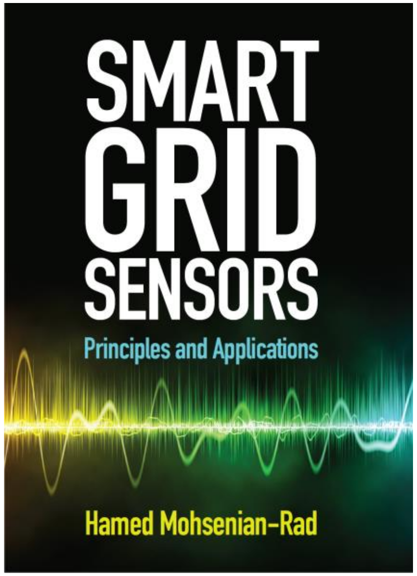
Chapter 5: Power and Energy Measurements and Their Applications



Smart Grid Sensors: Principles and Applications
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Overview

- We covered measuring voltage and current in the previous chapters, whether through RMS representation (**Chapter 2**), phasor representation (**Chapter 3**), and raw waveform representation (**Chapter 4**).

- In this chapter, we will discuss measuring power and energy.

- This is an important topic; because many components in the power grid are currently monitored based only on measuring their power and energy consumption/generation, such as the load of most utility customers.

- Further, there are some smart grid components that are characterized only (or primarily) based on their power and energy characteristics.

- Further, there are smart grid applications that only use power and energy measurements, even if voltage and current measurements are available.

Overview

- Therefore, it is important to know:

  1) How power and energy are measured;

  2) How the measurements of power and energy can be used in various smart grid applications, either when they are the only type of available measurements or when they are available together with other types of measurements.

- In this chapter, we discuss the fundamentals of measuring power and energy and several applications of power and energy measurements.

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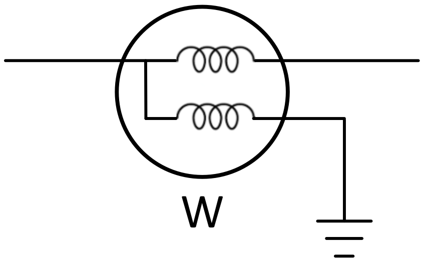
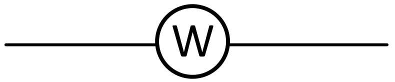
5.1. Measuring Power

- The instrument to measure electric power is the *wattmeter*.



(a)



(b)

- Recall from Section 1.2.4 in Chapter 1 that the *instantaneous power* is obtained by multiplying voltage and current. In an analog wattmeter, this multiplication is done implicitly by using a current coil that is connected in series to the circuit, the top coil in Figure (a), and a potential coil that is connected in parallel to the circuit, the bottom coil in Figure (a).

5.1. Measuring Power

- In a digital wattmeter, the multiplication is done rather explicitly, by measuring voltage using a voltmeter subsystem and measuring current using an ammeter subsystem, then conducting the multiplication.

- The symbol for wattmeters may not always show its internal sub-systems, such as the symbol in Figure (b) on the previous slide.

- Depending on the current rating of the wattmeter and the voltage rating of the wattmeter, we may need to use CTs to *step down current* and/or PTs to *step down voltage* to the levels that work for the wattmeter.

5.1. Measuring Power

- **Example 5.1**: Consider the voltage and current waveforms in Example 3.1 in Chapter 3. Recall that we have:

$$v(t) = 120\sqrt{2}\cos(\omega t)$$

$$i(t) = 1.63\sqrt{2}\cos(\omega t - 0.7532)$$

- The instantaneous power that is delivered to the load is obtained as

$$\begin{aligned} p(t) &= v(t)\, i(t) \\ &= 391.2\cos(\omega t)\cos(\omega t - 0.7532) \\ &= 142.69 + 195.6\cos(2\omega t - 0.7532). \end{aligned}$$

- The above instantaneous power waveform is *purely sinusoidal*. Its frequency is 120 Hz, which is *twice* the frequency of $v(t)$ and $i(t)$.

5.1. Measuring Power

- **Example 5.1 (Cont.)**: Next, consider the voltage and current waveforms in Example 4.1 in Chapter 4. Recall that $v(t)$ is the same but we have:

$$\begin{aligned} i(t) = 1.60\sqrt{2}\cos(\omega t - 0.7532) \\ + 0.27\sqrt{2}\cos(3\omega t - 0.4323) \\ + 0.15\sqrt{2}\cos(5\omega t + 3.3058). \end{aligned}$$

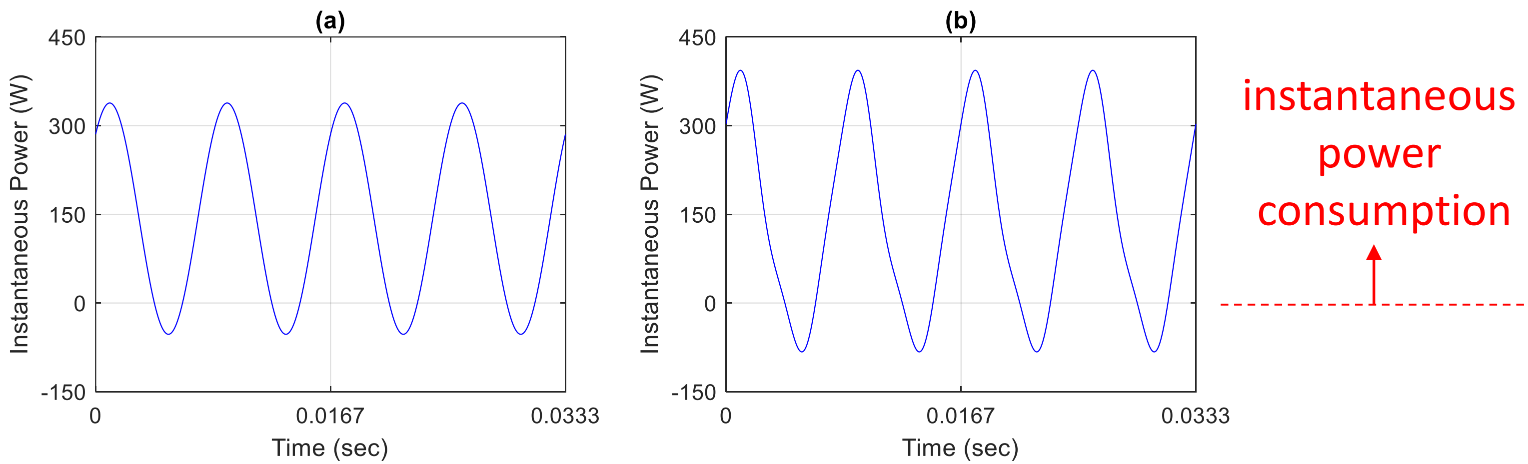
- The instantaneous power that is delivered to the load is obtained as

$$\begin{aligned} p(t) &= v(t)\, i(t) \\ &= 384\cos(\omega t)\cos(\omega t - 0.7532) \\ &\quad + 64.8\cos(\omega t)\cos(3\omega t - 0.4323) \\ &\quad + 36\cos(\omega t)\cos(5\omega t + 3.3058). \end{aligned}$$

- The above instantaneous power waveform is *non-sinusoidal* but periodic, and its frequency is again 120 Hz.

5.1. Measuring Power

- **Example 5.1 (Cont.)**: The instantaneous power waveforms in these two examples are shown below. Note that, in both examples, $p(t)$ has negative values at certain sub-intervals. This is due to the partly inductive (and partly resistive) nature of the motor load in both examples.

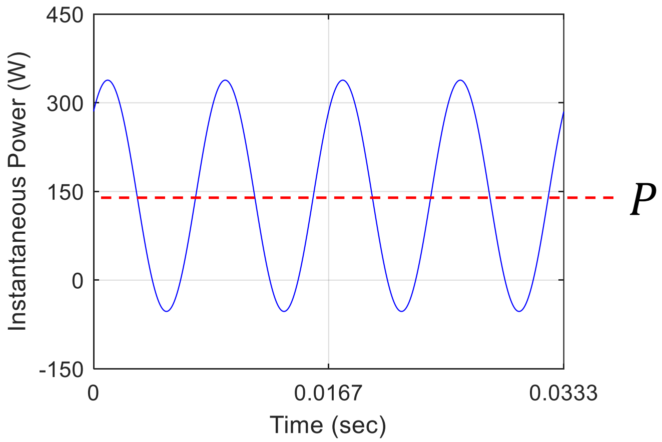


5.1.1. Active Power

- Measuring instantaneous power is rarely of practical use. A wattmeter rather reports the *average* of the instantaneous power across one or multiple cycles, i.e., it measures *active power*; see (1.21) in Chapter 1:

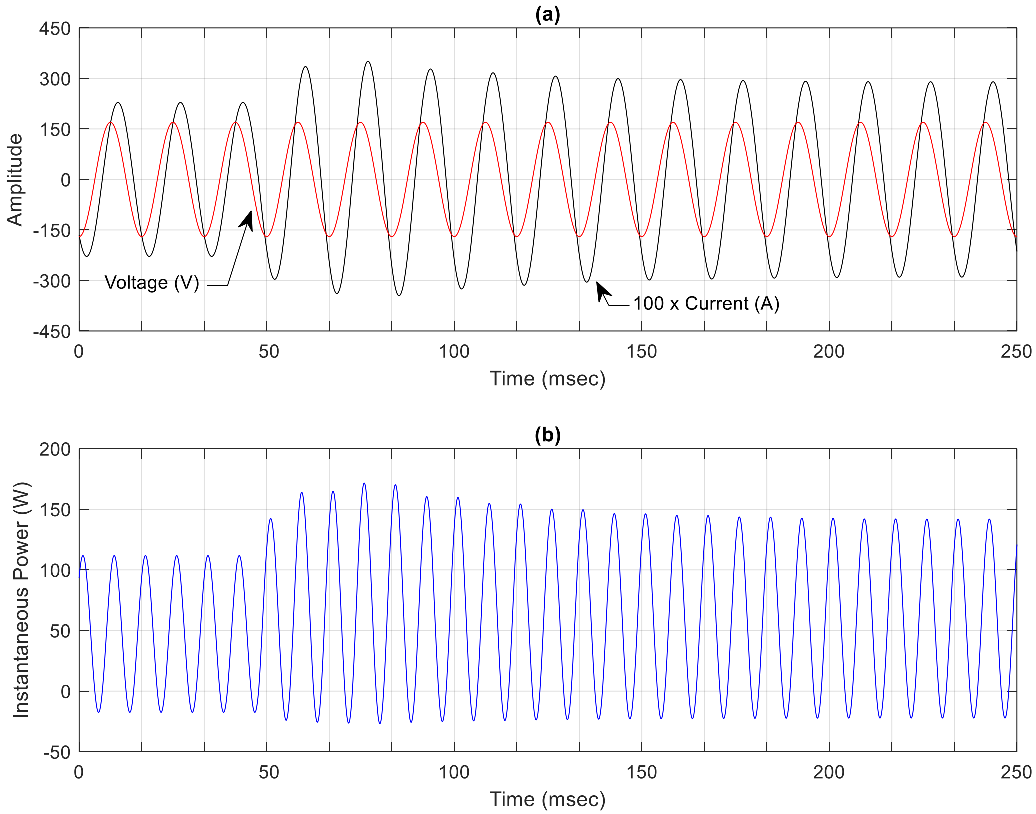
$$P = \frac{1}{T} \int_0^T p(t),$$

where $T = 1/f$. In Example 5.1, we have $P = 142.7\ W$ for the instantaneous power in the first case; and $P = 140.1\ W$ in the second case.



5.1.1. Active Power

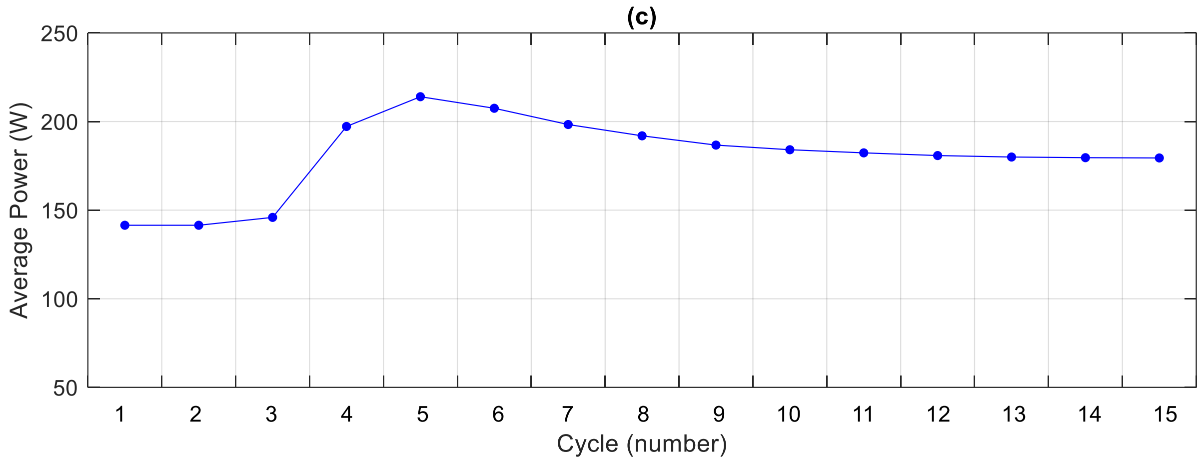
• In practice, the average power itself varies over time. This happens due to the changes in current and/or voltage waveforms. Here is an example.



5.1. Measuring Power

5.1.1. Active Power

• The voltage waveform does not change in this example. But there are changes in the current waveform. The average power changes accordingly.



5.1.1. Active Power

Relationship to Voltage and Current Phasors

- Under *steady-state* conditions, the active power can be obtained for purely sinusoidal voltage and current waveforms based on the voltage and current phasor measurements, see (1.22) in Chapter 1:

$$P = V I \cos(\theta - \phi),$$

where the voltage phasor measurement is denoted by $V\angle\theta$ and the current phasor measurement is denoted by $I\angle\phi$.

5.1.1. Active Power

Relationship to Voltage and Current Phasors

• We can also obtain the average power for voltage and current waveforms that contain *steady-state harmonic distortions* based on the voltage and current harmonic phasor measurements; see Section 4.1.1 in Chapter 4:

$$P = \sum_{h=1}^{\infty} V_h I_h \cos(\theta_h - \phi_h),$$

where at each harmonic order h, the voltage harmonic phasor measurement is denoted by $V_h \angle \theta_h$ and the current harmonic phasor measurement is denoted by $I_h \angle \phi_h$.

5.1. Measuring Power

5.1.1. Active Power

Reporting Rate

- The reporting interval of wattmeters is often much longer than one AC cycle; therefore, active power is *averaged* across several cycles.

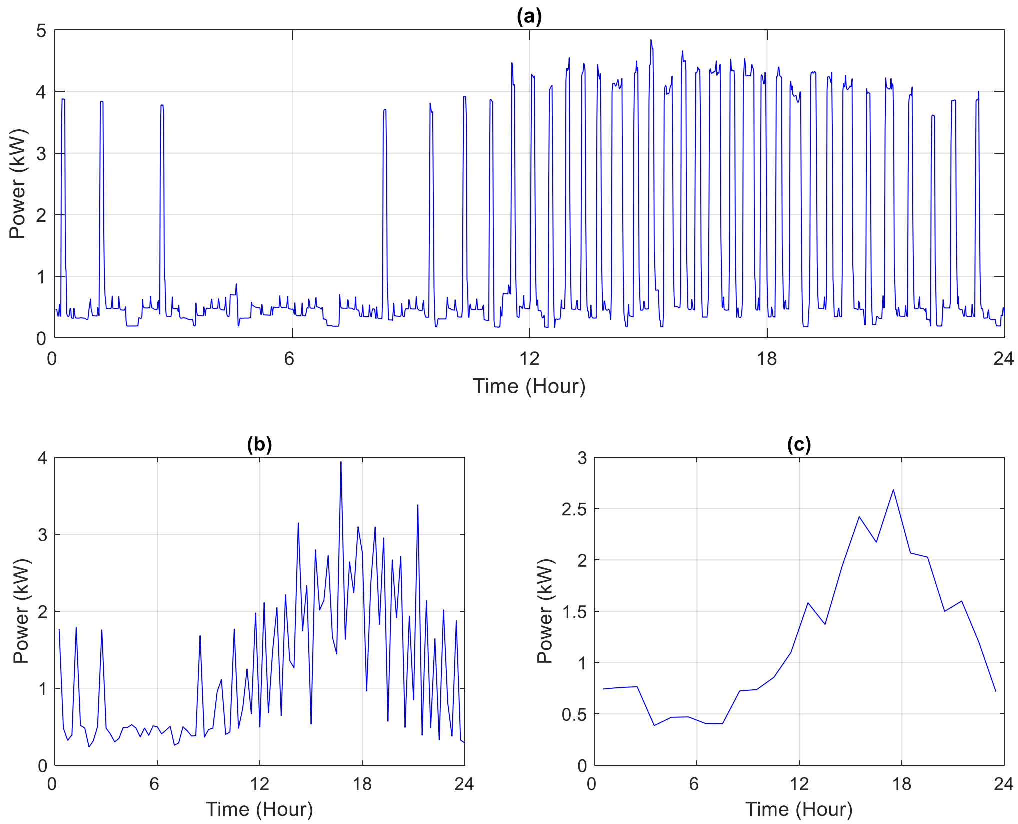
- Power measurements are typically reported once every few seconds to once every few minutes (e.g., every five minutes or every 15 minutes).

- **Example 5.2**: The figure on the next slide shows the power consumption measurements based on *different reporting rates* at a house during a day.
    - See the figure on the next slide.

5.1.1. Active Power

Reporting Rate

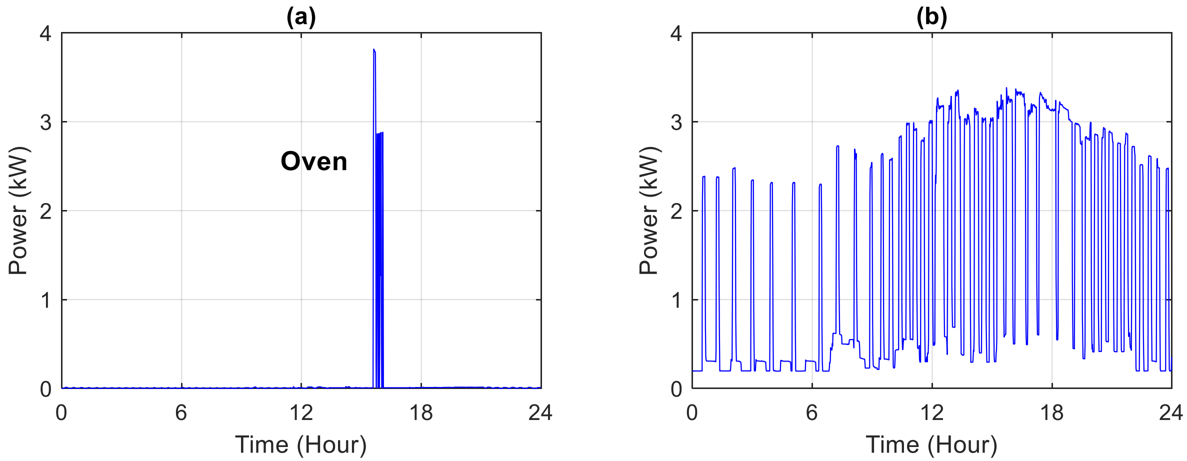
- **Example 5.2 (Cont.)**: A higher reporting rate is more informative and can show more details about power usage: **(a)** one reading per minute [268]; **(b)** one reading per 15 minutes; **(c)** one reading per hour.



5.1.2. Power Profile

- It is informative to examine the daily profiles ofn power measurements.

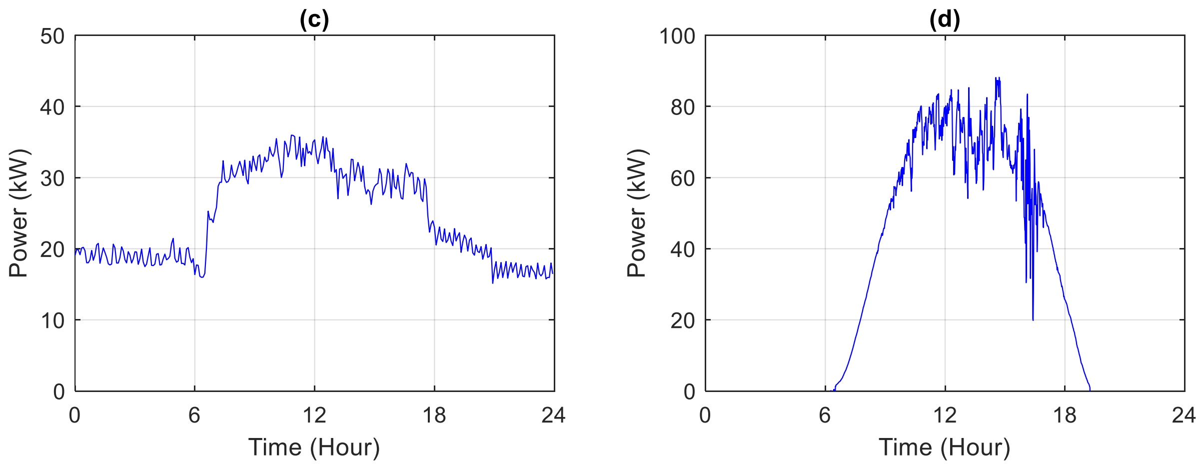
- Figure **(a)**: the daily profile for power consumption at *one* appliance, which is an oven. The appliance operates only once in late afternoon.



- Figure **(b)**: the daily profile for power consumption at a house. It is essentially the *aggregation* of the power profiles for *all* appliances.

5.1.2. Power Profile

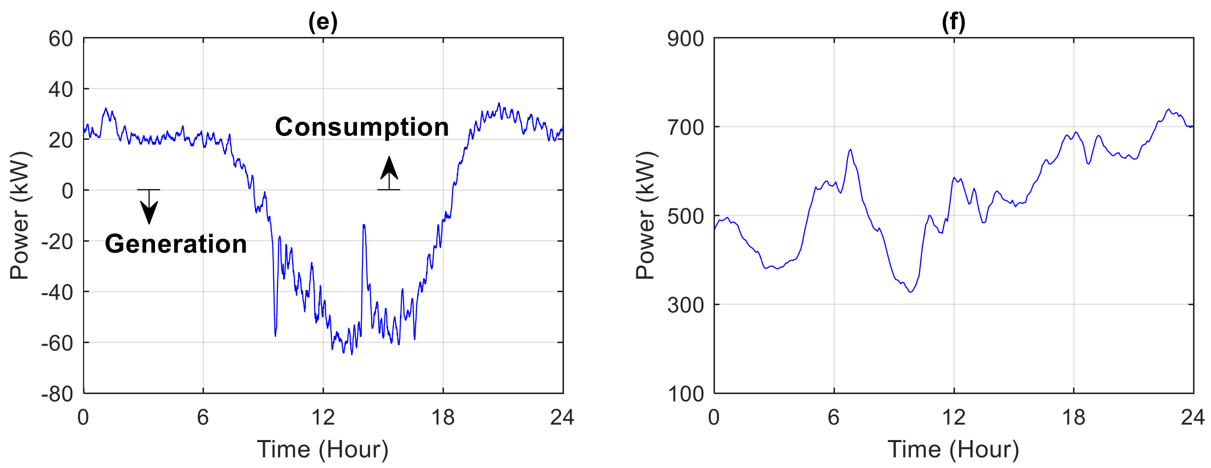
• Figure **(c)**: the daily profile for power consumption at a commercial building on a weekday. The load highly increases during *business hours*.



• Figure **(d)**: the daily profile for power generation at a solar power generation unit during a partially cloudy day. Power generation varies due to the variations in solar irradiance and the *movement of the clouds*.

5.1.2. Power Profile

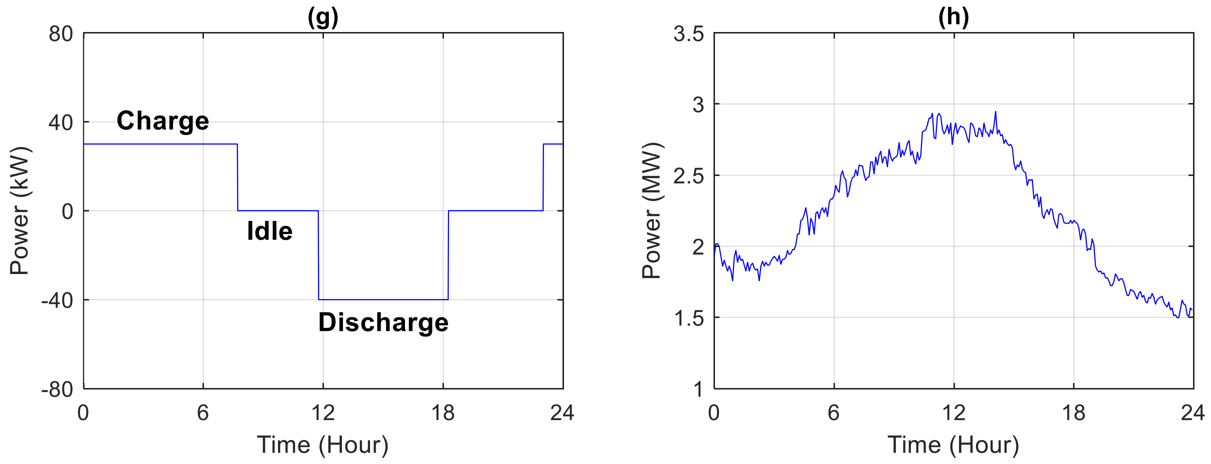
• Figure **(e)**: the daily profile for a mix of power consumption and power generation at a commercial building with on-site solar power generation. The building acts as a *load at night* and as a *generator during the day*.



• Figure **(f)**: the daily profile for power generation at a *wind turbine*, which varies based on weather conditions and the direction and speed of wind.

5.1.2. Power Profile

• Figure **(g)**: the daily profile based on the *charge and discharge cycles* of a grid-connected battery energy storage system.



• Figure **(h)**: the daily profile for the power flow at a power distribution line. The variations are due to the *changes in the load of the feeder*.

5.1.2. Power Profile

Load Factor

- Given a power consumption profile, its load factor is obtained as:

$$\text{Load Factor} = \frac{\text{Maximum of the Power Profile}}{\text{Average of the Power Profile}}.$$

- For example, if the peak load and the average load of a customer during a day is 4 kW and 1.6 kW, respectively, then the customer’s load factor is 2.5. Load factor indicates *how balanced (over time)* the load power profile is during a given time-period of interest.

- Load factor of 1 means a flat load profile.

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5.2. Measuring Reactive Power and Power Factor

- The (traditional) instrument to measure reactive power is a *varmeter*.

- Given the measurements of active power (from a wattmeter) and reactive power (from varmeter), one can calculate power factor.

- However, to increase accuracy, a separate electromechanical instrument, called a *power factor meter*, is used to measure power factor.

- These instrumentation distinctions are of concern mainly in electromechanical instruments. A *single digital sensor* is often capable of serving as a wattmeter, a varmeter, and a power factor meter.

5.2. Measuring Reactive Power and Power Factor

Relationship to Voltage and Current Phasors

- Both reactive power and power factor are primarily defined based on *purely sinusoidal* voltage and current waveforms under *steady-state* conditions; see Section 5.2.3 for the non-sinusoidal case.

- From (1.25) in Chapter 1 we have:

$$Q = V\, I \sin(\theta - \phi)$$

$$\mathrm{PF} = \cos(\theta - \phi)$$

5.2. Measuring Reactive Power and Power Factor

Relationship to Voltage and Current Phasors

• If the voltage or current waveforms are *distorted*, i.e., not purely sinusoidal, then the common approach is to obtain Q and PF based on the fundamental components of the voltage and current waveforms:

$$Q = V_1 I_1 \sin(\theta_1 - \phi_1)$$

$$\mathrm{PF} = \cos(\theta_1 - \phi_1)$$

where $V_1 \angle \theta_1$ and $I_1 \angle \phi_1$ are the phasors of the fundamental components of the voltage and the current waveforms, respectively.

• There is also an *alternative approach* to work directly with the distorted voltage or current waveform that we will see in Section 5.2.3.

5.2. Measuring Reactive Power and Power Factor

Relationship to Voltage and Current Phasors

- In Example 5.1 (Slide 10), we have:

$$\begin{aligned} i(t) = 1.60\sqrt{2}\cos(\omega t - 0.7532) \\ + 0.27\sqrt{2}\cos(3\omega t - 0.4323) \\ + 0.15\sqrt{2}\cos(5\omega t + 3.3058). \end{aligned}$$

Therefore, the fundamental component of the current waveform is $1.60\sqrt{2}\cos(\omega t - 0.7532)$. Therefore, we have $I_1 \angle \phi_1 = 1.6 \angle - 0.7532$. Since the voltage is purely sinusoidal, we have $V_1 \angle \theta_1 = 120\angle 0$.

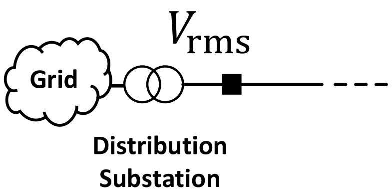
Therefore, we can obtain:

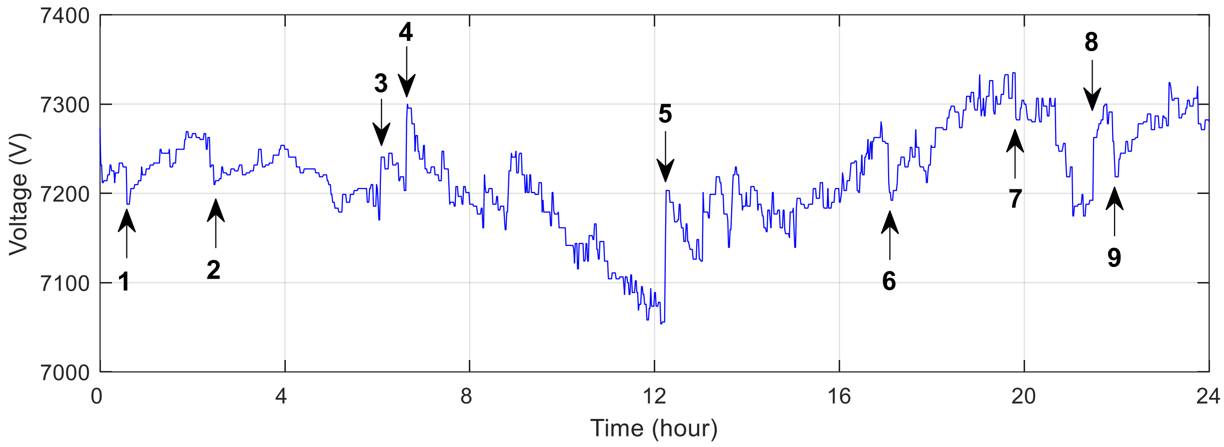
$Q = 131.3\ \mathrm{VAR}$ and $PF = 0.7295.$

5.2.1. Reactive Power and Power Factor Profiles

• The daily profiles of reactive power and power factor can often reveal useful information about the operation of the power system.

• For instance, recall the events that we had identified in Example 2.9 in Chapter 2 in the daily RMS voltage and current profiles at a power distribution feeder. The voltage profile is copied here for your reference:



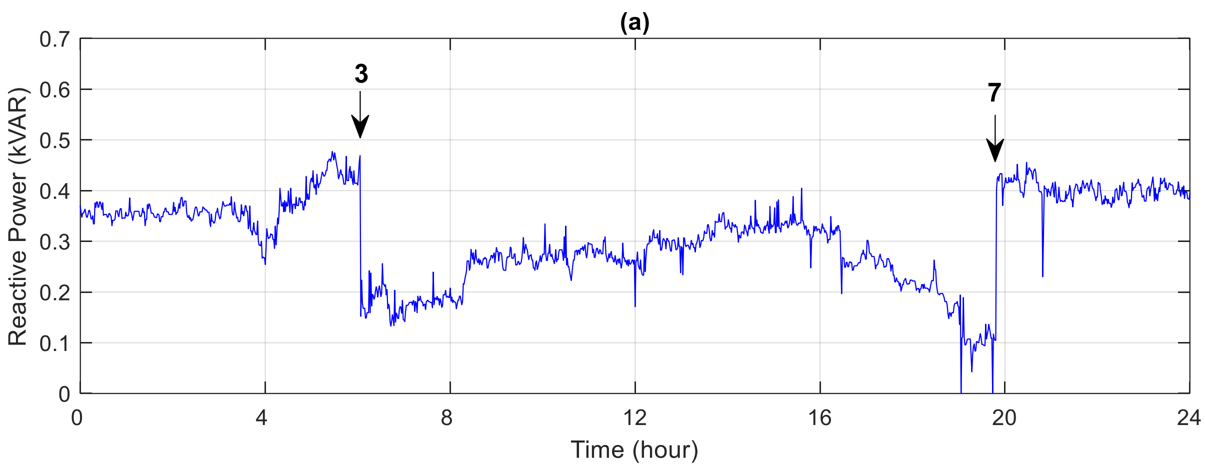


5.2. Measuring Reactive Power and Power Factor

5.2.1. Reactive Power and Power Factor Profiles

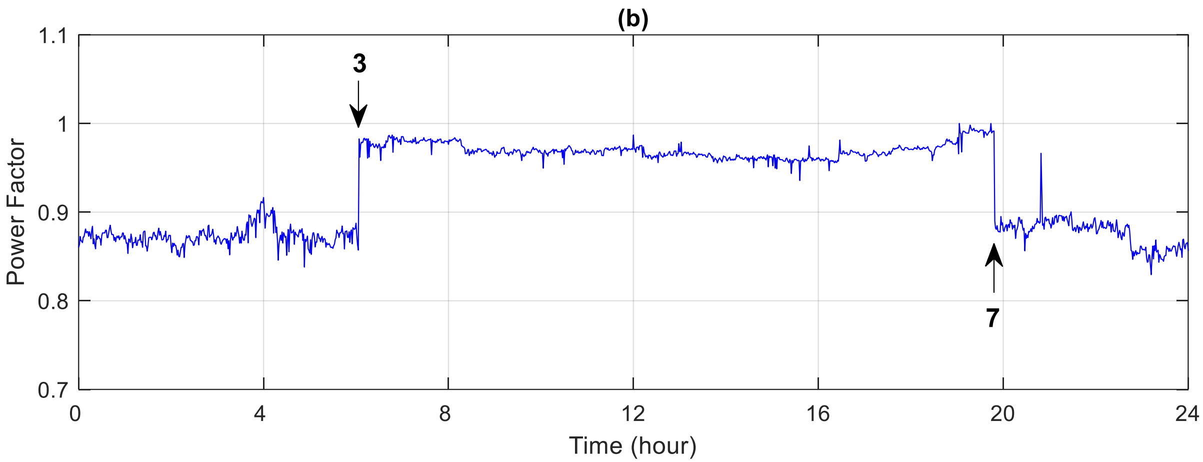
• Similar (or additional) information can be extracted also from the daily reactive power profile or daily power factor profile.

• **Example 5.3**: The daily reactive power load profile on the *same day* and at the *same feeder* as in Example 2.9 in Chapter 2 is shown below.



5.2.1. Reactive Power and Power Factor Profiles

• **Example 5.3 (Cont.)**: The daily power factor profile is also shown below.



• The causes for events 3 and 7 are now clear. At event 3, there is a sudden increase in the power factor of the feeder (due to the supply of reactive power by turning on a capacitor bank on the feeder). At event 7, there is a sudden decrease in power factor (due to turning off the capacitor bank).

5.2.2. Apparent Power

- Apparent power can be measured in different ways:

  - Measuring the RMS voltage (voltmeter) and RMS current (ammeter):

$$S = V_{\mathrm{rms}}\, I_{\mathrm{rms}}$$

  - Measuring active power (wattmeter) and reactive power (varmeter):

$$S = \sqrt{P^2 + Q^2}$$

  - Measuring active power (wattmeter) and power factor (power factor meter) as well measuring reactive power (varmeter) and power factor:

$$S = P/\mathrm{PF} = Q/\sqrt{1 - \mathrm{PF}^2}$$

5.2. Measuring Reactive Power and Power Factor

5.2.3. True Power Factor

- If the voltage or current waveforms are distorted, then one option is to define power factor based on the fundamental component of the distorted voltage or current. We already saw this definition on Slide 28:

$$\mathrm{PF} = \cos(\theta_1 - \phi_1)$$

- The above is often referred to as the *displacement power factor* (DPF).

- We can also measure the *true power factor* (TPF), which is obtained as:

$$\mathrm{TPF} = \frac{P}{S}$$

Average Power

$V_{\mathrm{rms}}\, I_{\mathrm{rms}}$

5.2.3. True Power Factor

- **Example 5.4**: Consider the voltage and current waveforms in Example 4.1 in Chapter 4. Recall that the current waveform is distorted. We have:

$$V_{\mathrm{rms}} = 120 \text{ V}, \quad I_{\mathrm{rms}} = 1.63 \text{ A}, \quad S = 195.6 \text{ VA}$$

- The average power in this example is P = 140.1 W. Therefore, we have:

$$\mathrm{TPF} = \frac{140.1}{195.6} = 0.7163$$

- We can see that TPF is smaller than DPF, which is 0.7295 in this example.

5.2.3. True Power Factor

• In practice, harmonic distortion is very small in voltage compared to in current. Thus, the contribution of the harmonics to the delivery of active power is not significant. Thus, it is reasonable to approximate $V_{\text{rms}} \approx V_1$ and $P \approx V_1 I_1 \cos(\theta_1 - \phi_1)$. From these, together with the relationship between I_1, I_{rms}, and THD in (4.6) in Chapter 4, we can derive:

$$\text{TPF} = \frac{P}{S} \approx \frac{V_1 I_1 \cos(\theta_1 - \phi_1)}{V_1 I_{\text{rms}}} = \frac{\text{DPF}}{\sqrt{1 + \text{THD}^2}},$$

where THD is associated with the current waveform

From the above relationship, we can show that TPF is always *less than or equal* to DPF. If $\text{THD} = 0$, i.e., if there is no distortion, then $\text{TPF} = \text{DPF}$.

5.2.3. True Power Factor

Distortion Power: There are different ways to define reactive power in presence of non-sinusoidal voltage and current signals.

For example, *six different definitions* for reactive power in this context are surveyed in [270, Section 2.4]. One option is to follow the definition of active power on Slide 16 and define reactive power as

$$Q = \sum_{h=1}^{\infty} V_h I_h \sin(\theta_h - \phi_h).$$

The triangular equality does not hold among S, P, and Q. In fact, we can show that S_2 is always greater than or equal to $P^2 + Q^2$; see Exercise 5.5.

5.2.3. True Power Factor

Distortion Power: Therefore, a quantity named distortion power, denoted by D, is defined as follows:

$$D^2 = S^2 - P^2 - Q^2,$$

which yields the equation $S^2 = P^2 + Q^2 + D^2$. If the voltage and current waveforms are purely sinusoidal, then $D = 0$ and $S^2 = P^2 + Q^2$.

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5.3. Measuring Energy

- The instrument to measure electric energy is a *watthour meter*.

- Energy is measured over a certain time interval by taking the integral of instantaneous power on the interval of interest. The time interval can be one hour, one day, or one month. For the interval between time t_1 and time t_2, energy E is calculated as the integral of instantaneous power $p(t)$ from $t = t_1$ to $t = t_2$, as shown below:

$$E = \int_{t=t_1}^{t_2} p(t)dt.$$

5.3. Measuring Energy

- In a traditional *analog watthour meter*, the integral is taken implicitly by counting the rotations of a metal disc, which is made to rotate at a speed proportional to the power passing through the meter.

- In a *digital watthour meter*, the integral is calculated explicitly, by measuring power and using discrete summation while taking into account the sampling rate for accurate computation.

- The unit of measuring E is watt hour.

- For the two scenarios in Example 5.1, the energy that is delivered to the motor load over a one-minute interval is obtained as 2.38 Wh and 2.34 Wh, respectively. The energy that is delivered over a one-hour interval is obtained as 142.8 Wh and 140.4 Wh, respectively.

5.3. Measuring Energy

Reporting Interval: The reporting interval for measuring energy could be fixed, or it could vary. This is because the energy measurements can be reported in *fixed intervals*, or they can be reported in *fixed increments*.

5.3.1. Fixed Intervals

- It is typical for energy measurements to be reported in fixed intervals.

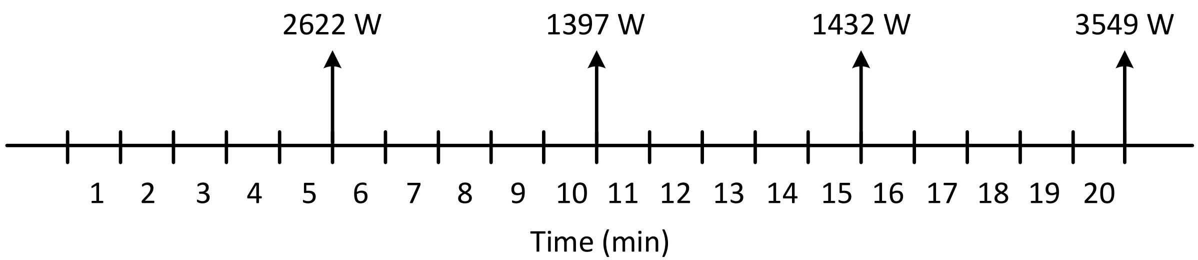
- For instance, suppose the minute-by-minute energy usage of a building over a period of 20 minutes is as shown in the Table.

Minute	Energy Usage	Minute	Energy Usage
1	383	11	160
2	502	12	267
3	630	13	404
4	446	14	281
5	661	15	320
6	378	16	549
7	269	17	648
8	330	18	805
9	298	19	718
10	122	20	829

5.3. Measuring Energy

5.3.1. Fixed Intervals

- Suppose the watthour meter reports the measurements *once every five minutes*. Figure (a) shows the reported measurements.



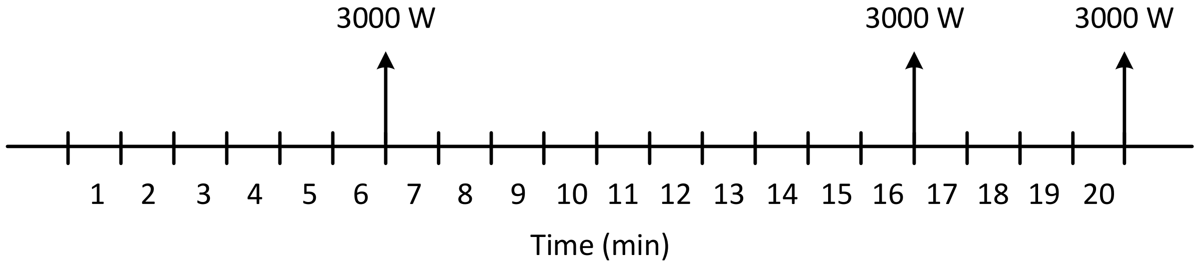
(a)

- The location of each arrow indicates when each measurement is reported. The number above each arrow indicates the energy measurement that is reported by the watthour meter. *The amount that is reported can vary, but the reporting is done at fixed intervals.*

5.3.2. Fixed Increments

• Some watthour meters operate as *pulse meters* and report energy usage in fixed increments. Each pulse corresponds to a certain amount of energy in kWh. Accordingly, one can calculate the amount of energy usage by counting the number of pulses in an interval of interest.

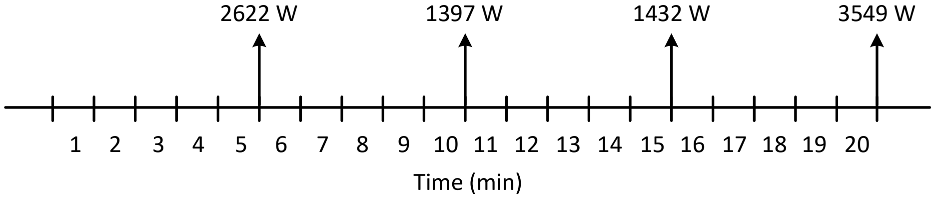
• Again, consider the minute-by-minute energy usage on Slide 43. Suppose the watthour meter reports the measurements once every 3000 W, i.e., at fixed increments. Figure (b) shows the reported measurements.



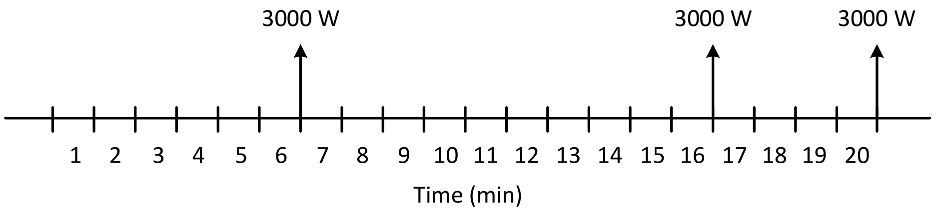
(b)

5.3.1. Fixed Intervals

• Let us now compare the two graphs on slides 44 and 45. In each case, the location of each arrow indicates when each measurement is reported.



(a)



(b)

• The number above each arrow indicates the energy measurement that is reported by the watthour meter. The amount that is reported in Figure (b) is always fixed, but the reporting is done at varying intervals.

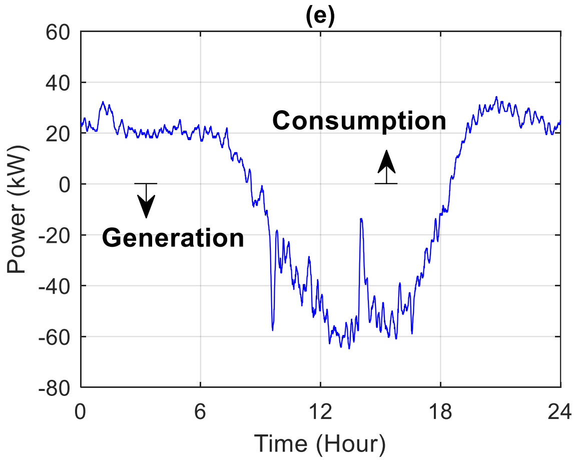
5.3. Measuring Energy

5.3.1. Fixed Intervals

- Notice that, based on the Table of Slide 43:
  - The sum of the energy usage during the first 6 minutes is 3000 W.
  - The sum of the energy usage during the next 10 minutes is 3000W.
  - The sum of the energy usage during the last 4 minutes is 3000 W.

5.3.2. Fixed Increments

• Recall from Figure (e) on Slide 21 in Section 5.1.2 that a building with behind-the-meter solar power generation sometimes acts as a power consumer and at other times acts as a power producer.



• The consumers who also produce and share surplus energy with the power grid are often referred to as *prosumers*; e.g., see [271–273].

5.3.3. Net Energy Metering and Feed-In Energy Metering

• Measuring energy for prosumers can be done in two different ways.

• One option is *net energy metering* (NEM).

• In this option, energy measurement is *bidirectional*.

• Energy measurement is positive when energy is consumed by the prosumer; and it is negative when energy is produced by the prosumer.

• Energy measurements from NEM can be used to bill prosumers. In this scenario, when the prosumer acts as an energy "producer", it is credited by the utility for its excess energy generation *at the same price* that the it is charged for its energy consumption when it acts as a "consumer".

5.3. Measuring Energy

5.3.3. Net Energy Metering and Feed-In Energy Metering

• Another option is *feed-in energy metering* (FIEM).

• In this option, energy measurement is *unidirectional*; energy usage is measured separately, and energy generation is also measured separately.

• This option often requires using two watthour meters, one to measure energy usage and one to measure energy generation. The second watthour meter is installed at the local source of energy generation.

• FIEM can be used to bill prosumers based on Feed-in Tariffs (FIT).

• In this billing scenario, the prosumer is credited by the utility for its excess energy generation at a price that is different from the price that the prosumer is charged for its energy consumption; e.g., see [275].

5.3.3. Net Energy Metering and Feed-In Energy Metering

- **Example 5.5**: A prosumer consumes 298.7 kWh energy during one day. On that same day, this prosumer generates 384.1 kWh solar energy.

First, suppose the prosumer is charged at 12 ¢/kWh for its energy usage and it is credited at the same price for its energy generation. By using *net energy metering*, this prosumer is credited (384.1−298.7)×0.12 = **$10.25**.

Next, suppose the prosumer is charged at 12 ¢/kWh for its energy usage and it is credited at 27 ¢//kWh for its energy generation [276]. By using *feed-in energy metering*, this prosumer is credited 384.1×0.27−298.7×0.12 = **$67.86**. In this example, an FIT pays *more* than the retail electricity rate for renewable power generation; which is intended to provide incentives to consumers to install solar panels.

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5.4. Smart Meters and Their Applications

• Electric meters are watthour meters that are used by utilities to measure the amount of electric energy consumed by a customer, such as a residence or a business, for billing purposes.

• Traditionally, electric meters have been *read manually* by the utility personnel, once per billing period, such as one every month.

• The next generation of electric meters are called “smart meters”.

• Smart meters are digital energy meters that record customer energy usage *much more frequently*, such as once every 15 minutes, and may they also provide near real-time automated meter reading (AMR) to the utility, through a two-way communications infrastructure.

5.4. Smart Meters and Their Applications

- Smart meters record not only the energy usage but also the *time* and *date* of energy usage (i.e., they provide time-stamped energy usage). This is in sharp contrast to traditional mechanical meters that record only the incremental electricity usage and do not record the time or date of usage.

- In addition to measuring energy usage, some smart meters also measure *RMS voltage*, *RMS current*, *power factor*, and *power quality*.

- Greater clarity on their consumption behavior for customer

- Helping the utility with enhanced system monitoring and the ability to implement different billing mechanisms, such as time-of-use pricing.

- Smart meters are a widely deployed smart grid technology to date.

5.4. Smart Meters and Their Applications

5.4.1. Price-Based Demand Response

• Demand response (DR) programs enable consumers to *reduce* their electricity usage during peak periods, or *shift* part of their usage to off-peak periods, in response to time-of-use rates or other financial incentives.

• Two categories of DR programs: 1) *price-based;* and 2) *incentive-based*.

• Smart meters enable DR programs in both categories.

• In this section, we discuss the use of smart meters to facilitate price-based DR. We will discuss incentive-based DR in Section 5.4.2.

5.4. Smart Meters and Their Applications

5.4.1. Price-Based Demand Response

- **Time-of-Use Pricing**: Traditionally, utility customers have been charged with *flat rates*. If a flat rate is used, then all its usage during a given period of time, such as a monthly billing cycle, is charged with the *same rate*.

- For instance, if the flat rate is 12 ¢/kWh, and the customer’s usage is 914 kWh in one month, then this customer is charged 914 × 0.12 = $109.68.

- It does not matter at what time during the day or night the energy was consumed. It also does not matter whether the energy consumption occurred during weekdays or weekends. The rate is the same in all cases.

5.4.1. Price-Based Demand Response

• **Time-of-Use Pricing (Cont.)**: However, flat rates do not reflect the variations in the cost of electricity generation, such as the higher cost of generation during peak demand hours [279].

• Smart meters can keep track of the energy usage during different times of a day and different days of a week. Therefore, they can facilitate billing customers based on time-of-use (ToU) pricing.

• Under a ToU rate plan, price of electricity varies depending on the time of day, day of week, and season. Prices are higher during *peak demand hours* and lower during *low demand hours*.

5.4.1. Price-Based Demand Response

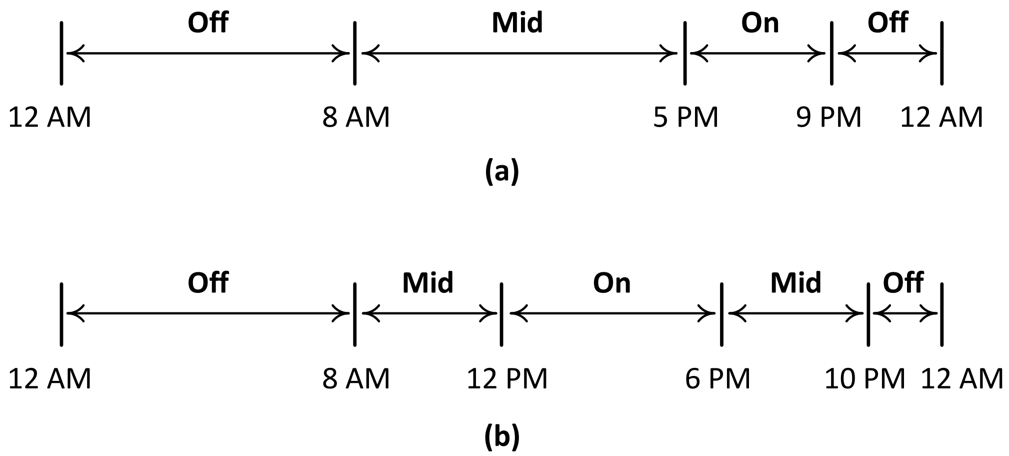
• **Time-of-Use Pricing (Cont.)**: Here are the rates for a real-world example for ToU prices that was used by a utility in California [280]:

- On-Peak Hours: 10.33 ¢/kWh
- Mid-Peak Hours: 8.28 ¢/kWh
- Off-Peak Hours: 7.27 ¢/kWh.

• Notice that the price of electricity is 42% higher during on-peak hours compared to the prices during the off-peak hours.

5.4.1. Price-Based Demand Response

- Figures (a) and (b) show the on-peak, mid-peak, and off-peak hours for this utility during *winter* and *during* summer, respectively.



5.4.1. Price-Based Demand Response

• **Example 5.6**: Suppose the hourly electricity usage of a utility customer is reported by a smart meter during a day in summer, as shown below:

Hour	Energy Usage	Hour	Energy Usage	Hour	Energy Usage
1	0.7428	9	0.7240	17	2.1726
2	0.7575	10	0.7372	18	2.6853
3	0.7650	11	0.8557	19	2.0673
4	0.3867	12	1.0984	20	2.0270
5	0.4673	13	1.5828	21	1.4991
6	0.4711	14	1.3717	22	1.5997
7	0.4069	15	1.9402	23	1.2005
8	0.4048	16	2.4208	24	0.7182

• The customer is charged based on the ToU rates on slides 59 and 60. The energy use of this customer during the on-peak hours is 12.173 kWh, which is the sum of its hourly usages at hours 13, 14, 15, 16, 17, and 18.

5.4.1. Price-Based Demand Response

• **Example 5.6 (Cont.)**: The energy usage during the mid-peak hours is 10.608 kWh. The energy usage during the off-peak hours is 6.321 kWh. Based on the information on Slides 59 and 60, the energy usage charges during different periods of the day are obtained as follows:

$$\text{On-Peak Charge} = 0.1033 \times 12.1734 = \$1.26,$$

$$\text{Mid-Peak Charge} = 0.0828 \times 10.6084 = \$0.88,$$

$$\text{Off-Peak Charge} = 0.0727 \times 6.3208 = \$0.46.$$

• Accordingly, the total energy usage charge to this customer during this day is obtained as $2.60 dollars, i.e., the sum of the above numbers.

5.4. Smart Meters and Their Applications

5.4.1. Price-Based Demand Response

- **Time-Shiftable Loads**: ToU pricing encourages customers to *shift* their energy use from on-peak hours to off-peak or mid-peak hours.

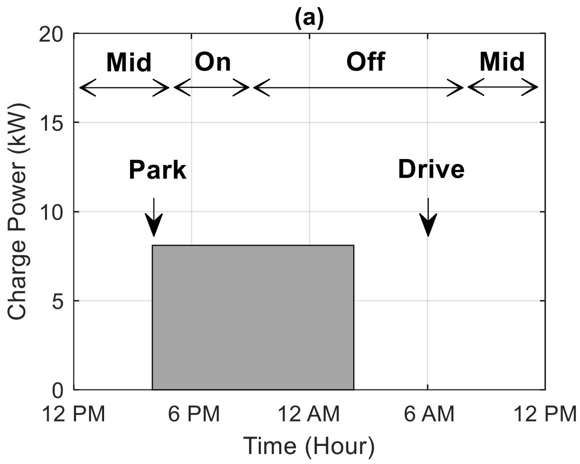
- If customers have energy usage that can be shifted from peak hours to off-peak hours, then they can reduce their energy bill.

- **Example 5.7**: An electric vehicle (EV) is parked at home and plugged in to an EV Charger from 4:00 PM till 6:00 AM. This EV needs 75 kWh to be charged. The rate of charge is 7.7 kW and the charge efficiency is 95%. Accordingly, this EV needs to be charged for

  - (75/0.95)/7.7 = 10.25 hours, i.e., 10 hours and 15 minutes.

5.4.1. Price-Based Demand Response

- **Example 5.7 (Cont.)**: If the customer is charged for its energy usage at a *flat rate*, then it is natural for the customer to start charging as soon as the EV is plugged in. In this scenario, the EV is charged from 4:00 PM till 2:15 AM; see Figure (a).

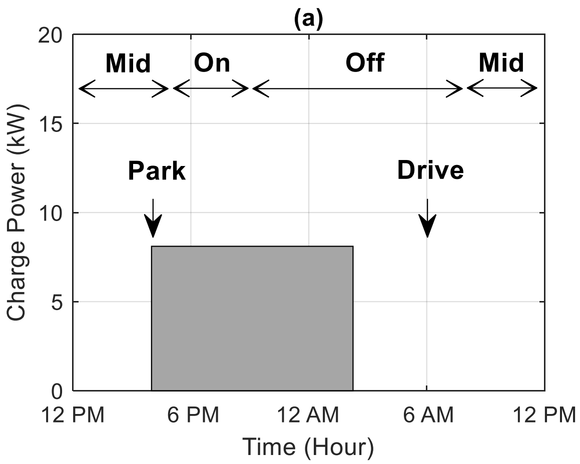


However, if the customer is charged for its energy usage based on ToU rates, then the customer can *schedule* charging the EV to lower its cost.

5.4. Smart Meters and Their Applications

5.4.1. Price-Based Demand Response

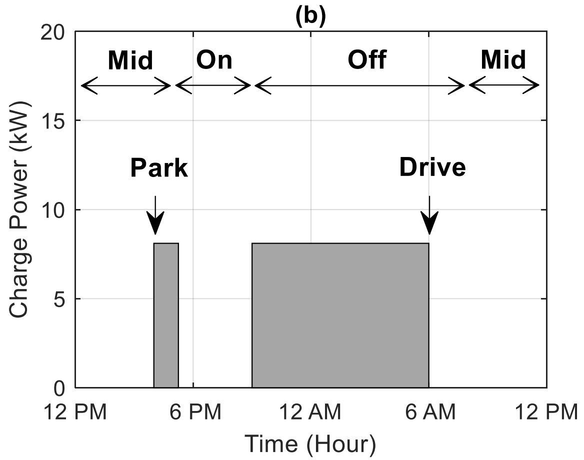
- **Example 5.7 (Cont.)**: If the customer is charged for its energy usage at a flat rate, then it is natural for the customer to start charging as soon as the EV is plugged in. In this scenario, the EV is charged from 4:00 PM till 2:15 AM; see Figure (a).



5.4. Smart Meters and Their Applications

5.4.1. Price-Based Demand Response

- **Example 5.7 (Cont.)**: Suppose the ToU rates are 10.33 ¢/kWh, 8.28 ¢/kWh, and 7.27 ¢/kWh during the on-peak, mid-peak, and off-peak hours, respectively. Suppose the on-peak, mid-peak, and off-peak hours are as in the top figure on Slide 60, which indicates operation during the winter. The energy usage cost to charge this EV is minimized if the charging is scheduled in two intervals, first from 4:00 PM till 5:15 PM, and then from 9:00 PM till 6:00 AM; see Figure (b).



5.4.1. Price-Based Demand Response

• **Example 5.7 (Cont.)**: In this scenario, the EV is charged for one hour during the mid-peak period, for 15 minutes during the on-peak period, and for nine hours during the off-peak period. This *schedule* results in

$$0.1033 \times 0.25 + 0.0828 \times 1 + 0.0727 \times 9 = 76¢$$

as the total energy usage cost to charge this EV.

In contrast, if the EV is charged as soon as it is plugged in, then the total energy usage cost of charging becomes:

$$0.1033 \times 4 + 0.0828 \times 1 + 0.0727 \times 5.25 = 88¢$$

5.4.1. Price-Based Demand Response

• An EV is a *time-shiftable* load, also known as a *deferrable* load. A time-shiftable load requires a certain amount of energy within a given time period; however, the timing of operation is flexible.

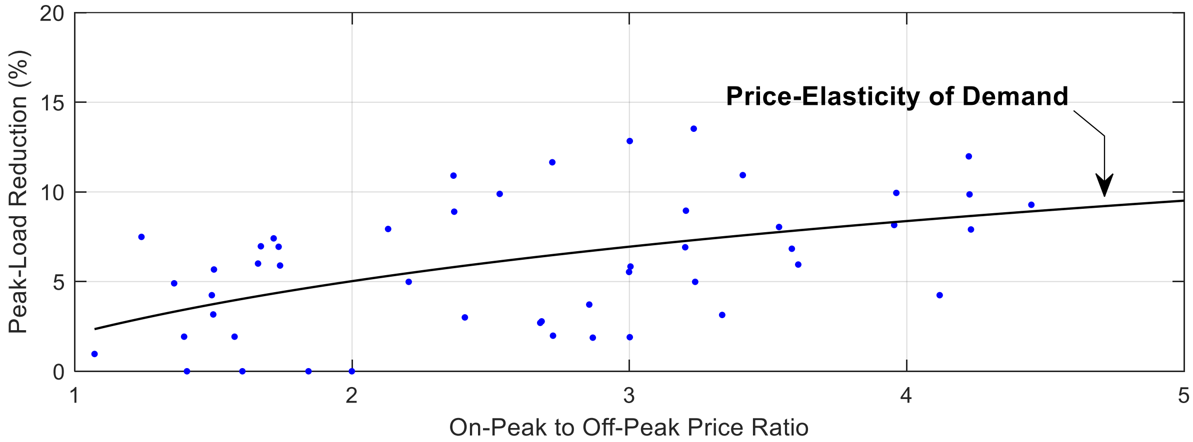
• Other examples of time-shiftable loads include various home appliances such as washing machines, dryers, and dishwashers [281, 282], water heaters [283], industrial equipment in process control and manufacturing [284, 285], batch processes in data centers and computer servers [286], irrigation pumps [287], and swimming pools [288].

• Some time-shiftable loads are *interruptable*, meaning that their operation can be interrupted and then later resumed; such as in the case of charging EVs as we saw in Example 5.7. Some time-shiftable loads are *uninterruptable*, such as some industrial processes, e.g., see [289].

5.4.1. Price-Based Demand Response

• **Price Elasticity of Demand**: The detailed measurements from smart meters can help evaluate the impacts of ToU prices in reducing peak load.

• An example is shown below based on the analysis in [290], which is based on different pilot projects. The key parameter here is the ratio between the price during on-peak hours and the price during off-peak hours.



5.4.1. Price-Based Demand Response

• **Price Elasticity of Demand (Cont.)**: In general, a higher ratio is more forceful to encourage customers to shift their load to off-peak hours; thus contributing to reducing the overall peak load in the power system.

• The curve is the result of an approximate curve fitting to the points in the figure. The curve shows the price-elasticity of the demand. It shows how much load shifting, and thus peak-load reduction, can be achieved by using different levels of the price ratio between the on-peak and off-peak hours.

5.4.1. Price-Based Demand Response

- **Other Pricing Methods:** Smart meters can also facilitate using other types of pricing methods in price-based demand response.

- One example is *peak-load pricing*, which requires customers to pay peak-load charges, also known as demand charges, based upon their highest amount of power consumption during any given time interval, typically 15 minutes, during the billing period.

- Another example is *real-time pricing*. It requires the customers to be charged for their electricity usage based on the price of electricity in wholesale electricity. See Section 7.3.2 in Chapter 7 for a related discussion.

5.4. Smart Meters and Their Applications

5.4.2. Incentive-Based Demand Response

• In incentive-based demand response, customers receive *financial incentives* for their *participation* in the DR program.

• Their participation is rather explicit because they are expected to reduce their load upon receiving a notification from the utility. In other words, customers are paid to be available to reduce their demand when needed, i.e., when they are informed that a *demand response event* has occured.

• DR events occur occasionally, such as 5–10 times a year. They take place at times when wholesale electricity market prices are high or when the reliability in the power system is jeopardized.

• The amount of incentive payments to a customer depends on how much the customer is capable of reducing its load when a DR event occurs.

5.4.2. Incentive-Based Demand Response

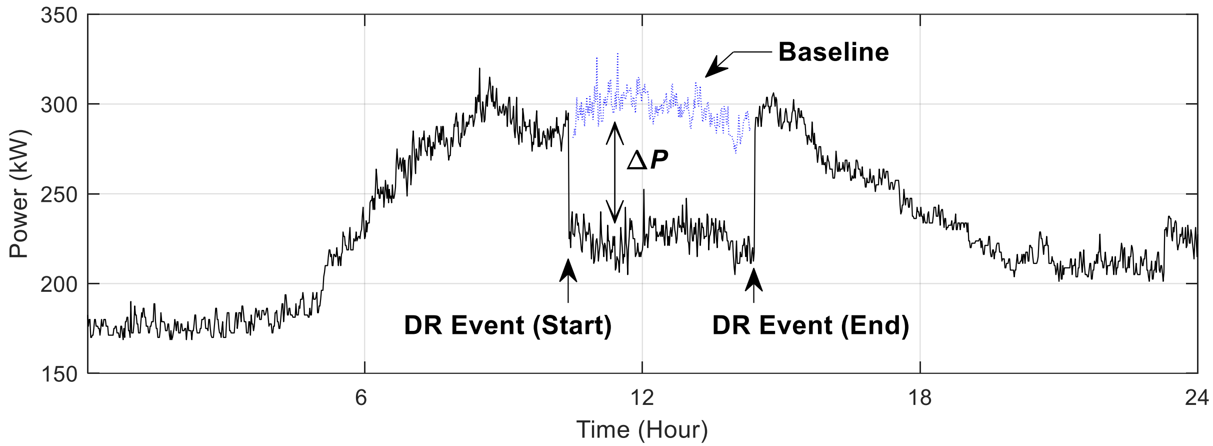
- **Baseline Calculation**: Once a customer that is enrolled to an incentive-based DR program receives the notification for a DR event, it *must curtail* its load accordingly.

- However, it is *not easy to calculate how much curtailment* the customer actually makes in response to the DR event. The process of making such a calculation is called baseline calculation.

- The key in baseline calculation is to *compare* the load of the customer with a baseline load, also known as the "*business as usual*" load, which is the load that the customer would have in case the customer had not responded to the DR event.

5.4.2. Incentive-Based Demand Response

- **Baseline Calculation (Cont.)**: An illustrative example is shown below. The DR event occurs at 10:25 AM. It lasts for four hours, ending at 2:25 PM.



It is clear that the customer did respond to the DR event. However, due to its inherent volatility, the customer's load fluctuates during the DR event.

5.4.2. Incentive-Based Demand Response

• **Baseline Calculation (Cont.)**: To evaluate the customer's performance during the DR event, we need to figure out how much of the customer's new load was the result of its curtailment efforts, and how much was due to its normal load variations.

• Therefore, it is necessary to *establish a baseline load profile* for the duration of the DR event, as shown in the figure.

• On the previous slide, ΔP is meant to indicate the *actual curtailment* in the customer's load during the DR event.

5.4.2. Incentive-Based Demand Response

• Baseline calculation is done often by examining the load of the customer over the past few days, i.e., over the baseline window.

• In this regard, the *baseline window* is defined as the window of time prior to the DR event, typically a number of days, over which the customer load data is collected in order to establish the baseline.

• A common choice in practice for the baseline window is the *previous 10 (non-event) business days* [291]. Using a 10-day time window is considered an appropriate choice because it is short enough to account for near-term trends and long enough to limit opportunities for gaming the system [292].

5.4.2. Incentive-Based Demand Response

• **High 5 of 10 Method**: A class of baseline calculation methods look only at a few highest load days within the baseline window.

• For example, in the "High 5 of 10" method, baseline is calculated based only on the five days, out of the 10 days in the baseline window, that have the highest average load for the corresponding duration of the DR event.

• The other five days are *excluded* from baseline calculation [291].

5.4.2. Incentive-Based Demand Response

- **Example 5.8**: Consider a customer that is enrolled in an incentive-based DR program. Suppose a DR event occurs at 1:15 PM and lasts for 90 minutes. It ends at 2:45 PM. Table below provides the average power usage of this customer during 15-minute intervals from 1:15 PM till 2:45 PM over the past 15 days.

Day	Day of Week	Interval 1	2	3	4	5	6	Average Usage	DR Event
1	Tue	308	309	302	296	297	301	302	No
2	Wed	293	297	289	291	281	278	288	No
3	Thu	282	277	281	283	285	284	282	No
4	Fri	301	300	296	291	288	290	294	No
5	Sat	165	165	164	161	158	156	162	No
6	Sun	144	141	136	135	135	136	138	No
7	Mon	295	295	294	289	268	272	286	Yes
8	Tue	292	289	286	295	298	296	293	No
9	Wed	307	303	305	302	305	304	304	No
10	Thu	307	314	314	308	306	307	309	No
11	Fri	314	318	308	304	300	302	308	No
12	Sat	173	169	170	170	165	166	169	No
13	Sun	148	145	143	146	145	143	145	No
14	Mon	303	306	304	309	309	307	306	No
15	Tue	291	307	305	310	288	297	300	No

Yesterday → 15

5.4.2. Incentive-Based Demand Response

• **Example 5.8 (Cont.)**: After we exclude the four weekends and also the day of the previous DR event, we obtain the baseline window which includes the following ten days: 1, 2, 3, 4, 8, 9, 10, 11, 14, 15.

• Next, consider the average power usage across all of the six intervals, i.e., the number on the second to the last column. The following five days have the *highest average power usage* within the baseline window: 1, 9, 10, 11, and 14. Accordingly, we can obtain the baseline as

$$\text{Interval 1: } (308 + 307 + 307 + 314 + 303)/5 = 308 \text{ kW},$$
$$\text{Interval 2: } (309 + 303 + 314 + 318 + 306)/5 = 310 \text{ kW},$$
$$\text{Interval 3: } (302 + 305 + 314 + 308 + 304)/5 = 307 \text{ kW},$$
$$\text{Interval 4: } (296 + 302 + 308 + 304 + 309)/5 = 304 \text{ kW},$$
$$\text{Interval 5: } (297 + 305 + 306 + 300 + 309)/5 = 303 \text{ kW},$$
$$\text{Interval 6: } (301 + 304 + 307 + 302 + 307)/5 = 304 \text{ kW}.$$

5.4.2. Incentive-Based Demand Response

• **Example 5.8 (Cont.)**: Suppose the load of this customer during the DR event period of interest is as follows:

$$\text{Interval 1: 289 kW,} \quad \text{Interval 2: 291 kW,} \quad \text{Interval 3: 290 kW,}$$
$$\text{Interval 4: 288 kW,} \quad \text{Interval 5: 285 kW,} \quad \text{Interval 6: 287 kW.}$$

• By subtracting the above numbers from the corresponding numbers on the previous slide, we estimate that the customer curtailed **19 kW**, **19 kW**, **17 kW**, **16 kW**, **18 kW**, and **17 kW** at intervals 1–6, respectively.

5.4.2. Incentive-Based Demand Response

• **Other Baseline Calculation Methods**: It is generally preferred to have a baseline calculation method that is *simple* and therefore *easy to understand by the customer*. However, given the challenges in calculating the baseline load profile, there are also many advanced methods that use statistical and machine learning techniques to calculate baseline load profiles, e.g., see [293–296].

• Of course, in all these methods, ultimately the key to success is having access to detailed power and energy usage data, i.e., the type of measurements which are provided by smart meters.

5.4.3. Energy Usage Clustering

- The load profiles that are obtained by smart meters can be used by utilities to do *customer energy usage clustering*, also known as usage segmentation, i.e., classifying customers based on their load profiles [297, 298]. The results can be used, for example, to *calculate baselines* in demand response programs, see Section 5.4.1; or to enhance accuracy in *load forecasting*, e.g., see [299–301].

5.4.3. Energy Usage Clustering

• ***k*-Means Clustering**: Classification of load profiles can be done by using techniques such as k-means clustering.

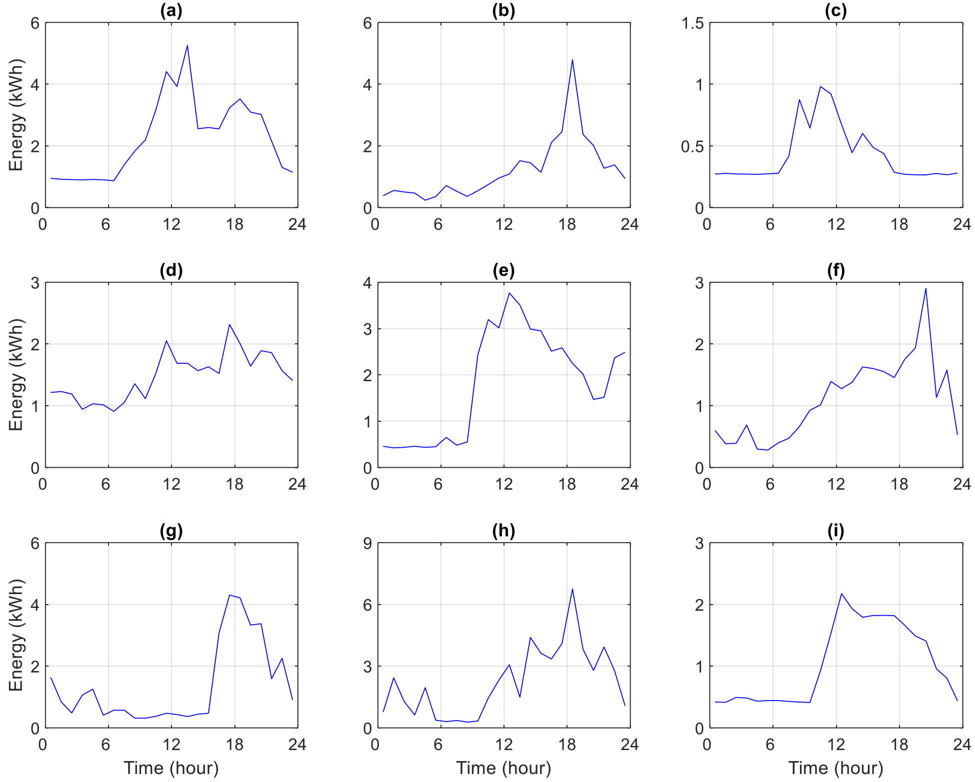
• As an example, consider the n = 9 hourly load profiles in the figure below that are reported by nine smart meters on the same day.

5.4.3. Energy Usage Clustering

• ***k*-Means Clustering**: Classification of load profiles can be done by using techniques such as k-means clustering.

• As an example, consider the $n = 9$ hourly load profiles in the figure that are reported by nine smart meters on the same day.

• We seek to *divide* these nine load profiles into $k = 2$ *clusters*, based on their similarities.



5.4.3. Energy Usage Clustering

• ***k*-Means Clustering (Cont.)**: The first step is to introduce adequate features to quantitatively represent each load profile. Suppose we define two features for each load profile:

- Feature 1: Peak-to-average magnitude divided by 3.
- Feature 2: Peak-time hour divided by 24.

• The features are normalized by 3 and 24 in order to take values around the same range. For instance, for the first load profile, the peak-to-average magnitude is 2.3472 and the peak hour is 14; thus, Feature 1 is 0.7824 and Feature 2 is 0.5833.

5.4.3. Energy Usage Clustering

- ***k*-Means Clustering (Cont.)**: The features of all nine load profiles that we saw in the figure on Slide 84 are given in the table below.

Load Profile #	Feature 1	Feature 2
1	0.7824	0.5833
2	1.3252	0.7917
3	0.7616	0.4583
4	0.5233	0.7500
5	0.6947	0.5417
6	0.8845	0.8750
7	1.0406	0.7500
8	1.0064	0.7917
9	0.6970	0.5417

5.4.3. Energy Usage Clustering

• ***k*-Means Clustering (Cont.)**: Let $\mathbf{x}_i$ denote the 2×1 vector of features for load profile i, that is:

$$\mathbf{x}_1 = \begin{bmatrix} 0.7824 \\ 0.5833 \end{bmatrix}, \quad \ldots, \quad \mathbf{x}_9 = \begin{bmatrix} 0.6970 \\ 0.5417 \end{bmatrix}.$$

• We seek to partition $n = 9$ load profiles into $k = 2$ sets, denoted by S_1 and S_2, so as to *minimize* the *within-cluster sum-of-squares*:

$$\sum_{i \in S_1} \|\mathrm{x}_i - \boldsymbol{\mu}_{S_1}\|^2 + \sum_{i \in S_2} \|\mathrm{x}_i - \boldsymbol{\mu}_{S_2}\|^2,$$

where $\boldsymbol{\mu}_{S_1}$ and $\boldsymbol{\mu}_{S_2}$ denote the *average vectors* of set S_1 and set S_2.

5.4.3. Energy Usage Clustering

- ***k*-Means Clustering (Cont.)**: Once we apply the k-means clustering method to the features in the table on Slide 86, we can classify the nine load profiles into the following two sets:

$$\mathrm{S}_1 = \{1,3,4,5,9\}, \qquad \mathrm{S}_2 = \{2,6,7,8\}.$$

- The corresponding average vectors for the two clusters are obtained as

- The within-cluster sum-of-squares is 0.0881 and 0.1125, respectively.

$$\mu_{S_1} = \begin{bmatrix} 0.6918 \\ 0.5750 \end{bmatrix}, \quad \mu_{S_2} = \begin{bmatrix} 1.0642 \\ 0.8021 \end{bmatrix}.$$

5.4.3. Energy Usage Clustering

- ***k*-Means Clustering (Cont.)**: The proper choice of features is critical in order to have an effective clustering. In fact, *feature extraction* and *feature selection* often require statistical characterization of the load profiles and also considering external factors; e.g., see [302].

- The above classification method that is based on clustering is considered an *unsupervised learning* method in the field of machine learning. Here, we do not need to first manually label a few training samples into set S_1 or set S_2. In contrast, in classification methods that are based on *supervised learning*; we must do prior manual labeling. An example for supervised learning is classification based on *support vector machines*; see Section 3.7.2 in Chapter 3 for more details.

5.4.3. Energy Usage Clustering

• **Baseline Calculation by Using Load Clustering**: Recall from Section 5.4.2 that the baseline for a customer that participates in an incentive-based DR program is to look at the recent load of that *same customer*. Alternatively, one can calculate the baseline by looking at the recent load of not only the customer in question itself but also the *other similar customers*. This can be done by using customer clustering.

• For instance, suppose we would like to calculate the baseline for Customer #1. From Slide 88, the load profile of this customer is classified to be in set S1. Thus, we may calculate the baseline for Customer #1 by looking at the recent load of not only Customer #1, but also Customers #3, #4, #5, and #9, since they belong to the same class of loads. In this option, we can select the five days with the highest average power usage based on the load of *all five of these customers* [291].

5.4.4. Other Applications of Smart Meter Measurements

• Besides what we have discussed so far, smart meter measurements may support many other smart grid applications. Next, we briefly discuss some of those applications.

• **Remote Service Connection and Disconnection**: In addition to the ability to read the smart meter measurements remotely, utilities can also remotely connect or disconnect service through smart meters, without the need to send their crew to the customer location [303].

5.4.4. Other Applications of Smart Meter Measurements

• **Outage Notification and Outage Management**: Many smart metering systems offer a "*last gasp*" message transmission capability to tell the utility that the customer has lost power.

• Therefore, unlike in traditional metering systems, the customer no longer needs to call the utility to report the outage.

• The outage notification is sent by the smart meter when it *detects a zero voltage event* lasting more than a programmed period of time [304].

• The outage notifications that are sent by the group of affected smart meters can also help the utility's *Outage Management System* (OMS) to properly respond to outage conditions, such as by helping to identify the approximate location of the fault that may have caused the outage [305].

5.4.4. Other Applications of Smart Meter Measurements

- **Electricity Theft Detection**: Non-technical losses, i.e., losses due to electricity theft, account for billions of dollars of revenue loss for utilities around the world as individuals may tamper with the electric meters to slow or stop the accumulation of energy usage [306].

- The data from smart meters can be used to *detect* electricity theft.

- For example, we may detect electricity theft by detecting *abnormal energy consumption patterns* in the load profile of customers [307].

- Another option is to examine the energy losses, i.e., to check the balance between the energy that is *supplied* by the utility transformers and the total energy that is *consumed* by the customers that are served by that transformer as reported by the smart meters [308].

5.4.4. Other Applications of Smart Meter Measurements

- **Identifying Customers with DR Potential**: Smart meters are necessary to facilitate various demand response programs, as we discussed in Sections 5.4.1 and 5.4.2. They can also help *identify* the customers that are good fit to participate in demand response based on their load patterns.

- This can be done by evaluating factors such as the *variability in usage*, *sensitivity* of electricity usage to *temperature*, *occupancy status*, and *inter-temporal usage dynamics*; see [309–311].

- Note that, besides smart meters, other types of sensors can also help with the identification and participation of customers in DR programs, such as different types of occupancy sensors; see Section 7.2 in Chapter 7.

5.4.4. Other Applications of Smart Meter Measurements

- **Distribution System State Estimation**: The measurements by smart meters can help with solving the state estimation problem in distribution systems; see [312–315], and also Section 5.9.3.

5.4.4. Other Applications of Smart Meter Measurements

• **Topology and Phase Identification**: We previously discussed the problem of topology identification in power distribution systems in Section 4.5.2 in Chapter 4. We also previously discussed the problem of phase identification in power distribution systems in Section 2.8.3 in Chapter 2, and Section 3.6.4 in Chapter 3.

• Smart meter measurements can help with both applications.

• For example, see the analysis in [316], where the authors inferred the topology of the distribution system from smart meter energy measurements. We will discuss the application of smart meter measurements in phase identification in Section 5.9.2.

5.4.4. Other Applications of Smart Meter Measurements

• **Load Modeling and Load Forecasting**: Smart meter measurements can be used in load modeling, as we will discuss in Section 5.7.1.

• They can also be used in load forecasting. Load forecasting at the customer level can be aggregated to help with the typical system-wide operation needs of the utility; they can also be used for the operation of a specific substation or a specific distribution feeder [317].

5.4.4. Other Applications of Smart Meter Measurements

• **Customer Reports**: Smart meters provide consumers with greater clarity on their electricity bills and also their own consumption behavior.

• Customer reports can break down the usage of the customer during different days and different on-peak, mid-peak, and off-peak hours.

• Customers may access their detailed load profile on the utility's website.

• The customer reports may help the customers identify opportunities to reduce their electricity cost, such as by coordinating the operation of their *programmable communicating thermostats* and *smart appliances* with the data from smart meters [318].

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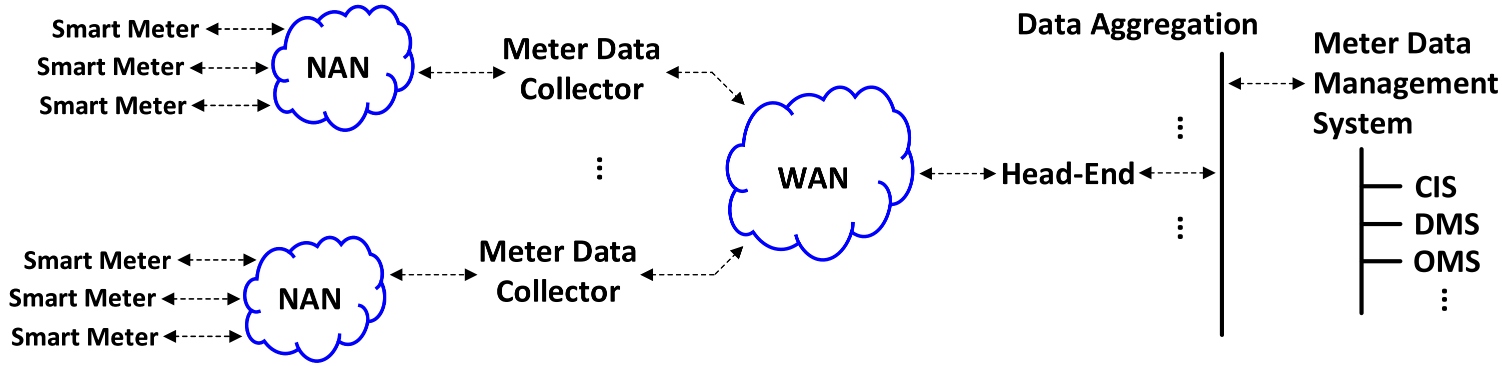
5.5. Advanced Metering Infrastructure

- Advanced metering infrastructure (AMI) is an integrated system of three main components: smart meters, communications networks, and data management systems. These components are shown below.



- We have already discussed smart meters and their applications in the previous sections. In this section, we will briefly discuss the *communications networks* and the *data management systems*.

5.5. Advanced Metering Infrastructure

5.5.1. AMI Communications Networks

- The AMI communication system typically includes two basic layers [319]:
    - A *neighborhood area network* (NAN) that provides the last-mile communication system between the smart meters to meter data collectors.
    - A *wide area network* (WAN) that serves as the backhaul communication system between the meter data collectors in the field and the head-end system.

5.5. Advanced Metering Infrastructure

5.5.1. AMI Communications Networks

- Different utilities have different geographical conditions, different load densities, and different legacy communication facilities.

- Therefore, the communication system to be implemented in each layer of the AMI system can be different for each utility [319].

- In Europe, *power line communication* (PLC) is the common choice for the communication technology in a NAN.

- PLC is a *wired* (as opposed to *wireless*) communication technology that reuses power lines as the media for the purpose of data transmission. We will discuss PLC further in Section 6.5 in Chapter 6.
    - Also see Section 4.1.2 for a discussion related to PLC.

5.5.1. AMI Communications Networks

• In North America, wireless communications are the more popular choice for the communication technology in a NAN.

• Two different architectures are often considered. In the *mesh architecture*, the NAN is in the form of a *wireless mesh network* (WMN). In this architecture, the smart meters that are near the meter data collector communicate with the meter data collector directly. However, smart meters that are away from the meter data collector *use other meters as repeaters* to communicate with the meter data collector. This is due to the short-range communication capability in WMNs, i.e., from 1 to 5 miles. In the *star architecture*, all smart meters communicate with the meter data collector directly. Different *radio frequency* (RF) technologies are used. RF technologies provide longer-range communication capability, i.e., from 5 to 15 miles [319].

5.5.1. AMI Communications Networks

- Other NAN technologies may include:

  - fiber-optic communications,
  - wireless broadband communications,
  - satellite communications (in remote areas).

- The second layer of communications, i.e., the WANs, offer a mix of different technologies, such as fiber, microwave, public/private cellular networks, and satellite links [320]. In recent years, *cellular communications* have been particularly popular in WANs around the world, due to their immediate availability from cellular phone service providers. By leveraging the *third-party cellular networks*, this option requires a relatively low capital investment by the utility.

5.5.2. AMI Data Management Systems

• Meter Data Management System (MDMS) is a database software application that interfaces with the AMI head-ends to collect, store, and analyze the smart meter readings [321].

• MDMS also interfaces with all *smart meter applications* and other *smart grid information systems*, such as the **Consumer Information System** (CIS), which also includes billing; **Geographic Information System** (GIS); **Demand Response Management System** (DRMS); **Distribution Management System** (DMS); **Distribution Automation System** (DAS); **Outage Management System** (OMS); **Fault Location Isolation and Service Restoration** (FLISR); **Power Quality Management System** (PQMS); and **Load Forecasting System** (LFS) [319].

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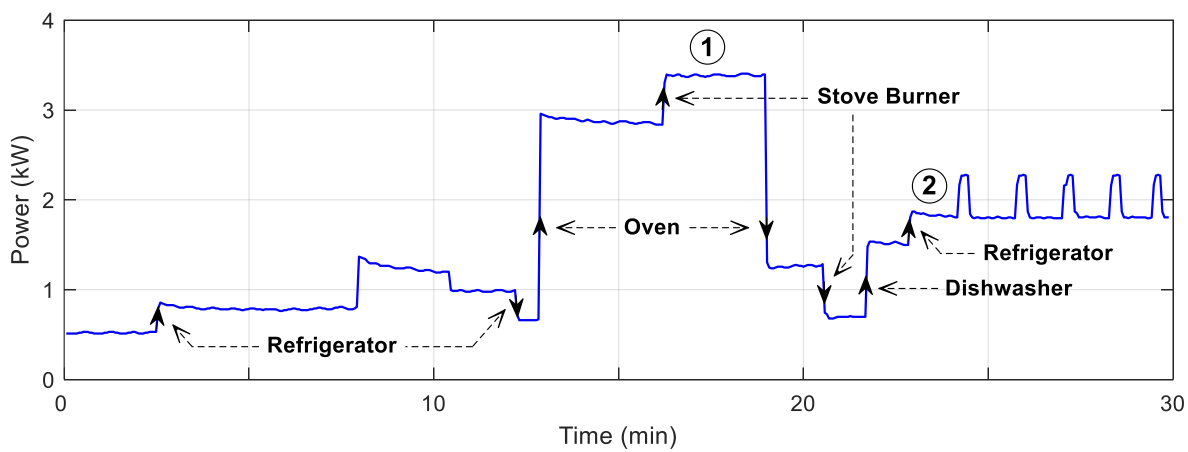
5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

• Load disaggregation refers to the problem of extracting the power or energy usage of individual appliances from their aggregated power and energy usage measurements, such as from the whole-building power or energy usage measurements [322]. For instance, consider the load profile of a residential customer as shown below, over a period of 30 minutes.



5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

• At any point in time, the power consumption of this customer is a *summation* of the power consumption of *multiple* appliances.

• For example, at point 1, the total load is the summation of the load of the oven, the load of the stove burner, and some other background loads.

• As another example, at point 2, the total load is the summation of the load of the dishwasher, the load of the refrigerator, and some other background (smaller) loads.

• In load disaggregation, we seek to identify the load of each of these major appliances without having direct access to those appliances.

5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

- Load disaggregation is a *non-intrusive* technique to monitor appliance-level loads because this technique does not require placing sensors on individual appliances in the customer's property [323].

- The *alternative* is *sub-metering*, which is an intrusive technique. We will discuss sub-metering in Section 5.6.3.

- Applications of load disaggregation may include:

    - demand response [324], load forecasting [301], appliance and equipment health monitoring [325], and household appliance marketing by the utility or third-party businesses [326, 327].

5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

• **Example 5.9**: Consider a residential customer whose *total* power usage is measured by a wattmeter. The power usage of this customer's largest loads are known to be as follows:

- **Water Heater**: 4500 W
- **Central Air Conditioning**: 3250 W
- **Clothes Dryer**: 2300
- **Dishwasher**: 1700 W
- **Oven**: 1200 W
- **Clothes Washer**: 800 W.

• If *none* of the above major appliances and equipment is on, then the power usage of this customer is *somewhere between* 200 W and 500 W.

5.6.1. Load Disaggregation

• **Example 5.9 (Cont.)**: Based on this information, we want to disaggregate the total load of this customer.

• First, assume that the total power usage of this customer is measured at **13.2 kW**. In that case, we can conclude that all the major loads, except for the clothes washer, are on; and the load of the remaining (non-major) appliances is 250 W:

$$4500\text{ W} + 3250\text{ W} + 2300\text{ W} + 1700\text{ W} + 1200\text{ W} + 250\text{ W} = 13.2\text{ kW}.$$

5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

- **Example 5.9 (Cont.)**: However, if the total power usage is measured at **11.7 kW**, then there are *two possibilities*.

Possibility 1) all the major loads, except for the clothes dryer, are on; and the load of the remaining (non-major) appliances is 250 W:

$$4500 \text{ W} + 3250 \text{ W} + 1700 \text{ W} + 1200 \text{ W} + 800 \text{ W} + 250 \text{ W} = 11.7 \text{ kW}.$$

Possibility 2) all major loads, except the dishwasher and clothes washer, are on; and the load of the remaining (non-major) appliances is 450 W:

$$4500 \text{ W} + 3250 \text{ W} + 2300 \text{ W} + 1200 \text{ W} + 450 \text{ W} = 11.7 \text{ kW}.$$

5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

• The basic idea that we examined in Example 5.9 has been considered in the literature, in form of a combinatorial optimization to identify which combination of appliances can match the total load; e.g., see the binary optimization problem formulation in [323].

• However, this approach is *not* always effective and it can be prone to uncertain solutions, as we saw in the second case in Example 5.9.

  • We could not decide which option (1 or 2) is correct.

5.6.1. Load Disaggregation

- **Analysis of Load Signatures**: Another approach is to identify the signatures of different types of appliances, when they switch on or off.

- Accordingly, in this approach, the focus is on load switching events.

- For instance, again consider the load profile on Slide 109.

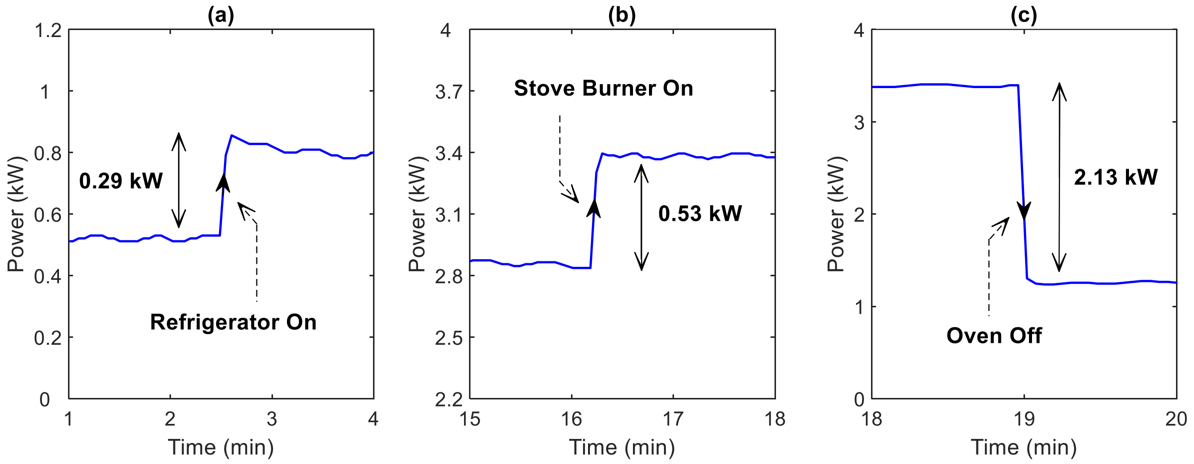
- If we focus on the *sharp edges* in the total load, they correspond to the switching events of the individual appliances.

  - A *positive sharp edge* indicates that a major appliance *switches on*;

  - A *negative sharp edge* indicates that a major appliance *switches off*.

5.6.1. Load Disaggregation

- **Analysis of Load Signatures (Cont.)**: Three examples of such events are shown below. When the refrigerator *switches on*, the total load suddenly increases by 0.29 kW; see Figure **(a)**. When the stove burner *switches on*, the total load increases by 0.53 kW; see Figure **(b)**. When the oven element *switches off*, then the total load decreases by 2.13 kW; see Figure **(c)**.



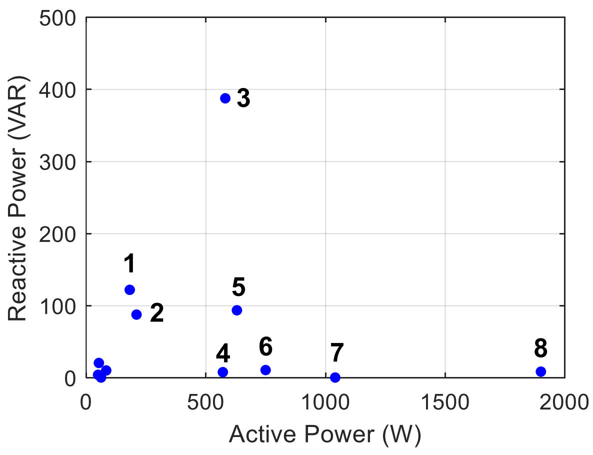
5.6.1. Load Disaggregation

• **Analysis of Load Signatures (Cont.)**: Once we identify *which appliance switches on* at each positive sharp edge and *which appliance switches off* at each negative sharp edge, we can identify all the appliances that are "on" at any time, thus solving the load aggregation problem.

• The analysis of load signatures during load switching events is in principle similar to the analysis of other types of events that we have studied throughout this book. For example, we can *detect* a switching event in power measurements by using similar methods that we learned for *event detection* in Section 2.7.2 in Chapter 2.

5.6.1. Load Disaggregation

• **Features - Active and Reactive Power**: The changes in the total active power and total reactive power are among the most common features that are looked at when load disaggregation is done based on the analysis of load signatures. Figure below shows the scatter plot for these two features for several appliances in a house [328]. The points that are due to the switching of the *major* appliances are marked from 1 to 8.



1: Refrigerator
2: Clothes Washer
3: Dehumidifier
4: Dishwasher
5: Vacuum Cleaner
6: Coffee Machine
7: Iron
8: Water Heater

5.6.1. Load Disaggregation

- **Features - Active and Reactive Power (Cont.)**: While the smaller loads are difficult to distinguish, the larger loads have *distinct signatures* in the active and reactive power measurements that can help identify them.

- The dehumidifier and the dishwasher have *similar active power* consumption, but they have *different reactive power* consumption.
    - This can help distinguish them.

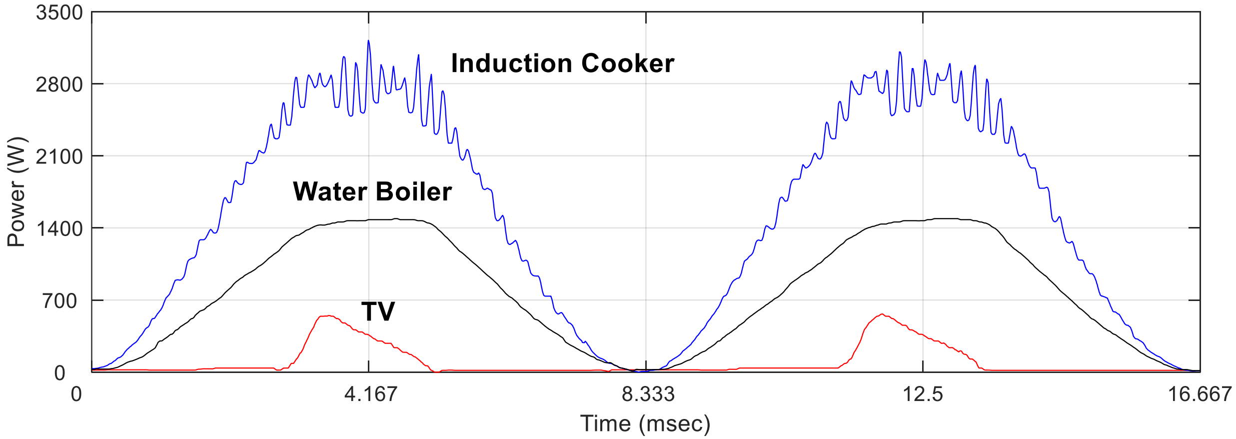
- Also, the clothes washer and the vacuum cleaner have *similar reactive power* consumption, but they have *different active power* consumption.
    - This can help distinguish them.

5.6. Disaggregation and Sub-Metering

5.6.1. Load Disaggregation

• **Features - Instantaneous Power and Harmonics**: Instantaneous power waveform can serve as another feature in load disaggregation.

• A few examples are shown below during one cycle of the AC power signal at 60 Hz. Recall from Section 1.2.4 in Chapter 1 that the frequency of the instantaneous power waveform is *twice the frequency* of the voltage and current waveforms. That is why we see *two cycles* of the instantaneous power waveform within 1/60 Hz = 16.667 msec.



5.6.1. Load Disaggregation

- **Features - Instantaneous Power and Harmonics (Cont.)**: The shape of the instantaneous power waveform is very different among the three types of loads that are shown in this figure.

- If the instantaneous power waveform measurements are available, then we can detect when these load types switch on, as soon as we notice *the presence* of these specific instantaneous power waveforms; and detect when these load types switch off, as soon as we notice *the absence* of these specific instantaneous power waveforms.

5.6.1. Load Disaggregation

- **Features - Instantaneous Power and Harmonics (Cont.)**: If instantaneous power waveform measurements are not available, then we may still use harmonics in current measurements.

- The induction cooker in the example on Slide 121 generates the *89th and 91st harmonics* in the current; and the television in that example generates the *3rd and 5th harmonics* in the current.

- These additional features can help distinguish different load types that have *similar active and reactive power consumption*.

- For instance, it is quite possible that two load types have similar active power and reactive power signatures; but *different harmonic signatures*.

5.6.1. Load Disaggregation

• **Features - Time, Day, and Other External Factors**: The performance of load disaggregation may improve by considering various other features.

• Time and day might add a factor of *likelihood of operation*. For example, an oven is less likely to be operated at 3:00 AM versus at 6:00 PM.

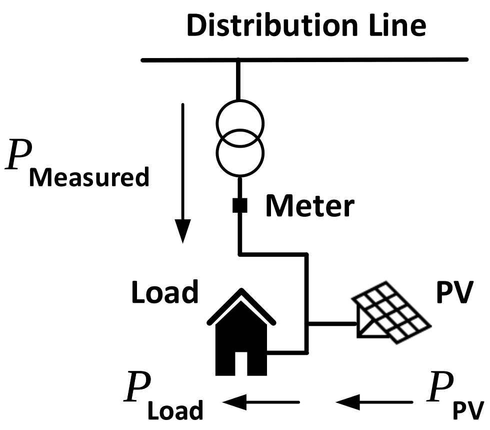
• The *sequence of operation* among appliances can also be considered, such as the sequence between the clothes washer and the clothes dryer; or some likely sequential operation of certain kitchen appliances [330].

5.6. Disaggregation and Sub-Metering

5.6.2. Net Load Disaggregation

• The increasing penetration of behind-the-meter renewable power generation resources, such as rooftop PV resources in residential customers, has direct impact on the problem of load disaggregation. The reason is that the utility's meters measure the *net aggregate* of the customer's load, i.e., the customer's load minus the customer's generation.

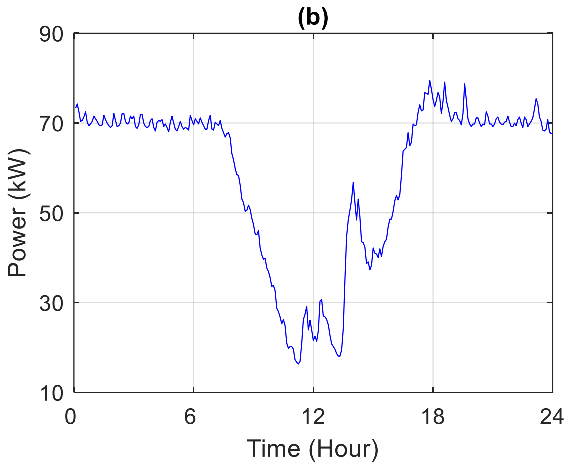
• An example is shown in the figure: a commercial building with a behind-the-meter solar power generation unit. What the utility measures at its meter is



$$P_{\text{Measured}} = P_{\text{Load}} - P_{\text{PV}}.$$

5.6.2. Net Load Disaggregation

- The net load measurements in this example would include the inherent impact of the solar generation at the building (notice the major decrease in the net load during the middle of the day).



- This new paradigm is transforming the traditional load disaggregation problem (Section 5.6.1) into the new problem of *net load disaggregation*.

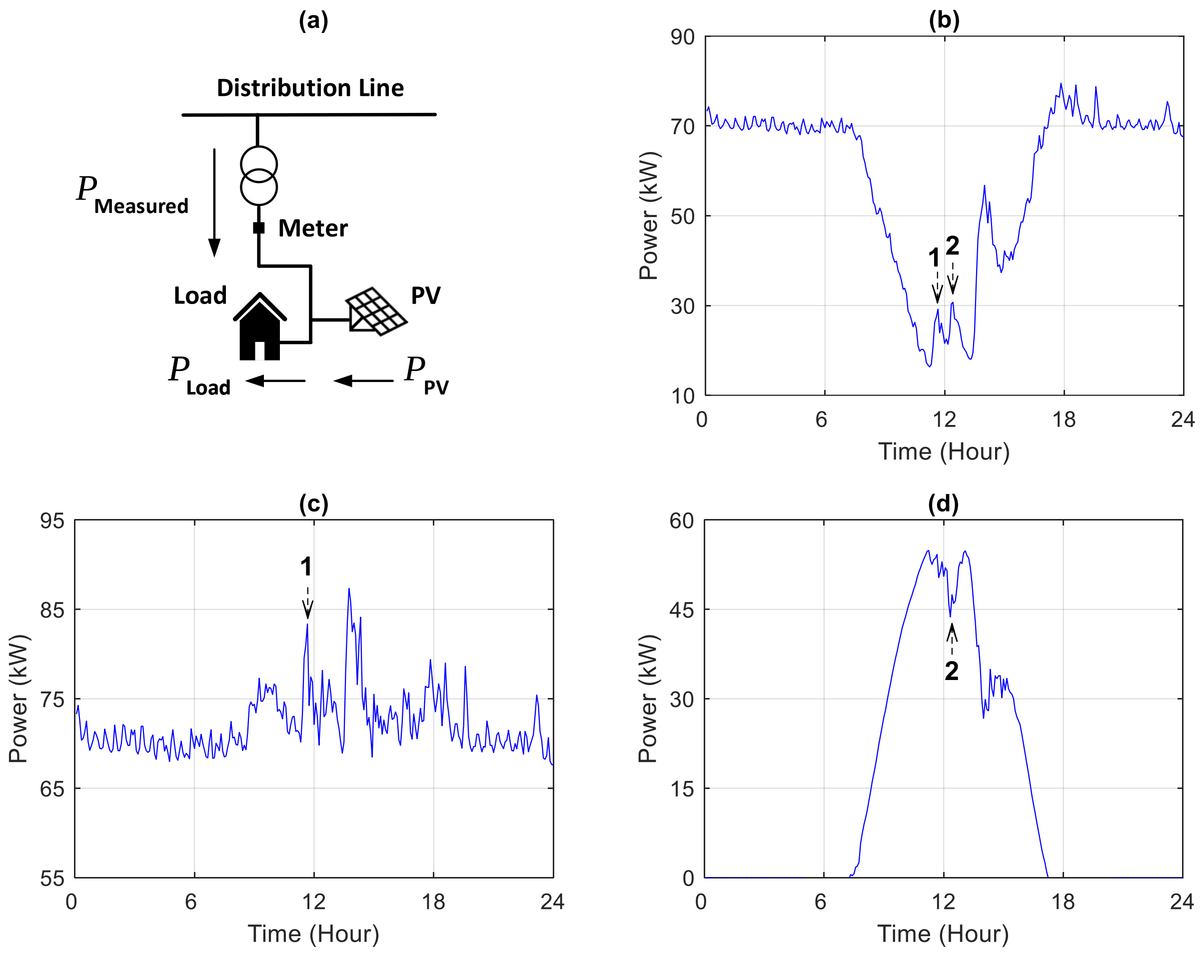
5.6.2. Net Load Disaggregation

- Apart from the initial goal of identifying the appliance-level load of a customer, net load metering can also help with *estimating* the customer’s behind-the-meter renewable power generation.

- This by itself is an important task, in particular, when it comes to estimating behind-the-meter solar power generation. This task is often referred to as *solar generation disaggregation* [331, 332].

5.6.2. Net Load Disaggregation

- The disaggregated load profile and the power generation profile are shown in Figures (c) and (d), respectively.



5.6.2. Net Load Disaggregation

• Two events in form of sudden increases in the net load of the customer are marked in Figure (b) with numbers 1 and 2.

• These two events have two different causes. The *first event* was caused by a *sudden increase* in the load of the customer, as marked in Figure (c).

• The *second event* was caused by a *sudden decrease* in solar power generation, as marked in Figure (d). The sudden decrease in solar power generation could be due to cloud passing. Therefore, the variations in solar power generation can create their own events in the net load, which may not be easy to distinguish from the signature of loads.

5.6. Disaggregation and Sub-Metering

5.6.2. Net Load Disaggregation

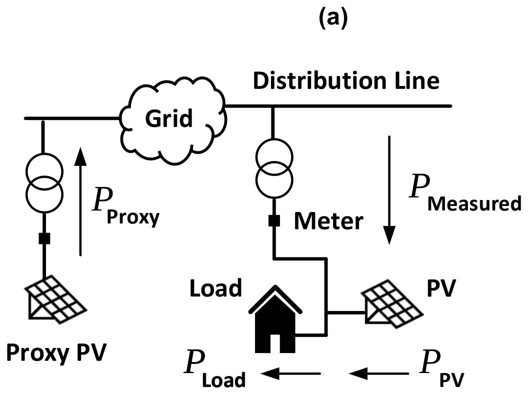
- **Proxy Measurements**: One option to consider in solar generation disaggregation, e.g., in [332, 333], is to use proxy measurements.

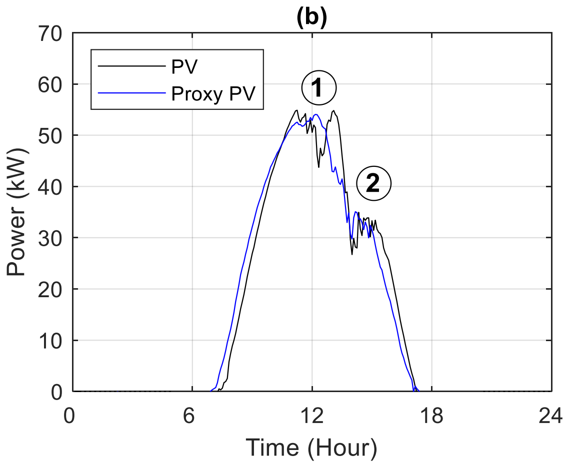
- A PV *proxy* is a PV system that is in a close-by location, and its power generation is measured directly. The *solar power generation at the PV proxy* can be used to estimate the solar power generation portion of a net load at the location where we need net load disaggregation.

5.6.2. Net Load Disaggregation

- **Proxy Measurements (Cont.)**: For example, consider the PV on Slide 128. Another PV is located about *two miles away* from this customer, as shown in Figure (a), and its solar power generation is measured directly.

- Therefore, this other PV system can be used as a PV proxy. Figure (b) shows the solar power generation at the original PV system and also at the solar power generation at the proxy PV system.



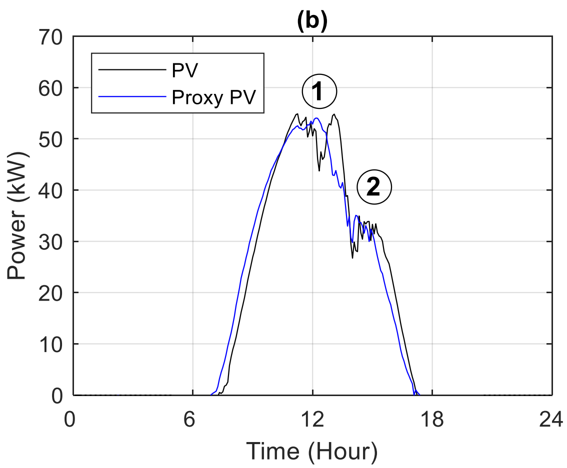


5.6.2. Net Load Disaggregation

• **Proxy Measurements (Cont.)**: Overall, the two power generation profiles are *similar*, because the two locations are *close to each other*.

• Two points, denoted by 1 and 2, are marked on this figure at the times when solar power generation is volatile.

• There are some considerable differences at 1, but the two solar power generation profiles are mostly similar at 2.



5.6.2. Net Load Disaggregation

- **Proxy Measurements (Cont.)**: We can disaggregate the net load as

$$P_{\text{PV}} \approx P_{\text{Proxy}}, \qquad P_{\text{Load}} \approx P_{\text{Measured}} - P_{\text{Proxy}}.$$

- This is a very rough approximation based on the proxy measurements.

- The accuracy of the above approximate may improve by factoring in some *physical characteristics* of the two PV systems, taking into account the information about *clouds movement in the region* [334], or training a model to use a proper *mixture of measurements from multiple PV proxies*.

5.6.2. Net Load Disaggregation

- **Other Methods**: Different methods have been proposed in the literature to solve the solar generation disaggregation problem.

- Some methods use *fundamental physical models* and take into account relationships between *location*, *weather*, *physical characteristics*, and *solar irradiance*; e.g., see [331].

- Some other methods use *data-driven and statistical models*, such as extracting features to describe load profiles with PVs and load profiles without PVs; e.g., see [335].

- Some methods also use a *combination* of physical models and statistical models, such as in [336].

5.6.3. Sub-Metering

- The discussions in Sections 5.6.1 and 5.6.2 are concerned with *non-intrusive disaggregation*. They are called non-intrusive because they do not require placing sensors on individual appliances, individual equipment, or individual PV units.

- The alternative to non-intrusive disaggregation is *sub-metering*.

- In sub-metering, we install a *separate meter* to measure power or energy usage or power or energy generation of a subsystem or a component of interest; e.g., see [268, 337].

5.6. Disaggregation and Sub-Metering

5.6.3. Sub-Metering

• Sub-metering has recently received a technology boost due to the advent of Internet-of-Things (IoT) and their related technologies.

• IoT technologies have lowered the cost of sensor installation and sensor data collection (e.g., in buildings). They can help enhance energy efficiency and facilitate participation in demand response.

• An IoT-based sub-metering system in a building may include hundreds of meters at *every lighting fixture*, *every power outlet*, and *every variable-air-volume valve*, *compressor*, or other subsystems of the *heating*, *ventilation*, and *air conditioning system* [338–340].

  • An example is given in Section 7.2.1 in Chapter 7.

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5.7. Load Modeling

- Load modeling is essential to power system analysis, planning, and operation. The purpose of load modeling is to understand the *behavior of the load*, such as in response to *changes in voltage or frequency*.

- Load modeling can be done at the *transmission level* to model the aggregate load at different locations on an interconnected power system, e.g., see [341]; or it can be done at the *distribution level* to model the load of a power distribution feeder, the load of a single customer, or even the load of a single appliance, e.g., see [342–344].

- Load modeling can be *component-based*, where the knowledge of physical behavior of the load components, such as the physical parameters of a motor load, are used to model the functioning of load devices [345].

    - This approach is *not* our focus in this section.

5.7. Load Modeling

- Load modeling can also be *measurement-based*, to use measurements from various sensors in order to capture the behavior of the load.

- Measurement-based load modeling has several advantages over component-based load modeling.

- For example, it can be applied to a load, even when we have *no prior knowledge* about the physical behavior or physical parameters of the load.

- Moreover, measurements-based load modeling can update the model *over time* in order to *capture the changes in the load*.

5.7. Load Modeling

- In this section, we discuss only measurement-based load modeling. A detailed review of different load modeling techniques is available in [347].

- There are two types of measurement-based load models:

  - *Static* load models

  - *Dynamic* load models

- Static load models will be discussed in Section 5.7.1.

- Dynamic load models will be discussed in Section 5.7.2.

5.7.1. Static Load Model

• **ZIP Model**: It is a popular measurement-based load model. It represents the relationship between (active and reactive) power consumption and voltage in a polynomial equation that combines ***constant impedance*** (Z), ***constant current*** (I), and ***constant power*** (P) components of the load:

$$P = P_0 \left[\alpha_Z \left(\frac{V}{V_0} \right)^2 + \alpha_I \left(\frac{V}{V_0} \right) + \alpha_P \right],$$

$$Q = Q_0 \left[\beta_Z \left(\frac{V}{V_0} \right)^2 + \beta_I \left(\frac{V}{V_0} \right) + \beta_P \right],$$

where V is the operating voltage; and P_0 and Q_0 are active power and reactive power consumption at rated voltage V_0, respectively.

5.7.1. Static Load Model

- **ZIP Model (Cont.)**: A ZIP model has six parameters:
    - α_Z, α_I, α_P are the coefficients for the model for active power;
    - β_Z, β_I, β_P are the coefficients for the model for reactive power.

5.7.1. Static Load Model

• **ZIP Model (Cont.)**: Suppose P_i and V_i denote the measured active power and the measured voltage at the load that we seek to model, where $i = 1, \ldots, n$, and n is the total number of measurements.

• We can obtain the unknown coefficients of the ZIP model for active power by solving the following least square (LS) optimization problem:

$$\min_{\boldsymbol{\alpha}} \ \|\mathbf{b} - \mathbf{M}\boldsymbol{\alpha}\|_2 \, ,$$

where the *unknown coefficients* are:

$$\boldsymbol{\alpha} = \begin{bmatrix} \alpha_Z \\ \alpha_I \\ \alpha_P \end{bmatrix} .$$

5.7.1. Static Load Model

- **ZIP Model (Cont.)**: The parameters in this LS optimization problem are:

$$\mathbf{b} = \begin{bmatrix} P_1/P_0 \\ P_2/P_0 \\ \vdots \\ P_n/P_0 \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} (V_1/V_0)^2 & (V_1/V_0) & 1 \\ (V_2/V_0)^2 & (V_2/V_0) & 1 \\ & \vdots & \\ (V_n/V_0)^2 & (V_n/V_0) & 1 \end{bmatrix},$$

which are obtained from the n measurements.

5.7.1. Static Load Model

- **ZIP Model (Cont.)**: We can solve the above LS problem by using a Least Square solver; such as $lsqlin$ in MATLAB [102].

- Alternatively, we can obtain the solution in closed-form as [103]:

$$\boldsymbol{\alpha} = (\mathbf{M}^T \mathbf{M})^{-1} \mathbf{M}^T \mathbf{b}.$$

- Given Q_i and V_i as the measured reactive power and the measured voltage at the load, where $i = 1, \ldots, n$, we can obtain the coefficients β_Z, β_I, and β_P by formulating and solving a similar LS optimization problem.

5.7.1. Static Load Model

- **Example 5.10**: Voltage, active power consumption, and reactive power consumption are measured at a load.

- The measurements are shown below, where $n = 20$.

i	V (Volt)	P (Watt)	Q (VAR)	i	V (Volt)	P (Watt)	Q (VAR)
1	120.6	1120	491	11	124.2	1184	510
2	119.3	1097	484	12	122.5	1154	501
3	123.7	1175	507	13	115.7	1036	462
4	122.1	1146	499	14	122.4	1152	501
5	120.3	1115	489	15	115.3	1029	460
6	116.4	1048	467	16	118.1	1076	477
7	121.6	1138	496	17	121.7	1139	497
8	124.9	1196	514	18	125.1	1200	515
9	120.6	1120	491	19	126.3	1222	521
10	115.1	1025	459	20	126.4	1223	521

5.7.1. Static Load Model

- **Example 5.10 (Cont.)**: The rated voltage, the rated active power, and the rated reactive power of the load are $V0 = 120\ V$, $P0 = 1109\ W$, and $Q0 = 487\ VAR$, respectively [348]. By solving the LS optimization problem that we introduced on Slide 144, we can obtain:

$$\alpha_Z = 0.7294, \;\; \alpha_I = 0.4279, \;\; \alpha_P = -0.1568.$$

- Similarly, we can obtain:

$$\beta_Z = -1.5544, \;\; \beta_I = 4.487, \;\; \beta_P = -1.9325.$$

5.7. Load Modeling

5.7.1. Static Load Model

• If $P = P_0$ and $V = V_0$, then from the model on Slide 142, we have:

$$\alpha_Z + \alpha_I + \alpha_P = 1.$$

• The above equation is sometimes added to the minimization problem on Slide 144 as a *constraint*; e.g., see [348].

5.7.1. Static Load Model

- **Exponential Model**: The ZIP load model may also be represented in exponential form, with the following formulation:

$$P = P_0 \left(\frac{V}{V_0}\right)^{\gamma_P}, \qquad Q = Q_0 \left(\frac{V}{V_0}\right)^{\gamma_Q}.$$

where γ_P and γ_Q are the parameters of the model. These parameters vary between 0 and 2. Accordingly, we can identify three special cases:

- If $\gamma_P = 2$, then the above model reduces to a *constant impedance* load.
- If $\gamma_P = 1$, then the above model reduces to a *constant current* load.
- If $\gamma_P = 0$, then the above model reduces to a *constant power* load.

5.7.1. Static Load Model

- **Exponential Model (Cont.)**: If the rated voltage V_0 and the rated power P_0 are known, then we can measure V and P to obtain

- If several measurements are available, then we can formulate an LS optimization problem to estimate γ_P, such as the following; see *Exercise 5.18*:

$$\min_{\gamma_P} \left\| \mathbf{b} - \mathbf{M}\gamma_P \right\|_2,$$

where

$$\mathbf{b} = \begin{bmatrix} \log\left(P_1/P_0\right) \\ \log\left(P_2/P_0\right) \\ \vdots \\ \log\left(P_n/P_0\right) \end{bmatrix}, \qquad \mathbf{M} = \begin{bmatrix} \log\left(V_1/V_0\right) \\ \log\left(V_2/V_0\right) \\ \vdots \\ \log\left(V_n/V_0\right) \end{bmatrix}.$$

5.7. Load Modeling

5.7.1. Static Load Model

• **Exponential Model (Cont.)**: If V_0 and P_0 are *not* known, then we can estimate γ_P with two pairs of voltage and active power measurements, such as during a voltage event; e.g., see [343].

• **Frequency-Dependant Model**: Power consumption of certain types of loads can be affected by the *frequency* of the power system.

• For example, some motor loads *slow down* when the *frequency* of the system *drops*; see a related discussion under the subject of frequency response of interconnected power systems in Section 2.9 in Chapter 2.

5.7.1. Static Load Model

• **Frequency-Dependant Model (Cont.)**: Both the ZIP model and the exponential model can be extended to incorporate the impact of frequency. The ZIP model can be extended to:

$$P = P_0 \left[\alpha_Z \left(\frac{V}{V_0} \right)^2 + \alpha_I \left(\frac{V}{V_0} \right) + \alpha_P \right] \left[1 + \alpha_f (f - f_0) \right],$$

$$Q = Q_0 \left[\beta_Z \left(\frac{V}{V_0} \right)^2 + \beta_I \left(\frac{V}{V_0} \right) + \beta_P \right] \left[1 + \beta_f (f - f_0) \right],$$

where α_f and β_f are the coefficients of the model, f is the operation frequency, and f_0 is the nominal frequency. Parameter α_f is positive; but parameter β_f can be positive or negative [349].

5.7.1. Static Load Model

- **Frequency-Dependant Model (Cont.)**: We can similarly extend the exponential model to incorporate the impact of frequency:

$$P = P_0 \left(\frac{V}{V_0} \right)^{\gamma_P} \left[1 + \alpha_f \, (f - f_0) \right],$$

$$Q = Q_0 \left(\frac{V}{V_0} \right)^{\gamma_Q} \left[1 + \beta_f \, (f - f_0) \right].$$

5.7. Load Modeling

5.7.1. Static Load Model

- **Example 5.11**: Consider an agricultural pump with $\alpha_f = 5.6$ and $\beta_f = 4.2$ [349]. If the frequency drops by 0.09 Hz (as in Example 2.23 in Chapter 2), then we can obtain the frequency term in the load model as

$$1 + \alpha_f\,(f - f_0) = 1 + 5.6 \times -0.09 = 0.496.$$

$$1 + \beta_f\,(f - f_0) = 1 + 4.2 \times -0.09 = 0.622.$$

- Active power load drops by 50%, and reactive power load drops by 38%.

- Such load reduction (as a result of drop in frequency) can contribute to the *inertial response of the system*, as we saw in Example 2.23.

5.7. Load Modeling

5.7.2. Dynamic Load Model

• Static loads are *static functions* of voltage and/or frequency.

• The power consumption of a static load at any instant depends on only the voltage and/or frequency at that same instant.

• Accordingly, static load models are in form of *algebraic equations*, such as in the formulations that we discussed in Section 5.7.2.

5.7. Load Modeling

5.7.2. Dynamic Load Model

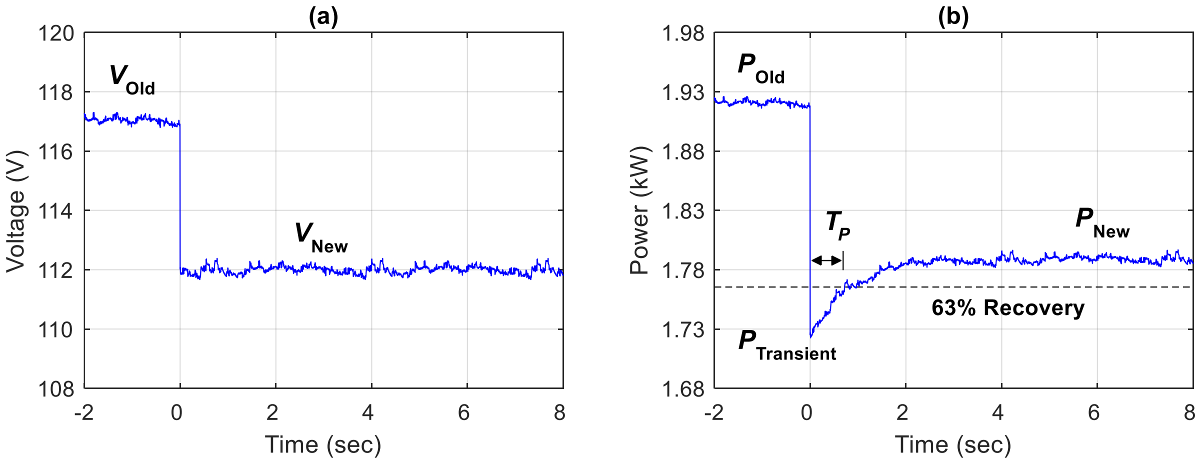
• Dynamic loads, such as induction motors, on the other hand, take some time in *transient conditions* before they reach steady-state conditions.

• This is because the power consumption of a dynamic load at any instant depends on not only the voltage and/or frequency at that same instant, but also the internal state variables of the load at previous instances.

• Accordingly, dynamic loads are modeled as a combination of *algebraic equations* and also *differential equations* [350, 351].

5.7.2. Dynamic Load Model

- **Recovery Model**: This model is a popular model in voltage stability analysis [352]. It captures the *recovery response* of a load to a *sudden voltage drop*. That is, it captures how the active power consumption and the reactive power consumption of the load changes before it reaches steady-state conditions. This model is illustrated in the figure below.



5.7.2. Dynamic Load Model

- **Recovery Model (Cont.)**: In the figure on the previous slide:

    - Voltage suddenly drops at time $t_0 = 0$ from $V_{\text{Old}} = 117\ V$ to $V_{\text{New}} = 112\ V$; see Figure (a) on the previous slide.

    - In response, the active power consumption of the load drops from $P_{\text{Old}} = 1.92\ kW$ to $P_{\text{Transient}} = 1.72\ kW$; and then it gradually increases to $P_{\text{New}} = 1.79\ kW$, see Figure (b).

- The primary focus of the recovery model is to capture the transient response of the load that takes place due to the dynamics of the load.

5.7.2. Dynamic Load Model

• **Recovery Model (Cont.)**: The recovery mode is described by the following *combination of algebraic and differential equations* [352]:

$$P(t) = P_0 \left(\frac{V(t)}{V_0} \right)^{\zeta_P} + \frac{x(t)}{T_P}$$

$$\frac{dx(t)}{dt} = -\frac{x(t)}{T_P} + P_0 \left(\frac{V(t)}{V_0} \right)^{\gamma_P} - P_0 \left(\frac{V(t)}{V_0} \right)^{\zeta_P}.$$

• Here, $x(t)$ is the state-variable of the load; γ_P is the static coefficient, which is the same parameter as in the exponential load model; ζ_P is the transient recovery coefficient; and T_P is the recovery time constant.

• We can obtain a similar recovery model for reactive power consumption.

5.7.2. Dynamic Load Model

- **Recovery Model (Cont.)**: We can measure parameters V_{Old}, V_{New}, P_{Old}, $P_{\text{Transient}}$, and P_{New}. If the rated voltage V_0 and the rated active power consumption P_0 are known, then we can obtain:

$$\gamma_P = \log\left(P_{\text{New}}/P_0\right) \Big/ \log\left(V_{\text{New}}/V_0\right),$$

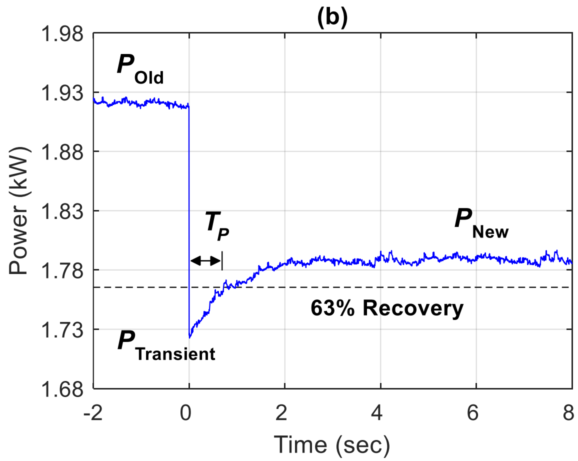
$$\zeta_P = \log\left(P_{\text{Transient}}/P_0\right) \Big/ \log\left(V_{\text{New}}/V_0\right).$$

- Notice that γ_P is obtained in the first line by placing $P(t) = P_{\text{New}}$, $V(t) = V_{\text{New}}$, and $dx(t)/dt = 0$ in the model on Slide 160.

- Also, ζ_P is obtained by placing $P(t) = P_{\text{Transient}}$, $V(t) = V_{\text{New}}$, and $x(t) = 0$ in the first line in the model on Slide 160.

5.7.2. Dynamic Load Model

- **Recovery Model (Cont.)**: Parameter T_P can be obtained by determining the time instance at which $P(t)$ in Figure (b) crosses the 63% threshold line during its recovery from $P_{\text{Transient}}$ to P_{New}. This threshold is based equation on Slide 160 and the fact that 100 × (1 − exp(−1)) = 63%; see [352] for more details. For the example in Figure (b), we have T_P = 0.824.



5.7. Load Modeling

5.7.2. Dynamic Load Model

- **Other Dynamic Load Models**: Other dynamic load models that are used in measurement-based load modeling include:

  - the induction motor (IM) model [353],
  - a combination of the ZIP model and the IM model [354],
  - and load models that are based on training neural networks [355].

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5.8. State Estimation Using Power Measurements

- We previously discussed the state estimation problem in Section 3.8 in Chapter 3, where we assumed that the measurements are voltage and current synchrophasors that are obtained by PMUs.

- Recall that a common choice for the states of a power system is the voltage magnitudes and voltage phase angles at all buses.

- Therefore, when synchrophasor measurements are available, we are able to directly measure a subset of the state variables.

- The use of synchrophasor measurements also results in formulating a state estimation problem that is inherently linear; see the relationship in equation (3.76) in Section 3.8 in Chapter 3.

5.8. State Estimation Using Power Measurements

- However, prior to the development of PMUs, the states of the power system could not be measured directly. In particular, the voltage phase angles could be only inferred from the power flow measurements [356].

- Accordingly, the traditional state estimation problem is the problem of using *active power and reactive power measurements* to solve the power flow equations that we saw in Section 1.3.1 in Chapter 1, so as to estimate the voltage magnitudes and voltage phase angles at all buses.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

- In this section, we discuss the basic formulation of the traditional state estimation problem, which uses active and reactive power measurements.

- This problem is a nonlinear and non-convex optimization problem.

    - We will solve the original nonlinear problem in this section.

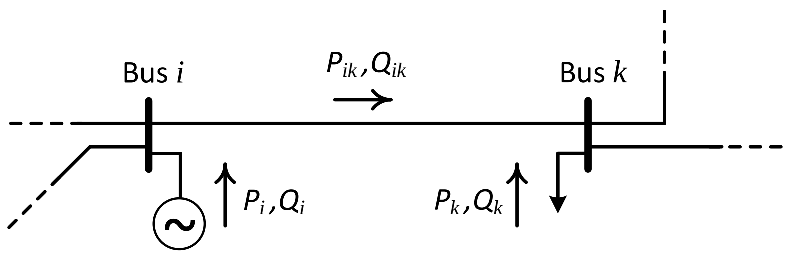
    - We will discuss a linearized approximate formulation of the state estimation problem in Section 5.8.2.

    - We will also briefly discuss other formulations in Section 5.8.3.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

• Let $\mathbf{x}$ denote the vector of all states of the power system. Let $\mathbf{z}$ denote the vector of all active and reactive power measurements, whether at buses or on power lines. An example is shown below.



• Here, P_i and Q_i denote the active power and reactive power at bus i; P_k and Q_k denote the active power and reactive power at bus k; and P_{ik} and Q_{ik} denote the active power and reactive power on line (i, k).

5.8.1. Basic Nonlinear Formulation

• The vector of state variables and the vector of measurements for the example in this figure are:

$$\mathbf{x} = \begin{bmatrix} \vdots \\ V_i \\ V_k \\ \vdots \\ \theta_i \\ \theta_k \\ \vdots \end{bmatrix}, \quad \text{and} \quad \mathbf{z} = \begin{bmatrix} \vdots \\ P_i \\ Q_i \\ P_k \\ Q_k \\ \vdots \\ P_{ik} \\ Q_{ik} \\ \vdots \end{bmatrix},$$

where $X_i\angle\theta_i$ and $X_k\angle\theta_k$ denote the voltage phasors at buses i and k.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

• From Section 1.3.1 in Chapter 1, the following relationships hold between the voltage phasors and nodal power injections and line power flows:

$$P_i = \sum_{k=1}^{n} V_i V_k \left(G_{ik}^{\text{bus}} \cos(\theta_i - \theta_k) + B_{ik}^{\text{bus}} \sin(\theta_i - \theta_k) \right)$$
$$Q_i = \sum_{k=1}^{n} V_i V_k \left(G_{ik}^{\text{bus}} \sin(\theta_i - \theta_k) - B_{ik}^{\text{bus}} \cos(\theta_i - \theta_k) \right)$$

$$P_{ik} = -V_i^2 G_{ik}^{\text{bus}} + V_i V_k \left(G_{ik}^{\text{bus}} \cos(\theta_i - \theta_k) + B_{ik}^{\text{bus}} \sin(\theta_i - \theta_k) \right)$$
$$Q_{ik} = V_i^2 B_{ik}^{\text{bus}} + V_i V_k \left(G_{ik}^{\text{bus}} \sin(\theta_i - \theta_k) - B_{ik}^{\text{bus}} \cos(\theta_i - \theta_k) \right),$$

where $G_{ik}^{\text{bus}}$and B_{ik}^{bus} denote the real part and the imaginary part of the entry in row i and column j of the Y-bus matrix; see (1.52) in Chapter 1.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

• From the equations on the previous slide, we can relate the measurements to the state variables as follows:

$$\mathbf{z} = \mathbf{h}(\mathbf{x}) + \epsilon,$$

where $\mathbf{h}(\mathbf{x})$ is the vector of nonlinear functions of the forms on Slide 171; and $\boldsymbol{\epsilon}$ is the vector of measurement noise. The state estimation problem can be formulated as the following LS optimization problem:

$$\min_{\mathbf{x}} \|\mathbf{z} - \mathbf{h}(\mathbf{x})\|_2 \, .$$

• The number of state variables is $2n$, where n is the number of buses; and the number of measurements is m.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

• Bus 1 can be assumed to be the *reference bus*.

• We assume to know the *voltage magnitude* at the *reference bus*.

• We also set the voltage phase angle at the reference bus to be *zero*, i.e., $\theta_1 = 0$. Recall from Section 3.1.2 in Chapter 3 that we can rotate the same phasors and represent them differently based on different references; therefore, we need to use a *reference phase angle* in order to avoid ambiguity in defining the phase angles.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

• The optimization problem in (5.61) is non-convex.

• There do exist some solvers to deal with non-convex least-square optimization problems, such as *lsqnonlin* in MATLAB [357].

• We can also use iterative algorithms such as the Gauss–Newton method that we will discuss in the next sub-section.

• Another option is to solve a linearized approximation of the non-linear problem on Slide 172 using the standard LS method; see Section 5.8.2.

• We will discuss some other problem formulations in Section 5.8.3.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

• **Gauss–Newton Iterations**: As mentioned earlier, the optimization problem on Slide 172 is non-convex and generally difficult to solve. One option is to use the Gauss–Newton method. Each iteration in the Gauss–Newton algorithm is formulated as [358]:

$$\mathbf{x} \leftarrow \mathbf{x} + \left[\mathbf{H}(\mathbf{x})^T \mathbf{H}(\mathbf{x})\right]^{-1} \mathbf{H}(\mathbf{x})^T \left(\mathbf{z} - \mathbf{h}(\mathbf{x})\right),$$

where

$$\mathbf{H} = [\partial \mathbf{h} / \partial \mathbf{x}]$$

is the *measurement Jacobian*. The entry in row l and column j of the measurement Jacobian matrix $\mathbf{H}$ is the partial derivative of row l of function $\mathbf{h}(\mathbf{x})$ with respect to the state variable in row j of vector $\mathbf{x}$.

5.8.1. Basic Nonlinear Formulation

- **Gauss–Newton Iterations (Cont.)**: The iterations on the previous slide start with a given *initial value* for $\mathbf{x}$. Note that the phase angle at the reference bus should always be kept at zero, i.e., $\theta_1 = 0$. The iterations continue until the norm of the measurement residues is less than a given threshold δ, i.e., until the following inequality holds:

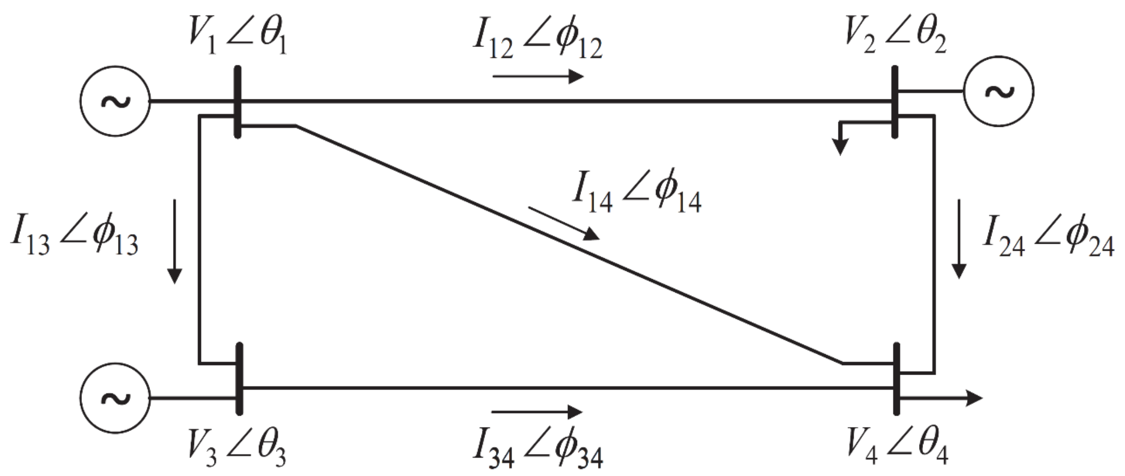
$$\|\mathbf{z} - \mathbf{h}(\mathbf{x})\|_2 \leq \delta.$$

- Note that matrix $\mathbf{H}$ has to be updated in each iteration.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

- **Example 5.12**: Again, consider the 4-bus transmission network in Example 3.20 in Chapter 3. This same network is also shown below.



- Suppose we measure the power injection at each bus.

5.8.1. Basic Nonlinear Formulation

- **Example 5.12 (Cont.)**: The true versus measured power values are:

Bus #	True Apparent Power (p.u.)	Measured Apparent Power (p.u.)
1	$3.5622 + j0.8768$	–
2	$-0.7795 + j0.0000$	$-0.7786 + j0.0000$
3	$1.2622 + j1.0203$	$1.2600 + j1.0206$
4	$-4.0000 - j1.0000$	$-3.9927 - j0.9982$

Line #	True Apparent Power (p.u.)	Measured Apparent Power (p.u.)
1,2	$1.3420 + j0.3884$	$1.3394 + j0.3879$
1,3	$0.3369 - j0.1848$	–
1,4	$1.8832 + j0.6734$	$1.8858 + j0.6731$
2,1	$-1.3329 - j0.2060$	–
2,4	$0.5534 + j0.2060$	–
3,1	$-0.3362 + j0.1986$	–
3,4	$1.5984 + j0.8217$	$1.5975 + 0.8200$
4,1	$-1.8646 - j0.2996$	–
4,2	$-0.5517 - j0.1710$	–
4,3	$-1.5838 - j0.5294$	$-1.5850 - j0.5295$

5.8.1. Basic Nonlinear Formulation

• **Example 5.12 (Cont.)**: But 1 is the reference bus.

• At the reference bus, the voltage phase angle is assumed to be zero and the voltage magnitude is measured directly. The true and the measured voltage magnitudes at bus 1 are 1.0332 p.u. and 1.0297 p.u., respectively.

5.8.1. Basic Nonlinear Formulation

• **Example 5.12 (Cont.)**: The state variables that need to be estimated are the voltage magnitude and the voltage phase angle at buses 2, 3, and 4. Accordingly, where $\mathbf{x}$ is a 6×1 vector and $\mathbf{z}$ is a 12×1 vector.

• The measurement matrix $\mathbf{H}$ is 12×6.

• The initial value for all unknown states is set to one for magnitude and zero for phase angle. The Gauss–Newton algorithm is based on the iterations on Slide 175. The iterations stop when the norm of the measurement residue is less than $\delta = 0.005$ p.u.

5.8. State Estimation Using Power Measurements

5.8.1. Basic Nonlinear Formulation

- **Example 5.12 (Cont.)**: The state estimation results are obtained as

$$\begin{aligned} V_1\angle\theta_1 &= 1.0297\,\angle 0^\circ, \\ V_2\angle\theta_2 &= 0.9938\,\angle{-7.4071^\circ}, \\ V_3\angle\theta_3 &= 1.0464\,\angle{-1.8296^\circ}, \\ V_4\angle\theta_4 &= 0.9718\,\angle{-10.6214^\circ}. \end{aligned}$$

- The Gauss–Newton algorithm converges after seven iterations.

- If we rotate all phase angles by 38.8884° clockwise, they can be presented equivalently as 38.8884°, 31.4813°, 37.0588°, and 28.2670°. Note that, these new values are comparable with the phase angles that are obtained in Example 3.20 in Chapter 3.

5.8.1. Basic Nonlinear Formulation

• **Other Iterative Algorithms**: Other iterative algorithms have been used to solve the state estimation optimization problem on Slide 172.

• Some of these algorithms include the Gauss–Seidel method, the Newton-Rophson method, and various decoupling methods; see the state estimation textbooks such as in [356, 359–361].

• Another important topic that is often discussed in these textbooks is about the methods that can *detect and discard bad data* in the state estimation problem, i.e., erroneous measurements, to enhance the accuracy of the state estimation solution.

5.8. State Estimation Using Power Measurements

<u>**5.8.1. Basic Nonlinear Formulation**</u>

• **Using Both Power Measurements and Phasor Measurements**: The state estimation problem may also involve a combination of both power measurements and synchronized phasor measurements.

• Such a problem can be solved in a way similar to the traditional state estimation problem that we have discussed throughout Section 5.8.1.

• However, one challenge is that, while the power flow equations are formulated and solved based on a locational reference, where the voltage phase angle is zero at a reference bus, synchronized phasor measurements are obtained based on a temporal reference, where the voltage phase angle is zero for the reference waveform. Therefore, it is necessary to *choose a reference bus that is equipped with a PMU*.

5.8.2. Linearized Approximate Formulation

• In this alternative formulation, we define the state estimation problem based on the *linearized power flow equations* in (1.65) and (1.66), instead of the original nonlinear power flow equations in (1.56) and (1.57).

• Accordingly, the following approximate relationships are considered between the voltage phase angles and nodal power injections, and the line power flows; see Section 1.3.1 in Chapter 1:

$$P_i = \sum_{k=1}^{n} B_{ik}^{\text{bus}} (\theta_i - \theta_k)$$

$$P_{ik} = B_{ik}^{\text{bus}} (\theta_i - \theta_k) \, .$$

5.8.2. Linearized Approximate Formulation

- Given the type of equations that are used in this formulation; the state variables are defined to include *only the voltage phase angles* at all buses.

- The magnitude of voltage phasors is assumed to be 1 per unit at all buses; see the explanation regarding this assumption in Section 1.3.1 in Chapter 1.

- As for the measurements, they include only active power measurements.

    - This is because reactive power does not appear in the linearized power flow equations in the previous slide.

5.8.2. Linearized Approximate Formulation

- The vector of *state variables* is defined as:

$$\mathbf{x} = \begin{bmatrix} \vdots \\ \theta_i \\ \theta_k \\ \vdots \end{bmatrix}.$$

- The vector of *measurements* is defined as:

$$\mathbf{z} = \begin{bmatrix} \vdots \\ P_i \\ P_k \\ \vdots \\ P_{ik} \\ \vdots \end{bmatrix}.$$

5.8.2. Linearized Approximate Formulation

• The measurements are related to the state variables as follows:

$$\mathbf{z} = \mathbf{\Psi}\mathbf{x} + \epsilon,$$

$$\min_{\mathbf{x}} \; \|\mathbf{z} - \mathbf{\Psi}\mathbf{x}\|_2 \,.$$

• The above is a standard LS optimization problem. It can be solved by using the command *lsqlin* in MATLAB [102]. As in the original formulation on Slide 172, we assume that the *voltage phase angle* at the *reference bus* is zero. The *voltage magnitude* is assumed to be 1 at all buses, including at the *reference bus*.

5.8.2. Linearized Approximate Formulation

• **Example 5.13**: We can solve the state estimation problem in Example 5.12 also by using the approximate linearized formulation. The vector of state variables and the vector of measurements are as follows:

$$\mathbf{x} = \begin{bmatrix} \theta_1 \\ \theta_2 \\ \theta_3 \\ \theta_4 \end{bmatrix}, \qquad \mathbf{z} = \begin{bmatrix} -0.7786 \\ 1.2600 \\ -3.9927 \\ 1.3394 \\ 1.8858 \\ 1.5975 \\ -1.5850 \end{bmatrix}.$$

5.8.2. Linearized Approximate Formulation

- **Example 5.13 (Cont.)**: The measurement matrix is obtained as

$$\mathbf{H} = \begin{bmatrix} -10 & 20 & 0 & -10 \\ -10 & 0 & 20 & -10 \\ -10 & -10 & -10 & 30 \\ 10 & -10 & 0 & 0 \\ 10 & 0 & 0 & -10 \\ 0 & 0 & 10 & -10 \\ 0 & 0 & -10 & 10 \end{bmatrix}.$$

5.8.2. Linearized Approximate Formulation

- **Example 5.13 (Cont.)**: The state estimation results are obtained as

$$\begin{aligned} \theta_1 &= 0^\circ, \\ \theta_2 &= -7.6439^\circ, \\ \theta_3 &= -1.7514^\circ, \\ \theta_4 &= -10.7790^\circ. \end{aligned}$$

- By comparing the above results with those in Example 5.12, we can see that the obtained phase angles in the two methods are generally similar. If we rotate the above phase angles by 38.8884° clockwise, then they can be presented equivalently as 38.8884°, 31.2445°, 37.1370°, and 28.1094°. These new values are comparable with the phase angles that are obtained in Example 3.20 in Chapter 3.

5.8.3. Convex Relaxation and Other Formulations

• Besides the approximate linearized formulation that we used in Section 5.8.2, there are also other methods that use other linearization techniques or other approximation techniques in formulating and solving the power flow equations.

• An overview of some of these methods is provided in [362].

• In general, any alternative formulation of the power flow equations can potentially be used also for formulation of the state estimation problem.

5.8.3. Convex Relaxation and Other Formulations

• Besides the approximate linearized formulation that we used in Section 5.8.2, there are also other methods that use *other linearization techniques* or *other approximation techniques* in formulating and solving the power flow equations.

• An overview of some of these methods is provided in [362].

• In general, any *alternative formulation of the power flow equations* can potentially be used also for formulation of the state estimation problem.

5.8.3. Convex Relaxation and Other Formulations

• Furthermore, there have been important advancements in recent years in the field of power flow analysis by using different methods for *convex relaxation*; e.g., see [363–365].

• The idea is to start from the original power flow equations in complex domain, such as (1.54) and (1.55) in Chapter 1, and formulate the power flow analysis as a *nonconvex quadratic optimization* problem in *complex domain*.

• Then use techniques for convex relaxation, such as relaxation of the original problem formulation to a *semidefinite program* (SDP), to solve the power flow equations.

5.8.3. Convex Relaxation and Other Formulations

• In general, the solutions that are obtained by applying convex relaxation techniques are *approximate*.

• However, under certain conditions, such as in a *balanced symmetric radial distribution feeder*, the solutions can be exact; e.g., see the summary discussions in [366, 367].

• These new approaches have already been used in state estimation. Some examples include the studies in [368–371].

5.8.3. Convex Relaxation and Other Formulations

• Before we end this section, it should be noted that the state estimation problems that we discussed here and throughout this book are about *static states*, where the state variables at any instance depend only on the measurements at that same instance.

• However, state estimation problems can also be defined with respect to *dynamic states* of the system, such as generator rotor angles and speeds.

• Tools such as Kalman Filters are used to conduct dynamic state estimation; e.g., see [372, 373].

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5.9. Three-Phase Power and Energy Measurements

- The *total power* in a three-phase power system is the *summation* of the power that is measured separately at *each* of the three phases:

$$P = P_A + P_B + P_C,$$
$$Q = Q_A + Q_B + Q_C.$$

- Similarly, the total energy in a three-phase power system is the sum of the energy that is measured separately at each of the three phases:

$$E = E_A + E_B + E_C.$$

- As for apparent power and power factor, they can be measured *individually on each phase*. In general, it is not common to express them as overall quantities in three-phase systems.
    - We will further discuss this subject in Section 5.9.4.

5.9. Three-Phase Power and Energy Measurements

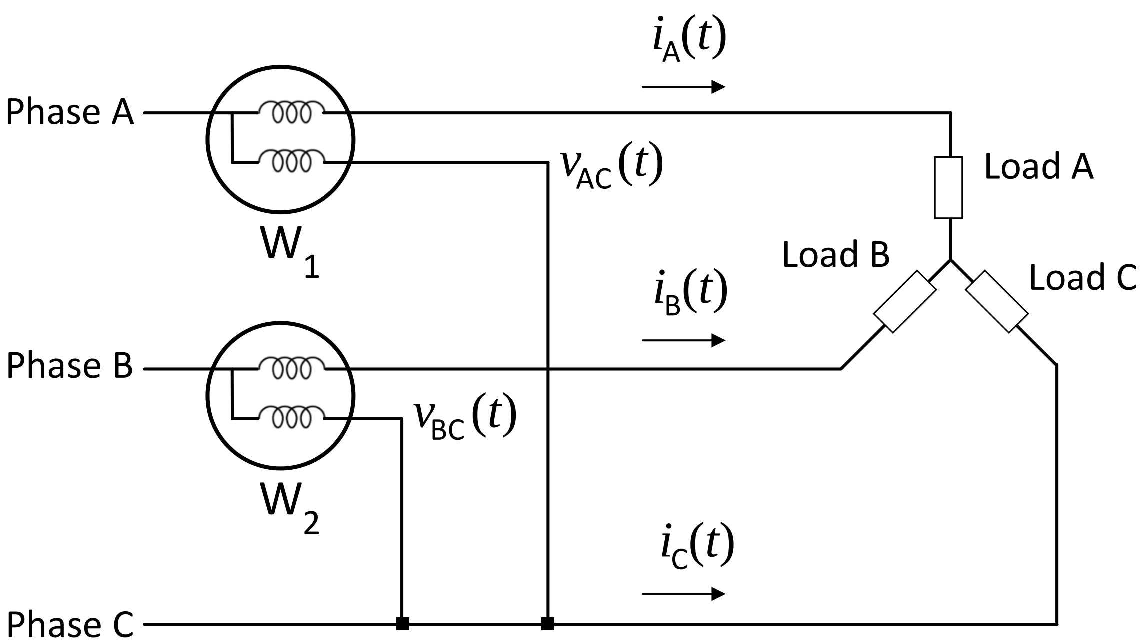
5.9.1. Two-Wattmeter Method

• If the three-phase power system is balanced, then it is sufficient to use only one wattmeter to measure active power on *one phase*, and we can multiply the measurements by three to obtain the total active power for the overall three-phase system.

• However, if the three-phase power system is unbalanced, then we usually need three wattmeters to make separate measurements on *each phase*.

5.9.1. Two-Wattmeter Method

• In some special cases, we may need fewer sensors. An example is shown below. Here, the three-phase load has a three-wire star topology.



5.9. Three-Phase Power and Energy Measurements

5.9.1. Two-Wattmeter Method

• In the three-wire star topology, we can measure the total power usage of the three-phase load by using only two (instead of three) wattmeters.

• Wattmeter W1 is connected to Phase A and wattmeter W2 is connected to Phase B [374]. Unlike in the standard connection in Figure (a) on Slide 7, where the potential coil of the wattmeter has a ground connection, the *potential coils* of wattmeters W1 and W2 in the figure on the previous slide are connected to *Phase C*.

• Thus, W1 measures the *line-to-line voltage* between Phase A and Phase C; and W2 measures the *line-to-line voltage* between Phase B and Phase C.

5.9.1. Two-Wattmeter Method

- Next, we show why W_1 and W_2 are sufficient to measure the total power.

- The instantaneous power usage that is measured by W_1 is

$$i_A(t)\, v_{AC}(t) = i_A(t)\, (v_A(t) - v_C(t))\,.$$

- Similarly, the instantaneous power usage that is measured by W_2 is

$$i_B(t)\, v_{BC}(t) = i_B(t)\, (v_B(t) - v_C(t))\,.$$

- The sum of the instantaneous power that is measured by W_1 and W_2 is

$$i_A(t)\, v_A(t) + i_B(t)\, v_B(t) - (i_A(t) + i_B(t))\, v_C(t).$$

5.9. Three-Phase Power and Energy Measurements

5.9.1. Two-Wattmeter Method

• However, we know that in a three-wire star topology, we have:

$$i_A + i_B + i_C = 0 \quad \Rightarrow \quad i_C = -\left(i_A + i_B\right).$$

• By replacing the above equation in the last equation on the previous slide, we can see that the sum of the instantaneous power usage that is measured by wattmeters W_1 and W_2 is equal to the *total* instantaneous power usage across all three phases:

$$i_A(t)\, v_A(t) + i_B(t)\, v_B(t) + i_C(t)\, v_C(t) = p_A(t) + p_B(t) + p_C(t).$$

5.9. Three-Phase Power and Energy Measurements

5.9.1. Two-Wattmeter Method

• It should be noted that the above two-wattmeter method can also be applied to a three-phase load with *delta connections*; see Exercise 5.21.

• Also, if we replace the two wattmeters with *two varmeters* or *two watthour meters*, then we can similarly measure the total reactive power usage or the total energy usage of this three-phase load, respectively, still using only two sensors in each case.

5.9.2. Phase Identification by Power and Energy Measurements

• A fundamental problem in any three-phase system is phase identification.

• We previously discussed solving the phase identification problem in power distribution systems by using different types of measurements:

- using *voltage or current measurements* in Section 2.8.3 in Chapter 2
- using *phase angle measurements* in Section 3.6.4 in Chapter 3.

• In this section, we seek to solve the phase identification problem by using power and energy measurements.

5.9.2. Phase Identification by Power and Energy Measurements

• The basic idea is to use the *principle of conservation of electric charge*, i.e., the fact that energy that is supplied by a power distribution feeder *must be equal* to the energy that is consumed by the loads, *plus losses and measurement errors*.

• Accordingly, one can transform the phase identification problem into the problem of identifying the phase connections to *minimize the mismatch* between the measured supplied energy on each phase and the summation of the measured consumed energy by all the loads that are connected to that phase, as we explain next.

5.9.2. Phase Identification by Power and Energy Measurements

- Suppose all loads are single phase.

- Let N denote the number of loads.

- Suppose the energy usage of each load is measured *periodically* at equal-length time intervals, such as once every five minutes.

- The total number of the measurement intervals is denoted by T.

5.9.2. Phase Identification by Power and Energy Measurements

• Let $E_n[t]$ denote the energy usage of load n during time interval t.

• For each load n, let us define ζ_n^A, ζ_n^B, and ζ_n^C as binary phase identification variables corresponding to phases A, B, and C, respectively.

• If load n is connected to phase A, then $\zeta_n^A = 1$.

• Otherwise $\zeta_n^A = 0$.

• Variables ζ_n^B and ζ_n^C are defined similarly.

5.9.2. Phase Identification by Power and Energy Measurements

- Accordingly, the following equality must hold because *each single-phase load* must be connected to *exactly one phase*:

$$\zeta_n^A + \zeta_n^B + \zeta_n^C = 1.$$

- The total load that is identified on phase A at time interval t is obtain as

$$\sum_{n=1}^{N} \zeta_n^A E_n[t].$$

5.9.2. Phase Identification by Power and Energy Measurements

- Next, suppose $E_A[t]$, $E_B[t]$, and $E_C[t]$ denote the total energy usage of the entire power distribution feeder that is *measured* at the distribution substation during time interval t on phases A, B, and C, respectively.

- If phase identification is done correctly, i.e., if ζ_n^A, ζ_n^B, and ζ_n^C are set correctly for all loads $n = 1, \ldots, N$, then the total load in the second equation on Slide 209 would match $E_A[t]$ at all time intervals $t = 1, \ldots, T$.

- A similar statement is true for phases B and C.

5.9.2. Phase Identification by Power and Energy Measurements

- In order to identify the unknown phase connections, we seek to select ζ_n^A, ζ_n^B, and ζ_n^C for all loads $n = 1, \ldots, N$ such that we *minimize* the following expression subject to the constraint that $\zeta_n^A + \zeta_n^B + \zeta_n^C = 1$:

$$\sum_{t=1}^{T} \left[\left(\sum_{n=1}^{N} \zeta_n^A E_n[t] - E^A[t] \right)^2 + \left(\sum_{n=1}^{N} \zeta_n^B E_n[t] - E^B[t] \right)^2 + \left(\sum_{n=1}^{N} \zeta_n^C E_n[t] - E^C[t] \right)^2 \right].$$

5.9.2. Phase Identification by Power and Energy Measurements

- Hence, we can formulate the phase identification problem as follows:

$$\begin{aligned} \underset{\zeta}{\text{minimize}} \quad & \|\mathbf{f} - \mathbf{E}\,\boldsymbol{\zeta}\|_2\,, \\ \text{subject to} \quad & \zeta_n^A + \zeta_n^B + \zeta_n^C = 1, \quad n = 1, \ldots, N. \end{aligned}$$

which is a *binary* LS problem. $\mathbf{f}$ is the $3T \times 1$ vector of per-phase feeder load measurements, $\boldsymbol{\zeta}$ is the $3N \times 1$ vector of per-load phase identification variables, $\mathbf{E}$ is the $3T \times 3N$ matrix of single-phase load measurements.

- The above constrained binary LS problem can be solved in MATLAB by using any convex optimization toolbox that supports binary or integer variables, such as CVX [178, 376] or CPLEX [377].

5.9.2. Phase Identification by Power and Energy Measurements

- **Example 5.14**: Consider the energy measurements as shown in the table below over $T = 2$ time intervals. We would like to identify how each of the $N = 5$ loads is connected to a phase.

Time	Phase (kWh)			Loads (kWh)				
Interval	A	B	C	1	2	3	4	5
$t = 1$	5.5	3.2	7.1	2.1	3.3	3.0	2.6	4.3
$t = 2$	6.2	8.7	12.5	2.9	3.1	8.6	5.5	7.1

- The vector of unknown phase identification variables is formulated as

$$\zeta = \begin{bmatrix} \zeta_1^A & \zeta_1^B & \zeta_1^C & \ldots & \zeta_5^A & \zeta_5^B & \zeta_5^C \end{bmatrix}^T.$$

5.9.2. Phase Identification by Power and Energy Measurements

• **Example 5.14 (Cont.)**: The other parameters of the phase identification problem are as follows:

$$\mathrm{f} = \begin{bmatrix} 5.5 & 3.2 & 7.1 & 6.2 & 8.7 & 12.5 \end{bmatrix}^T,$$

$$\mathbf{E} = \begin{bmatrix} 2.1 & 0 & 0 & 3.3 & 0 & 0 & 3.0 & 0 & 0 & 2.6 & 0 & 0 & 4.3 & 0 & 0 \\ 0 & 2.1 & 0 & 0 & 3.3 & 0 & 0 & 3.0 & 0 & 0 & 2.6 & 0 & 0 & 4.3 & 0 \\ 0 & 0 & 2.1 & 0 & 0 & 3.3 & 0 & 0 & 3.0 & 0 & 0 & 2.6 & 0 & 0 & 4.3 \\ 2.9 & 0 & 0 & 3.1 & 0 & 0 & 8.6 & 0 & 0 & 5.5 & 0 & 0 & 7.1 & 0 & 0 \\ 0 & 2.9 & 0 & 0 & 3.1 & 0 & 0 & 8.6 & 0 & 0 & 5.5 & 0 & 0 & 7.1 & 0 \\ 0 & 0 & 2.9 & 0 & 0 & 3.1 & 0 & 0 & 8.6 & 0 & 0 & 5.5 & 0 & 0 & 7.1 \end{bmatrix}.$$

5.9.2. Phase Identification by Power and Energy Measurements

- **Example 5.14 (Cont.)**: The solution is obtained as

$$\zeta_1^A = \zeta_2^A = \zeta_3^B = \zeta_4^C = \zeta_5^C = 1$$

$$\zeta_3^A = \zeta_4^A = \zeta_5^A = \zeta_1^B = \zeta_2^B = \zeta_4^B = \zeta_5^B = \zeta_1^C = \zeta_2^C = \zeta_3^C = 0.$$

That is, loads 1 and 2 are connected to phase A, load 3 is connected to phase B, and loads 4 and 5 are connected to phase C.

The minimum l_2 norm of the measurement residues is obtained as 0.3873 kWh, which corresponds to *power loss and measurement errors*.

5.9. Three-Phase Power and Energy Measurements

5.9.2. Phase Identification by Power and Energy Measurements

• As the number of loads increases, more time intervals need to be considered in order to collect enough information to identify the phases.

• In principle, the above phase identification problem can be formulated based on not only *active power* measurements but also *reactive power* measurements to utilize more information.

• The problem formulation can also be adjusted to include not only single-phase but also two-phase and three-phase loads.

• When the relevant data is available, the above phase identification problem can also be reinforced by power measurements at several *load transformers across* the power distribution feeder.

5.9.3. Other Applications of 3-Phase Power and Energy Measurements

• Many of the applications of power and energy measurements that we discussed in this chapter can be extended to three-phase measurements.

• **Load Modeling**: Load modeling can be done at each of the three phases. All the load models that we discussed in Section 5.7 can also be used to model two-phase or three-phase loads. For example, for a three-phase load, we can extend the exponential load model on Slide 150 to:

$$P = P_{A,0}\left(\frac{V_A}{V_{A,0}}\right)^{\gamma_{A,P}} + P_{B,0}\left(\frac{V_B}{V_{B,0}}\right)^{\gamma_{B,P}} + P_{C,0}\left(\frac{V_C}{V_{C,0}}\right)^{\gamma_{C,P}},$$

where the parameters are γ_A, P, γ_B, P, and γ_C, P.

5.9.3. Other Applications of 3-Phase Power and Energy Measurements

- **Disaggregation**: Load disaggregation can also be done at each of the three phases, or across a combination of single-phase, two-phase, and three-phase loads [323]. The methods that we learned in Section 5.6.1 can also be used in these cases. For example, switching of a three-phase load can be recognized based on its signature on all three phases, i.e., the changes that it causes in active power consumption and reactive power consumption, as in the figure on Slide 119, but on each phase.

- Net load disaggregation, such as solar generation disaggregation, can also be done at each phase. One potentially helpful note is that, in practice, most three-phase PV and wind inverters are balanced, with equal power generation per phase across the three phases [378]; therefore, we can assume that the generation component of the net load is balanced across the three phases, while the load is unbalanced.

5.9.3. Other Applications of 3-Phase Power and Energy Measurements

- **Per-Phase Smart Pricing**: Pricing methods are usually designed based on system-wide considerations in the power system, such as with respect to the overall load profile of the utility and the price of power procurement in the wholesale electricity market.

- However, there is a growing interest in also developing pricing methods that reflect the operation challenges at the power distribution level, such as with respect to such as voltage regulation, balancing load across phases, or integrating distributed energy resources.

- Depending on the purpose of the pricing mechanism, some of these pricing methods are defined on each phase, i.e., they are per-phase prices; e.g., see the studies in [379–381].

5.9.3. Other Applications of 3-Phase Power and Energy Measurements

• **Three-Phase DSSE**: Three-phase state estimation is often not necessary at the transmission level; because the power systems at the level of the power transmission are mostly balanced.

• In contrast, the power system at the level of power distribution networks is usually unbalanced; therefore, there is a need in practice to develop efficient three-phase DSSE solutions.

• The convex relaxation methods that we briefly mentioned in Section 5.8.3 may not result in exact solutions when the three-phase power distribution system is unbalanced.

5.9.3. Other Applications of 3-Phase Power and Energy Measurements

• **Three-Phase DSSE (Cont.)**: One option is to approximately decompose the three-phase DSSE problem into three *separate* single-phase DSSE problems, e.g., by ignoring the *mutual impedance* across different phases of the distribution lines.

• In that case, each single-phase DSSE problem can be solved separately by using a convex relaxation method.

• Another option is to solve the original three-phase DSSE problem by either using the standard methods such as the Gauss–Newton method that we learned in Section 5.8.1 [382], or using some advanced optimization techniques such as those in [383–385].

5.9.4. Three-Phase Apparent Power and Power Factor

- There are different ways to define apparent power in 3-phase systems:

- Arithmetical apparent power [387]:

$$S = V_{\mathrm{A,rms}}\, I_{\mathrm{A,rms}} + V_{B,\mathrm{rms}}\, I_{B,\mathrm{rms}} + V_{C,\mathrm{rms}}\, I_{C,\mathrm{rms}}.$$

- Geometrical apparent power, where P and Q are defined on Slide 197 :

$$S = \sqrt{P^2 + Q^2}\,.$$

- Here is another definition from [388]:

$$S = \sqrt{V_{\mathrm{A,rms}}^2 + V_{B,\mathrm{rms}}^2 + V_{C,\mathrm{rms}}^2}\,\sqrt{I_{\mathrm{A,rms}}^2 + I_{B,\mathrm{rms}}^2 + I_{C,\mathrm{rms}}^2}.$$

- These different definitions result in different values; see *Exercise 5.22*.

5.9.4. Three-Phase Apparent Power and Power Factor

• As for the power factor, the common approach is either to look at the power factor at *each phase individually*, or to simply use the *average* of the power factors across the three phases.

• Alternatively, the overall power factor may also be defined by dividing the total active power P as defined on Slide 198 to S, where S can be any of the definitions of apparent power on the previous slide.

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5.10. Accuracy in Power and Energy Measurements

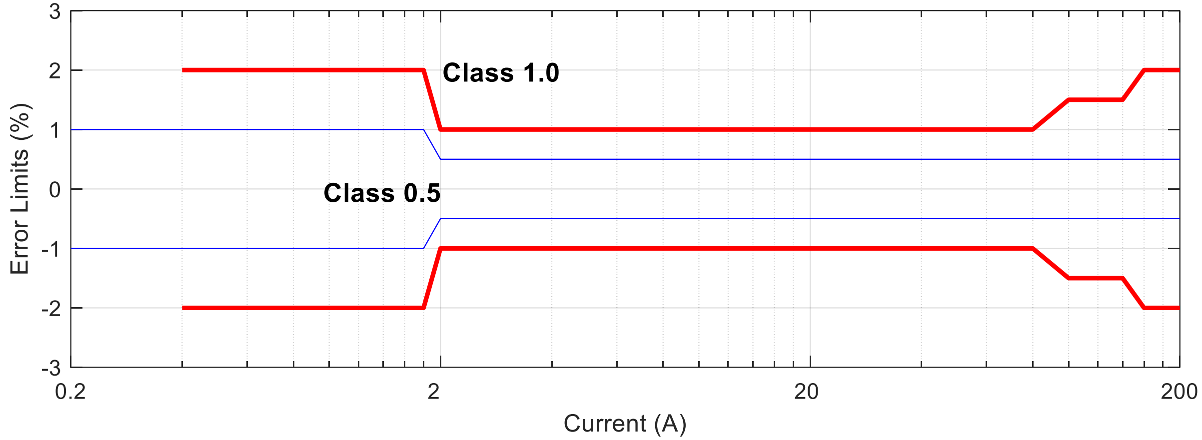
5.10.1. Accuracy Classes

• One of the standards that defines the accuracy classes for electricity metering is ANSI C12. In this standard, the accuracy is expressed in terms of limits on measurement error; which is limited to 0.1%, 0.2%, 0.5%, and 1% for Accuracy Classes 0.1, 0.2, 0.5, and 1.0, respectively [389].

• These accuracy levels are defined at the *normal* operating load current, i.e., between 2 A and 100 A. The error limits are typically higher at both low load and high load current conditions.

5.10.1. Accuracy Classes

• Two ANSI C12 accuracy classes for power and energy metering are compared in the figure below. The accuracy is expressed in terms of limits on measurement error, which are shown versus the operating load current, ranging from 0.2 A to 200 A on the logarithmic scale.



5.10. Accuracy in Power and Energy Measurements

5.10.1. Accuracy Classes

- Besides providing a better metering accuracy at normal loads, an Accuracy Class 0.5 meter is saturated at a higher load current, beyond 100 A which is the saturation level at Accuracy Class 1.0; it also continues to meter down to 0.1 A, whereas an Accuracy Class 1.0 meter may stop metering below 0.3 A.

    - See the right side of the figure on the previous slide.

5.10. Accuracy in Power and Energy Measurements

5.10.1. Accuracy Classes

• It should be noted that the error limits shown in the figure on Slide 227 are under the assumption that the power factor is one, i.e., the load is purely resistive.

  • If the power factor reduces, then error limits may increase.

• For example, the error for an Accuracy Class 0.5 meter may increase up to ±2.5% if the power factor reduces to 0.25.

• This level of error is much higher than the maximum error limit of ±1% that we can see in the figure on Slide 227 using unity power factor.

5.10.2. Meter Accuracy versus System Accuracy

• The accuracy of a power measurement system or an energy measurement system depends on the accuracy of all its components, which includes the meter itself and any instrument transformer that is being used; see Section 2.1 in Chapter 2.

• Instrument transformers can affect the accuracy in measuring voltage and current. They can also affect the accuracy in measuring the phase shift between voltage and current, i.e., the accuracy in measuring power factor.

5.10.2. Meter Accuracy versus System Accuracy

- In order to obtain the total accuracy in the metering system, it is often assumed that the error that is caused by each component in the system has a Gaussian distribution.

- Therefore, the total accuracy of the system is obtained as [390, 391]:

$$\epsilon = \sqrt{\epsilon_{\mathrm{M}}^2 + \epsilon_{\mathrm{CT}}^2 + \epsilon_{\mathrm{PT}}^2 + \epsilon_{\mathrm{PF}}^2},$$

where ϵ_M, ϵ_{CT}, ϵ_{PT}, and ϵ_{PF} denote the meter accuracy, the CT accuracy, the PT accuracy, and the power factor (phase shift) accuracy, respectively.

Note that, ϵ, ϵ_M, ϵ_{CT}, ϵ_{PT}, and ϵ_{PF} are all expressed in *percentage error*.

5.10.2. Meter Accuracy versus System Accuracy

- **Example 5.15**: A Class 0.5 meter, i.e., with an accuracy level of $\epsilon_M = 0.5\%$, is used to measure power consumption of a load. A CT with an accuracy level of $\epsilon_{CT} = 0.75\%$ is used in this measurement system. No PT is used; because voltage level of the load is already within the operating range of the meter. Therefore, $\epsilon_{PT} = 0\%$.

- The power factor of the load is around 0.75. At this level of power factor, the CT can cause $\epsilon_{PF} = 1.16\%$ error in measuring power factor.

- The total accuracy level of this measurement system is obtained as

$$\epsilon = \sqrt{0.5^2 + 0.75^2 + 0^2 + 1.16^2} = 1.47\%.$$

- Thus, if the meter indicates 4 kW load, the true load is 4 kW ± 59 W.

5.10.2. Meter Accuracy versus System Accuracy

• **Phase Shift**: The error in phase shift is sometimes expressed in minutes, where 60 minutes equal one degree and 30 minutes equal 0.5 degrees. This information is useful because it allows us to obtain the accuracy in measuring power factor at different power factor levels.

• Let ϵ_{Shift} denote the error level in phase shift. We have [392]:

$$\epsilon_{\text{PF}} = 100\% \times \left| 1 - \frac{\cos(\theta - \phi + \epsilon_{\text{Shift}})}{\cos(\theta - \phi)} \right|,$$

where θ−ϕ denotes the difference between the phase angle of voltage, i.e., θ, and the phase angle of current, i.e., ϕ. For instance, suppose ϵ_{Shift} is 45 minutes, i.e., 0.75∘.

5.10.2. Meter Accuracy versus System Accuracy

• **Phase Shift (Cont.)**: If power factor is 0.75, i.e., $\theta - \phi = 41.41°$, then from the equation on the previous slide, we have:

$$\epsilon_{\mathrm{PF}} = 100\% \times \left| 1 - \frac{\cos(41.41^\circ + 0.75^\circ)}{\cos(41.41^\circ)} \right| = 1.16\%,$$

which is the same number that we used in Example 5.15. If power factor is 1, then ϵ_{PF} = 0.009%; and if power factor is 0.5, then ϵ_{PF} = 2.28% [392].

5.10.2. Meter Accuracy versus System Accuracy

• **Revenue Meters**: Error in revenue metering is of concern because of its impact on billing and financial transactions.

• Any major error in the recording of energy usage or power usage can result in a loss to the utility, when understating, or to the customer, when overstating.

• Of course, higher accuracy metering does cost more in equipment and maintenance; however, the higher cost can often be justified when compared with the reduced level of uncertainty that it can offer in billing, especially for larger customers.

5.10.2. Meter Accuracy versus System Accuracy

- **Example 5.16**: Consider a commercial customer that is required to pay *peak-load charges* based on its peak power usage, measured in kW, during on-peak hours, mid-peak hours, and off-peak hours; see the paragraph on Other Pricing Methods in Section 5.4.1. The rates for calculating the peak-load charges are

  - On-Peak Hours: 6.88 $/kW
  - Mid-Peak Hours: 2.74 $/kW
  - Off-Peak Hours: 1.31 $/kW.

The revenue meter indicates that the peak power usage is 11,760 kW during on-peak hours, 10,467 kW during mid-peak hours, and 8,732 kW during off-peak hours.

5.10.2. Meter Accuracy versus System Accuracy

- **Example 5.16 (Cont.)**: First, assume that a *high accuracy* power measurement system is used, with the following accuracy levels:

$$\epsilon_{\mathrm{M}} = 0.5\%, \;\; \epsilon_{\mathrm{CT}} = 0.3\%, \;\; \epsilon_{\mathrm{PT}} = 0.3\%, \;\; \epsilon_{\mathrm{PF}} = 0.77\%.$$

- Accordingly, from the equation on Slide 231, the overall accuracy of the metering system is 1.01%. This introduces

$$0.0101 \times 11760 = 119 \text{ kW},$$
$$0.0101 \times 10467 = 106 \text{ kW},$$
$$0.0101 \times 11760 = 88 \text{ kW}$$

uncertainty in measuring peak demand during *on-peak* hours, mid-peak hours, and *off-peak* hours, respectively.

5.10.2. Meter Accuracy versus System Accuracy

- **Example 5.16 (Cont.)**: This results in a total of

$$119 \times 6.88 + 106 \times 2.74 + 88 \times 1.31 = \$1{,}224$$

uncertainty in the *monthly* peak-load charges for this customer.

- Next, assume that a *low accuracy* power measurement system is used:

$$\epsilon_{\mathrm{M}} = 1\%, \;\; \epsilon_{\mathrm{CT}} = 1.2\%, \;\; \epsilon_{\mathrm{PT}} = 1.2\%, \;\; \epsilon_{\mathrm{PF}} = 1\%.$$

- Thus, the overall accuracy of the metering system is 2.29%.

5.10. Accuracy in Power and Energy Measurements

5.10.2. Meter Accuracy versus System Accuracy

- **Example 5.16 (Cont.)**: This introduces 269 kW, 240 kW, and 200 kW uncertainty in measuring peak demand during on-peak hours, mid-peak hours, and off-peak hours, respectively.

- This results in a total of

$$269 \times 6.88 + 240 \times 2.74200 \times 1.31 = \$2{,}770$$

uncertainty in the monthly peak-load charges for this customer.

The lower accuracy of the second power measurement system creates an *additional $18,552 uncertainty* in the peak-load charges for this customer over the course of a year.

5.10.3. Other Factors that Affect Accuracy

• The most common mode of failure for the traditional electromechanical energy meters is *reduced registration*.

• Anything that increases the drag on the rotating disk can cause a meter to run slow, resulting in reduced billing amounts.

• Failure modes also exist that could cause an electromechanical meter to run fast, but they are less common.

• Digital energy meters are also prone to error due to sampling. Furthermore, they are susceptible to line voltage transient events. Traditional electromechanical meters are generally more immune to standard surge events [393].

5.10.3. Other Factors that Affect Accuracy

• The accuracy in measuring energy depends also on clock accuracy.

• Due to cost considerations, the internal clock in customer meters has limited accuracy, certainly much less than the time accuracy in D-PMUs.

• For example, if a meter reports 56 Wh energy usage for the 6:00:00 PM to 6:15:00 PM time interval, and its internal clock lags the true clock by one second, then in reality, the reported 56 Wh was consumed from 6:00:01 PM to 6:15:01 PM. Such error in time may not cause a major issue for billing purposes. However, it may have impact in certain applications, such as state estimation and phase identification.

5.10.3. Other Factors that Affect Accuracy

- Ambient temperature can also affect measurement accuracy.

- The accuracy levels that are shown in the figure on Slide 227 are at 23°C as the reference temperature. At higher (or lower) temperatures, the error limits can be higher [394].