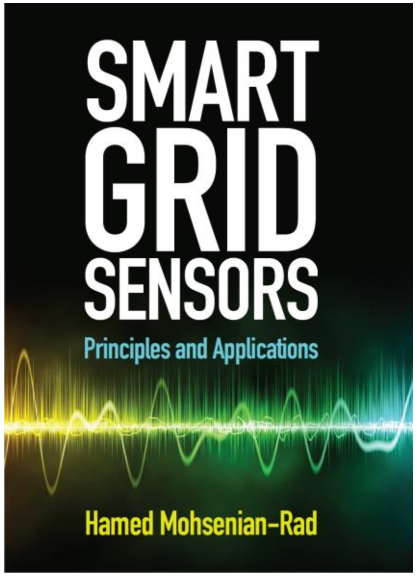
Chapter 3: Phasor and Synchrophasor Measurements and Their Applications



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3.1. Measuring Voltage and Current Phasors

- The instrument to measure phasors is *phasor measurement unit* (PMU).

- PMUs have received great attention over the past two decades. They have been deployed widely in many countries for various applications.

- Recall from Section 1.2.2 that a *sinusoidal wave* can be represented by a phasor, which is a *complex number* with magnitude X and phase angle θ:

$$x(t) = \sqrt{2}\, X \cos(\omega t + \theta) \quad \longrightarrow \quad X\angle\theta$$

- Notice that the frequency (ω) is *implicit* in defining the phasor.

3.1. Measuring Voltage and Current Phasors

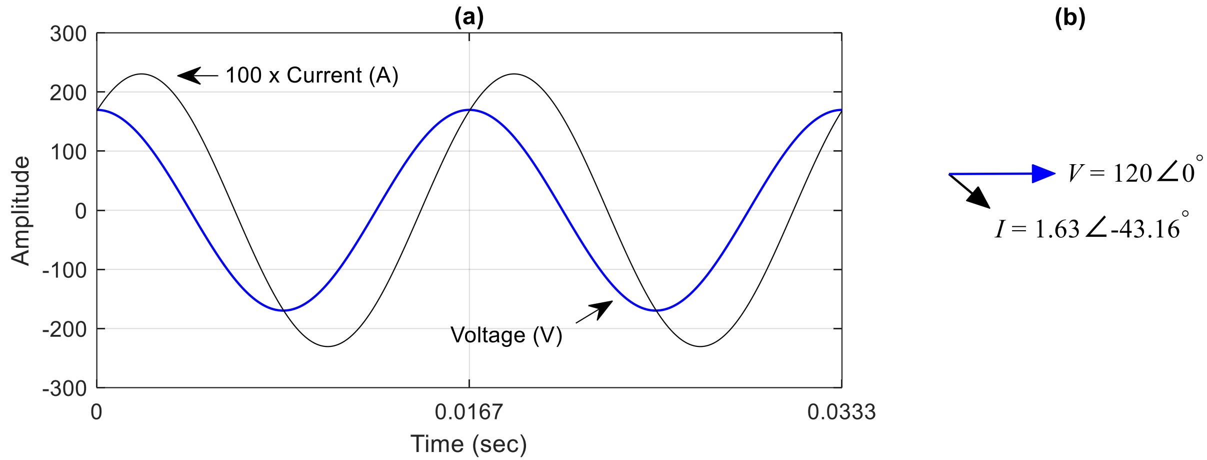
- **Example 3.1**: The voltage at a motor load is measured as

$$v(t) = 120\sqrt{2}\cos(\omega t)$$

and the current is measured as

$$v(t) = 1.63\sqrt{2}\cos(\omega t - 0.7532)$$

The angular frequency is ω = 2π × 60. These voltage and current waves and their phasor representations are shown below.



3.1. Measuring Voltage and Current Phasors

- If voltage and current waveforms are sinusoidal, as in Example 3.1, then the magnitude of the phasor is equal to the RMS value of the waveform.

- However, in practice, voltage and specially current waveforms are often distorted and may not have purely sinusoidal waveform (see Chapter 4).

- Therefore, voltage and current phasors are defined based on the *fundamental component* of their respective waveforms.

- Phasor measurements are obtained by applying *Discrete Fourier Transform* (DFT) to sampled data of voltage and current measurements.

3.1. Measuring Voltage and Current Phasors

3.1.1. Phasor Calculation Using Discrete Fourier Transform

• Consider a window of *N* samples taken from one cycle of periodic signal $x(t) = \sqrt{2}\, X \cos(\omega t + \theta)$. The samples are denoted by $x(0), \ldots, x(N-1)$. Phasor $X\angle\theta$ corresponding to these *N* samples are obtained as

$$\mathrm{Re}\{X\angle\theta\} = \frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x(n) \cos(2\pi n/N) \qquad \text{(Real Part)}$$

$$\mathrm{Im}\{X\angle\theta\} = -\frac{\sqrt{2}}{N} \sum_{n=0}^{N-1} x(n) \sin(2\pi n/N) \qquad \text{(Imaginary Part)}$$

• PMUs must have enough computation power to do the DFT calculation within each *reporting interval* (see Section 2.3). For example, to support 10 phasor readings per second, a PMU may internally sample voltage or current at a *much higher rate*, such as at 48 samples per cycle [2].

3.1.2. Time Reference to Measure Phase Angle

- The value of the phase angle θ depends on the *time reference*.

- One can change the time reference and obtain a different phasor representation for the same sinusoidal waveform, which has the *same magnitude* but a *different phase angle*.

- In Example 3.1, let us *shift* the time axis to the right by

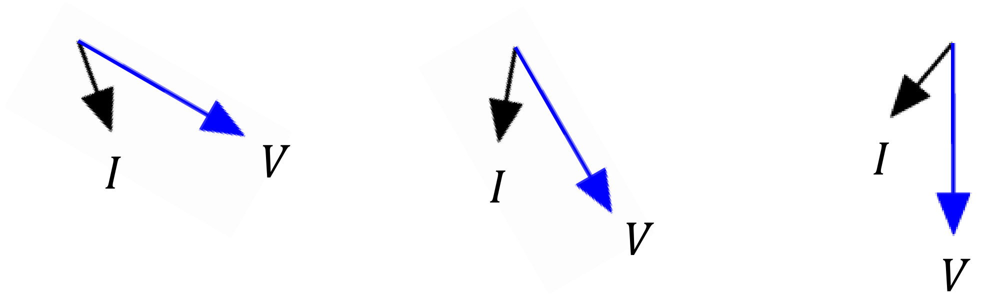
(π/6)/ω seconds: $V = 120\angle 30^\circ,\ I = 1.63\angle{-13.16^\circ}$

(π/3)/ω seconds: $V = 120\angle 60^\circ,\ I = 1.63\angle 16.84^\circ$

(π/2)/ω seconds: $V = 120\angle 90^\circ,\ I = 1.63\angle 46.84^\circ$

3.1. Measuring Voltage and Current Phasors

• These phasor representations can be obtained by rotating the phasors in Example 3.1 by $30°$, $60°$, and $90°$ *counterclockwise*, respectively.



• Any of the above phasor representations correctly captures the *relative* relationship between the voltage phasor and the current phasor.

• However, if we are using *multiple PMUs*, then we must use the *same time reference* so that the phasor measurements from different PMUs can be comparable with each other to be used in the *same analysis*.

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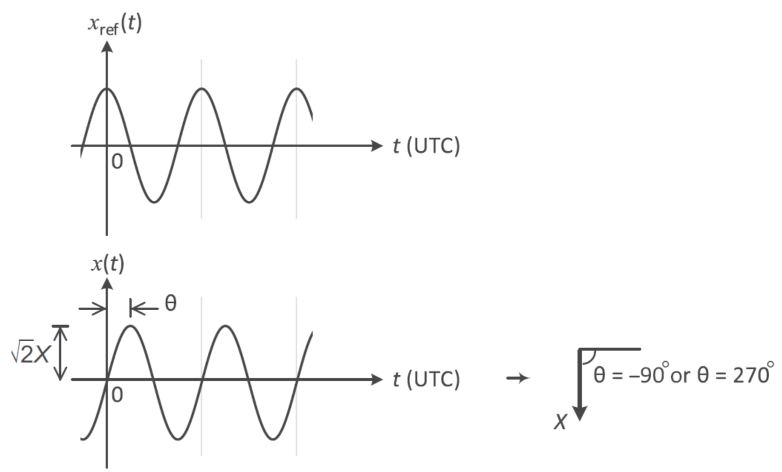
3.2. Time Synchronization and Synchrophasors

• These PMUs measure what is called *synchrophasors*, i.e., phasors whose phase angles are measured relative to a *common reference*.

• The common reference is a sinusoidal function, denoted by $x_{ref}(t)$ at the nominal system frequency which is synchronized to the Universal Time Coordinated (UTC) standard time.

• In this example, the reference signal $x_{ref}(t)$ has a maximum at time $t = 0$. Signal $x(t)$ has a positive zero crossing at time t = 0. The offset is $-90°$. Thus, we have $\theta = -90°$, or alternatively, $\theta = 270°$.



3.2. Time Synchronization and Synchrophasors

3.2.1. Precise Time Synchronization

• According to the IEEE C37.118 Standard, the clock must be accurate to better than *one microsecond* [141]. This is to ensure that the error in synchrophasor measurements is less than *1%*; see Section 3.9.

• The most common way of time synchronization across PMUs is to use the Global Positioning System (GPS). It uses *satellite-synchronized clocks*.

• Each PMU is equipped with a GPS receiver. In this figure, two different types *GPS receiver antennas* are shown that are mounted outside a substation.

3.2. Time Synchronization and Synchrophasors

- All phasors that are measured by a group of GPS-synchronized PMUs are on the same time reference; hence, they are *comparable*.

- **Time Stamps**: For each synchrophasor, a PMU reports the *magnitude*, the *phase angle*, and the *time stamp*. An example is shown below.

UTC Time (micro-second)	Magnitude (V)	Phase Angle (°)
1579101467916666	39832.582	183.680175
1579101467933333	39830.183	183.729736
1579101467950000	39831.321	183.780303
1579101467966666	39830.669	183.832092
1579101467983333	39831.177	183.887145
1579101468000000	39832.093	183.939575
1579101468016666	39833.123	183.991912
1579101468033333	39832.088	184.046417
1579101468050000	39830.748	184.098785
1579101468066666	39831.515	184.145889
1579101468083333	39833.174	184.197860

3.2. Time Synchronization and Synchrophasors

- The UTC time stamp in the example on Slide 12 is in microseconds.

- The reporting rate is 60 Hz, i.e., one reading every 16.667 msec:

$$\begin{array}{r} -\,1579101467916666 \\ 1579101467933333 \\ \hline 16667 \end{array}$$

1579101467916666 ⟵ Second Row

1579101467933333 ⟵ First Row

- Time stamps can be converted to actual date and time:

1579101468000000 ⟶ January 15, 2020 at 3:17:48.000 PM

- **Note**: The phase angles in the third column in this example are given in the 0° to 360° range, as opposed to the -180° to 180° range.

- Since all PMUs are time-synchronized, they can provide us with the desired synchrophasor measurements, which come from *different measurement locations* across the power system.

- An example for time-stamped measurements from three PMUs:

Time (UTC)	Phasor
1579101467916666	...
1579101467933333	...
1579101467950000	...
1579101467966666	...
1579101467983333	...
1579101468000000	...

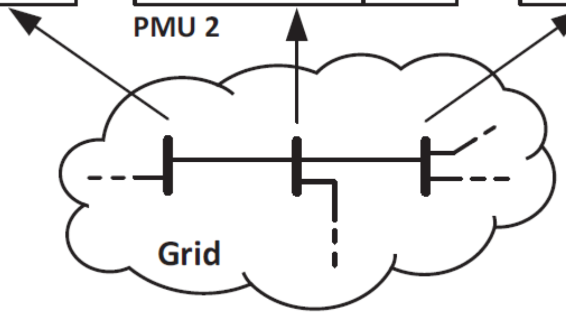
PMU 1

Time (UTC)	Phasor
1579101467916666	...
1579101467933333	...
1579101467950000	...
1579101467966666	...
1579101467983333	...
1579101468000000	...

PMU 2

Time (UTC)	Phasor
1579101467916666	...
1579101467933333	...
1579101467950000	...
1579101467966666	...
1579101467983333	...
1579101468000000	...

PMU 3



3.2. Time Synchronization and Synchrophasors

- **Frame Per Second**: The reporting rate of PMUs is sometimes stated in frames per second (fps), where each frame refers to one reading of the PMU, such as each row in the table on Slide 12.

- PMUs are required by the IEEE C37.118 Standard to report at least 10 readings per second, i.e., at 10 fps, for a 60 Hz system [141].

- Different types of PMUs have different typical reporting rates; see Section 3.2.3. Some may report up to 120 fps.

3.2.2. Application of Time Synchronization

• PMUs have three desirable features when compared with the traditional voltage and current sensors that we discussed in Chapter 2:

1) They can measure *phase angle*; in addition to the *magnitude*.

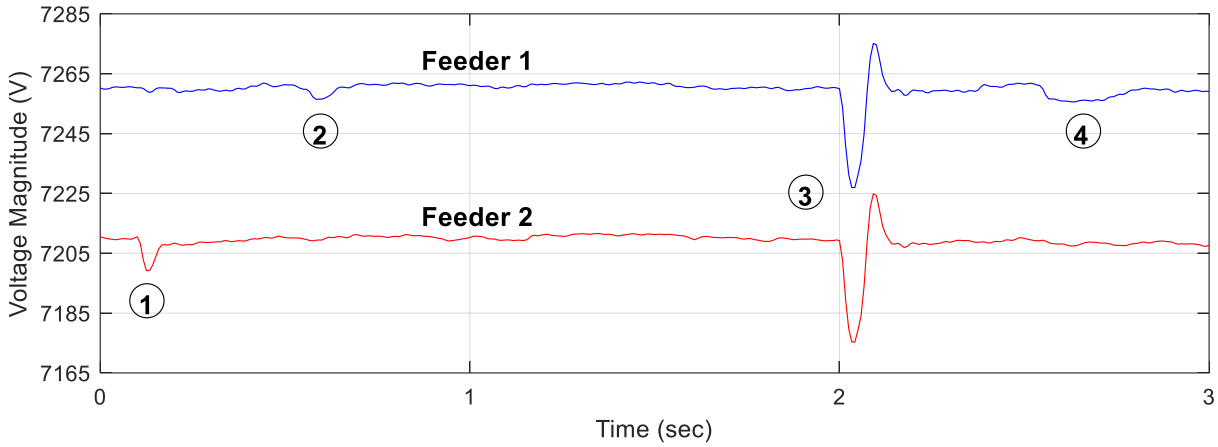
2) They have *high reporting rates*.

3) They have *precise time stamps*.

• Items 2 and 3 have important applications even if we are not concerned with phase angle and only focus on measuring the magnitude.

3.2. Time Synchronization and Synchrophasors

- **Example 3.2**: Suppose we seek to extend the analysis in Example 2.9 in Chapter 2 to identify whether a voltage event, *lasting only a few milliseconds*, is caused by a *system-wide issue* at the transmission level or a *local issue* at the distribution level. In this regard, we compare voltage measurements at two nearby feeders, as shown below:



- Given the small time scale of the analysis, even **1** or **2** seconds drift between the clocks of the two sensors may create misleading results.

3.2. Time Synchronization and Synchrophasors

- **Example 3.2 (Cont.)**: Therefore, PMUs with precise time synchronization are used for this analysis.

- Four transient voltage events are marked on the figure.

- Event 1 is visible in Feeder 2 but not in Feeder 1.

- Events 2 and 4 are visible in Feeder 1 but not in Feeder 2.

- Event 3 is visible in both feeders.

- Therefore, event 3 is caused in transmission or sub-transmission systems while events 1, 2, and 4 are caused in distribution systems.

3.2.3. Different Types of PMU Technologies

• We will discuss different Performance Classes of PMUs in Section 3.9.1.

• However, in terms of applications, we can informally divide the existing phasor measurement technologies into different groups.

• Traditionally, PMUs have been installed at high voltage locations, such as at the transmission-level substations, to support system-wide monitoring of the power grid. Their typical reporting rate is 30 fps or 60 fps [146].

• Distribution-level PMUs (D-PMUs), also known as micro-PMUs, are rather installed at medium voltage distribution-level substations and low voltage load transformers. Their typical reporting rate is 120 fps [147].

3.2. Time Synchronization and Synchrophasors

- There are other types of PMU technologies that could potentially be placed as separate groups with respect to their applications.

- For example, Frequency Disturbance Recorders (FDRs) can be seen as a special type of PMUs that are are installed at ordinary 120 V outlets. They often serve as low-cost *synchronized frequency sensors* to analyze system-wide frequency disturbances, see Section 3.3.3.

3.2.4. Synchrophasor Data Concentration

• Several applications require access to data from multiple PMUs at remote locations. Also, PMUs have limited local data storage capacity.

• In practice, *phasor data concentrator* (PDC) devices are used to gather data from several PMUs, identify and reject bad data, align the time stamps, and create a coherent record of all collected synchrophasor data.

• A PDC often does *data aggregation* to align the data that it receives from multiple PMUs or other PDCs and transmits the combined data.

3.2. Time Synchronization and Synchrophasors

- **Note**: The incoming data may have different reporting rates.

- The reporting rate at the output of the PDC may have to be adjusted accordingly. If the reporting rate in the PDC's output stream is lower than the reporting rate in one of its input streams, then the PDC must conduct *down-conversion* on input data. If the reporting rate in the PDC's output stream is higher than the reporting rate in one of its input streams, then the PDC may conduct *up-conversion* (extrapolation) on input data.

- Other functions of PDCs may include latency calculation, data validation, bad data detection, bad data correction, managing data communications, cyber-security, etc. [148].

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3.3. Nominal and Off-Nominal Frequencies

- A phasor is defined for a *given* frequency $\omega = 2\pi f$.

- Ideally, the frequency should be the same and stay constant, at its rated value, all across the same power grid. For example, in North America, the frequency in each interconnection is rated at f = 60 Hz.

- However, in practice, the frequency constantly changes, as we saw in Section 2.9 in Chapter 2. Such changes are often considered normal.

- Q: How does the changes in frequency affect measuring the phasors by PMUs, because *phasors are defined for a given frequency*.

    - The impact is particularly on measuring *phase angle*.

3.3. Nominal and Off-Nominal Frequencies

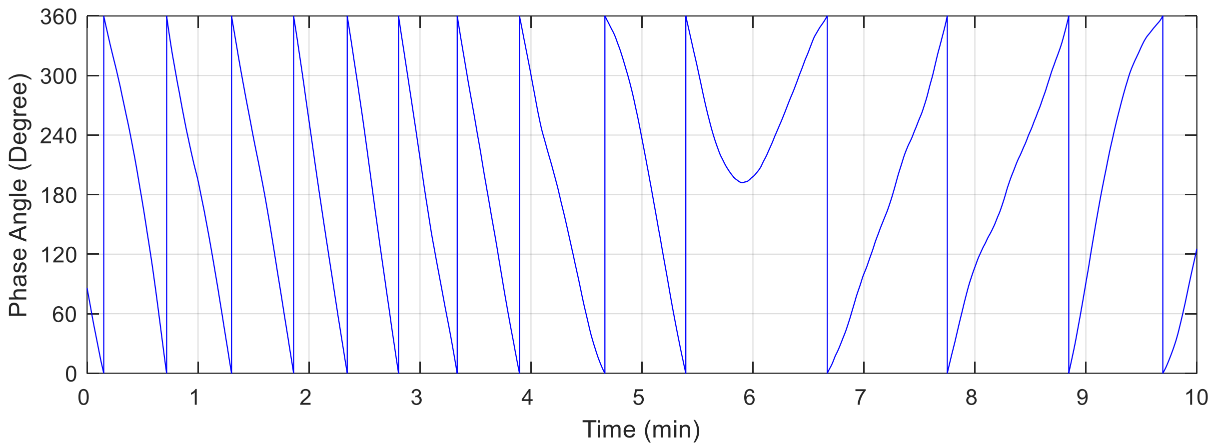
3.3.1. Impact of Frequency on Measuring Phase Angle

• The phase angle measurements in the table on Slide 12 change at every reading. In fact, the appear to constantly increase at a small pace.

UTC Time (micro-second)	Magnitude (V)	Phase Angle (°)	
1579101467916666	39832.582	183.680175	+0.049561
1579101467933333	39830.183	183.729736	
1579101467950000	39831.321	183.780303	
1579101467966666	39830.669	183.832092	+0.055053
1579101467983333	39831.177	183.887145	
⋮	⋮	⋮	

• These changes are mostly due to the fact that real-world power systems seldom operate at the *nominal frequency* $f_0 = 60\ Hz$. Instead, they may operate at an *off-nominal frequency* $f = f_0 + \Delta f$, where $\Delta f \neq 0$.

- **Example 3.3**: Consider the voltage phase angle measurements for a duration of 10 minutes as shown in the figure below.



- For the first few minutes, the phase angle decrements in cycles that run from 360° to 0° before *wrapping around* and jumping back to 360°.

3.3. Nominal and Off-Nominal Frequencies

- **Example 3.3 (Cont.)**: At around minute six, there is a *reverse* in this pattern of the curve in the figure, and the phase angle starts increasing.

- For the last few minutes, the phase angle increments in cycles that run from $0°$ to $360°$ before *wrapping around* and jumping back to $0°$.

- Q: What can we tell about the frequency of the system based on the above different patterns in the curve in this figure?

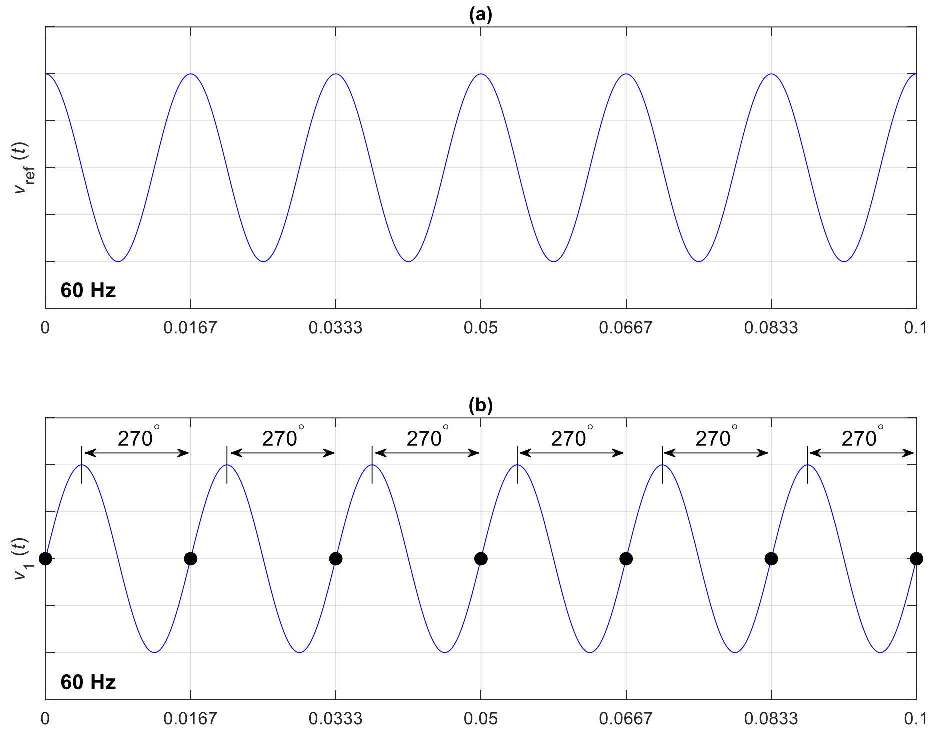
3.3. Nominal and Off-Nominal Frequencies

- To understand the impact of frequency on phase angle measurements, first consider the case where a PMU with reference signal $v_{\mathrm{ref}}(t)$ is used to measure the phase angles for signal $v_1(t)$ at frequency $f_1 = 60\ \mathrm{Hz}$:

Reference Signl:

Measured Voltage:

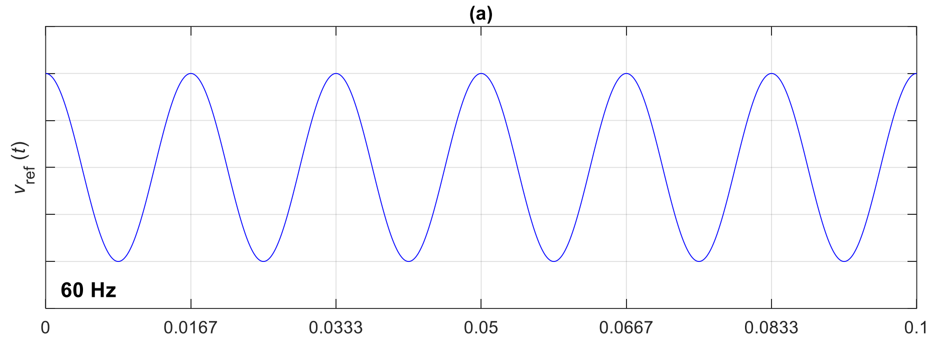
$$v_1(t) = \sqrt{2}\cos(2\pi f_1 t + 270°)$$



3.3. Nominal and Off-Nominal Frequencies

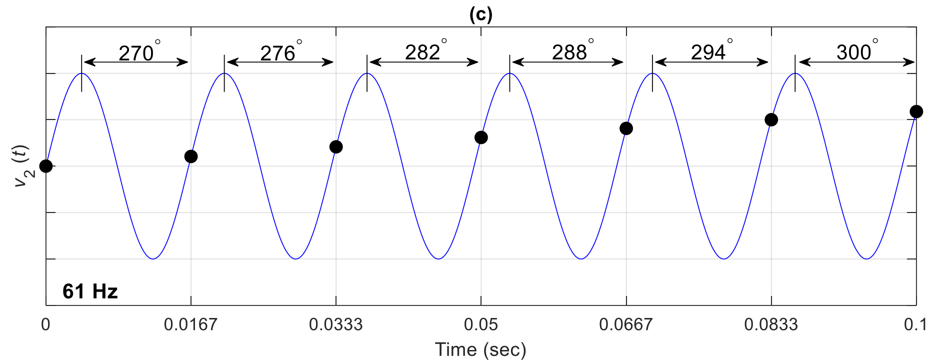
- Next consider the case where a PMU with reference signal $v_{\text{ref}}(t)$ is used to measure the phase angles for signal $v_2(t)$ at frequency $f_1 = 61\ \text{Hz}$:

Reference Signl:



Measured Voltage:

$$v_2(t) = \sqrt{2}\cos(2\pi f_2 t + 270°)$$



- Pay attention to the *angular location* of the *black dots*.

3.3. Nominal and Off-Nominal Frequencies

- In the first case, we have:

$$v_1(t) = \sqrt{2}\cos(2\pi f_1 t + 270°) = \sqrt{2}\cos(2\pi f_0 t + \underbrace{270°}_{\text{Measured Phase Angle}})$$

- Accordingly, the black dot in the figure is always at $270°$; and the PMU reports $270°$ at all the reporting instances at $t = 0$, $t = 0.0167$, $t = 0.0333$, $t = 0.0500$, $t = 0.0667$, $t = 0.0833$, and $t = 0.1000$.

3.3. Nominal and Off-Nominal Frequencies

- In the second case, we have:

$$v_2(t) = \sqrt{2}\cos(2\pi f_2 t + 270°) = \sqrt{2}\cos(2\pi f_0 t + \underline{2\pi\Delta f t + 270°})$$

Measured Phase Angle

- Accordingly, the black dot in the figure keeps changing; because it changes over time due to the *time varying* term $2\pi\Delta f t$, where $\Delta f = 1\text{ Hz}$.

- The PMU reports at $270°$, $276°$, $282°$, $288°$, $294°$, and $300°$ at $t = 0$, $t = 0.0167$, $t = 0.0333$, $t = 0.0500$, $t = 0.0667$, $t = 0.0833$, and $t = 0.1000$, respectively. The *increments* in phase angle measurements are:

$$360° \times \Delta f \times \Delta t = 360°/60 = 6°.$$

3.3. Nominal and Off-Nominal Frequencies

- Based on Slides 29-32, if the frequency is *fixed*, or it *changes very slowly*, then frequency can be estimated from the measured phase angles as:

$$f = f_0 + \Delta f = f_0 + \frac{1}{2\pi}\frac{\Delta\theta}{\Delta t}$$

($\Delta\theta$ is in radian)

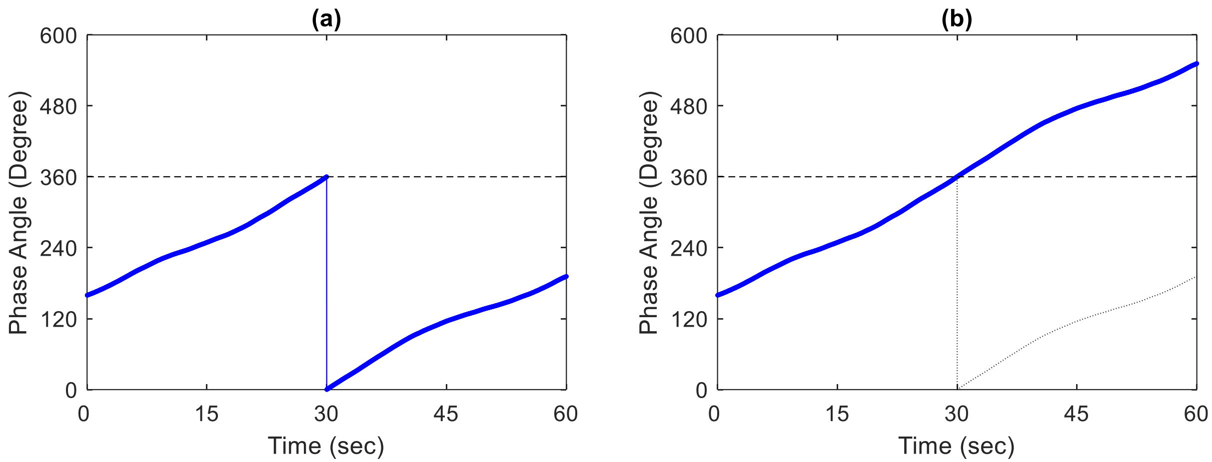
or

$$f = f_0 + \Delta f = f_0 + \frac{1}{360}\frac{\Delta\theta}{\Delta t}$$

($\Delta\theta$ is in degrees)

3.3. Nominal and Off-Nominal Frequencies

- **Unwrapping Phase Angle Measurements**: If $\Delta\theta$ is calculated at a point where the phase angle measurement wraps around, then the phase angle measurement must be *unwrapped* before it is used to estimate frequency.



Unwrapped

Wrapped

- In this example, the phase angle measurements are unwrapped by *adding* $360°$ to all measurements at time $t = 30$ sec and beyond.

3.3.2. Rate of Change of Frequency

- The formulations on Slide 33 are valid when the frequency is fixed.

- Q: What if we consider frequency as a function of time, denoted by $f(t)$?

- Let's use the first two terms in the Taylor series approximation to write:

$$f(t) = f_0 + \Delta f + \underbrace{\frac{df}{dt}}_{\text{Rate of Change of Frequency (ROCOF)}} t$$

3.3. Nominal and Off-Nominal Frequencies

- To find a way to estimate the ROCOF (and the time-varying frequency), consider measuring the phase angle of this sinusoidal signal:

$$x(t) = \sqrt{2} X cos(\vartheta(t))$$

- From the analysis of power system dynamics, we know that [149]:

$$\omega(t) \;=\; d\vartheta(t)/dt,$$

where $\omega(t) = 2\pi f(t)$ is the *instantaneous angular frequency*. Thus:

$$\begin{aligned} \vartheta(t) &= 2\pi \int_0^t f(\tau) d\tau + \vartheta(0) \\ &= 2\pi f_0 t + \left(\underline{2\pi \Delta f t + \pi \text{ ROCOF } t^2 + \vartheta(0)}\right) \end{aligned}$$

Phase Angle, *relative* to $x_{\text{ref}}(t)$ with frequency f_0

3.3. Nominal and Off-Nominal Frequencies

- From the last equation on the previous slide, the *relative* phase angles with respect to the reference signal that are reported by the PMU at time instances $t = 0, \Delta t, 2\Delta t, \ldots$ are obtained as

$$\begin{aligned}
\theta(0) &= \vartheta(0), \\
\theta(1) &= 2\pi\Delta f \times (\Delta t) + \pi\text{ROCOF} \times (\Delta t)^2 + \vartheta(0), \\
\theta(2) &= 2\pi\Delta f \times (2\Delta t) + \pi\text{ROCOF} \times (2\Delta t)^2 + \vartheta(0), \\
&\vdots
\end{aligned}$$

- As a special case, if ROCOF = 0, then we can use the above equations to obtain $\Delta\theta = 2\pi\Delta f\Delta t$, which after reordering the terms would result in the same expression for the estimation of frequency as in Slide 33.

- However, in general, ROCOF is often *not* zero and *not* known in advance.

3.3. Nominal and Off-Nominal Frequencies

- The system of equations in the previous slide can be used to *simultaneously* estimate Δf and ROCOF. This is done by using a window of N phase angle measurements. We can derive:

$$\underbrace{\begin{bmatrix} \theta(1)-\theta(0) \\ \theta(2)-\theta(0) \\ \theta(3)-\theta(0) \\ \vdots \\ \theta(N-1)-\theta(0) \end{bmatrix}}_{\substack{\mathbf{\Phi} \\ (N-1)\times 1}} = \underbrace{\begin{bmatrix} 2\pi\Delta t & \pi\Delta t^2 \\ 4\pi\Delta t & 4\pi\Delta t^2 \\ 6\pi\Delta t & 9\pi\Delta t^2 \\ \vdots & \vdots \\ 2(N-1)\pi\Delta t & (N-1)^2\pi\Delta t^2 \end{bmatrix}}_{\substack{\mathbf{A} \\ (N-1)\times 2}} \underbrace{\begin{bmatrix} \Delta f \\ ROCOF \end{bmatrix}}_{2\times 1}$$

- **Note**: matrix A depends only on Δt. Therefore, for a given N, it can be pre-calculated and stored as a constant for use in real time.

3.3. Nominal and Off-Nominal Frequencies

- The unknowns can now be estimated using the LS method:

$$\begin{bmatrix} \Delta f \\ ROCOF \end{bmatrix} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{\Phi}$$

- Once Δf and ROCOF are estimated, the frequency itself is estimated as

$$f(t) = f_0 + \Delta f + \mathrm{ROCOF}\ t$$

3.3. Nominal and Off-Nominal Frequencies

- **Example 3.4**: One can use the phase angle measurements in the first $N = 10$ rows in the table on Slide 13 to estimate

$$\Delta f \approx 0.008528$$

and

$$\text{ROCOF} \approx 0.002193$$

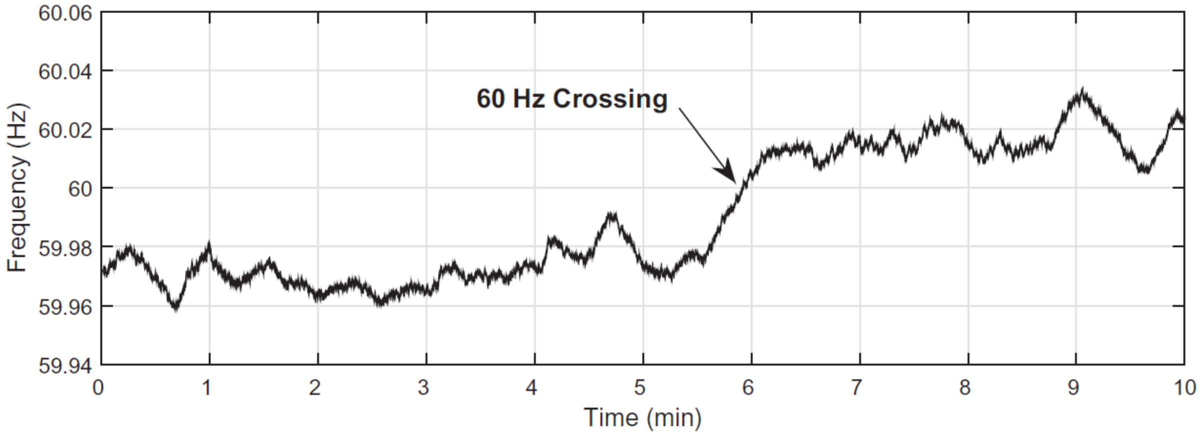
Accordingly, we can estimate the frequency as

$$f \approx 60 + 0.008528 + 0.002193 \times \frac{1}{60} \approx 60.0085645.$$

Here, $\mathbf{\Phi}$ is a 9 × 1 vector and $\mathbf{A}$ is a 9 × 2 matrix.

3.3. Nominal and Off-Nominal Frequencies

• The frequency estimations corresponding to the phase angle measurements on Slide 27 are shown below for $N = 120$ (window size). Frequency crosses 60 Hz at around minute six, i.e., the time when there is a *reverse* in the pattern of changes in phase angle measurements.



• Increasing the window size N can result in more smooth frequency estimations. However, one should not use excessively long windows to avoid degrading the accuracy of the estimated frequencies.

3.3. Nominal and Off-Nominal Frequencies

3.3.3. Synchronized Frequency Measurements

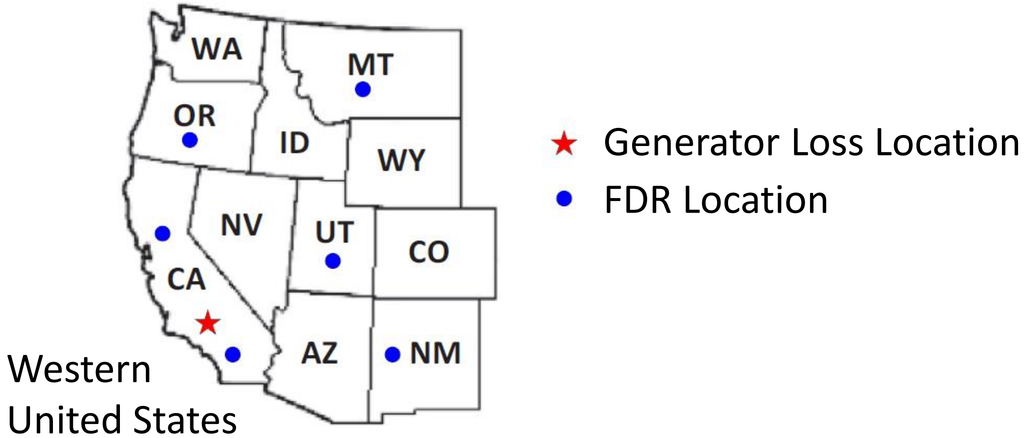
- Major events can cause sudden changes in the frequency of the power system all across an entire interconnection.

- Time synchronization across PMUs, together with the fact that PMUs can estimate frequency, can result in another application for PMUs, namely their ability to provide synchronized frequency measurements.

- This can helps us observe the *system-wide* impact of frequency events.

3.3. Nominal and Off-Nominal Frequencies

- **Example 3.5**: A group of six FDRs (see Slide 21) are installed at six different locations on the on the Western Interconnection in the United States as part of the FNET/GridEye project [133].

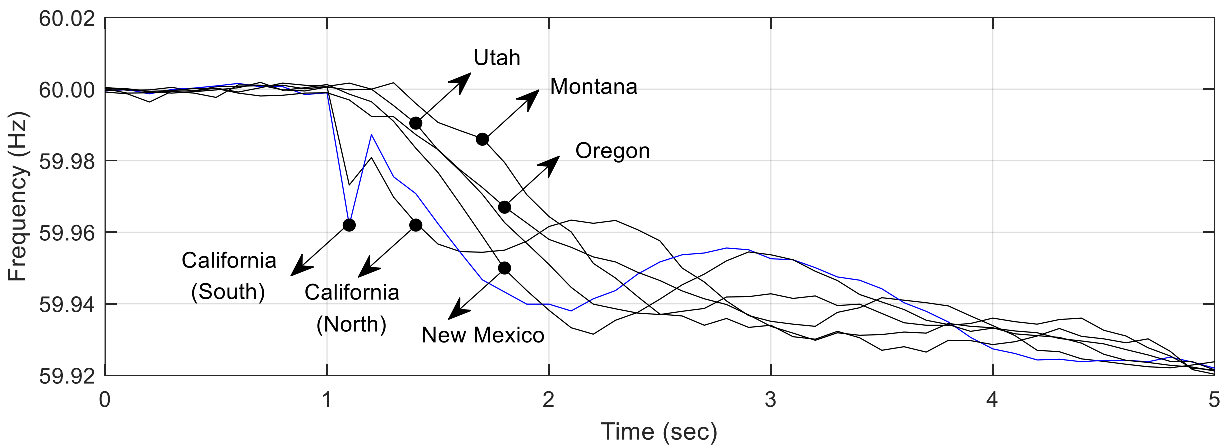


- They are used to obtain *time-synchronized frequency measurements* (based on the analysis of phasor measurements) at these six locations.

- The frequency suddenly drops due to a major generator loss event.

3.3. Nominal and Off-Nominal Frequencies

- **Example 3.5 (Cont.)**: The lost generator is located in Southern California. As a result, frequency changes are first seen by the FDRs in Southern and Northern California. The relative timing of the frequency events at different locations can help estimate the location of the lost generator.



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3.4. Relative Phase Angle Difference

- The Relative Phase Angle Difference (RPAD) is the difference between the phase angles in two synchrophasors on the same interconnection.

- For example, given the voltage synchrophasors $V_1\angle\theta_1$ and $V_2\angle\theta_2$ at locations 1 and 2 on the same interconnection, such as at the two ends of a transmission line, the corresponding RPAD is obtained as:

$$\text{RPAD} = \theta_1 - \theta_2.$$

- *Time synchronization* is critical to calculate RPAD. If the two phasors are not synchronized, then RPAD calculation is practically useless.

- RPAD is almost always calculated for *voltage synchrophasors* but not for current synchrophasors. Also, phase angle measurements must be *unwrapped* before they can be used to calculate RPAD; see Slide 34.

3.4. Relative Phase Angle Difference

3.4.1. Approximate Relationship Between RPAD and Power Flow

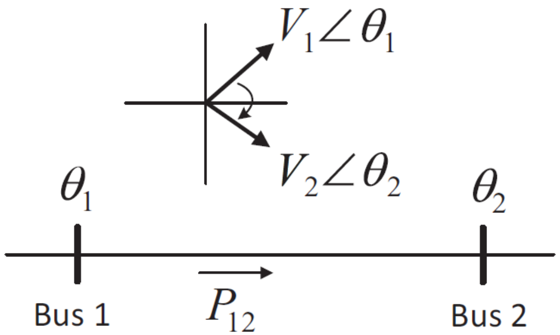
• Recall from Section 1.3.1 in Chapter 1 that the flow of real power between two buses on an AC power network depends on the voltage angle difference between the two buses. When the line impedance is dominantly inductive, as in the case of most transmission lines, the real power flow between bus 1 and bus 2 is approximated as

$$P_{12} = \frac{V_1 V_2}{X_{12}} \sin(\theta_1 - \theta_2)$$

• where X12 is the line inductance, $V_1\angle\theta_1$ is the voltage phasor at location 1, and $V_1\angle\theta_1$ is the voltage phasor at location 2.

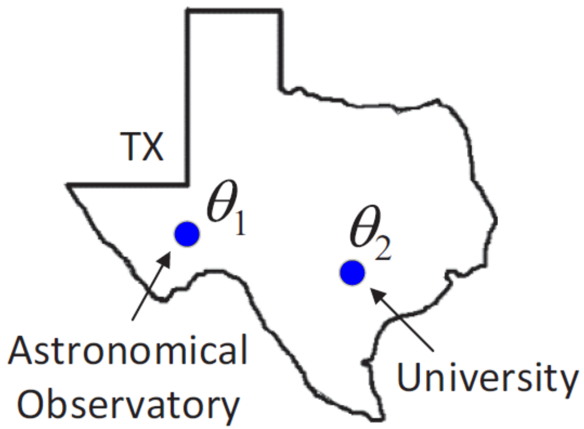
3.4. Relative Phase Angle Difference

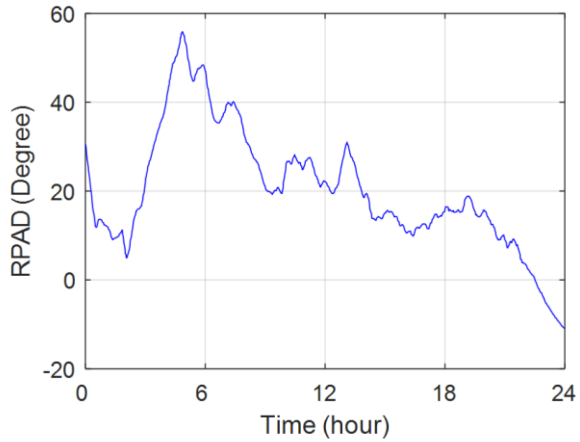
• From the equation on the previous slide, active power flow in a power transmission line is nearly proportional to the sine of RPAD of voltages at the two terminals of the line. Since many of the planning and operational considerations in a power network are concerned with the flow of active power, measuring RPAD across transmission lines is of interest.



3.4. Relative Phase Angle Difference

• RPAD may also serve as a rough *approximation* of the active power flow between two *regions*. An example is shown below for the case of a study based on the data from the Texas synchrophasor network [113].

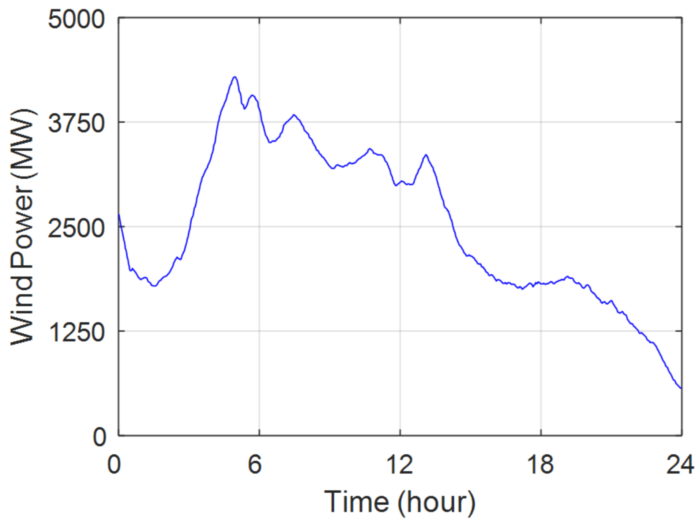




• Location 1 is an astronomical observatory in West Texas, a region which has a relatively small population but is home to some of the largest wind farms in the United States. Location 2 is a university campus in East Texas, a region which is home to the state's largest population centers.

3.4. Relative Phase Angle Difference

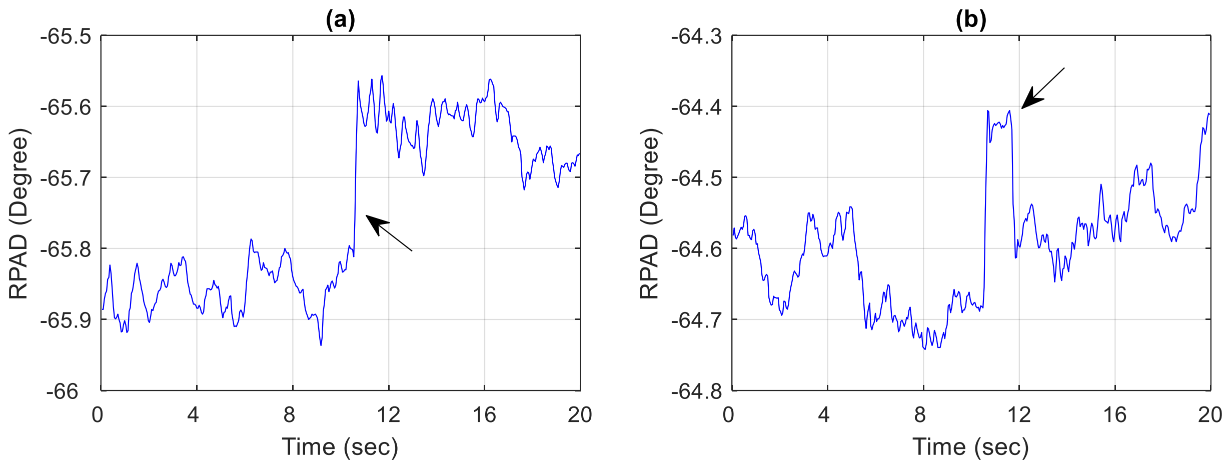
- We can compare the daily profile of RPAD on the previous slide with the daily profile of the amount of wind power generation in West Texas:



- Since the majority of the wind power that is generated in West Texas is consumed in East Texas, the *overall shape* of the wind power generation profile on this slide is roughly approximated by the overall shape of the active power flow (and RPAD) between the two regions on Slide 49.

3.4.2. Sustained and Transient Events in RPAD

• RPAD measurements can be used to identify various system-wide events in power systems. Two examples are shown below [113].



• The event in Figure (a) is a *sustained step change* in RPAD at $0.24°$ due to transmission line tripping. A line tripping results in *redirecting power flow* to surrounding transmission lines, thus causing a change in RPAD.

3.4. Relative Phase Angle Difference

- The event in Figure (b) on the previous slide includes a sudden change in RPAD at 0.25◦; however RPAD *returns* to its initial value. This is the result of a transmission line *reclosing* event [113].

- The duration of the event is about 1.5 seconds.

- Here, the fault is rather *temporary*; therefore, the transmission line reclosing operation is successful, and the power flow returns to its original value within a short period of time.

- RPAD also returns to its pre-fault value.

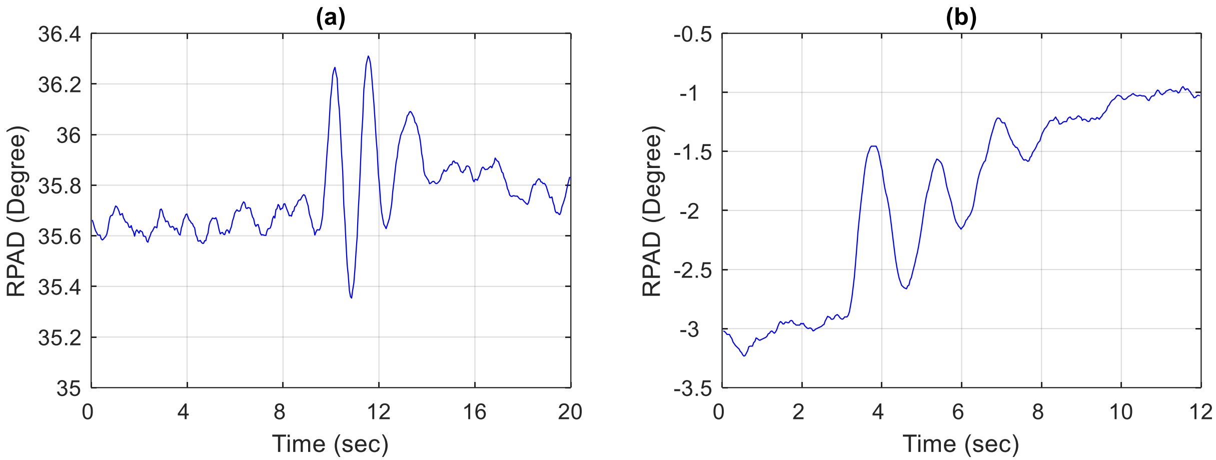
3.4.3. Impact of Inter-Area Oscillations

• RPAD measurements can identify and characterize inter-area oscillations.

• Recall from Section 2.6.1 in Chapter 2 that an inter-area oscillation indicates an *unintentional periodic exchange of power* across *different regions* of a power grid. Based on the approximate relationship between RPAD and active power exchange that we saw in Section 3.4.1, oscillations in RPAD measurements across two regions can be considered as an indication of inter-area oscillations between the two regions.

• Inter-area oscillations often carry a frequency between 0.15 Hz and 1 Hz, or sometimes up to 2 Hz; depending on the interconnection.

• **Example 3.6**: Two oscillation events in RPAD measurements are shown below. For the *damped transient event* in Figure (a), the duration of the event is 4.4 seconds and the largest swing is 0.96◦. The *frequency of the oscillation*, i.e., for its dominant mode, is 0.59 Hz.



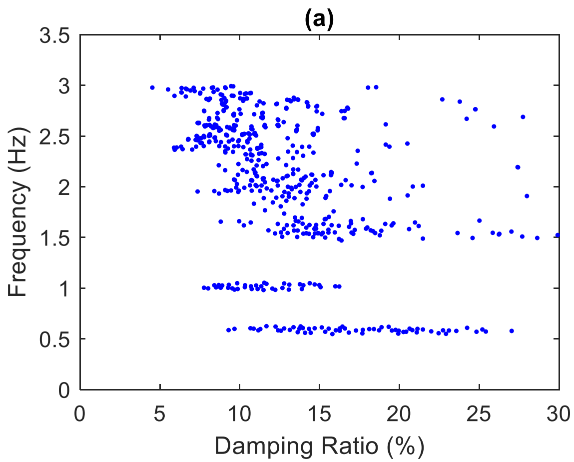
• **Example 3.6 (Cont.)**: For the *damped transient event* in Figure (b), the duration is 4.8 seconds and the largest swing is 1.33◦. The oscillation has a strong mode at 0.64 Hz. This oscillation in this event was known to be caused by a sudden loss of a 810 MW power generation unit [113].

• The frequencies of both of the oscillatory events in Figures (a) and (b) fall within the typical range of inter-area oscillations.

• **Note**: By extending the above analysis and similarly examining all the oscillation events in RPAD measurements *over several hours or days*, we can obtain the *statistical characteristics* of the inter-area oscillations and other types of low-frequency oscillations in the power system.

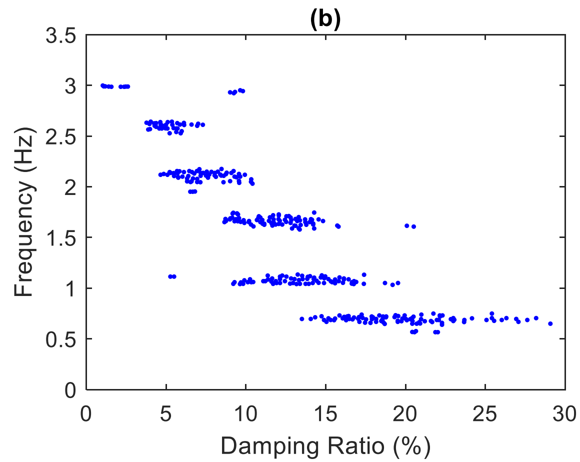
3.4. Relative Phase Angle Difference

- **Example 3.7**: Figures below are two *scatter plots* for the *frequency versus damping ratio* of the oscillation modes in the RPAD measurements across the two PMUs in Texas that we previously saw on Slide 49.



Low-Wind Conditions

(Wind power penetration: 4.6% ~ 5.0%)



High-Wind Conditions

(Wind power penetration: 15% ~ 18%)

3.4. Relative Phase Angle Difference

- **Example 3.7 (Cont.)**: We can see that the points in Figure (b) are *more tightly clustered* at certain frequencies, while the points in Figure (a) are *more scattered*, specially at frequencies above 1.5 Hz.

- Furthermore, some of the points in Figure (b) are *shifted* toward the left, which indicates that they have *smaller damping ratios*, compared to the points in Figure (a). This means that some of the oscillation modes in the system are *not* damped as quickly during high-wind conditions as they do during low-wind conditions.

- The type of scatter plots that we saw on Slide 57 can be used to characterize the oscillation modes in the system under different operating conditions; see the detailed study in [113].

Table of Contents

3.5. Phasor Differential and Differential Synchrophasors

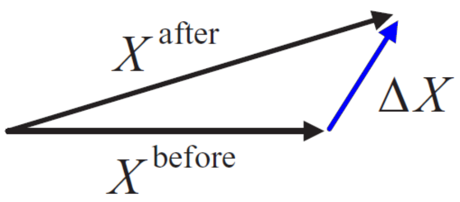
3.5.1. Phasor Differential Calculation

• Phasor Differential (PD) is the difference between two phasor measurements that are obtained from the *same* PMU.

• PD is often obtained when an *event* occurs.

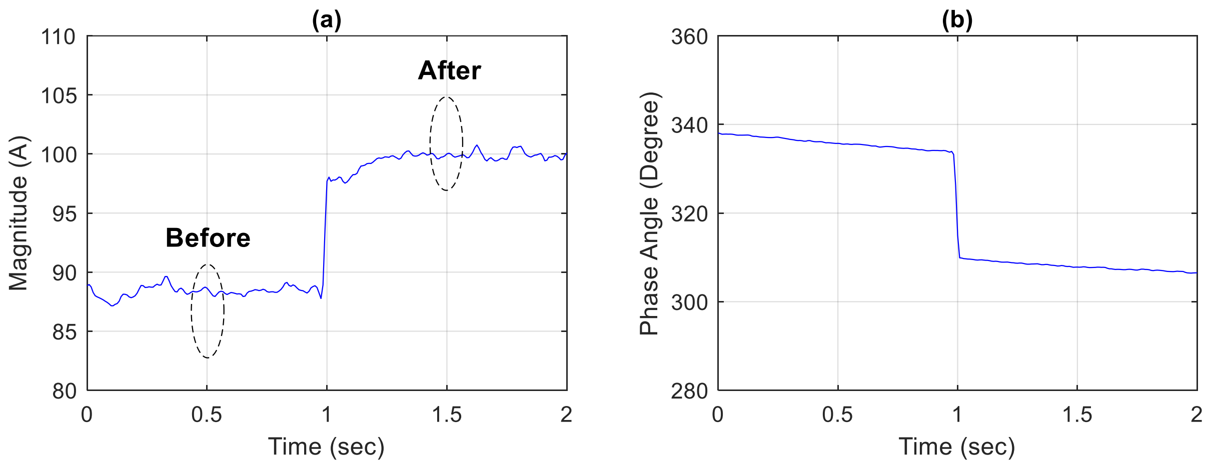
• Suppose X^{before} and X^{after} denote the phasor measurements *before* and *after* the event. The PD corresponding to the event is obtained as:

$$\Delta X = X^{\text{after}} - X^{\text{before}}$$



• **Example 3.8**: Consider a capacitor bank switching event that is captured by a PMU that measures the phasor of current.

The measurements for magnitude and the measurements for phase angle are shown in Figures (a) and (b) below, respectively.



3.5. Phasor Differential and Differential Synchrophasors

- **Example 3.8 (Cont.)**: The phasors of current before and after the event are measured as (as marked on the figure on the previous slide):

$$I^{\text{before}} = 88.64\angle 335.73^\circ$$

and

$$I^{\text{after}} = 100.01\angle 307.81^\circ.$$

- The PD corresponding to the above current phasors is obtained as

$$\begin{aligned} \Delta I &= 100.01\angle 307.81^\circ - 88.64\angle 335.73^\circ \\ &= 46.83\angle -114.6^\circ \end{aligned}$$

3.5. Phasor Differential and Differential Synchrophasors

- PD can also be defined based on voltage phasor measurements.

- For the capacitor bank switching event in Example 3.8, the voltage phasors before and after the event are measured as

$$V^{\text{before}} = 7266.7\angle 339.31^\circ$$
$$V^{\text{after}} = 7218.6\angle 335.55^\circ$$

respectively. Accordingly, we can obtain:

$$\begin{aligned}\Delta V &= 7218.6\angle 335.55^\circ - 7266.7\angle 339.31^\circ \\ &= 477.6\angle - 118.35^\circ.\end{aligned}$$

- Together, ΔI and ΔV model the impact of the event at the current and voltage phasor measurements at the location of the PMU.

3.5. Phasor Differential and Differential Synchrophasors

- **Comparison between PD and RPAD**: They have several differences:

  1) RPAD is a scalar; PD is a phasor.

  2) RPAD is measured between two PMUs; PD is measured for one PMU.

  3) RPAD uses only *phase angle* measurements; PD uses both phase angle measurements and magnitude measurements.

  4) RPAD is usually measured for voltage phasors only; PD is measured for both voltage phasors and current phasors.

3.5.2. Impact of Off-Nominal Frequency on PD Calculation

• Calculating PD can be affected by the operation of the power grid at off-nominal frequencies. From Section 3.3, if $\Delta f \neq 0$, then there is an *offset* in the measured phase angles across any two subsequent phasor readings.

• This offset can cause error in measuring the PD corresponding to an event. Therefore, we may want to *remove* it before the PD is calculated.

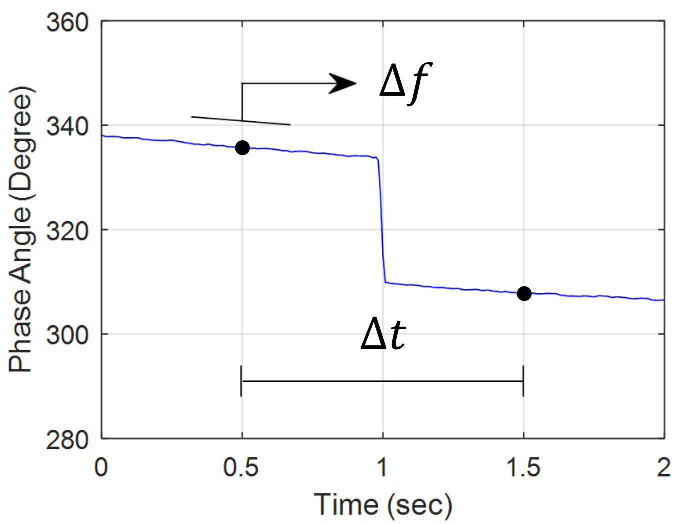
• From Slide 33 and after reordering the terms:

$$\Delta\theta = 2\pi\ \Delta t\ \Delta f$$

• Here, Δt denotes the time between the two measurements, such as the time between measuring X^{before} and measuring X^{after}.

- **Example 3.9**: Consider the phase angle measurements of current phasor in Example 3.8. There is a major *sudden change* in the phase angle due to the event. There is also a *relatively small but continuous change* in the phase angle that is *not* related to the event, but rather related to Δf.

By following the analysis in Section 3.3.1, we can check the changes in the phase angle during the first 0.5 seconds, i.e., before the event occurs, and obtain the deviation in the off-nominal frequency as:



$$\Delta f = \frac{\Delta\theta}{360 \times \Delta t} = -0.0133 \text{ Hz}$$

where $\Delta t = 0.5$ seconds and $\Delta\theta = 335.7^\circ - 338.1^\circ = -2.4^\circ$.

- **Example 3.9 (Cont.)**: Next, we can use the equation on Slide 64 to make the following "*correction*" in the phase angle measurement for I^{after}:

$$307.81^\circ - 360 \times 1 \times -0.0133 = 312.61^\circ,$$

where $\Delta t = 1$ second is the time difference between I^{after} and I^{before}.

Thus, the actual change in the phase angle that is caused by the event in this example is less than what the initial measurement had suggested:

$$\begin{aligned}\Delta I &= 100.01\angle 312.61^\circ - 88.6434\angle 335.73^\circ \\ &= 39.4\angle -109.41^\circ.\end{aligned}$$

3.5.3. Differential Snchrophasors

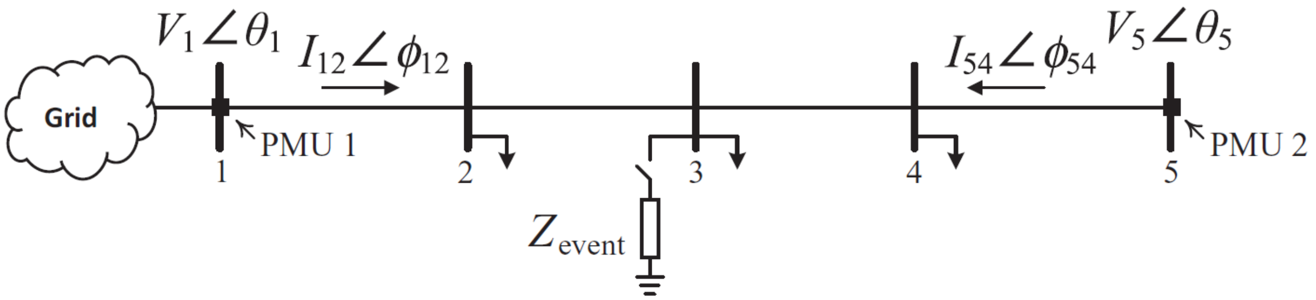
• Recall that a PD is a phasor. It can be obtained at every PMU.

• Since the PMU measurements are time-stamped, the PDs that are obtained at multiple PMUs can be *time-synchronized*.

• If X^{before} is measured at the same time at all PMUs, and X^{after} is measured at the same time at all PMUs, then the set of ΔX calculations that are provided by the PMUs form *differential synchrophasors*.

• Differential synchropasors capture how the voltage and current phasors are *simultaneously affected* across the interconnected power system as the result of the same event.

- **Example 3.10**: Consider a power distribution system with five buses and four lines. Suppose two PMUs are installed at bus 1 and bus 5 to measure voltage phasors and current phasors at these two buses.



- Suppose an impedance switching event, such as a capacitor bank switching event or a motor load switching event, happens at Bus 3.

- **Example 3.10 (Cont.)**: The PDs at the PMU at bus 1 are obtained as:

$$\Delta V_1 = V_1^{\text{after}} \angle \theta_1^{\text{after}} - V_1^{\text{before}} \angle \theta_1^{\text{before}},$$
$$\Delta I_{12} = I_{12}^{\text{after}} \angle \phi_{12}^{\text{after}} - I_{12}^{\text{before}} \angle \phi_{12}^{\text{before}}.$$

Similarly, the PDs at the PMU at bus 5 are obtained as

$$\Delta V_5 = V_5^{\text{after}} \angle \theta_5^{\text{after}} - V_5^{\text{before}} \angle \theta_5^{\text{before}},$$
$$\Delta I_{54} = I_{54}^{\text{after}} \angle \phi_{54}^{\text{after}} - I_{54}^{\text{before}} \angle \phi_{54}^{\text{before}}.$$

If the “before” and “after” instances are time-synchronized, then the above measurements result in a vector of differential synchrophasors:

$$\begin{bmatrix} \Delta V_1 & \Delta V_5 & \Delta I_{12} & \Delta I_{54} \end{bmatrix}^T.$$

3.5.4. Application in Event Location Identification

• In Example 3.10, suppose the location of the event is unknown, i.e., we do not know the fact that the change in impedance occured at bus 3.

• The question is: *Can we use the synchronized phasor measurements at PMU 1 and PMU 2 to identify the location of the event?*

• Recall that the impedance switching event in Example 3.10 was represented by the following differential synchrophasors:

$$\begin{bmatrix} \Delta V_1 & \Delta V_5 & \Delta I_{12} & \Delta I_{54} \end{bmatrix}^T.$$

• By using these differential synchrophasors and by taking the following four steps, we can identify the location of the event.

3.5. Phasor Differential and Differential Synchrophasors

- **Step 1: Forming the Equivalent Circuit Model**

- According to the *compensation theorem* in circuit theory [156], once an element changes in a circuit, such as due to a switch operation (event), the amount of changes in nodal voltages and branch currents on the circuit can be obtained by forming an *equivalent circuit*.

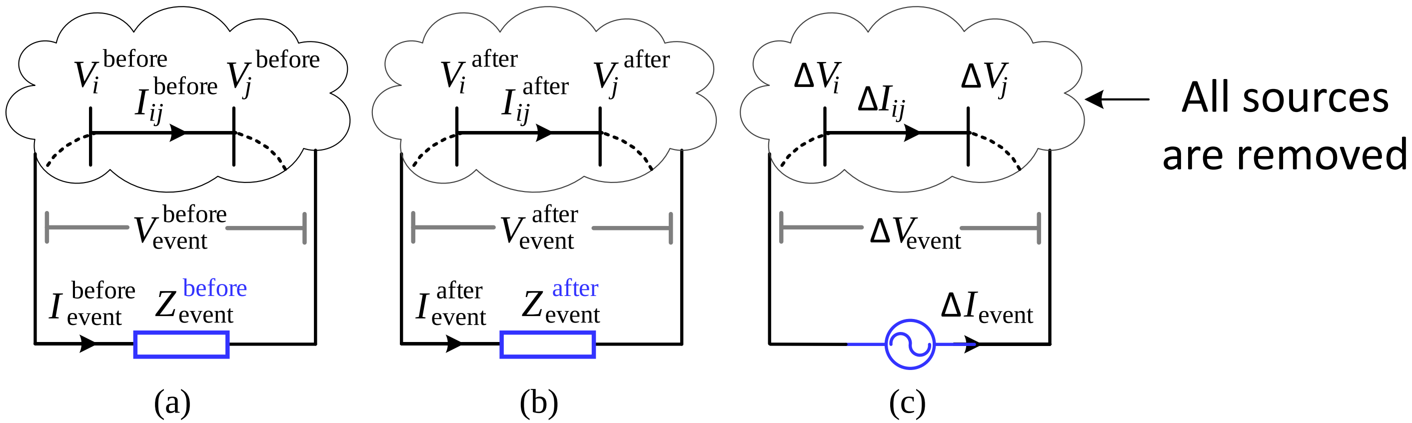
- In the equivalent circuit, the element that has changed is replaced by a *current source* that injects a current phasor equal to the *difference* in the current phasor that goes through the element *before* and *after* the event.

  - **Note**: This is directly related to the concept of PD.

- In the equivalent circuit, any other voltage source or current source in the circuit is replaced by its internal impedance.

- **Step 1: Forming the Equivalent Circuit Model (Cont.)**

- The construction of the equivalent circuit is shown below:

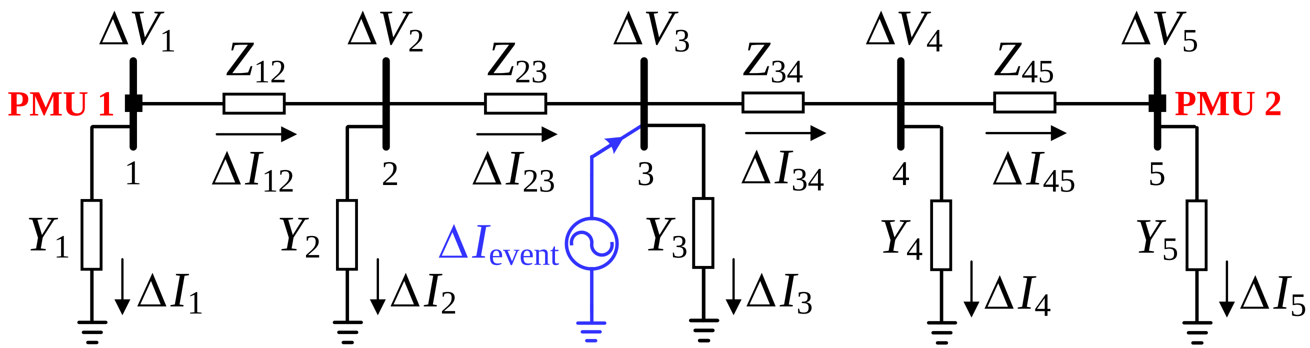


- Here, the event is a *change* in impedance Z_{event}.

- The voltage and current phasors *before* and *after* the event are shown above, in Figures (a) and (b), respectively. The equivalent circuit based on the compensation theorem is shown above in Figure (c).

- **Step 1: Forming the Equivalent Circuit Model (Cont.)**

- We can similarly construct the equivalent circuit corresponding to the event in Example 3.10. The equivalent circuit is shown below:



- The only source in this equivalent circuit is the current source at bus 3 (marked in blue) that represents the event.

- All nodal voltages are represented in terms of their phasor differences before and after the event; denoted by ΔV_1, ΔV_2, ΔV_3, ΔV_4, and ΔV_5.

3.5. Phasor Differential and Differential Synchrophasors

- **Step 1: Forming the Equivalent Circuit Model (Cont.)**

- Similarly, all branch currents are represented in terms of their phasor differences before and after the event: ΔI_{12}, ΔI_{23}, ΔI_{34}, and ΔI_{45}.

- The differential synchrophasors from PMU 1 and PMU 2 give us four measurements on the equivalent circuit: ΔV_1, ΔV_5, ΔI_{12}, and $\Delta I_{45} = -I_{54}$.

- The differential synchrophasors for ΔV_2, ΔV_3, ΔV_4, ΔI_{23}, ΔI_{34}, ΔI_1, ΔI_2, ΔI_3, ΔI_4, ΔI_5, and ΔI_{event} are still *unknown*.

- Furthermore, the location of current source ΔI_{event} is also *unknown*.

- The known parameters (pseudo-measurements) in this analysis are line impedances Z_{12}, Z_{23}, Z_{34}, Z_{45} and node admittances Y_1, Y_2, Y_3, Y_4, Y_5.

3.5. Phasor Differential and Differential Synchrophasors

- **Step 2: Nodal Voltage Calculation Starting from PMU 1**

- Let k denote the bus number where the switching event occurs, which is *not* known. In Example 3.10, we have $k = 3$.

- As a *hypothetical assumption*, suppose we know parameter k; i.e., we
- know the location of the event. Suppose we even know phasor ΔI_{event}.

- Under this hypothetical assumption, we can use the measurements in PMU 1, together with the known parameters, and successively apply the KVL and KCL to the equivalent circuit to obtain $\Delta V_1, \Delta V_2, \Delta V_3, \Delta V_4, \Delta V_5$.

  - The formulations are shown on the next slide.

3.5. Phasor Differential and Differential Synchrophasors

- **Step 2: Nodal Voltage Calculation Starting from PMU 1 (Cont.)**

- Analysis of equivalent circuit starting from PMU 1:

$$\begin{aligned}
\Delta V_1 &= \Delta V_1 \quad \leftarrow \quad \text{Direct Measurement from PMU 1} \\
\Delta V_2 &= \Delta V_1 - \Delta I_{12}\, Z_{12} \\
\Delta V_3 &= \Delta V_2 - (\Delta I_{12} - \ Y_2 \Delta V_2)\, Z_{23} \\
\Delta V_4 &= \Delta V_3 - (\Delta I_{12} - Y_2 \Delta V_2 - Y_3 \Delta V_3 + \Delta I_{\text{event}})\, Z_{34} \\
\Delta V_5 &= \Delta V_4 - (\Delta I_{12} - Y_2 \Delta V_2 - Y_3 \Delta V_3 - Y_4 \Delta V_4 + \Delta I_{\text{event}})\, Z_{45}
\end{aligned}$$

- **Note 1**: ΔI_{event} is used in the calculation of ΔV_4 and ΔV_5.

- **Note 2**: In obtaining ΔV_3, we used the fact that $Y_2 \Delta V_2 = \Delta I_2$ and the fact that $\Delta I_{12} - \Delta I_2 = \Delta I_{23}$. Similar facts are used in obtaining ΔV_4 and ΔV_5.

- **Step 2: Nodal Voltage Calculation Starting from PMU 1 (Cont.)**

- However, since we do *not* actually know the location of the event, i.e., parameter k, and we also do *not* know ΔI_{event}, we cannot use the equations on Slide 76 in their current form.

- If we drop ΔI_{event} from the previous equations, they become:

$$
\begin{aligned}
\Delta V_1 &= \Delta V_1 \quad \leftarrow \quad \text{Direct Measurement from PMU 1} \\
\Delta V_2 &= \Delta V_1 - \Delta I_{12}\, Z_{12} \\
\Delta V_3 &= \Delta V_2 - (\Delta I_{12} - \ Y_2 \Delta V_2)\, Z_{23} \\
\Delta V_4 &\neq \Delta V_3 - (\Delta I_{12} - Y_2 \Delta V_2 - Y_3 \Delta V_3)\, Z_{34} \\
\Delta V_5 &\neq \Delta V_4 - (\Delta I_{12} - Y_2 \Delta V_2 - Y_3 \Delta V_3 - Y_4 \Delta V_4)\, Z_{45}
\end{aligned}
$$

- Notice that the *equality* signs in the last two lines are now replaced with the *inequality* signs due to not being able to include ΔI_{event}.

3.5. Phasor Differential and Differential Synchrophasors

- **Step 3: Nodal Voltage Calculation Starting from PMU 2**

- Similar to the analysis in Step 2, we can use the measurements in PMU 2 and successively apply the KVL to obtain the following:

$$\begin{aligned}
\Delta V_5 &= \Delta V_5 \quad \leftarrow \quad \text{Direct Measurement from PMU 2} \\
\Delta V_4 &= \Delta V_5 + \Delta I_{45}\, Z_{45} \\
\Delta V_3 &= \Delta V_4 + (\Delta I_{45} + Y_4 \Delta V_4)\, Z_{34} \\
\Delta V_2 &\neq \Delta V_3 + (\Delta I_{45} + Y_4 \Delta V_4 + Y_3 \Delta V_3)\, Z_{23} \\
\Delta V_1 &\neq \Delta V_2 + (\Delta I_{45} + Y_4 \Delta V_4 + Y_3 \Delta V_3 + Y_2 \Delta V_2)\, Z_{12}
\end{aligned}$$

- Again, notice the *inequality* signs in the last two lines.

- **Step 4: Minimum Discrepancy**

- Let us denote the expressions on the right hand side on Slide 77 as

$$\widehat{\Delta V}_1, \quad \widehat{\Delta V}_2, \quad \widehat{\Delta V}_3, \quad \widehat{\Delta V}_4, \quad \widehat{\Delta V}_5.$$

- Let us denote the expressions on the right hand side on Slide 78 as

$$\widecheck{\Delta V}_1, \quad \widecheck{\Delta V}_2, \quad \widecheck{\Delta V}_3, \quad \widecheck{\Delta V}_4, \quad \widecheck{\Delta V}_5.$$

- *Discrepancy* between the nodal voltage calculations in Step 2 at each bus and the nodal voltage calculations in Step 3 at the same bus:

$$\Phi_i = \widehat{\Delta V}_i - \widecheck{\Delta V}_i, \quad i = 1, \ldots, 5.$$

- **Step 4: Minimum Discrepancy (Cont.)**

- When it comes to calculating ΔV_3, the corresponding expression in Step 2 and the corresponding expression in Step 3 are both equal to ΔV_3; thus, $\Phi_3 = 0$. However, that is *not* the case at *any other bus*. At any bus other than bus 3, we face an inequality either in Step 2 or in Step 3.

- Thus, the discrepancy index Φ_i is *not* zero at any bus other than bus 3:

$$
\begin{aligned}
\Phi_1 &\neq 0 \\
\Phi_2 &\neq 0 \\
\Phi_3 &= 0 \\
\Phi_4 &\neq 0 \\
\Phi_5 &\neq 0
\end{aligned}
$$

3.5. Phasor Differential and Differential Synchrophasors

- **Step 4: Minimum Discrepancy (Cont.)**

- In practice, the discrepancy may not be precisely zero at the event bus because of the error in phasor measurements and the inaccuracy in the known parameters. Therefore, instead of looking for a zero discrepancy at a bus, we look for the *minimum absolute value* of the discrepancy across all the buses to identify the location of the event.

- In this regard, the location of the event, i.e., parameter k is obtained as

$$k = \underset{i=1,\ldots,5}{\arg\min} \left|\Phi_i\right|.$$

- The magnitude of Φ_i is used here, because discrepancy is a *phasor*.

- **Example 3.11**: Consider the impedance switching event in Example 3.10 in Section 3.5.3. Suppose the voltage and current phasors *before* and *after* the event are measured as shown in the table below:

Phasor	**Before**	**After**
$V_1\angle\theta_1$	$0.9926\angle 29.194^\circ$	$0.9973\angle 29.178^\circ$
$I_{12}\angle\phi_{12}$	$3.1738\angle 1.753^\circ$	$2.9169\angle 19.046^\circ$
$V_5\angle\theta_5$	$0.9570\angle 27.536^\circ$	$0.9709\angle 27.228^\circ$
$I_{45}\angle\phi_{45}$	$0.7566\angle 9.101^\circ$	$0.7676\angle 8.793^\circ$

- The line impedances are $Z_{12} = Z_{23} = Z_{34} = Z_{45} = 0.0025 + j0.0050$ per unit. The admittances at buses 2, 3, and 4 in the equivalent circuit are $Y_2 = 0.7 - j0.4$, $Y_3 = 0.5 - j0.3$, and $Y_4 = 1.0 - j0.5$ per unit.

3.5. Phasor Differential and Differential Synchrophasors

- **Example 3.11 (Cont.)**: The differential synchrophasors are obtained as

$$\Delta V_1 = 0.004708\angle 25.803^\circ,$$

$$\Delta I_{12} = 0.950233\angle 115.902^\circ,$$

$$\Delta V_5 = 0.014834\angle 6.937^\circ,$$

$$\Delta I_{45} = 0.011738\angle -11.480^\circ.$$

By conducting Steps 2 and 3, we can obtain:

$$\widehat{\Delta V_1} = 0.0047\angle 25.8029^\circ, \qquad \widecheck{\Delta V_1} = 0.0154\angle 8.598^\circ,$$

$$\widehat{\Delta V_2} = 0.0098\angle 11.7582^\circ, \qquad \widecheck{\Delta V_2} = 0.0152\angle 7.987^\circ,$$

$$\widehat{\Delta V_3} = 0.0150\angle 7.4890^\circ, \qquad \widecheck{\Delta V_3} = 0.0150\angle 7.505^\circ,$$

$$\widehat{\Delta V_4} = 0.0204\angle 5.5197^\circ, \qquad \widecheck{\Delta V_4} = 0.0149\angle 7.116^\circ,$$

$$\widehat{\Delta V_5} = 0.0258\angle 4.5440^\circ, \qquad \widecheck{\Delta V_5} = 0.0148\angle 6.937^\circ,$$

3.5. Phasor Differential and Differential Synchrophasors

- **Example 3.11 (Cont.)**: The discrepancy indexes are obtained as

$$\begin{aligned} &\left|\Phi_1\right| = 0.0110, \quad \left|\Phi_2\right| = 0.0055, \quad \left|\Phi_3\right| = 0.0000, \\ &\left|\Phi_4\right| = 0.0055, \quad \left|\Phi_5\right| = 0.0110. \end{aligned}$$

Accordingly, the minimum discrepancy is obtained at bus 3.

Therefore, bus 3 is the event bus, i.e., we have $k = 3$.

- **Other Methods**:

- The method in Steps 1 to 4 is important because it shows one of the key applications of differential synchrophasor measurements.

- However, there are also many other methods that one can use to identify the location of events. In particular, there is a rich literature on identifying the location of faults by using phasor measurements.

- A common approach is to try to estimate the distance between the location of the sensor and the location of the fault [157–159]. Other examples for the methods to identify the location of a fault include [160–164]. There are also few methods to identify the location of events that are benign, yet they can reveal how different components operate in the power system. For example, there are studies on how to identify the location of a capacitor bank in distribution systems, e.g., in [165, 166].

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3.6. Three-Phasor and Unbalanced Phasor Measurements

- Traditionally, most PMUs are installed on power transmission networks.

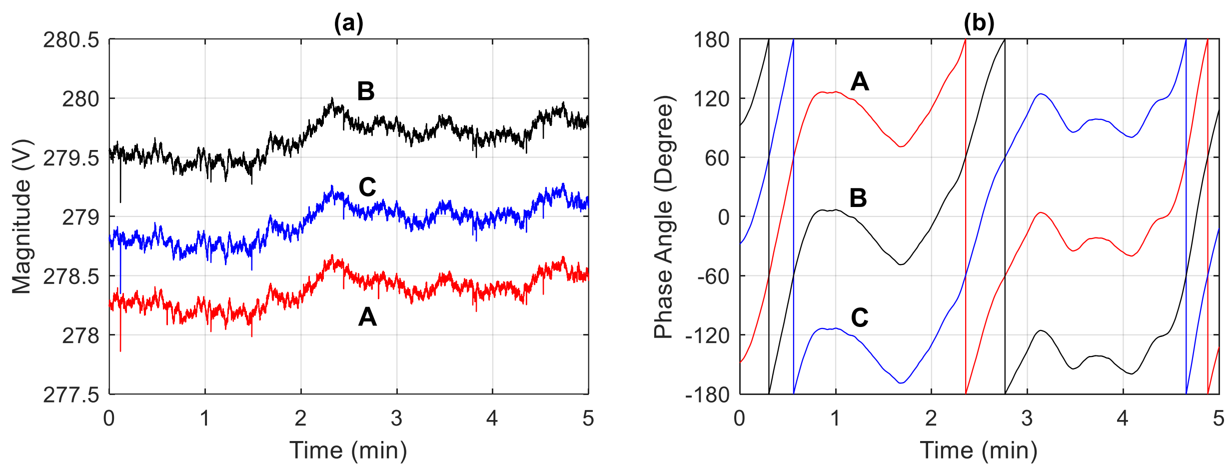
- Since the power system is mostly balanced at the transmission level, it is often not necessary for the PMUs to report phasor measurements on all three phases. They may measure one phase only; or they may report only the *positive sequence* of the three-phase voltage and current phasors.

- However, when it comes to power distribution systems; voltage and current phasors are commonly *unbalanced*; therefore, it is beneficial to measure the voltage and current phasors on *all three phases*.

- Hence, D-PMUs often report phasors measurements on all phases [167].

3.6. Three-Phasor and Unbalanced Phasor Measurements

- An example for voltage phasor measurements at a three-phase load is shown below. The unbalance in magnitude is evident in Figure (a). There is also slight unbalance in phase angle in Figure (b); at a fraction of a degree.



- **Note**: To check the unbalance in phase angle, we need to use the measurements in Figure (b) to calculate the *instantaneous phase angle difference* between any two phases to see how much it deviates from 120◦.

3.6.1. Symmetrical Components

- Suppose V_A, V_B, and V_C denote the *unbalanced* three-phase voltage phasor measurements. The *symmetrical components* are defined as

$$V_A^0 = V^0 \qquad V_A^+ = V^+ \qquad V_A^- = V^-$$

$$V_B^0 = V^0 \qquad V_B^+ = \alpha^2\, V^+ \qquad V_B^- = \alpha\, V^-$$

$$V_C^0 = V^0 \qquad V_C^+ = \alpha\, V^+ \qquad V_C^- = \alpha^2\, V^-$$

(*Zero* Sequence) (*Positive* Sequence) (*Negative* Sequence)

- **Note**: Each of the above set of phasors includes three *balanced* phasors.

- Vectors V^0, V^+, and V^- are defined on the next slide.

3.6. Three-Phasor and Unbalanced Phasor Measurements

- Vectors V^0, V^+, and V^- are obtained from V_A, V_B, and V_C as follows:

$$V^0 = (V_A + V_B + V_C)/3$$

$$V^+ = \left(V_A + \alpha V_B + \alpha^2 V_C\right)/3$$

$$V^- = \left(V_A + \alpha^2 V_B + \alpha V_C\right)/3$$

Vector $\alpha = e^{j2\pi/3}$ is a *phase shift operator*, $1 + \alpha + \alpha^2 = 0$, and $\alpha^3 = 1$. By multiplying α to a phasor, we shift (increase) its phase angle by 120^0. By multiplying α^2 to a phasor, we shift (increase) its phase angle by 240^0.

- **Note**: If a set of three-phase measurements are balanced, then it has only the positive sequence. Negative and zero components can *characterize the unbalanced nature* of the three original phasors.

3.6. Three-Phasor and Unbalanced Phasor Measurements

- The original unbalanced three-phase phasors can be expressed as the sum of three sets of symmetrical components:

$$V_A = V_A^0 + V_A^+ + V_A^-$$

$$V_B = V_B^0 + V_B^+ + V_B^-$$

$$V_C = V_C^0 + V_C^+ + V_C^-$$

- **Example 3.12**: Consider the following voltage phasor measurements:

$$V_A = 278.0574\angle -73.1170^\circ$$

$$V_B = 276.8067\angle 167.2848^\circ$$

$$V_C = 278.5330\angle 47.1288^\circ$$

- **Example 3.12 (Cont.)**: From Slide 90, we can obtain:

$$V^0 = 0.3469\angle{-76.9327^\circ}$$
$$V^+ = 277.7978\angle{-72.9014^\circ}$$
$$V^- = 1.0257\angle{-167.8488^\circ}$$

Zero components are obtained as

$$V_A^0 = V_B^0 = V_C^0 = 0.3469\angle{-76.9327^\circ}$$

- **Example 3.12 (Cont.)**: Positive components are obtained as

$$V_A^+ = 277.7978\angle{-72.9014^\circ}$$
$$V_B^+ = 277.7978\angle 167.0986^\circ$$
$$V_C^+ = 277.7978\angle 47.0986^\circ$$

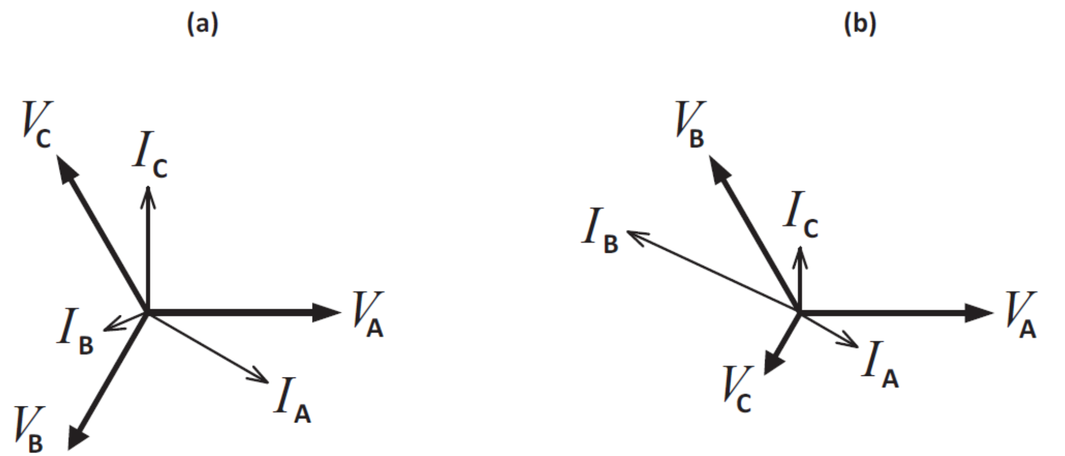
Negative components are obtained as

$$V_A^- = 1.0257\angle{-167.8488^\circ}$$
$$V_B^- = 1.0257\angle{-47.8488^\circ}$$
$$V_C^- = 1.0257\angle 72.1512^\circ$$

3.6. Three-Phasor and Unbalanced Phasor Measurements

• **Analysis of Faults**: Symmetrical components can be used to analyze severe unbalanced conditions, such as those that are caused by faults.

• For example, in an *open phase system*, see Figure (a), or in a *phase-to-ground fault*, see Figure (b), there are often major negative sequence currents and possibly major zero sequence currents; see [168–170].

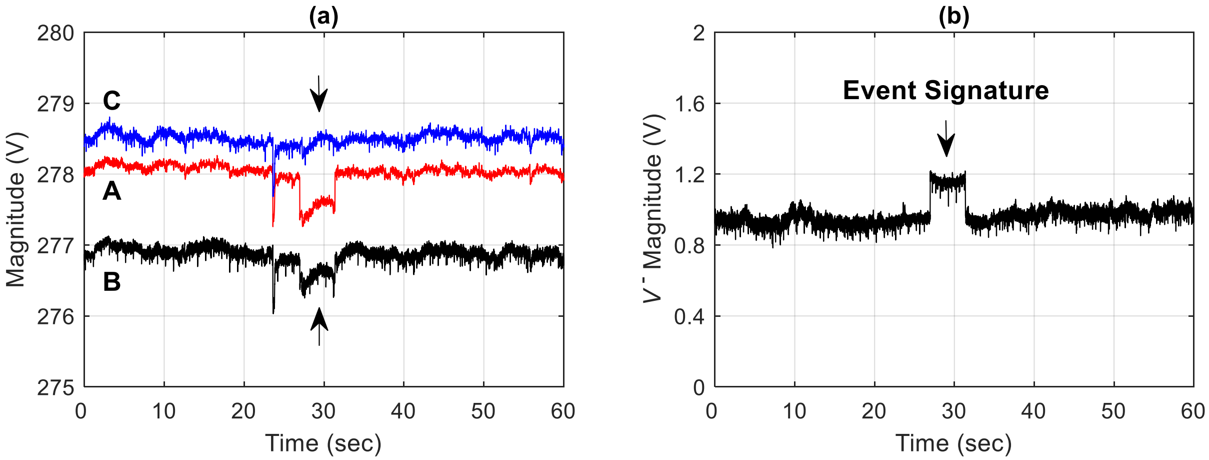


3.6.2. Unbalanced Events

- Apart from faults that are often severely unbalanced, some *benign* events may also be *unbalanced* and affect only one or two phases.

- Unbalanced events can be *detected* by applying various event detection methods that we learned in Section 2.7.2 in Chapter 2 to
  - The negative-sequence or
  - The zero-sequence measurements (or both).

- **Example 3.13**: Consider the three-phase voltage measurements in Figure (a). Only the magnitudes of the measured voltage phasors are shown here. An *unbalanced event* is marked with two arrows. This event affects only two phases, A and B. It does *not* affect Phase C.



- The magnitude of the corresponding negative sequence voltage phasors, i.e., the magnitude of V^-, is shown in Figure (b).

3.6. Three-Phasor and Unbalanced Phasor Measurements

• **Example 3.13 (Cont.)**: We can see in Figure (b) that the event has a very clear signature in this figure on the negative sequence.

• Any event detection method, such as the methods that we learned in Section 2.7.2 in Chapter 2, can be applied to the time series in Figure (b) in order to detect this unbalanced event.

• **Note**: Even though the unbalanced event is not at all severe (i.e., unlike most faults), the analysis of symmetrical components is still very useful.

3.6.3. Voltage Unbalance Factor

• Recall from Section 2.8.2 in Chapter 2 that phase unbalance can negatively affect the operation of induction motors.

• We previously discussed *PU* (Phase Unbalance) in Chapter 2 as a *metric* to quantify the extent of phase unbalance. PU can also be calculated based on phasor measurements; see *Exercise 3.18*.

• Since motors are influenced mostly by the negative-sequence voltage, another metric is the *Unbalance Factor* (UF), which is defined as [128]:

$$\mathrm{UF} = \frac{|V^-|}{|V^+|} \times 100\%.$$

• Here, we divide the magnitude of the negative sequence to the magnitude of the positive sequence.

- **Example 3.14**: Again, consider the voltage phasor measurements in Example 3.12. From Slide 93 and the definition of UF, we can obtain

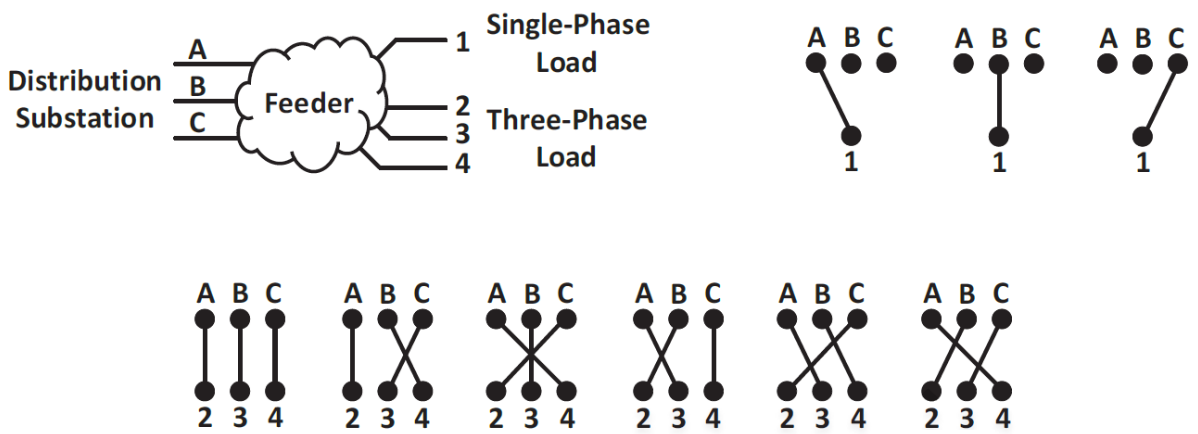
$$\text{UF} = \frac{1.0257}{277.7978} \times 100\% = 0.37\%.$$

- **Note**: We may also use $|V^0|/|V^+|$ as yet another metric to examine unbalanced phasors. While negative-sequence voltages impact motors and other line-to-line connected loads the most, zero-sequence unbalance can affect line-to-ground connected three-phase loads.

3.6. Three-Phasor and Unbalanced Phasor Measurements

3.6.4. Phase Identification

• Recall from Section 2.8.3 in Chapter 2 that phase identification is an important and challenging problem in power distribution systems.

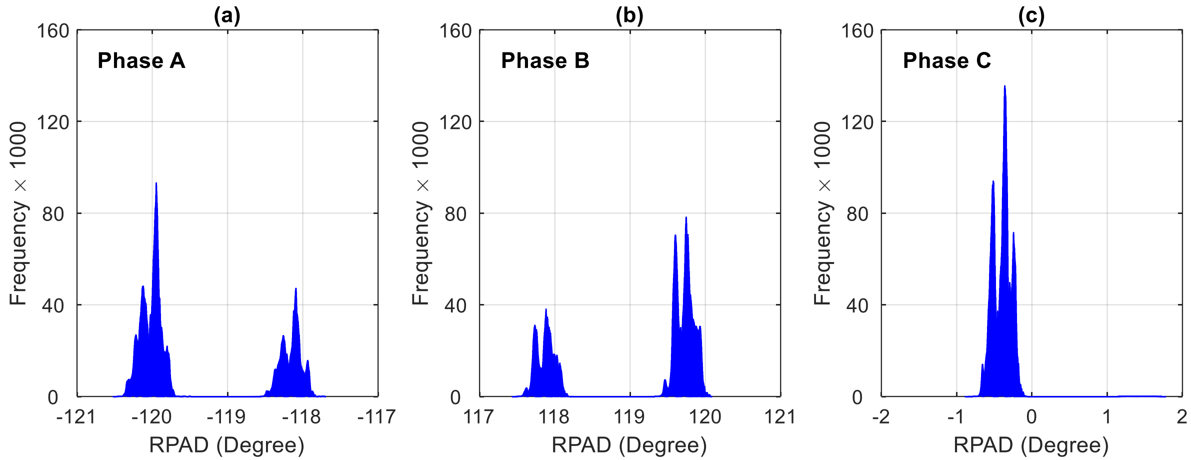


• It is the problem of identifying the correct phase connection labeling at each load, equipment, etc. on a power distribution network.

- However, if phasor measurements are available at distribution level, i.e., if we have *D-PMUs*, then phase identification is a rather *trivial* problem.

- Because *phase angle measurements* can directly identify the phases.

- Consider the below figure that shows the histogram of one day of *RPAD* measurements between the voltage phasor at a single-phase load and the voltage phasors at each of the three phases at the substation.



3.6. Three-Phasor and Unbalanced Phasor Measurements

- Notice in the figure that RPAD measurements concentrate around:
  - -120° on Phase A
  - 120° on Phase B
  - 0° on Phase C.

- It is clear that the load is connected to phase C, because there is only a *very small phase angle difference* between the unknown phase at the load location and Phase C (known) at the substation.

- By analyzing RPAD, any change in phase connections in a power distribution system can be identified by D-PMUs almost instantly.

3.6. Three-Phasor and Unbalanced Phasor Measurements

• **Note**: the reason to have two separate clusters of bars in Figures (a) and (b) on Slide 101 is the operation of a switched capacitor bank on this distribution feeder, which is energized for only a portion of the day [139].

Depending on whether or not the capacitor bank is energized, there can be some changes in the measured RPAD. However, these changes (about 2°) are very small compared to the roughly 120◦ phase angle difference between any two phases that are *not* connected to each other.

Thus, the operation of the capacitor bank in this example does *not* cause any ambiguity in phase identification based on RPAD.

3.6. Three-Phasor and Unbalanced Phasor Measurements

- Based on the analysis on Slides 101-103, it is clear that D-PMUs are ideal sensors to resolve the phase identification problem.

- However, given the *high cost* of D-PMU, utilities may choose *not* to install D-PMUs at *each* load location across the distribution feeder.

- If only a few D-PMUs are available, then a *combination* of the analysis of RPAD from D-PMUs, together with the methods in Section 2.8.3 in Chapter 2 can be used to solve the phase identification problem.

- We will also discuss phase identification in Section 5.9.2 in Chapter 5, and also in Sections 6.2 and 6.7 in Chapter 6.

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3.7. Events in Phasor Measurements

- We previously defined "events" in power systems as any change in any component that is worth studying; see Section 2.7 in Chapter 2.

- Events in power systems may affect *both* the magnitude and the phase angle in voltage or current phasors. However, when it comes to capturing the impact of an event on the *magnitude* of voltage or current, PMUs provide the *same* type of information that one can get from the conventional voltage and current sensors that we discussed in Chapter 2.

- What is truly unique about PMUs is their ability also to directly capture the *impact of events on the phase angle* of voltage or current.

- Thus, we focus on the analysis of events in *phase angle measurements*.

3.7. Events in Phasor Measurements

3.7.1. Analysis of Events in Phase Angle Measurements

• PMUs can reveal the impact of events on not only the magnitude but also the phase angle of voltage or current.

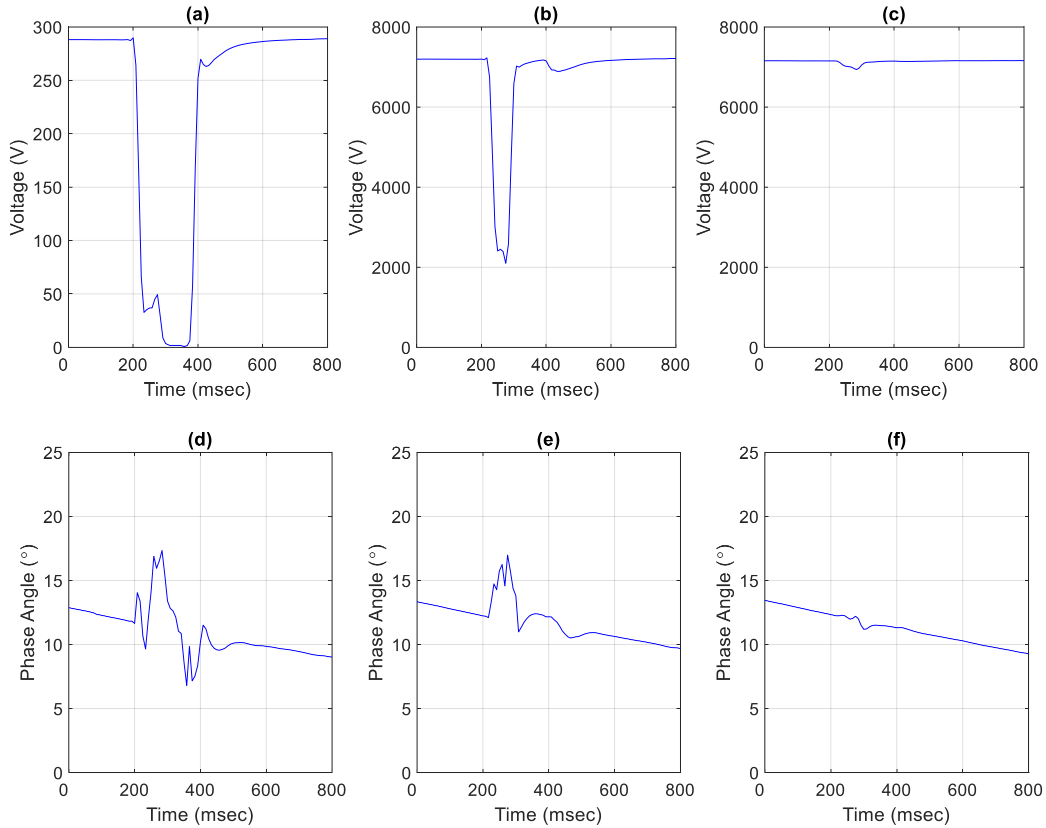
• An example is shown on the next slide. Here, we examine the same event that we previously studied in Example 2.12 in Section 2.5.1 in Chapter 2. Recall that the event in this example is a fault on a power distribution feeder. The fault is caused by animal contact.

• We previously looked at the voltage measurements during this event at three *different sensor locations*: **(1)** at a load location on the feeder where the fault occurs; **(2)** at the distribution substation that supplies the feeder where the fault occurs; and **(3)** at another substation that is located several miles away from the feeder where the fault occurs.

- The *magnitude* and the *phase angle* of the voltage phasors at the three sensor locations are shown in Figures (a)–(c) and (d)–(f), respectively.

Magnitude

Phase Angle



Location 1 **Location 2** **Location 3**

3.7. Events in Phasor Measurements

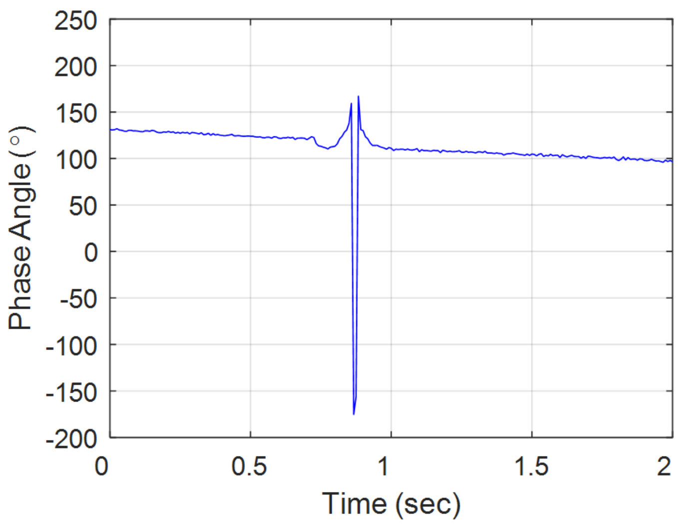
• We can see on the previous slide that the *signature of the event* is clearly visible on *both* magnitude and phase angle.

• Phase angle is affected not only at the first two sensor locations, which are close to the faulted location, but also at the third sensor location, which is relatively far from the faulted location.

• Adding the information about the signature of the event on the phase angle can bring *additional information* to the analysis of the event.

3.7. Events in Phasor Measurements

- **Unwrapping Measurements**: Consider the event that is captured in the phase angle measurements as shown in the figure below:

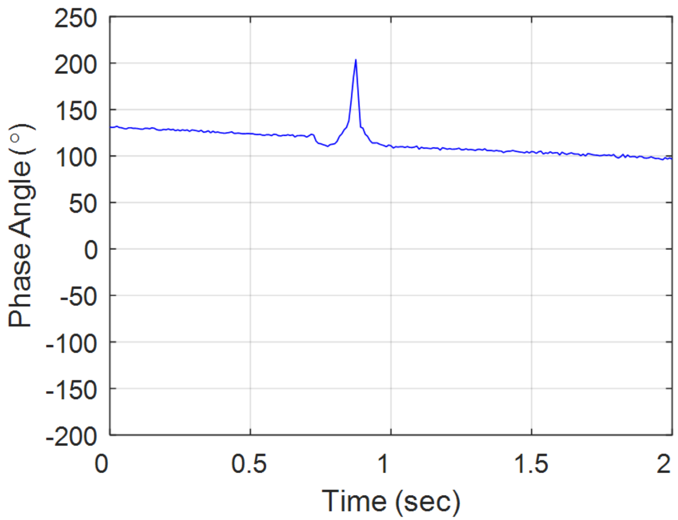


- It may appear that the phase angle drops *suddenly* and *drastically* during this event. However, that is *not* really the case. (see next slide)

3.7. Events in Phasor Measurements

• The sudden drop in phase angle in the figure on the previous slide is simply due to the fact that the phase angle measurements in this example are *wrapped around* at 180◦ down to −180◦.

• The *unwrapped version* of the same event is shown below.



• The event is still significant in its impact on phase angle, but it does *not* involve the type of sudden drop that was saw on the previous slide.

3.7. Events in Phasor Measurements

- **Removing the Impact of Off-Nominal Frequency**:

- From Section 3.3, the frequency of the power system directly affects how phase angle is measured. This creates some challenges when it comes to the analysis of events based on *phase angle* measurements.

- For example, consider the phase angle measurements on Slide 108. We can observe two different types of changes in the measurements:

  - Some changes in the phase angle measurements *are* caused by the event; between time $t = 200$ msec and time $t = 500$ msec.

  - Some changes in the phase angle measurements are *not* caused by the event; they take place *gradually* and for the entire period between time $t = 0$ msec and time $t = 800$ msec.

3.7. Events in Phasor Measurements

- **Removing the Impact of Off-Nominal Frequency**: From Section 3.3, the frequency of the power system directly affects how phase angle is measured. This creates some challenges when it comes to the analysis of events based on *phase angle* measurements.

- For example, consider the phase angle measurements on Slide 108. We can observe two different types of changes in the measurements:

  - Some changes in the phase angle measurements *are* caused by the event; between time $t = 200$ msec and time $t = 500$ msec.

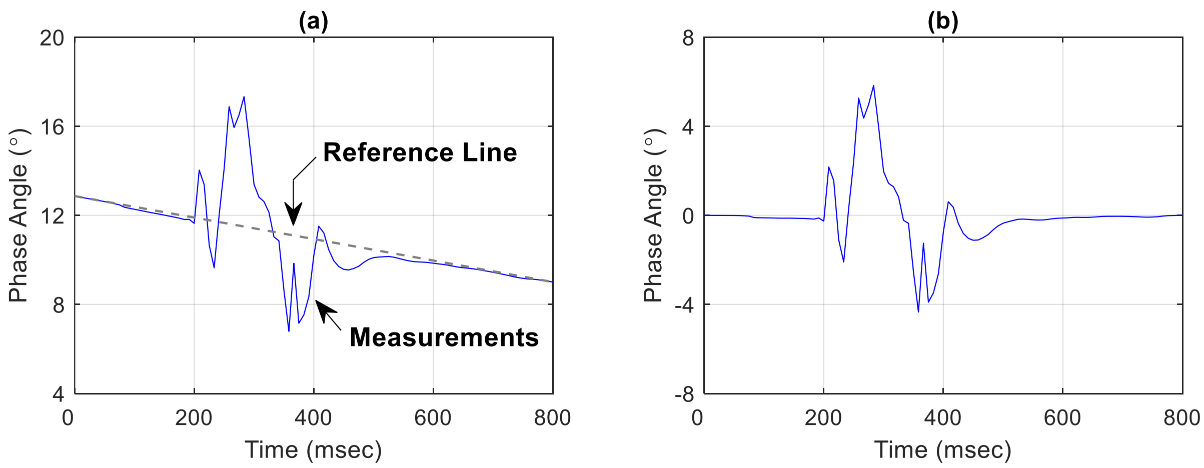
  - Some changes in the phase angle measurements are *not* caused by the event; they take place *gradually* and for the entire period between time $t = 0$ msec and time $t = 800$ msec.

- We need to *distinguish* these two types of events. (Q: Why?)

3.7. Events in Phasor Measurements

• **Note**: Only the former changes represent the event. The latter changes represent only the off-nominal frequency of the power system. The off-nominal frequency must *not* affect our analysis of the event.

• One option to resolve this issue is to *remove* the impact of the off-nominal frequency on the phase angle measurements during the event. This can be done by forming an approximate *reference line*:



3.7. Events in Phasor Measurements

• The reference line, which is represented by a dashed line, is the line that goes through the point corresponding to the *first measurement* at time $t = 0$ on the top left of the figure and the point corresponding to the *last measurement* at time $t = 800$ msec on the bottom right of the figure.

• The purpose of this reference line is to *approximate* the impact of the off-nominal frequency on phase angle measurements.

• Accordingly, by *subtracting* the reference line from the measurements, i.e., by subtracting the dashed line from the solid line, we can obtain the curve shown in Figure (b) on the previous slide.

• This curve is a good approximation of the changes in the phase angle that are due *solely* to the *event* and *not* to the off-nominal frequency.

3.7. Events in Phasor Measurements

• **Using RPAD and Power Factor:** An alternative to explicitly removing the impact of the off-nominal frequency is to use quantities that do involve phasor angle measurements but are *less sensitive* to the off-nominal frequency. One example is RPAD. Another example is power factor.

• The inherent subtraction in RPAD *automatically removes* the impact of the off-nominal frequency. Since the frequency is almost the same across an interconnected power system, subtracting the phase angle measurements at one location from the phase angle measurements at another location can cancel out the impact of the off-nominal frequency.

$$\mathrm{RPAD} = \theta_1 - \theta_2.$$

• It is as if the reference lines at the two locations cancel out each other.

3.7. Events in Phasor Measurements

- Depending on the type of event, such as when the event causes a change in the phase angle in the phasor of current, power factor can be used as a measure to capture the change in phase angle with little impact from the off-nominal frequency of the system. Suppose $V\angle\theta$ and $I\angle\varphi$ denote the phasor measurements for voltage and current, respectively.

- Recall that power factor is obtained as

$$\mathrm{PF} = \cos(\theta - \phi).$$

- Again, the inherent subtraction in the process of obtaining the power factor *automatically removes* the impact of the off-nominal frequency.

3.7.2. Event Classification

- Recall from Section 2.4.2 in Chapter 2 that scatter plots can help us *visually* classify events into different groups that have *similar features*.

- In this section, we learn to do event classification rather *automatically*.

- We will discuss the following topics:

  1) Feature Selection

  2) Using SVM Classifier for Event Classification

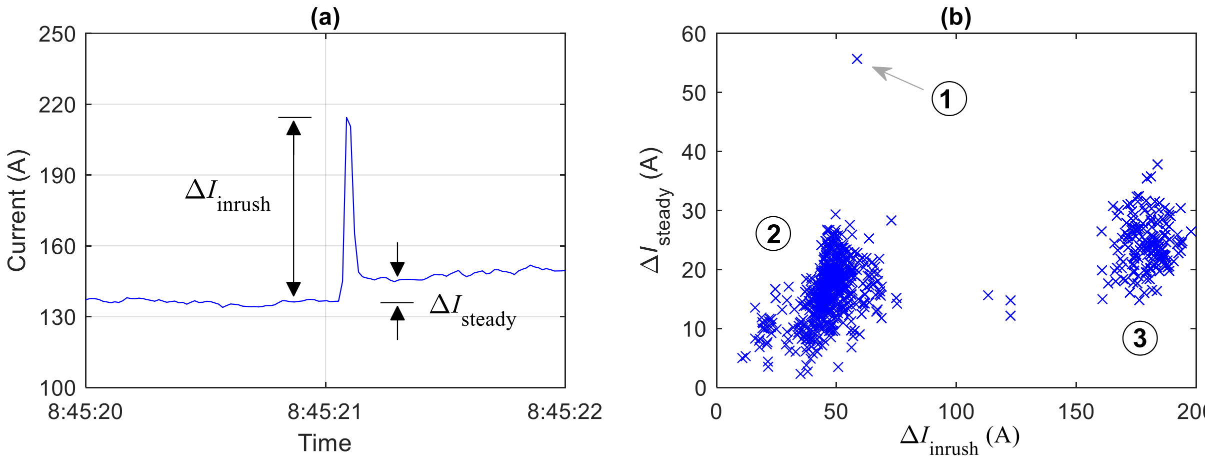
  3) Training SVM Classifier for Event Classification

  4) Other Classifiers

3.7. Events in Phasor Measurements

1) Feature Selection: A critical task in classification is to choose adequate *quantitative features* that can help us distinguish among different classes.

• For instance, recall that in Example 2.11 in Chapter 2, we created a scatter plot based on two features: $\triangle I_{\text{inrush}}$ and $\triangle I_{\text{steady}}$.



• We can represent *each event* by the *vector* of its features:

$$\mathbf{x} = \begin{bmatrix} \Delta I_{\text{inrush}} \\ \Delta I_{\text{steady}} \end{bmatrix}.$$

3.7. Events in Phasor Measurements

- When it comes to phasor measurements, the features could be [124]:

$$V, I, cos(\theta - \varphi).$$

These features include the magnitude of voltage, the magnitude of current, and the power factor, see the advantage of using power factor on Slide 117. The vector of features for each event is accordingly formed as

$$\mathbf{x} = \begin{bmatrix} V \\ I \\ \cos(\theta - \phi). \end{bmatrix}$$

- **Note**: Due to the impact of off-nominal frequency on measuring phase angles, phase angle is usually *not* taken as a feature by itself.

3.7. Events in Phasor Measurements

- An alternative is to use the following features [118]:

$$V, I, VI\ cos(\theta - \varphi), VI\ sin(\theta - \varphi).$$

where the power factor is replaced with *active power* and *reactive power*.

- RPAD between two locations may also be used as another feature.

$$V_1,\ \ V_2,\ \ \theta_1 - \theta_2.$$

- In all cases on this slide and the last slide, we try to define the features based on magnitudes of phasors as well as some functions of the phase angle of the phasors that can remove the impact off-nominal frequency.

3.7. Events in Phasor Measurements

- The features in the previous slide can be defined *separately* on *each phase* (A, B, and C), or as the *average* across all phases.

- Features in 3-phase events may also include *symmetrical components*.

- In fact, when it comes to the classification of unbalanced events, the magnitude of the *negative sequence*, i.e., V^-, and the magnitude of the *zero sequence*, i.e., V^0, can be useful features, because they reveal the unbalanced nature of the event; see Example 3.13 in Section 3.6.2.

3.7. Events in Phasor Measurements

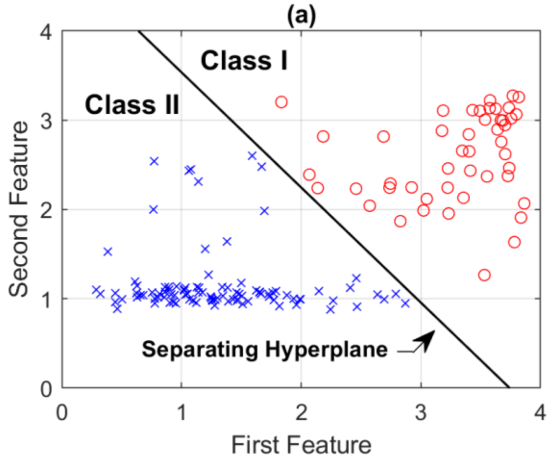
- Some *arithmetic* or *statistical calculations* based on the various quantities in the previous slides can also be used as features.

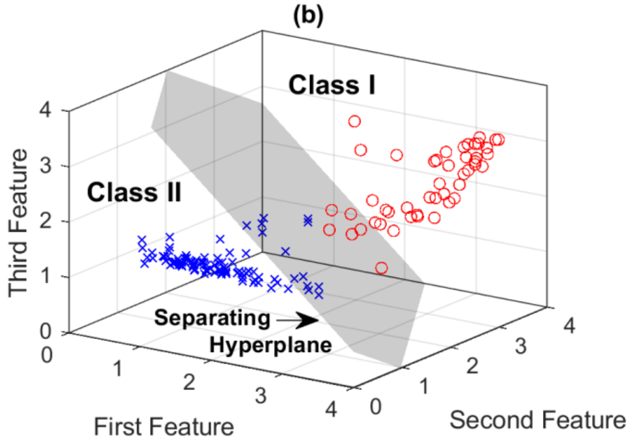
- For example, one can use the *sign*, the *absolute value*, or the *differential* of the previously mentioned features as alternative or additional features.

- One can also use the *minimum*, the *maximum*, the *mean*, the *median*, the *median absolute deviation*, or the *variance* of the mentioned features over a given window of time as alternative or additional features.

- Once all the features are selected, we can represent each event by the vector of its features, denoted by $\mathbf{x}$. The size of the vector of features is equal to the number of selected features.

2) Using SVM Classifier for Event Classification: Support vector machines (SVMs) are commonly used in machine learning as a tool for data classification [172–175]. its basic form, SVM is a *linear classifier*. It works based on obtaining a *separating hyperplane* to classify the data.





(Feature Vector $\mathbf{x}$ is 2×1) (Feature Vector $\mathbf{x}$ is 3×1)

- The hyperplane divides the data into two classes: Class I and Class II.

3.7. Events in Phasor Measurements

- All the points on one side of the hyperplane belong to the same class.

- The separating hyperplane has the following general formulation:

$$\mathbf{a}^T\mathbf{x} + b = 0$$

where $\mathbf{a}$ is the vector with the same size as vector $\mathbf{x}$; and b is a scalar.

- Suppose we have captured an event, and its vector of features is $\mathbf{x}$. Suppose we do *not* know whether this event belongs to Class I or Class II. We can use the separating hyperplane to classify this event:

  - If $\mathbf{a}^T\mathbf{x} + b < 0$, then the event belongs to Class I
  - If $\mathbf{a}^T\mathbf{x} + b > 0$, then the event belongs to Class II
  - If $\mathbf{a}^T\mathbf{x} + b = 0$, then the event can be placed in *either* class

- **Example 3.15**: Suppose we have captured four events. The number of features is two, i.e., each event is represented by two features.

These four events are represented by the following vectors of features:

$$\mathbf{x}_1 = \begin{bmatrix} 3.02 \\ 1.83 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 1.76 \\ 2.84 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 2.68 \\ 1.53 \end{bmatrix}, \quad \mathbf{x}_4 = \begin{bmatrix} 2.43 \\ 2.48 \end{bmatrix}$$

Each event may belong to either Class I or Class II.

Tollowing separating hyperplane is given to us for this classification:

$$\mathbf{a} = \begin{bmatrix} -0.7902 \\ -0.6129 \end{bmatrix}, \quad b = 3.2635.$$

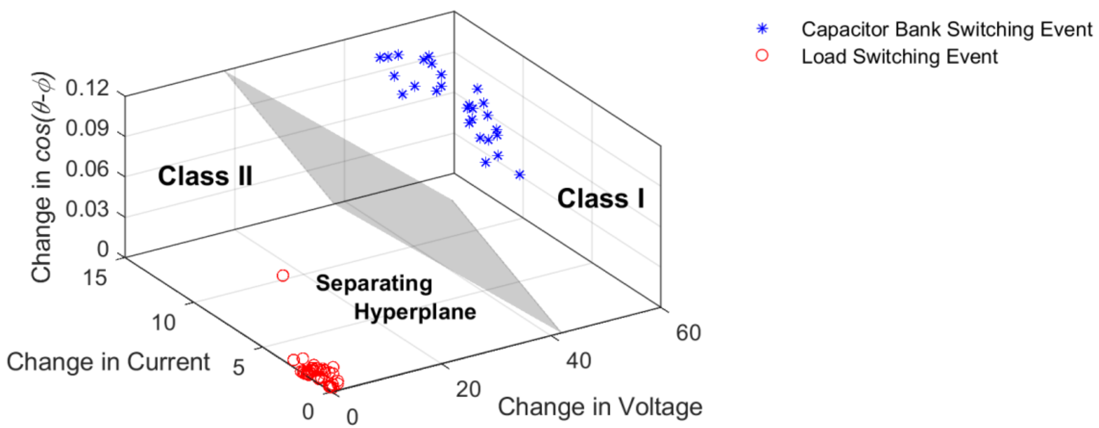
- **Example 3.15 (Cont.)**: Accordingly, we can obtain

$$\mathbf{a}^T \mathbf{x}_1 + b = -0.2444 < 0 \quad \rightarrow \quad \text{Class I},$$
$$\mathbf{a}^T \mathbf{x}_2 + b = 0.1322 > 0 \quad \rightarrow \quad \text{Class II},$$
$$\mathbf{a}^T \mathbf{x}_3 + b = 0.2081 > 0 \quad \rightarrow \quad \text{Class II},$$
$$\mathbf{a}^T \mathbf{x}_4 + b = -0.1766 < 0 \quad \rightarrow \quad \text{Class I}.$$

- Thus, we can conclude that the first and the fourth events belong to Class I, and the second and the third events belong to Class II.

• Event classification can help put similar events into the same class so that we can scrutinize them, characterize them, and make conclusions.

• **Example 3.16**: Figure below shows two different types of switching events in a power distribution system: Class I) capacitor bank switching, and Class II) load switching. Each event is characterized by three features.



• The features are defined on the next slide.

3.7. Events in Phasor Measurements

- **Example 3.16 (Cont.)**: The three features are defined based on the voltage and current phasor measurements at the substation:

  1) the *change* that is caused by the event in the magnitude of voltage;
  2) the *change* that is caused by the event in the magnitude of current;
  3) the *change* that is caused by the event in the power factor.

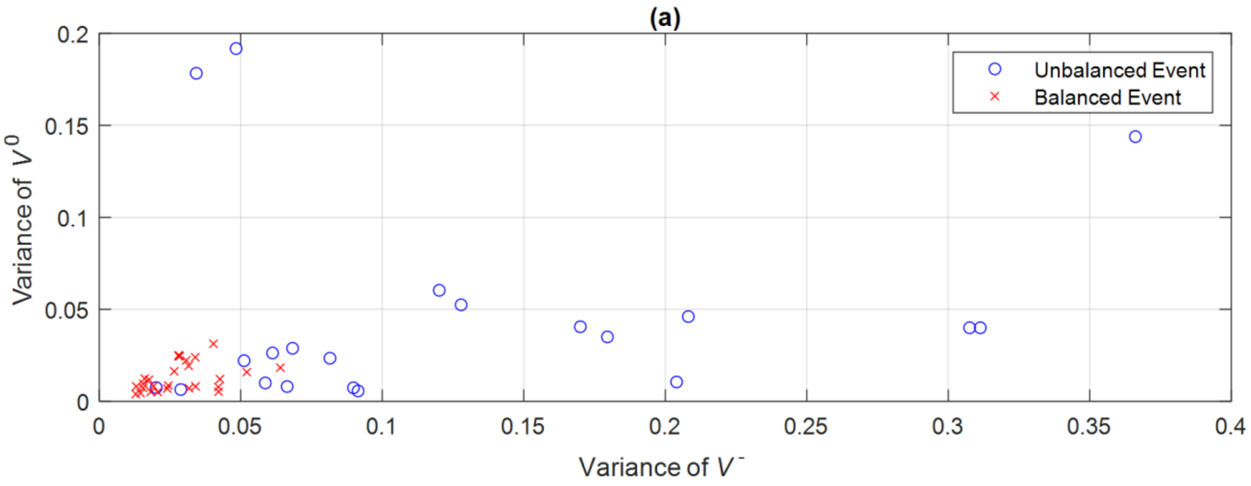
- The events in Class I can be used to monitor the operation and the *state of health* of the capacitor bank; e.g., see the analysis in [152].

- The events in Class II can be used to characterize and model the *different types of loads* in the system; e.g., see the analysis in [176].

3.7. Events in Phasor Measurements

• The vectors of features for the two classes in Example 3.16 are clearly *separable* from each other. However, that is *not* always the case.

• **Example 3.17**: Figure below shows two broad types of events in a three-phase power distribution system: Class I) balanced events; and Class II) unbalanced events. The *labeling* of these two types of events is done *manually* by a power system expert who looked at the voltage phasor measurements and decided the Class for each event.



3.7. Events in Phasor Measurements

- **Example 3.17 (Cont.)**: The events are characterized by two features:

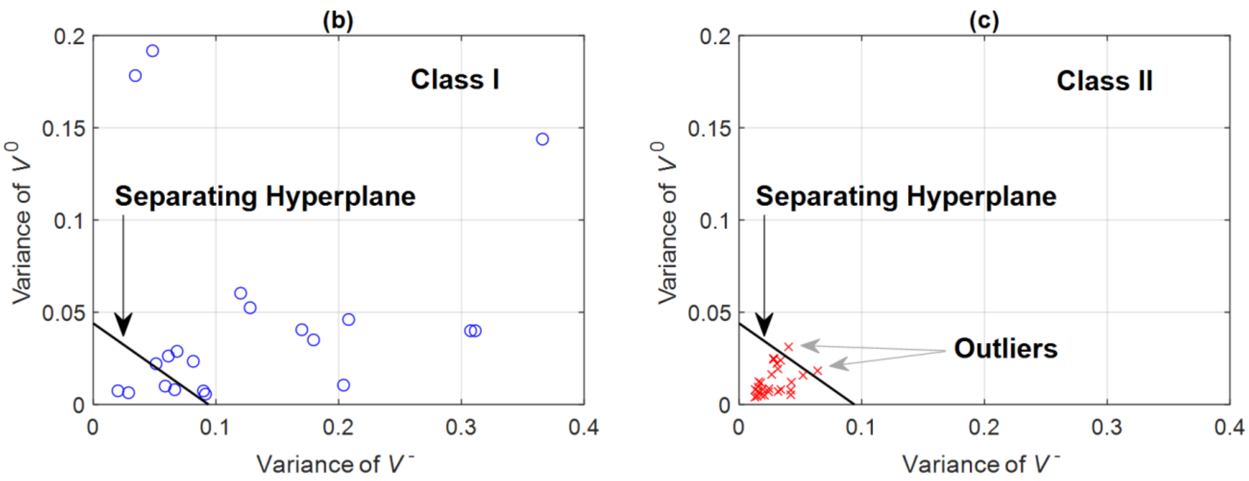
    1) the *variance* of the magnitude of the *negative sequence* of voltage over a window of two seconds;
    2) the *variance* of the magnitude of the *zero sequence* of voltage over a window of two seconds.

We can see in the figure on the previous slide that the events that are labeled as unbalanced are scattered away from the origin, while the events that are labeled as balanced are concentrated close to the origin.

However, unlike in Example 3.16, the two types of events are *not separable* based on their selected features.

- Notice that no single line can separate x points from o points.

• **Example 3.17 (Cont.)**: Nevertheless, we can still use a separating hyperplane to classify these events, as shown in Figures (b) and (c) below.



• Here, the classification is *not* precise, because some of the events show up on the *incorrect side* of the separating hyperplane. This is inevitable. These few events are considered to be *outliers*, as marked in Figure (c).

3.7. Events in Phasor Measurements

• **Note**: One subtle point about Example 3.17 is that *all* the events in this example had *some* level of nonzero magnitude for the negative or zero sequences (or both). That means, *in theory*, they are *all* unbalanced events.

However, *in practice*, many of these events are considered to be balanced events in the eyes of a power systems expert who does manual labeling.

The whole point of classification in this example is to figure out how to *mimic* the approach of the human judgement that was taken by the power systems expert to label these events *similarly*, but rather *automatically*.

3) Training SVM Classifier for Event Classification: The above examples show how separating hyperplanes can be *used* for event classification.

Q: But how can we obtain the separating hyperplane?

- The separating hyperplane has the following general formulation:

$$\mathbf{a}^T\mathbf{x} + b = 0$$

- In order to obtain $\mathbf{a}$ and b to model the separating hyperplane, we need to *first manually label and classify* a number of sample data.
    - This is similar to the manual labeling in Example 3.17.
    - After that, we can mathematically obtain proper $\mathbf{a}$ and b.

3.7. Events in Phasor Measurements

- A set of m events are *already labeled* as being in Class 1 or in Class 2. Each event i, where $i = 1, \ldots, m$, is represented by a vector of features $\mathbf{x}_i$.

- We define y_i as a *binary variable* to indicate the class of event i. If event i belongs to Class 1, then $y_i = -1$. If event i belongs to Class 2, then $y_i = 1$.

- We can use the following *optimization problem* to *train* the SVM classifier:

$$\begin{aligned} \underset{\mathbf{a},\, b}{\text{minimize}} \quad & \|\mathbf{a}\|_2 \\ \text{subject to} \quad & y_i \left(\mathbf{a}^T \mathbf{x}_i + b \right) \geq 1, \quad i = 1, \ldots, m. \end{aligned}$$

- The *unknown* optimization variables in the above problem are $\mathbf{a}$ and b.

- The *known* parameters are $\mathbf{x}_i$ and y_i, for all $i = 1, \ldots, m$.

3.7. Events in Phasor Measurements

- Consider the constraints in the optimization problem on Slide 137:

$$y_i\left(\mathbf{a}^T\mathbf{x}_i + b\right) \geq 1, \qquad\qquad i = 1, \dots, m.$$

- For each event i, based on the above constraint:

  - If $y_i = -1$, then it is requires that $\mathbf{a}^T\mathbf{x}_i + b \leq -1$;
    If $y_i = 1$, then it is requires that $\mathbf{a}^T\mathbf{x}_i + b \geq 1$.

- Here, we enforce a *hard margin* of length 1 on each side of the separating hyperplane, to make a clear distinction between the events in Class I and the events in Class II. This is illustrated on the next slide.

- **Note**: the optimization problem on Slide 135 is convex. It can be solved by using It can be solved by using the convex programming solvers, such as the command `quadprog` in MATLAB [177].

- **Example 3.18**: Suppose we are provided 13 sample events to train an SVM classifier. The events are already labeled as belonging to either Class I (seven events) or Class II (six events). The number of features is two.

The vectors of features for the seven events that belong to Class I are:

$$\mathbf{x}_1 = \begin{bmatrix} 0.373 \\ 0.826 \end{bmatrix}, \quad \mathbf{x}_2 = \begin{bmatrix} 0.572 \\ 0.856 \end{bmatrix}, \quad \mathbf{x}_3 = \begin{bmatrix} 0.439 \\ 0.680 \end{bmatrix},$$

$$\mathbf{x}_4 = \begin{bmatrix} 0.620 \\ 0.702 \end{bmatrix}, \quad \mathbf{x}_5 = \begin{bmatrix} 0.251 \\ 0.631 \end{bmatrix}, \quad \mathbf{x}_6 = \begin{bmatrix} 0.321 \\ 0.437 \end{bmatrix}, \quad \mathbf{x}_7 = \begin{bmatrix} 0.156 \\ 0.454 \end{bmatrix}.$$

The vectors of features for the six events that belong to Class II are:

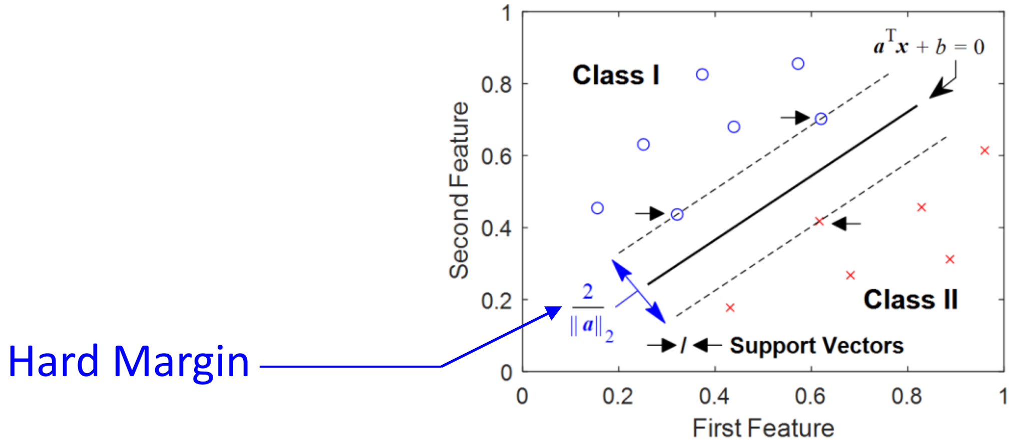
$$\mathbf{x}_8 = \begin{bmatrix} 0.617 \\ 0.418 \end{bmatrix}, \quad \mathbf{x}_9 = \begin{bmatrix} 0.960 \\ 0.614 \end{bmatrix}, \quad \mathbf{x}_{10} = \begin{bmatrix} 0.829 \\ 0.457 \end{bmatrix},$$

$$\mathbf{x}_{11} = \begin{bmatrix} 0.681 \\ 0.268 \end{bmatrix}, \quad \mathbf{x}_{12} = \begin{bmatrix} 0.888 \\ 0.312 \end{bmatrix}, \quad \mathbf{x}_{13} = \begin{bmatrix} 0.431 \\ 0.178 \end{bmatrix}.$$

- **Example 3.18 (Cont.)** Once we solve the optimization problem on Slide 135 based on the given 13 training samples, we can obtain the parameters of the separating hyperplane as

$$\mathbf{a} = \begin{bmatrix} 6.3004 \\ -7.1088 \end{bmatrix}, \quad b = 0.0841.$$

The resulting separating hyperplane ($\mathbf{a}^T\mathbf{x} + b = 0$) and the vectors of features for all the 13 training events are shown below.



3.7. Events in Phasor Measurements

- The requirement to have a *hard margin* (i.e., an empty area with no data point) *around* the separating hyperplane based on the optimization problem on Slide 135 may *not* always be achievable.

- For instance, recall from Example 3.17 that we *cannot* identify any line that can *completely separate* all the unbalanced events from all the balanced events in that example. In such cases, the optimization problem on Slide 135 becomes *infeasible*, i.e., without solution.

    - This happens any time that the two classes are not separable.

- The remedy is to replace the hard margins in the SVM classifer in the optimization problem on Slide 135 with *soft margins* (next slide).

- The following revised optimization problem incorporate the soft margin in training the SVM classifier:

$$\begin{aligned} \underset{\mathbf{a}, b, \zeta}{\text{minimize}} \quad & \|\mathbf{a}\|_2 + \lambda \sum_{i=1}^{m} \zeta_i \\ \text{subject to} \quad & y_i \left(\mathbf{a}^T \mathbf{x}_i + b \right) \geq 1 - \zeta_i, \quad i = 1, \ldots, m, \\ & \underline{\zeta_i} \geq 0, \quad i = 1, \ldots, m. \end{aligned}$$

Slack variable: measures the deviation from the initial hard in order to achieve a feasible solution.

- Based on the second term in the objective function, we seek to obtain the minimum for such a deviation.

- **Example 3.19**: Consider the same set of 13 training events in Example 3.18. Suppose we add the following two events to the training set:

$$\mathbf{x}_{14} = \begin{bmatrix} 0.546 \\ 0.426 \end{bmatrix}, \quad y_{14} = -1$$

$$\mathbf{x}_{15} = \begin{bmatrix} 0.579 \\ 0.578 \end{bmatrix}, \quad y_{15} = 1.$$

- Event #14 belongs to Class I, and event #15 belongs to Class II.

- If we use all the $13 + 2 = 15$ event samples to train the SVM classifier, then the optimization problem on Slide 135 would be *infeasible*.

- Instead, we should use the optimization problem on Slide 140.

- We set λ = 5. The new separating hyperplane is obtained as

- **Example 3.19 (Cont.)**: We set λ = 5. The new separating hyperplane is obtaining by solving the optimization problem on Slide 140:

$$\mathbf{a} = \begin{bmatrix} 4.9723 \\ -5.6102 \end{bmatrix}, \quad b = -0.1444.$$

- The slack variables are obtained as

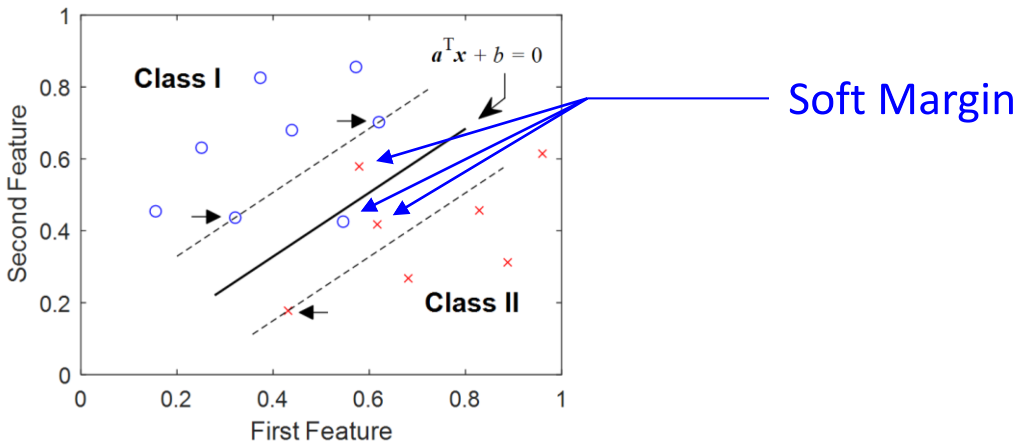
$$\zeta_8 = 0.4216, \quad \zeta_{14} = 1.1805, \quad \zeta_{15} = 1.5082$$

and

$$\zeta_i = 0, \quad i = 1, \ldots, 7, 9, \ldots, 13.$$

3.7. Events in Phasor Measurements

- **Example 3.19 (Cont.)**: The separating hyperplane is shown below:



- Notice that the points corresponding to events 8, 14, and 15 are on the *incorrect side* of the separating hyperplane.

- These three events are considered as outliers (allowed by a soft margin).

4) Other Classifiers:

• If the event classification involves more than two classes, then we can still use SVM classification to train *multiple separating hyperplanes*. Each separating hyperplane can separate one class from the *rest* of the classes.

• We can also use *nonlinear classification*, where we replace the separating hyperplane with polynomial functions, etc.; see [179–181].

• SVM classification is considered a *supervised learning* method; because it requires us to first *label* some events to serve as training samples to obtain the classifier. There is a rich literature on supervised learning methods that can be used for event classification; e.g., see [182–184].

3.7. Events in Phasor Measurements

- There are also some powerful *unsupervised classification* methods that can be considered in the analysis of events in various measurements.

- In unsupervised classification, we do *not* need to first manually identify the samples of the data that belong to each class.

  - Instead we need to group data based on *similarities in features*.

- We will see an example with a popular method for unsupervised classification (clustering) in Section 5.4.3 in Chapter 5.

Table of Contents

3.8. State and Parameter Estimation

• In this section, we discuss the basics of *state estimation* using phasor measurements. We also see a few examples for *parameter estimation*, including the estimation of the topology of the power system.

• Some of the problems that we discuss in this section will also be discussed in future chapters for other types of measurements.

• For example, we will discuss the state estimation problem also in Section 4.5.1 in Chapter 4 and in Section 5.8 in Chapter 5.

3.8. State and Parameter Estimation

3.8.1. State Estimation Using PMU Measurements

• State estimation is the problem of calculating the *states* of the system, i.e., voltage magnitudes and voltage phase angles at all buses.

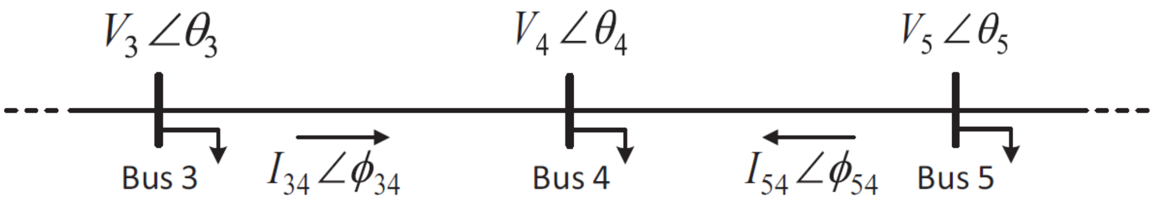
• In this section, we use PMU measurements to solve this problem. Later in Section 5.8, we will use active and reactive power measurements.

• If the PMU measurements that are available to us are limited to voltage synchrophasors (but *not* current synchrophasors), then there is practically *no* "estimation", because the states are just *measured directly*.

• In such a case, the measurements do *not* provide *redundancy* for each other. If one PMU fails, then the phasor measurements from other PMUs *cannot* be used to recover the troubled or missing data.

3.8. State and Parameter Estimation

- If *both* voltage and current synchrophasors are measured, then we can formulate a state estimation problem. For example, in this network:



the PMU at bus 3 measures V_3, θ_3, I_{34}, φ_{34}. The PMU at bus 5 measures V_5, θ_5, I_{54}, φ_{54}. No PMU is installed at bus 4; thus V_4 and θ_4 are unknown.

The following relationships hold:

$$I_{34}\angle\phi_{34} = (V_3\angle\theta_3 - V_4\angle\theta_4)\,(G_{34} + j\,B_{34})\,,$$

$$I_{54}\angle\phi_{54} = (V_5\angle\theta_5 - \underbrace{V_4\angle\theta_4}_{\text{Unknown}})\,(G_{54} + j\,B_{54})\,.$$

3.8. State and Parameter Estimation

- Any one of these two equations can be used to calculate V_4 and θ_4:

- However, together, the two equations in the previous slide can provide *redundancy* in obtaining V_4 and θ_4. In fact, with the availability of current synchrophasor measurements, there is now redundancy to also estimate V_3, θ_3, V_5, and θ_5, even though they are measured directly.

- Redundancy can help to address measurement noise.

  - True phasor: $\hat{X}\angle\hat{\theta}$

  - Measured phasor: $X\angle\theta$

  - The difference between $\hat{X}\angle\hat{\theta}$ and $X\angle\theta$ is due to measurement noise.

3.8. State and Parameter Estimation

- Let us define:

$$\hat{X}_r = \mathrm{Re}\{\hat{X}\angle\hat{\theta}\} = \mathrm{Re}\{X\angle\theta\} + \epsilon_r = X_r + \epsilon_r,$$
$$\hat{X}_i = \mathrm{Im}\{\hat{X}\angle\hat{\theta}\} = \mathrm{Im}\{X\angle\theta\} + \epsilon_i = X_i + \epsilon_i.$$

- Subscripts r and i stand for real and imaginary, respectively.

- We can now relate the measurements to the state variables as:

$$\mathbf{z} = \mathbf{A}\mathbf{x} + \boldsymbol{\epsilon},$$

where

$$\mathbf{z} = \begin{bmatrix} \hat{V}_{r3} & \hat{V}_{i3} & \hat{I}_{r34} & \hat{I}_{i34} & \hat{V}_{r5} & \hat{V}_{i5} & \hat{I}_{r54} & \hat{I}_{i54} \end{bmatrix}^T$$

$$\mathbf{x} = \begin{bmatrix} V_{r3} & V_{i3} & V_{r4} & V_{i4} & V_{r5} & V_{i5} \end{bmatrix}^T$$

$$\boldsymbol{\epsilon} = \begin{bmatrix} \epsilon^V_{r3} & \epsilon^V_{i3} & \epsilon^I_{r34} & \epsilon^I_{i34} & \epsilon^V_{r5} & \epsilon^V_{i5} & \epsilon^I_{r54} & \epsilon^I_{i54} \end{bmatrix}^T$$

and

$$\mathbf{A} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ G_{34} & -B_{34} & -G_{34} & B_{34} & 0 & 0 \\ B_{34} & G_{34} & -B_{34} & -G_{34} & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -G_{54} & B_{54} & G_{54} & -B_{54} \\ 0 & 0 & -B_{54} & -G_{54} & B_{54} & G_{54} \end{bmatrix}.$$

- The relationship between $\mathbf{z}$ and $\mathbf{x}$ in $\mathbf{z} = \mathbf{A}\mathbf{x} + \epsilon$ is *linear by construction*. It is *not* a linearized approximation of any originally nonlinear relationship.

3.8. State and Parameter Estimation

- The state estimation problem based on PMU measurements can be formulated as the following LS optimization problem:

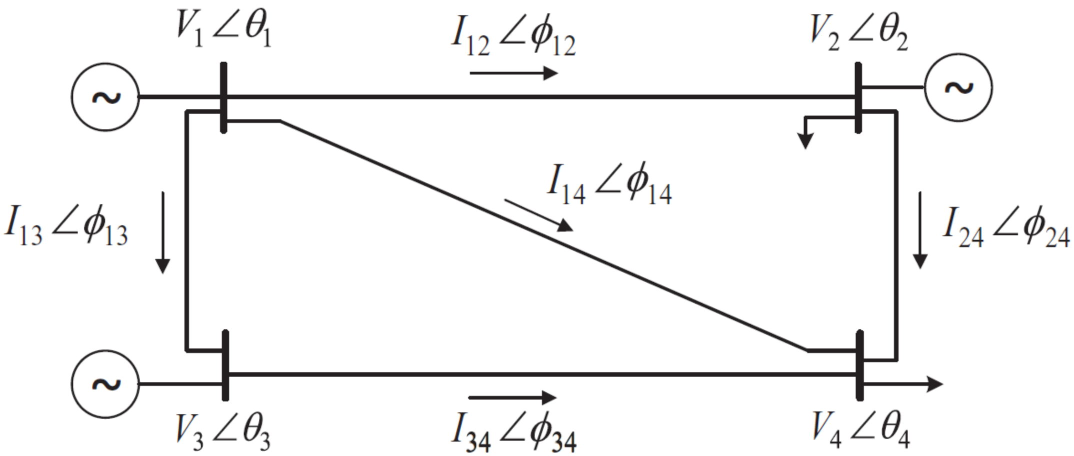
$$\min_{\mathbf{x}} \|\mathbf{z} - \mathbf{A}\mathbf{x}\|_2 \, .$$

- The solution of the above optimization problem is obtained as [103]:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{A})^{-1} \mathbf{A}^T \mathbf{z}.$$

- Any state estimation problem that is based solely on synchrophasor measurements can be formulated similarly as a standard LS problem also and solved similarly for any AC power network [140].

- **Example 3.20**: Consider the following 4-bus transmission network:



Suppose we measure voltage synchrophasors at buses 1, 2, and 4, and current synchrophasors on transmission lines (1,3), (1,4), and (3,4).

The true versus measured synchrophasors are shown on the next slide.

- **Example 3.20 (Cont.)**:

Bus #	True Voltage (p.u.)	Measured Voltage (p.u.)
1	$1.0332\angle 38.8884^\circ$	$1.0297\angle 39.1346^\circ$
2	$0.9974\angle 31.5332^\circ$	$0.9967\angle 31.3237^\circ$
3	$1.0499\angle 37.0645^\circ$	–
4	$0.9755\angle 28.3416^\circ$	$0.9749\angle 28.4696^\circ$

Line #	True Current (p.u.)	Measured Current (p.u.)
1,2	$1.3522\angle 22.7486^\circ$	–
1,3	$0.3719\angle 67.6409^\circ$	$0.3707\angle 67.2757^\circ$
1,4	$1.9357\angle 19.2137^\circ$	$1.9519\angle 19.3943^\circ$
2,4	$0.5920\angle 11.1179^\circ$	–
3,4	$1.7117\angle 9.5889^\circ$	$1.7093\angle 10.1346^\circ$

3.8. State and Parameter Estimation

- **Example 3.20 (Cont.)**: Vector $\mathbf{x}$ is 8×1, matrix $\mathbf{A}$ is 12×8, and vector $\mathbf{z}$ is 12×1. The admittance for each transmission line is $0.5 - j10\ p.u.$

- By using the formulation on Slide 153, the state estimation results are obtained as

$$
\begin{aligned}
V_1\angle\theta_1 &= 0.8002 + j0.6505 = 1.0312\angle 39.1081^\circ,\\
V_2\angle\theta_2 &= 0.8514 + j0.5181 = 0.9967\angle 31.3237^\circ,\\
V_3\angle\theta_3 &= 0.8337 + j0.6340 = 1.0474\angle 37.2495^\circ,\\
V_4\angle\theta_4 &= 0.8555 + j0.4641 = 0.9733\angle 28.4800^\circ.
\end{aligned}
$$

- **Note**: State estimation estimates not only the unknown state at bus 3, but also the states at buses 1, 2, and 4, which are also measured directly.

3.8. State and Parameter Estimation

- **Incorporating the Distribution of Measurement Noise**

- The state estimation problem formulation on Slide 153 is based on the *assumption* that all PMUs have the *same* level of measurement noise.

- However, if different PMUs have *different* probability distributions for their measurement noises, and if such different probability distributions for measurement noises happen to be *known* in advance, then we can incorporate them into the state estimation problem formulation.

- For example, suppose the measurement noise follows a Gaussian distribution with *zero mean* and *known variance*. (next slide)

3.8. State and Parameter Estimation

- We can reformulate the state estimation problem in form of the following *Weighted Least Square* (WLS) problem:

$$\min_{\mathbf{x}} \|\mathbf{W}(\mathbf{z} - \mathbf{A}\mathbf{x})\|_2 ,$$

where $\mathbf{W}$ denotes the noise variance matrix, which is diagonal:

$$\mathbf{W} = \begin{bmatrix} 1/\sigma_1^2 & & 0 \\ & \ddots & \\ 0 & & 1/\sigma_m^2 \end{bmatrix}.$$

- Here, m is the total number of PMUs and σ_i^2 is the variance for the measurement noise at PMU number i, where $i = 1, \ldots, m$.

3.8. State and Parameter Estimation

- If a PMU has a *large variance* in measurement noise, then it is given a s*maller weight* in the WLS problem. The solution of the WLS problem that is shown on Slide 158 is obtained as follows [103]:

$$\mathbf{x} = (\mathbf{A}^T \mathbf{W} \mathbf{A})^{-1} \mathbf{A}^T \mathbf{W} \mathbf{z}.$$

- **Note**: If we set $\mathbf{W}$ to be an identity matrix, then the above solution reduces to the same solution as in Slide 153.

3.8. State and Parameter Estimation

- **Observability**

- The ability to solve the state estimation problem for a given power system depends on the *number* and *location* of the measurements, i.e., whether or not the unknown states or the unknown parameters are observable based on the available measurements.

- For instance, in Example 3.20, if we do *not* have access to the phasor measurements of current on transmission lines (1,3) and (3,4), then we *cannot* estimate the voltage phasor at bus 3. In that case, the state estimation problem in Example 3.20 would *not* be observable.

- **Note**: When the states of the power system are not observable, it affects the rank of matrix A in the formulation on Slide 153; see *Exercise 3.26*.

3.8. State and Parameter Estimation

- Observability can be achieved by increasing the number of sensors and/or placing them at the right locations in the power system. For example, there is a rich literature on *sensor placement*; e.g., see [185–189].

- Alternatively, or in addition to increasing the number of sensors, one can use techniques such as probing and perturbation to enhance observability in a power system, as we will see in Chapter 6.

3.8.2. Parameter Estimation Using PMU Measurements

• PMU measurements may also be used for *parameter estimation*.

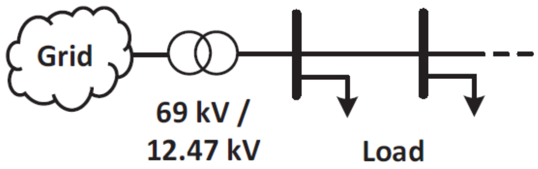
• Recall that the system parameters, such as the admittance of conductors, are *assumed to be known* in state estimation.

• However, in practice, certain parameters may not be known, or their values may change due to aging, atmospheric conditions, or grid conditions.

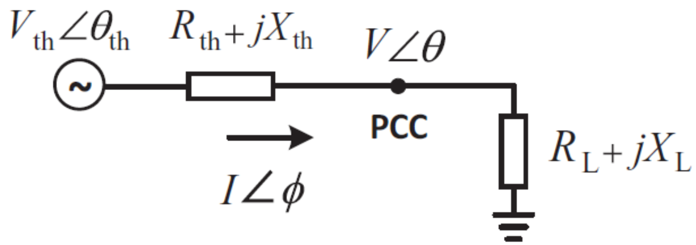
• Accordingly, parameter estimation might be necessary in order to estimate certain parameters of the power system by using measurements.

3.8. State and Parameter Estimation

- Consider the following power distribution network:



- The *load side* can be modeled as a load impedance $R_L + jX_L$. Also, the *utility side* can be modeled using its Thevenin equivalent circuit.



- Suppose a PMU is installed at the *point of common coupling* (PCC) to measure the voltage phasor $V\angle\theta$ and the current phasor $I\angle\varphi$.

3.8. State and Parameter Estimation

- On the *load side*, we can write:

$$V\angle\theta = (R_L + jX_L)\, I\angle\phi.$$

- The above equation, together with the PMU measurements at PCC, *are sufficient* to estimate the *load impedance*.

- On the utility side, we can write:

$$V_{\text{th}}\angle\theta_{\text{th}} = V\angle\theta + (R_{\text{th}} + jX_{\text{th}})\, I\angle\phi.$$

- The above equation, together with the PMU measurements at PCC, are *not sufficient* to estimate the *utility impedance*. This issue can be resolved if one monitor the network under major load events, as we will see next.

3.8. State and Parameter Estimation

- **Example 3.21**: Again, consider the network on Slide 163. Suppose there is an *abrupt change in the load*, e.g., due to a major load switching.

  - Suppose $V^{\text{before}}\angle\theta^{\text{before}}$ and $I^{\text{before}}\angle\phi^{\text{before}}$ denote the PMU measurements *before* the load event.

  - Suppose $V^{\text{after}}\angle\theta^{\text{after}}$ and $I^{\text{after}}\angle\phi^{\text{after}}$ denote the PMU measurements *after* the load event.

If the equivalent source Vth θth and the equivalent impedance $R_{\text{th}} + jX_{\text{th}}$ do not change during the load event, then one can express the equation on the utility side twice, i.e., based on the measurements *before* the load event and also based on the measurements *after* the load event.

  - This is shown on the next slide.

3.8. State and Parameter Estimation

- **Example 3.21 (Cont.)**: We have:

Before: $$V_{\text{th}}\angle\theta_{\text{th}} = V^{\text{before}}\angle\theta^{\text{before}} + (R_{\text{th}} + jX_{\text{th}})I^{\text{before}}\angle\varphi^{\text{before}}.$$

After: $$V_{\text{th}}\angle\theta_{\text{th}} = V^{\text{after}}\angle\theta^{\text{after}} + (R_{\text{th}} + jX_{\text{th}})I^{\text{after}}\angle\varphi^{\text{after}}.$$

The two equations can then be solved to obtain [190]:

$$R_{\text{th}} + jX_{\text{th}} = \frac{V^{\text{after}}\angle\theta^{\text{after}} - V^{\text{before}}\angle\theta^{\text{before}}}{I^{\text{after}}\angle\phi^{\text{after}} - I^{\text{before}}\angle\phi^{\text{before}}}.$$

which provides us with the impedance of the Thevenin equivalent circuit.

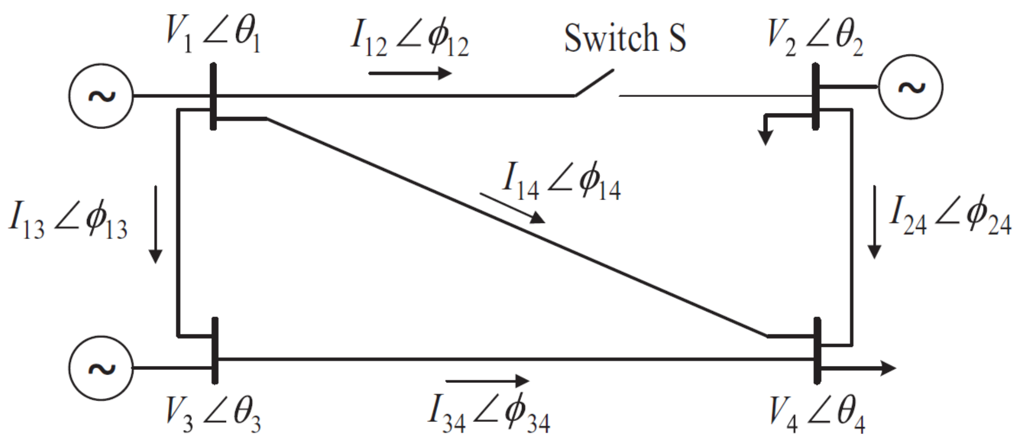
3.8.3. Topology Identification Using PMU Measurements

• Topology identification (TI) is the problem of identifying the *status* of the *switches* in the network; thus, determining the correct network topology.

• It is a special type of the parameter estimation problem, where the parameters to be estimated are the *closed* or *open* status for the switches.

• Again, consider the 4-bus power transmission network in Example 3.20 in Section 3.8.1. Suppose there is a switch, denoted by S, on transmission line (1,2), as shown in the figure on the next slide.

- We seek to estimate the status of Switch S:



- As in Example 3.20, suppose we measure *voltage phasors* at buses 1, 2, and 4 and *current phasors* on transmission lines (1,3), (1,4), and (3,4).

- Depending on the status of switch S, the phasor measurements can be very different. This is shown on the next slide.

3.8. State and Parameter Estimation

- If switch S is *closed*, then the voltage and current phasors are:

Bus #	**True Voltage (p.u.)**	**Measured Voltage (p.u.)**
1	$1.0332\angle 38.8884°$	$1.0297\angle 39.1346°$
2	$0.9974\angle 31.5332°$	$0.9967\angle 31.3237°$
3	$1.0499\angle 37.0645°$	–
4	$0.9755\angle 28.3416°$	$0.9749\angle 28.4696°$
Line #	**True Current (p.u.)**	**Measured Current (p.u.)**
1,2	$1.3522\angle 22.7486°$	–
1,3	$0.3719\angle 67.6409°$	$0.3707\angle 67.2757°$
1,4	$1.9357\angle 19.2137°$	$1.9519\angle 19.3943°$
2,4	$0.5920\angle 11.1179°$	–
3,4	$1.7117\angle 9.5889°$	$1.7093\angle 10.1346°$

- This table is the same as the one we saw on Slide 155.

3.8. State and Parameter Estimation

- If switch S is *open*, then the voltage and current phasors are:

Bus #	True Voltage (p.u.)	Measured Voltage (p.u.)
1	$1.0332\angle 38.8884^\circ$	$1.0309\angle 39.0827^\circ$
2	$0.9135\angle 16.9944^\circ$	$0.9140\angle 16.8693^\circ$
3	$1.0185\angle 34.5833^\circ$	–
4	$0.9217\angle 22.2936^\circ$	$0.9215\angle 22.4060^\circ$
Line #	**True Current (p.u.)**	**Measured Current (p.u.)**
1,2	–	–
1,3	$0.7855\angle 28.7818^\circ$	$0.7837\angle 28.4862^\circ$
1,4	$3.0331\angle 12.0857^\circ$	$3.0547\angle 12.2459^\circ$
2,4	$0.8533\angle -163.0056^\circ$	–
3,4	$2.2918\angle 6.4349^\circ$	$2.2932\angle 6.9070^\circ$

- **Note**: We assume that we do *not* have a PMU to measure the current on transmission line (1,2); otherwise we could simply check whether or not there is current flowing on this line to tell the status of switch S.

3.8. State and Parameter Estimation

- However, even *without* a direct measurement, we can identify the status of switch S to identify the network topology of the network.

- This can be done by conducting *state estimation* results based on the measurements that are already available.

- Suppose the *results of the state estimation problem* are as follows:

$$
\begin{aligned}
V_1\angle\theta_1 &= 1.0322\angle 39.0858^\circ,\\
V_2\angle\theta_2 &= 0.9140\angle 16.8693^\circ,\\
V_3\angle\theta_3 &= 1.0163\angle 34.7648^\circ,\\
V_4\angle\theta_4 &= 0.9203\angle 22.3802^\circ.
\end{aligned}
$$

3.8. State and Parameter Estimation

- If switch S is closed, then we expect the *current phasor* on line (1,2) to be:

$$I_{12}\angle\phi_{12} = (V_1\angle\theta_1 - V_2\angle\theta_2)\,(G_{12} + j\,B_{12})$$

- If switch S is open, then we expect that

$$I_{12}\angle\phi_{12} = 0\angle 0^\circ$$

- We need to decide between the above two possible cases.

- Suppose we know that the *capacity* for active power generation of the power plant at bus 1 is 5.00 p.u. Q: Can this information be used in order to decide between the above two cases?

3.8. State and Parameter Estimation

- Let us use the state estimation results on Slide 171 and estimate the amount of active power flow on transmission lines (1,2), (1,3), and (1,4).

  - If we *assume* that switch S is *closed*, then we estimate that

$$P_{12} = 3.66, \quad P_{13} = 0.80, \quad P_{14} = 2.81.$$

  - If we *assume* that switch S is *open*, then we estimate that

$$P_{12} = 0, \quad P_{13} = 0.80, \quad P_{14} = 2.81$$

- However, the first set of estimations *cannot* be correct because they require the power plant at bus 1 to generate 7.27 p.u. real power, which is way above its generation capacity at 5.00 p.u.

  - Thus, we conclude that switch S is *open*.

3.8. State and Parameter Estimation

• The topology identification problem can sometimes be formulated as an *optimization problem*, whether as a stand-alone problem or as an extension of the state estimation problem.

• The idea is to introduce optimization variables that are *binary* such that they can capture the status of each switch; e.g., see [191, 192].

• We will discuss the topology identification problem also in Section 4.5.2 in Chapter 4 and in Sections 6.2.1 and 6.7.1 in Chapter 6.

Table of Contents

3.9. Accuracy in Synchrophasor Measurements

- The assessment of the accuracy and performance of PMUs is an elaborate process. It may require taking into account:
  - The *performance class* of the PMU that is being used
  - The phasor measurements under *steady-state* operating conditions
  - The phasor measurements under *dynamic* operating conditions
- We will cover these aspects in this section. Furthermore, we will cover:
  - The accuracy of three-phase synchrophasors
  - The accuracy of RPAD and PD.

3.9. Accuracy in Synchrophasor Measurements

3.9.1. Performance Classes

- According to the IEEE C37.118 Standard, the accuracy and performance of PMUs should be evaluated based on their class of performance.

- The above standard currently defines two classes of performance:

  - **P Class** (Protection Applications)

  - **M Class** (Measurement Applications)

- In other words, the expected performance depends on the “application”.

3.9. Accuracy in Synchrophasor Measurements

- PMUs in P Class:
  - Are intended for applications that require *faster response*;
  - Have *minimal internal filtering* or other types of internal processing so as to speed up the reporting of the phasor measurements.
- These may come at the expense of degraded measurement accuracy.
- The typical application of PMUs in this class is in power system protection, where the reading of PMUs can lead to some protection actions, such as opening a circuit breaker [193, 194].

3.9. Accuracy in Synchrophasor Measurements

- PMUs in M Class:
  - Are intended for applications that require *higher accuracy*;
  - Have *enhanced internal filtering* and other internal processing so as to improve measurement accuracy, even if it may come at the expense of some delay in reporting the measurements.
- The typical application of PMUs in this class is in power system monitoring, which require more precise measurements [195, 196].

3.9. Accuracy in Synchrophasor Measurements

• An example for the type of internal filters that might be different between the above two classes of PMUs is the details in their *anti-aliasing filters*; see Section 2 in Chapter 2.3.

• Compared to the output of M Class PMUs, the output of P Class PMUs may contain additional aliasing components; see [141] for more details.

3.9.2. Steady-State Performance

• **Total Vector Error**:

• The accuracy of a phasor is most commonly evaluated in terms of its *total vector error* (TVE). Here, the *vector error* (VE) is the difference between the reported syncrhophasor and the true syncrhophasor.

• VE is a complex number.

• TVE is a fraction:

$$\mathrm{TVE} = \frac{|\hat{X}\angle\hat{\theta} - X\angle\theta|}{|X\angle\theta|}$$

- **Example 3.22**: A PMU reports a voltage phasor as

$$39657.214\angle 183.215^\circ.$$

The true voltage phasor is

$$39832.582\angle 183.681^\circ.$$

The TVE is obtained as

$$\begin{aligned}\text{TVE} &= \frac{|39657.214\angle 183.215782^\circ - 39832.582\angle 183.680175^\circ|}{|39832.582\angle 183.680175^\circ|} \\ &= \frac{366.7791}{39832.582} = 0.92\%.\end{aligned}$$

3.9. Accuracy in Synchrophasor Measurements

- Suppose $\hat{X} = X + \epsilon_m$ and $\hat{\theta} = \theta$, i.e., the error in phasor measurement is *solely* due to *error in measuring magnitude*.

- The TVE in this case is obtained as

$$\text{TVE} = \frac{|(X + \epsilon_m)\angle\theta - X\angle\theta|}{|X\angle\theta|} = \frac{|\epsilon_m|}{X}.$$

3.9. Accuracy in Synchrophasor Measurements

- Next, suppose $\hat{X} = X$ and $\hat{\theta} = \theta + \epsilon_p$, i.e., the measurement error is *solely* due to *error in measuring the phase angle*.

- The TVE in this case is obtained as

$$\text{TVE} = \frac{|X\angle(\theta + \epsilon_p) - X\angle\theta|}{|X\angle\theta|} = 2|\sin(\epsilon_p/2)|.$$

- The IEEE C37.118 Standard recommends limiting TVE at *steady state* to *1%*. This requirement is the same for both P Class and M Class PMUs [141].

3.9. Accuracy in Synchrophasor Measurements

- **Frequency Error and Rate of Change of Frequency Error**

- Two other metrics to assess accuracy of synchrophasor measurements:

  - Frequency error (FE): Error in measuring frequency

  - Rate of change of frequency error (RFE): Error in measuring ROCOF

- Both FE and RFE depend on the error in measuring the *phase angle*.

- Required limits during steady-state conditions:

  - **Class P**: FE $\leq$ 0.01 Hz and RFE $\leq$ 0.01 Hz/sec.

  - **Class M**: FE $\leq$ 0.005 Hz and RFE $\leq$ 0.01 Hz/sec.

3.9. Accuracy in Synchrophasor Measurements

- **Error in Time Synchronization**

- For accurate synchrophasor measurements, PMUs require reliable and accurate time synchronization, such as from the GPS satellites.

- Even a small error in time synchronization may considerably increase TVE.

- **Example 2.23**: At a 60 Hz power system, each cycle takes 1/60 seconds, i.e., 16.667 msec. Therefore, a 1 microsecond *error in time synchronization* results in the following error in measuring the phase angle:

$$\epsilon_p = 360^\circ \times 60 \times 10^{-6} = 0.022^\circ \,.$$

The corresponding TVE is about 0.04%; see Slide 183.

3.9.3. Dynamic Performance

• The performance of PMUs during dynamic operating conditions are tested based on their response to certain step changes and ramp changes.

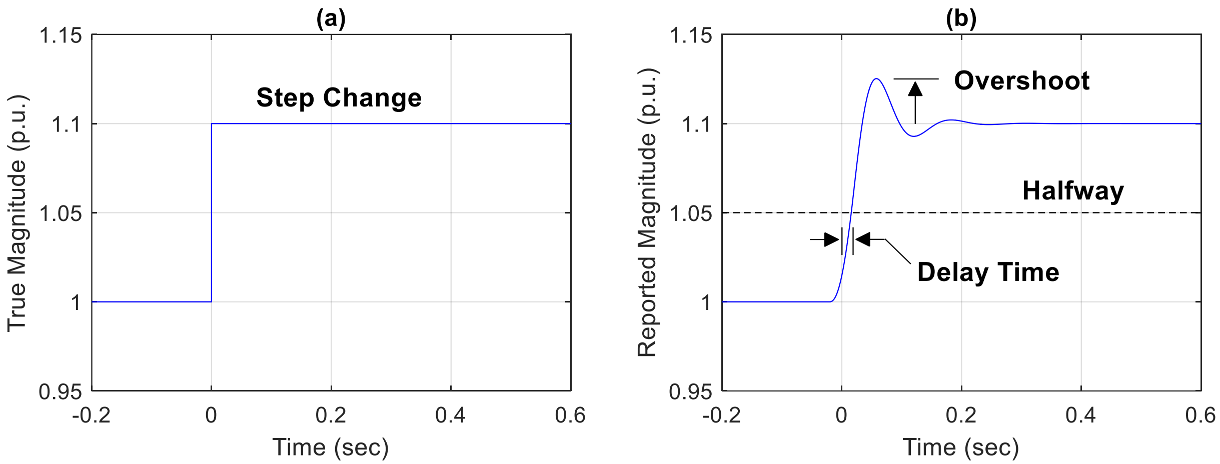
• **Step Changes in Magnitude and Phase Angle**

• In this test, we examine the phasor measurements that are reported by a PMU during a step change in either the magnitude or the phase angle of the phasor that is being measured.

• Here, we discuss only a step change in the magnitude; however, a step change in the phase angle can be studied similarly.

3.9. Accuracy in Synchrophasor Measurements

- Consider a *step change*, from 1.0 p.u. to 1.1 p.u., that is made in the magnitude of a true phasor at time $t = 0$, as shown in Figure (a) below:



- Due to the internal dynamics of the PMU, what the PMU reports is *not* an identical instantaneous step change in its measurement of magnitude.
  - It is rather a *dynamic response*, as shown in Figure (b).

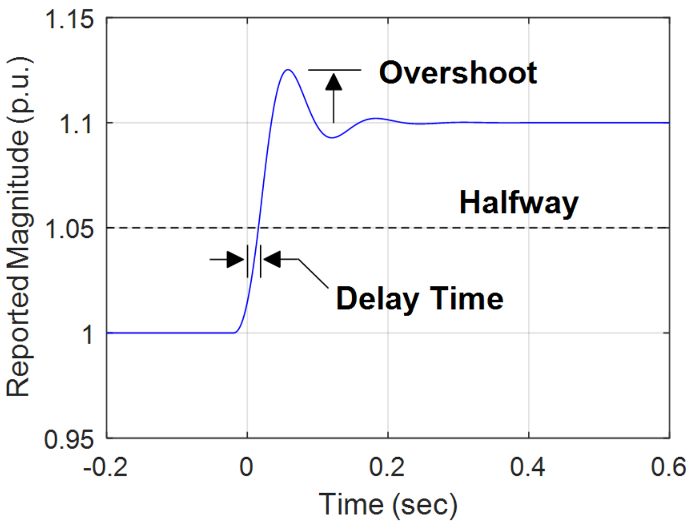
3.9. Accuracy in Synchrophasor Measurements

- The following three quantities are commonly examined in order to evaluate the dynamic response of a PMU under a step change [141]:
  - Delay time
  - Overshoot (or undershoot)
  - Response time
- These quantities are marked on Figure (b) on the previous slide.
- Their definitions are given in the next few slides.

3.9. Accuracy in Synchrophasor Measurements

- **Delay time** is the time interval between the instant that the step change occurs in the magnitude of the true phasor and the instant that the magnitude of the reported phasor measurement reaches a value that is halfway between the initial value and the final steady-state value.

- The delay time is the time interval between $t = 0$ and $t = 0.016$ seconds, where the latter is the time instant at which the magnitude of the reported phasor measurement reaches 1.05 p.u., which is *halfway* between 1.0 p.u. and 1.1 p.u.

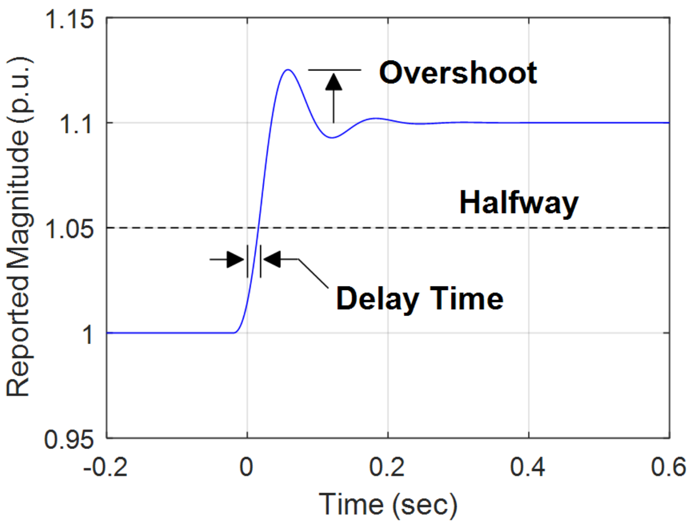


- Accordingly, the delay time in this example is 16 msec.

3.9. Accuracy in Synchrophasor Measurements

- **Overshoot** is the maximum magnitude of the reported phasor measurements after the step change occurs until we reach the steady-state conditions.

- The overshoot in this example is 0.125 p.u. Note that the step change here is *positive*, i.e., the magnitude of the true phasor *increases* during the step change. If the step change is negative, i.e., the magnitude of the true phasor *decreases* during the step change, then we must check the *undershoot* instead of the overshoot.



- **Response time** is the time it takes for the reported phasor measurements to reach the steady-state conditions after the step change.

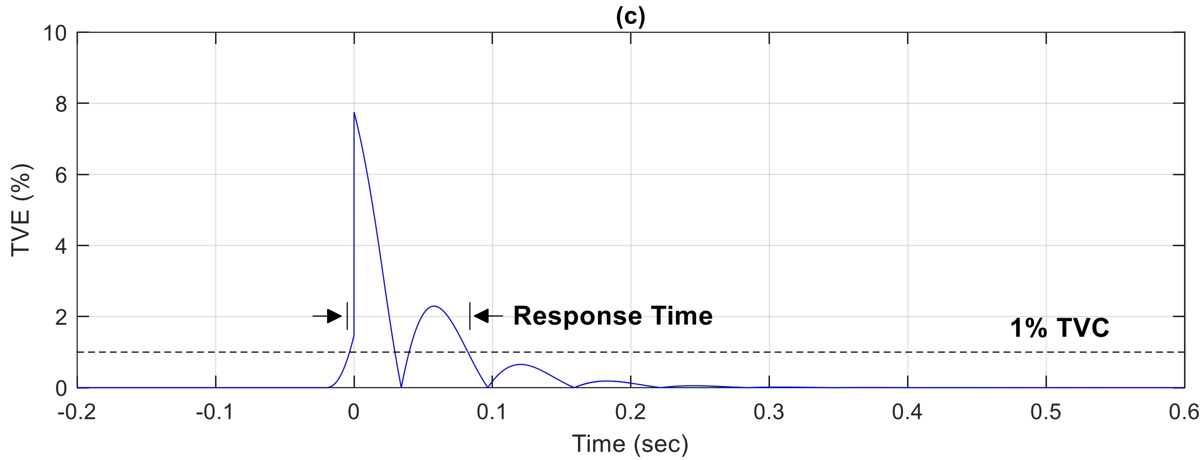
- Here, the steady-state conditions are defined based on a specified accuracy limit. For instance, recall from Section 3.9.2 that TVE under steady-state conditions must be limited to 1%.

- Accordingly, the response time with respect to TVE is defined as the time interval between the instant that the measurements leave the required 1% limit in TVE due to the step change and the instant that they reenter and stay within the 1% limit in TVE.

  - This is illustrated on the next slide.

3.9. Accuracy in Synchrophasor Measurements

- The TVE during the step change that we saw on Slide 188:



- The TVE is initially almost zero at steady state, well below the 1% limit.
- The step change causes the TVE to leave the 1% limit at $t = -0.003$.
- It takes until $t = 0.082$ for the TVE to go back to and stay below 1%.
- The response time for the TVE is 0.082 − (−0.003) seconds = 85 msec.

3.9. Accuracy in Synchrophasor Measurements

- **Note 1**: Due to the common use of averaging filters in PMUs, even some of the readings of the phasors that are associated with a few instances before the step change are also affected by the step change.

- That is why in the figure on the previous slide, the TVE starts increasing even for phasor measurements that are time stamped slightly *before* the step change occurs, i.e., at the negative time instances.

- **Note 2**: Each PMU class has its own requirement for dynamic performance. The details are shown on the next two slides.

3.9. Accuracy in Synchrophasor Measurements

- Dynamic Performance Requirements a P Class PMU:
  - Limit *overshoot* or *undershoot* to 5% of the step change
  - Limit *delay time* to 1 divided by four times the reporting rate.
  - Limit and the *response time* to 1.7 divided by the nominal frequency.

- For instance, consider the step change on Slide 188. Suppose reporting rate is 10 fps and the system frequency is 60 Hz. A P Class PMU must limit:
  - the overshoot to $0.05 \times 0.1 = 0.005$ p.u.,
  - the delay time to $1/(4\times10) = 25$ msec,
  - the response time to $1.7/60 = 28.3$ msec.

- Here, the response time is the *same* with respect to the TVE requirements, the FE requirements, and the RFE requirements.

3.9. Accuracy in Synchrophasor Measurements

- Dynamic Performance Requirements an M Class PMU:
  - *Overshoot* or *undershoot*: Same as P Class.
  - *Delay time*: Same as M Class.

  - Limit and the *response time* to (at 10 fps):
    - 595 msec for TVE
    - 869 msec for FE
    - 1.038 sec for RFE

- These numbers are much *higher than* the 25 msec limit on the response at the same reporting rate for a P Class PMU. This is one of the main differences between the P Class and M Class PMUs.

- For more details for M Class PMUs see Table 11 in [141].

3.9. Accuracy in Synchrophasor Measurements

- **Ramp Changes in System Frequency**

- The dynamic accuracy of phasor measurements also is usually tested during a ramp change in the true system frequency.

- The goal is to determine whether the TVE, FE, and RFE continue to stay within their required limits while the system frequency gradually changes.

- The details are available in [141].

3.9.4. Accuracy of Three-Phase Synchrophasors

• The accuracy assessment methods that we discussed in Sections 3.9.2 and 3.9.3 are often used based on the assumption that the true system is *balanced* or our focus is on measuring the *symmetrical components*.

• For example, both the step change and the ramp change that we discussed in Section 3.9.3 are explicitly required by the IEEE C37.118 Standard to be made by applying *balanced three-phase step* changes to *balanced three-phase true phasors*.

• However, in principle, the same methods can also be used to assess the accuracy of the phasor measurements on *each phase*, A, B, and C.

- **Example 3.24**: Suppose a PMU *reports* the following voltage phasors:

$$V_A = 278.0574\angle{-73.1170^\circ},$$
$$V_B = 276.8067\angle 167.2848^\circ,$$
$$V_C = 278.5330\angle 47.1288^\circ.$$

- Suppose the *true* voltage phasors are

$$V_A = 277.3742\angle{-73.1342^\circ},$$
$$V_B = 277.0118\angle 167.2712^\circ,$$
$$V_C = 279.2083\angle 47.1329^\circ.$$

- The true positive sequence component is calculated as:

$$V^+ = 277.8636\angle{-72.9100^\circ}\,.$$

3.9. Accuracy in Synchrophasor Measurements

- **Example 3.24 (Cont.)**: The TVE for the *positive sequence* is obtained as

$$\mathrm{TVE}^{+} = \frac{|277.7978\angle{-72.9014^\circ} - 277.8636\angle{-72.9100^\circ}|}{|277.8636\angle - 72.9100^\circ|} = 0.028\%.$$

The TVE for *individual phases* are obtained as

$$\mathrm{TVE}_A = \frac{|278.0574\angle{-73.1170^\circ} - 277.3742\angle{-73.1342^\circ}|}{|277.3742\angle{-73.1342^\circ}|} = 0.248\%,$$

$$\mathrm{TVE}_B = \frac{|276.8067\angle 167.2848^\circ - 277.0118\angle 167.2712^\circ|}{|277.0118\angle 167.2712^\circ|} = 0.078\%,$$

$$\mathrm{TVE}_C = \frac{|278.5330\angle 47.1288^\circ - 279.2083\angle 47.1329^\circ|}{|279.2083\angle 47.1329^\circ|} = 0.242\%.$$

It is possible that the TVE for the positive sequence is below the required limit in the standard; but the TVE for some phases do exceed such limit.

3.9.5. Accuracy of RPAD and PD

• The existing phasor measurement standards may *not* cover the accuracy of all the quantities that we discussed in this chapter.

• For example, TVE does *not* directly tell us what to expect for the accuracy of the RPAD measurements (see Section 3.4) or the accuracy of the PD measurements (see Section 3.5.1).

• The accuracy of RPAD depends on the accuracy of measuring phase angles at the two locations where the voltage phasors are measured.

• If we use two PMUs, each of which is guaranteed to limit the error in phase angle measurements by 1°, then the error in RPAD is guaranteed to be limited to $2^\circ = 1^\circ + 1^\circ$.

3.9. Accuracy in Synchrophasor Measurements

- As for the accuracy of PD, there is no direct relationship between the TVE for the two phasor measurements before and after the event and the TVE for the PD that is obtained from the two phasor measurements [197].

- **Example 3.25**: Suppose a PMU *reports* the voltage phasor before an event and the voltage phasor after an event as follows:

$$V^{\text{before}} = 7266.7\angle 339.31^\circ$$
$$V^{\text{after}} = 7218.6\angle 335.55^\circ$$

- Suppose the true voltage phasors are

$$V^{\text{before}} = 7289.3\angle 339.79^\circ$$
$$V^{\text{after}} = 7237.9\angle 335.01^\circ$$

3.9. Accuracy in Synchrophasor Measurements

- **Example 3.25 (Cont.)**: Accordingly, we have:
  - Measured Phasor Differential:

$$\Delta V = 7218.6\angle 335.55^\circ - 7266.7\angle 339.31^\circ = 477.6\angle - 118.35^\circ.$$

  - True Phasor Differential:

$$\Delta V = 7237.9\angle 335.01^\circ - 7289.3\angle 339.79^\circ = 607.98\angle{-117.45^\circ}.$$

while the TVE for the voltage phasor measurements, both *before* and *after* the event, is *small* (below 1%), the TVE for the phasor differential is *large*:

$$\mathrm{TVE}_{\Delta V} = \frac{|477.6\angle{-118.35^\circ} - 607.98\angle{-117.45^\circ}|}{|607.98\angle{-117.45^\circ}|} = 21.48\%.$$