \times P_1 + 0.001562 \times (P_1)^2 \quad 150 \text{ MW} \leq P_1 \leq 600 \text{ MW}$$

$$C_2(P_2) = 310 + 7.85 \times P_2 + 0.001940 \times (P_2)^2 \quad 100 \text{ MW} \leq P_2 \leq 400 \text{ MW}$$

$$C_3(P_3) = 78 + 7.97 \times P_3 + 0.004820 \times (P_3)^2 \quad 50 \text{ MW} \leq P_3 \leq 200 \text{ MW}$$

- Each power plant has some min and max generation levels.

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- Each power plant has some min and max generation levels.

# Economic Dispatch Problem

- We should select  $P_1$ ,  $P_2$ , and  $P_3$  to:
  - Meet **total load**  $P_{\text{load}} = 850$  MW
  - Minimize the **total cost** of generation
- Economic Dispatch Problem:

$$\underset{P_1, P_2, P_3}{\text{minimize}} \quad C(P_1) + C(P_2) + C(P_3)$$

$$\text{subject to} \quad 150 \leq P_1 \leq 600$$

$$100 \leq P_2 \leq 400$$

$$50 \leq P_3 \leq 200$$

$$P_1 + P_2 + P_3 = 850$$

# Economic Dispatch Problem

- Is the formulated problem a **convex program**? Why?
- Convex programs can be solved efficiently.
- An useful software is CVX for Matlab (<http://cvxr.com/cvx>).
- The **optimal** economic dispatch solution:

$$P_1 = 393.2 \text{ MW}$$

$$P_2 = 334.6 \text{ MW}$$

$$P_3 = 122.2 \text{ MW}$$



**Q:** Do they satisfy all constraints?

# Economic Dispatch Problem

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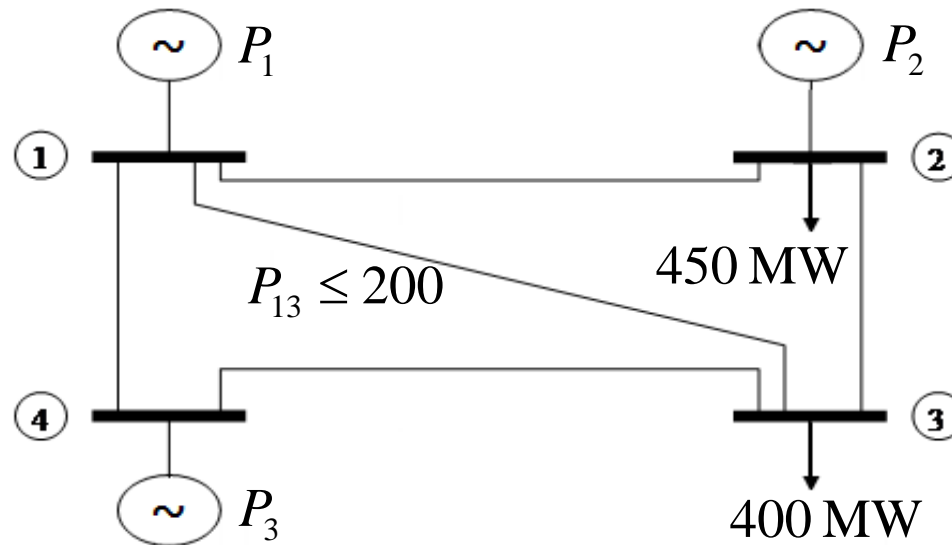
$$P_3 = 122.2 \text{ MW}$$



$$\begin{aligned} \text{Minimum Cost} &= 3916.6 + 3153.8 + 1123.9 \\ &= \mathbf{8194.3} \end{aligned}$$

# Economic Dispatch Problem

- What if we have to satisfy **topology constraints**?

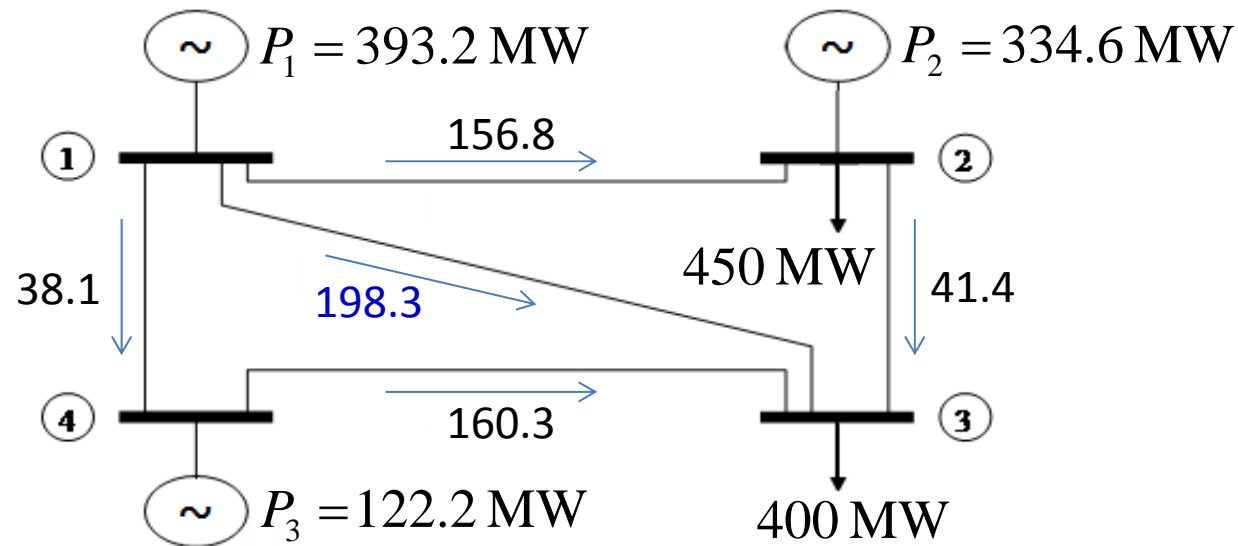


$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} 20 & -10 & 0 \\ -10 & 30 & -10 \\ 0 & -10 & 20 \end{bmatrix}^{-1} \begin{bmatrix} P_2 - 450 \\ -400 \\ P_3 \end{bmatrix}$$

$$P_{13} = B_{13}(\theta_1 - \theta_3) = -10\theta_3 \leq 20 \Rightarrow \theta_3 \geq -20$$

# Economic Dispatch Problem

- The **same** optimal solutions are still valid:



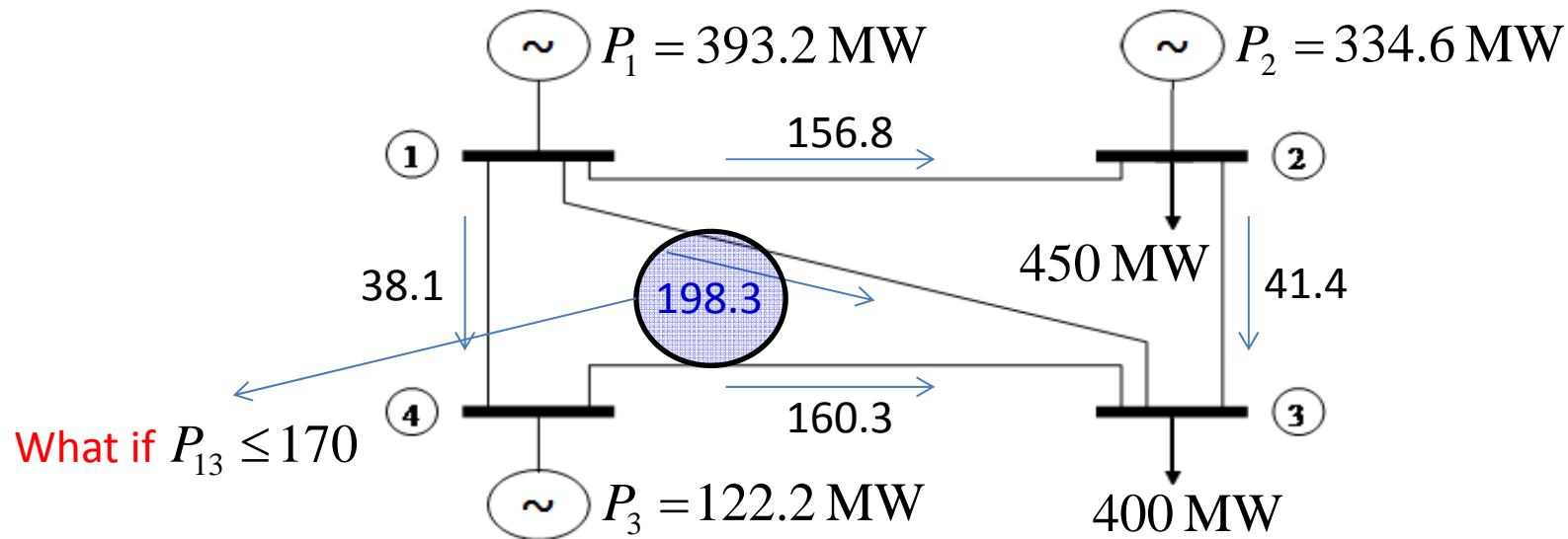
$$\begin{bmatrix} \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix} = \begin{bmatrix} -15.685 \\ -19.830 \\ -3.805 \end{bmatrix}$$

$$\theta_3 \geq -20$$

$$P_{13} \leq 200$$

# Economic Dispatch Problem

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$$\theta_3 \geq -20$$

$$P_{13} \leq 200$$

# Economic Dispatch Problem

- Then the economic dispatch problem becomes:

$$\underset{P_1, P_2, P_3, \theta_1, \theta_2, \theta_3}{\text{minimize}} \quad C(P_1) + C(P_2) + C(P_3)$$

$$\text{subject to} \quad 150 \leq P_1 \leq 600$$

$$100 \leq P_2 \leq 400$$

$$50 \leq P_3 \leq 200$$

$$P_1 + P_2 + P_3 = 850$$

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$$\theta_3 \geq -17$$



# Economic Dispatch Problem

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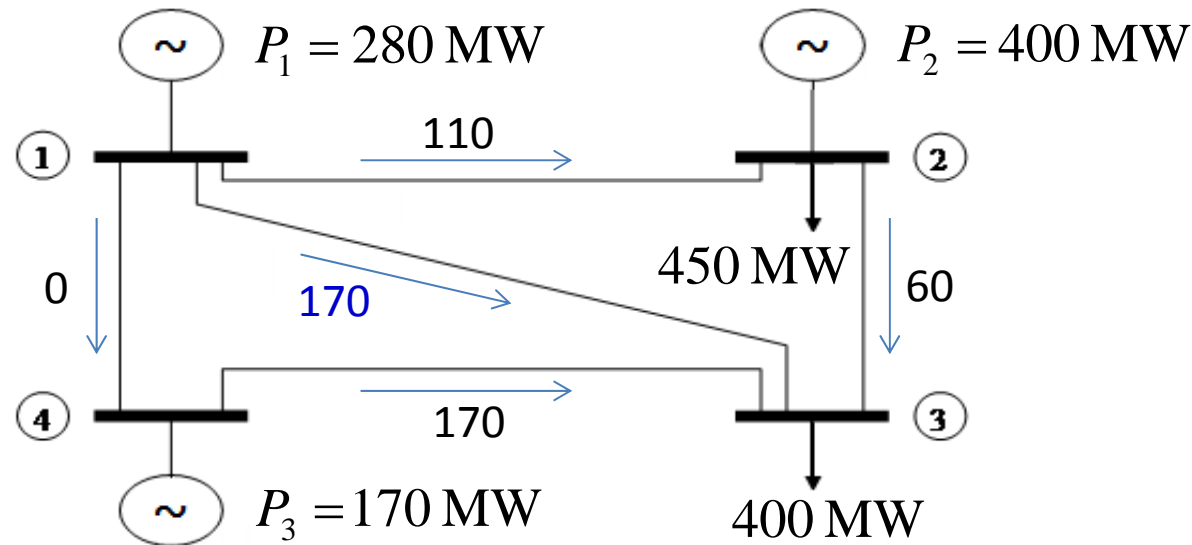
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$$\theta_3 \geq -17$$

Still a Convex Program?

# Economic Dispatch Problem

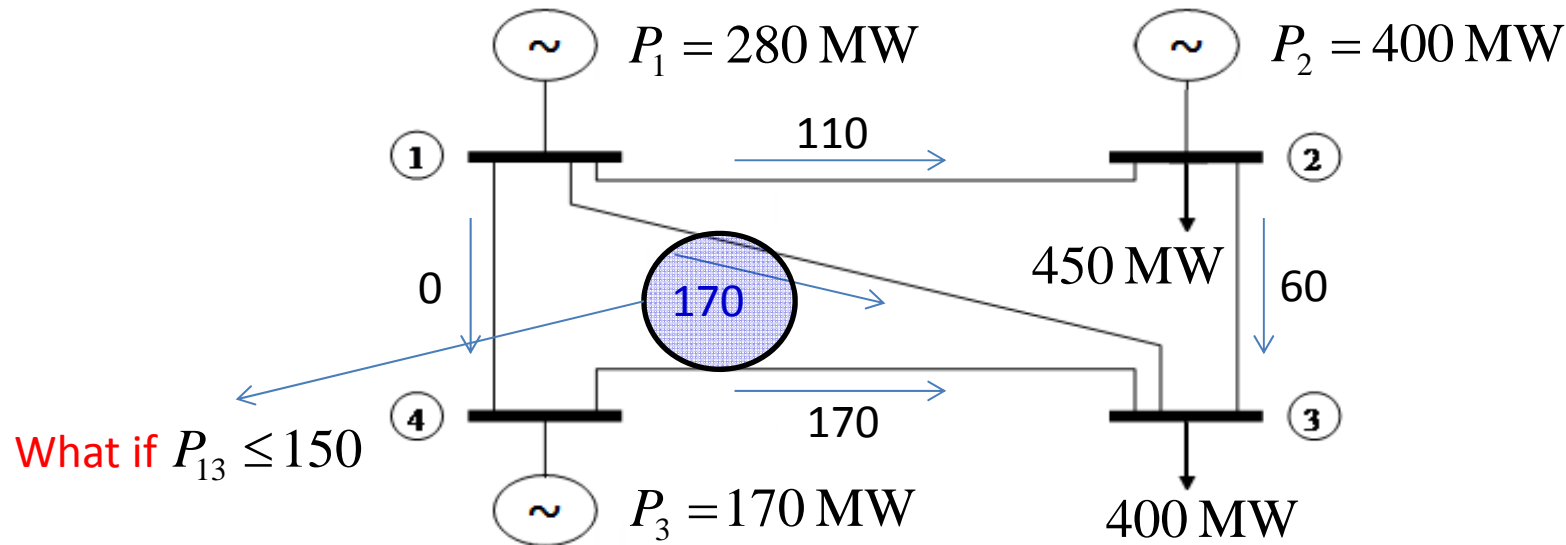
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# Unit Commitment

- Economic Dispatch is solved **a few hours ahead** of operation.
- On the other hand, we need to decide about the choice of power plants that we want to **turn on** for the **next day**.
- This is done by solving the **Unit Commitment** problem.
- We particularly decide on which **slow-starting** power plants we should turn on during the next day given various constraints.
- The mathematical concepts are similar to the E-D problem.

# References

- W. J. Wood and B. F. Wollenberg, *Power Generation, Operation, and Control*, John Wiley & Sons, 2<sup>nd</sup> Ed., 1996.
- J. McCalley and L. Tesfatsion, "Power Flow Equations", *Lecture Notes*, EE 458, Department of Electrical and Computer Engineering, Iowa State University, Spring 2010.