Chapter 2: Voltage and Current Measurements and their Applications

Smart Grid Sensors: Principles and Applications
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2.1. Instrument Transformers
2.2. Non-Contact Voltage and Current Sensors
2.3. Sampling Rate, Reporting Rate, and Accuracy
2.4. RMS Voltage and Current Profiles
2.5. RMS Voltage and Current Transient Responses
2.6. RMS Voltage and Current Oscillations
2.7. Events in RMS Voltage and Current Measurements
2.8. Three-Phase Voltage and Current Measurements
2.9. Measuring Frequency
## Table of Contents

1. Instrument Transformers
2. Non-Contact Voltage and Current Sensors
3. Sampling Rate, Reporting Rate, and Accuracy
4. RMS Voltage and Current Profiles
5. RMS Voltage and Current Transient Responses
6. RMS Voltage and Current Oscillations
7. Events in RMS Voltage and Current Measurements
8. Three-Phase Voltage and Current Measurements
9. Measuring Frequency
2.1. Instrument Transformers

- The basic measurement instrument to measure voltage is *voltmeter*.

- Voltmeter is wired in *parallel* with the two points of the circuit at which the voltage difference is intended to be measured.

- The set up to measure voltage \( v(t) \).

- An ideal voltmeter would have an infinite impedance such that it would take no current from the circuit.

![Diagram](image)
• The basic measurement instrument to measure voltage is \textit{ammeter}.

• Ammeter is wired in series with the circuit that carries the current that is intended to be measured.

• The set up to measure current $i(t)$.

• An ideal ammeter would have zero impedance such that it would create no voltage drop on the circuit.

(\textit{wired in series})
2.1. Instrument Transformers

- Voltmeters and ammeters have limited ranges for measurements.

- Example: Maximum Current Rating = 20 A
  Maximum Voltage Rating = 300 V

- Q: What if we need to measure higher voltages or currents?

A Typical Three-Phase Power Distribution System

\[
\begin{array}{c}
\text{A} \\
\text{B} \\
\text{C} \\
\text{N}
\end{array}
\]

- 12.47 kV
- 7200 V = 7.2 kV
2.1. Instrument Transformers

- The common solution is to use instrument transformers.

![Diagram of instrument transformers]

- The PT is used to measure voltage $v_2$ instead of voltage $v_1$; because $v_2$ is proportional to $v_1$, but it is significantly lower than $v_1$.

- The CT is used to measure current $i_2$ instead of current $i_1$; because $i_2$ is proportional to $i_1$, but it is significantly lower than $i_1$.

- PT: Potential Transformer
- CT: Current Transformer
2.1.1. Turn Ratio:

\[
\frac{i_2}{i_1} = \frac{N_1}{N_2} \quad \frac{v_2}{v_1} = \frac{N_2}{N_1}
\]

where \(N_1\) and \(N_2\) indicate the number of turns in the primary winding and the number of turns in the secondary winding, respectively.

• Some typical turn ratios:

  CT) 4000:5, 400:5, 100:5

  PT) 4500:1, 100:1, 60:1

• For example, a 100:5 CT transforms up to 100 A current on its primary winding to up to 5 A current on its secondary winding.
• **Example 2.1**: We use a CT with a turn ratio of 100:5 to measure current. If the ammeter shows 3.48 A, then the actual current is:

\[ 3.48 \times (100/5) = 69.6 \text{ A}. \]
2.1.2. Load Rating and Burden:

• Load rating is defined mostly for CTs. It indicates the maximum load current that can be applied to the primary windings of a CT. That is, for a CT, the load rating is a limit on the primary current $i_1$.

• Burden is associated with the secondary windings of a CT or PT. It indicates the amount of impedance made by the elements of the metering circuit, which may be connected to the secondary windings of the transformer, without causing a metering error greater than that specified by the instrument transformer's accuracy classification.

• Burden for each metering device may be indicated in terms of impedance or in terms of active and reactive power consumption at a given secondary current and frequency; see Exercise 2.2.
2.1.3. Three-Phase Systems:

- The setup on Slide 7 can be used to connect a PT and a CT to each phase of a three-phase power system to measure voltage and current at each phase. Three PTs and three CTs would be used in this setup.

- Voltage also can be measured between two phases.

- For example, one can use two identical PTs to measure the voltage difference across Phase A and Phase B, as shown on the next slide.

- Here, the ultimate goal is to measure the phase-to-phase voltage \( v_1^A - v_1^B \), also known as the line-to-line voltage.
2.1. Instrument Transformers

• Using two identical PTs to measure line-to-line voltage:

![Diagram of two PTs connected in a line-to-line configuration.](image)

• Other possible configurations to connect PTs to measure phase-to-phase voltages in three-phase power systems are explained in [75].
• **Example 2.2**: Suppose two PTs are used to measure line-to-line voltage on a three-phase power system. The turn ratio is 60:1 for both PTs. If the voltmeter shows 206.33 V, then the actual current line-to-line voltage $v_1^A - v_1^A$ is measured as

$$206.33 \times 60 = 12.38 \text{kV}.$$  

• **Q**: What is the problem if the two PTs are *not* identical?
# Table of Contents

2.1. Instrument Transformers

2.2. Non-Contact Voltage and Current Sensors

2.3. Sampling Rate, Reporting Rate, and Accuracy

2.4. RMS Voltage and Current Profiles

2.5. RMS Voltage and Current Transient Responses

2.6. RMS Voltage and Current Oscillations

2.7. Events in RMS Voltage and Current Measurements

2.8. Three-Phase Voltage and Current Measurements

2.9. Measuring Frequency
2.2. Non-Contact Voltage and Current Sensors

• Conventional ammeters and voltmeters are installed as integrated parts of the electric circuit. That is why they need CTs and PTs.

• An emerging alternative is to use sensors without electric contact:
  
  • Measure **magnetic** field $B$ $\rightarrow$ Current $I$
  
  • Measure **electric** field $E$ $\rightarrow$ Voltage $V$

• The relationships are proportional: ($B$ and $I$) and ($E$ and $V$).

• Line-mounted:  

  ![Power Line Sensor](www.sentientenergy.com)
2.2.1. Different Working Principles:

- Different non-contact sensors may use different principles to measure the magnetic and electric fields.

- For example, the current that goes through a conductor can be measured by a Rogowski coil that captures the magnetic field surrounding the wire and generates magnetic-field-induced voltage $v_s$, which is proportional to the rate of change of current. Therefore, by measuring $v_s$ we can obtain the intended current $i$; see Exercise 2.5.
• As another example, the voltage between the conductor and the ground, as denoted by $v$ in the figure below, can be measured by measuring the current $i_s$ between an isolated capacitor plate and a local ground, where the isolated capacitor plate is charged by the electric field surrounding the wire whose voltage is being measured. Therefore, by measuring $i_s$ we can obtain the intended voltage $v$. 

(capsacitor plate to measure voltage)
Another generation of non-contact voltage and current sensors has been emerging recently that work on principles in *optics*.

For example, a *magneto-optic current transducer* (MOCT) uses the Faraday effect. MOCT detects how the magnetic field surrounding the conductor rotates the polarization of plane-polarized light that is transmitted by a light-emitting diode (LED) through a single fiber-optic pass around the conductor. The amount of rotation is proportional to the strength of the magnetic field; thus it is proportional to current $i$; see Exercise 2.4.
2.2. Non-Contact Voltage and Current Sensors

- Also, voltage can be measured by an **electro-optic voltage transducer** (EOVT) that uses the Pockles effect. Here, the full voltage of the conductor is applied between the two end faces of a cylinder-shaped crystal, which has several rounds of fiber-optic winding on its circumferential surface. EOVT detects the impact of the electric field on the optical phase shift in the transmitted light. The detected phase modulation is proportional to the total voltage.

![Diagram of an electro-optic voltage transducer](image)
2.2. Non-Contact Voltage and Current Sensors

2.2.2. Power Harvesting:

• Most non-contact overhead line sensors are self-powered, harvesting power from the conductor’s magnetic fields in a non-contact fashion.

• This is a useful feature because it reduces the need for maintenance and therefore the cost of operation. Power harvesting starts at a minimum pick-up conductor current, such as at 12 A on 0.375-inch to 1.030-inch conductors for the line current sensors.

• The power-harvesting feature of non-contact sensors may also introduce some new smart grid monitoring applications due to the “binary nature” of the sleep mode (zero) versus the wake-up mode (one) caused by their self-powered operation. One such example application is discussed in Section 7.1.2 in Chapter 7.
2.1. Instrument Transformers
2.2. Non-Contact Voltage and Current Sensors
2.3. Sampling Rate, Reporting Rate, and Accuracy
2.4. RMS Voltage and Current Profiles
2.5. RMS Voltage and Current Transient Responses
2.6. RMS Voltage and Current Oscillations
2.7. Events in RMS Voltage and Current Measurements
2.8. Three-Phase Voltage and Current Measurements
2.9. Measuring Frequency
2.3. Sampling Rate, Reporting Rate, and Accuracy

• When addressing power grid modernization, it is natural to focus exclusively on digital sensors, as opposed to the traditional electromechanical meters. Two important specifications of digital sensors are the sampling rate and the reporting rate.

2.3.1. Sampling Rate:

• Digital sensors make discrete measurements at a fixed sampling rate.

• The sampling rate of a sensor is indicated in samples per second or samples per cycle of the AC signal. For example, the sampling rate can be 10 samples per cycle, i.e., $10 \times 60 = 600$ samples per second.
2.3. Sampling Rate, Reporting Rate, and Accuracy

- A continuous sinusoidal voltage wave \( v(t) \) versus its discrete (sampled) version \( v[k] \) under different sampling rates:

\[
\text{Continuous} \quad \begin{array}{c}
\text{10 Samples / Cycle} \\
\text{30 Samples / Cycle} \\
\text{100 Samples / Cycle}
\end{array}
\]

\[
\text{10 Samples / Cycle} \quad \begin{array}{c}
\text{30 Samples / Cycle} \\
\text{100 Samples / Cycle}
\end{array}
\]
• If the voltage or current wave is non-sinusoidal and distorted, then we may need a very high sampling rate, see Chapter 4.

• Even if the measured voltage or current wave is purely sinusoidal, its reconstruction requires a minimum sampling rate, namely twice the frequency of the signal, according to the Nyquist-Shannon sampling theorem. For example, the minimum required sampling rate for a 60 Hz purely sinusoidal voltage wave is 120 samples per second.

• Aliasing: Proper signal filtering is also needed in order to avoid measurement aliasing, where the measured signal can be mistaken to have a frequency that is very different from its true frequency.
**Example 2.3**: A sensor that takes eight samples per second *cannot distinguish* a 9 Hz sinusoidal signal from a 1 Hz sinusoidal signal. A 9 Hz signal may appear to be, i.e., may be aliased to, a 1 Hz signal. From the Nyquist-Shannon sampling theorem, the 8 Hz sampling rate for this sensor allows properly measuring periodic signals with up to $8/2 = 4$ Hz frequency. Thus, to avoid measurement aliasing, an *anti-aliasing filter* (a low-pass filter), must be used *before* the sampler to filter out any measurement with a frequency higher than 4 Hz.
2.3.2. Reporting Rate:

- It is the rate at which the sensor reports its measured data.

- It is often \textit{slower} than the sampling rate to allow any pre-processing of the measured data within each reporting interval.

- For example, a digital voltmeter may report the RMS value for voltage:

\[
V_{\text{rms}} = \sqrt{\frac{1}{N} \sum_{k=1}^{N} v[k]^2}
\]

Reporting can be \textit{per-cycle, second-by-second, minute-by-minute}, etc. However, we need a much higher sampling rate to accurately calculate \( V_{\text{rms}} \).
Example 2.4: Consider the RMS readings of the voltage at a commercial building. Only one phase is shown here. Two voltage sensors are used. One sensor reports 60 readings per second, i.e., one RMS value per AC cycle. The other sensor reports one reading per minute, i.e., it reports the average RMS value of 3600 AC cycles.
The momentary voltage sag in the previous example was due to the switching of a major load; which also caused an \textit{inrush current}:

\begin{itemize}
\item \textbf{Note 1:} These granular details can be seen only when the reporting rate of the sensor is high.
\item \textbf{Note 2:} Keep in mind that here we are looking at the (per cycle) RMS values of the sinusoidal waveforms.
\end{itemize}
2.3.3. Accuracy:

- The accuracy of a digital voltmeter or a digital ammeter is often given by the sensor manufacturer in the form of a percentage of the reading (rdg) and/or a percentage of the full scale (FS).

**Example 2.5:** A digital voltmeter has a measurement range of 0 V to 345 V. The accuracy of this voltmeter is ±0.05% rdg ± 0.05% FS. If the RMS reading of voltage is 275 V, then the accuracy of this reading is:

\[
±0.0005 \times 275 \pm 0.0005 \times 345 = ±0.31 \text{ V}.
\]

That means the voltage is within the range 247.69 V and 275.31 V.
• If a PT or CT is used, then the accuracy of the measurement depends not only on the accuracy of the measurement device but also on the *accuracy of the instrument transformer*.

• For example, accuracy of a CT is defined based on its rated primary current. For an IEC 60044-1 standard grade (Class 1.0) CT, accuracy is ±1.0% at 100% of the rated primary current, ±1.5% at 20% of the rated primary current, and ±3.0% at 5% of the rated primary current.

• Also, a *ratio-correction factor* (RCF) is often defined for CTs as the fraction of the true turn ratio over the nameplate turn ratio. RCF is often provided as a *curve* plotted against multiples of the rated primary or secondary current for a given constant burden (next slide).
• **Example 2.6**: If RCF is 1.01 at a certain primary current for a CT with a nameplate ratio of 100:5, then the true turn ratio is:

\[ 1.01 \times \frac{100}{5} = 20.2 \text{, as opposed to } \frac{100}{5} = 20 \]

In other words, the true turn ratio is 100:4.95.

• We will discuss "system accuracy", i.e., the measurement accuracy when involving all measurement components, in Section 5.10.2.
2.3.4. Impact of Averaging Filters:

- Most voltage and current sensors use some sort of averaging filters (including weighted averaging); these may include, among others, the low-pass anti-aliasing filter that we discussed in Section 2.3.1.

- As a result, each individual measurement that is reported by the sensor is the average or weighted average of multiple raw measurements over a finite time interval (a number of samples), known as the *measurement window*. The length of the measurement window varies among sensors and affects their class and performance.
2.3. Sampling Rate, Reporting Rate, and Accuracy

- Consider two different choices for the measurement window size (one cycle vs. four cycles) to measure a continuous voltage wave.
They result in different measurement outcome:

- If the window size is one, then the RMS value of the most recent voltage cycle is reported. If the window is four, then the average of the RMS values of the four most recent voltage cycles is reported.

- We can see that the use of averaging filters results in smoothing the measurements. When the measurement window is four, the reported voltage sag is smaller in magnitude and it takes longer to settle down.
• The details about the averaging filter and other internal mechanisms are often considered proprietary information and not released by sensor manufacturers. Therefore, caution is required when interpreting the measurement data that come from various sensor devices.

• **Example 2.7**: Consider the impact of a lightning strike on voltage measurements that are taken at a power line with 7200 V rated voltage:
• Example 2.7 (Cont.): The reporting rate of the voltage sensor in this example is one RMS value per one AC cycle, i.e., 60 readings per second. Based on the voltage measurements that are shown on the previous slide, the *lightning-induced voltage sag* lasted about 80 msec.

However, it is known that a lightning strike lasts only several microseconds. The discharge of the lightning surge current through a surge arrester also often lasts only a few milliseconds. Therefore, we should consider the possibility that the relatively slow response in the voltage sag that is seen in the figure on the previous slide could be due to the *filtering and other internal dynamics* of the sensor [87].
• The impact of averaging filters is *negligible* in a steady-state analysis, and even in a dynamic analysis if the transient response is relatively slow compared to the sensor’s reporting rate.

• However, if one tends to analyze very *fast* events, such as a lightning strike, one may have to use waveform sensors; see Chapter 4.
2.1. Instrument Transformers
2.2. Non-Contact Voltage and Current Sensors
2.3. Sampling Rate, Reporting Rate, and Accuracy
2.4. RMS Voltage and Current Profiles
2.5. RMS Voltage and Current Transient Responses
2.6. RMS Voltage and Current Oscillations
2.7. Events in RMS Voltage and Current Measurements
2.8. Three-Phase Voltage and Current Measurements
2.9. Measuring Frequency
2.4. RMS Voltage and Current Profiles

• Traditionally, the voltage and current measurements in the power industry have been mostly in terms of the RMS values of the AC voltage and current waves. This has been particularly the case in the traditional *supervisory control and data acquisition* (SCADA) systems.

• Therefore, it is important to learn the kinds of analysis that one can do with the RMS voltage and current measurements.

• For the rest of Chapter 2, we will focus on the RMS voltage and current measurements. We will discuss measuring voltage and current phasors in Chapter 3 and measuring voltage and current waveforms in Chapter 4, as they require using more advanced sensors.
2.4.1. Daily Profiles:

• It is often informative to examine the daily RMS profiles of voltage and current, e.g., at a substation, a PV inverter, or a load. In most cases, there is no need for a sensor with a very high reporting rate. A one-minute reporting interval is often sufficient to see the overall trends and to identify the major voltage and current events.

• Example 2.8: Two daily profiles of the RMS current measurements that are taken at the output terminals of a solar Photovoltaic (PV) inverter are shown on the next slide. The profile in Figure (a) is measured on a sunny day in summer. The profile in Figure (b) is measured on a cloudy and rainy day in winter.
• **Example 2.8 (Cont.):** The generation output highly depends on the time of day, season, and weather conditions.

![Graph showing current profiles for Summer and Winter](image-url)
2.4. RMS Voltage and Current Profiles

• **Example 2.9**: Consider the daily RMS profiles of voltage and current measurements at a power distribution feeder that are shown below.
• Example 2.9 (Cont.): The reporting rate is once per minute. The rated voltage is 7200 V. The RMS voltage profile is relatively stable, remaining between 7050 V, i.e., 0.979 p.u., and 7350 V, i.e., 1.021 p.u.

The RMS current profile is generally higher during the day and lower at night. Of interest are the sudden changes in the RMS voltage profile.

A subset of those sudden changes are marked with numbers 1 to 9.

Events 1, 2, 6, 7, and 9 demonstrate voltage sags.

Events 3, 4, 5, and 8 demonstrate voltage swells.
The voltage sags/swells in Example 2.9 are caused by different sources. Some of them could be due to a *local issue*, such as a sudden change in this feeder’s load, or the operation of a capacitor bank, a voltage regulator, or another device on this feeder. Some could be due to a *non-local issue*, i.e., a voltage sag or swell that occurred at the substation, sub-transmission system, or transmission system, which also showed up on this feeder’s voltage profile.

**Q**: For the nine events in Example 2.9, which one(s) do you think are caused by local issues and which one(s) are caused by non-local issues?
• Events 1, 2, 4, 5, 6, 8, and 9 on the voltage profile are less likely to be due to sudden changes in the feeder’s load, because they do not coincide with a major change in the current profile. For instance, the current profile is almost flat from midnight until 3:45 AM, yet there are major changes in the voltage in this period, including Events 1 and 2.

• Events 3 and 7 do coincide with some major changes in the current profile, which would be more visible if we zoom in on the figure. Thus, Events 3 and 7 could be load-induced or caused by other local issues. These two events will be further discussed in Section 5.2 in Chapter 5.
2.4. RMS Voltage and Current Profiles

• Next, suppose we also obtain the daily RMS voltage profile of a *neighboring power distribution feeder* on the *same* day. The same nine events that we identified in Example 2.9 are also marked here.
2.4. RMS Voltage and Current Profiles

• Events 1, 2, 4, 5, 6, and 9 also showed up quite noticeably in this figure. From this, together with the analysis in Example 2.9, we can conclude that all these events very likely have root causes at the substation, sub-transmission system, or transmission system, i.e., they are very likely not caused by local issues on either of the two feeders.

• Similarly, we can now conclude that events 3, 7, and possibly 8 are caused by some local issues in the first feeder.

• While the above analysis is intuitive and manual, one can use techniques from statistics and machine learning to extract the above patterns automatically and from large sets of measurement data; see Section 2.7 as well as Section 3.7 in Chapter 3.
2.4.2. Histograms and Scatter Plots:

• The time period of the daily RMS voltage and current profiles can be extended to several *days*, *weeks*, or *months*. However, for such longer periods, it is often beneficial to present the measurements in a way that can highlight some of their *statistical characteristics*. In this section, we discuss two common options.

• **Histograms** can be used to provide an approximate representation of the distribution of a large set of measurements. To construct a histogram, we divide the entire range of the measurements into a series of intervals, called *bins*, and we count the number of measurements whose values fall into each interval.
**Example 2.10:** Suppose the RMS voltage measurements are available on a minute-by-minute basis at two power distribution feeders for a period of one month. The histogram representation of the measurements at each of these two feeders is shown below.

![Histograms](image)

**Feeder 1**

**Feeder 2**

**Q:** What do we learn from these histograms?
2.4. RMS Voltage and Current Profiles

- **Example 2.10 (Cont.):** The range of the measurements is from 6900 V to 7400 V. This range is divided into 25 bins. The first bin includes all the measurements between 6900 V and 6920 V, the second bin includes all the measurements between 6920 V and 6940 V, and so on.

The histogram corresponding to Feeder 1 is almost evenly distributed around the rated voltage at 7200 V; see Figure (a). About 50.5% of the reported voltage measurements are below the rated voltage, and about 49.5% are above the rated voltage.

The histogram corresponding to Feeder 2 is wider and placed mostly to the left of the rated voltage; see Figure (b). About 85.8% of the reported voltage measurements are below the rated voltage, and about 14.2% are above the rated voltage.
2.4. RMS Voltage and Current Profiles

- The scatter plot of the minutely voltage measurements in Example 2.10 is shown below. Here, the voltage measurements at Feeder 2 are plotted against their corresponding voltage measurements at Feeder 1.

- Q: Explain ① and ②.
First, consider the dashed diagonal line, which is marked as $\bar{1}$.

The points below this line are associated with the minutes during which the voltage at Feeder 1 was higher than the voltage at Feeder 2. This accounts for 90.6% of the total points on the scatter plot.

Conversely, the points above this line are associated with the minutes during which the voltage at Feeder 1 was lower than the voltage at Feeder 2. That accounts for 8.4% of the points.

Note that 1.1% of the points are on the diagonal line. These are the cases where the voltages were equal on both feeders to the extent of the reading resolution of the sensors.
• Second, consider the four arrows, which are marked as 2.

• They point at a few outliers.

• During the minutes associated with these outliers, Feeder 2 had unusually major voltage sags that were likely only local to Feeder 2 because Feeder 1 did not similarly experience such major voltage sags.

• **Clustering**: Scatter plots can sometimes reveal clusters of points that can be considered for classification, i.e., grouping of measurements that have similar features.
Example 2.11: Recall from Example 2.4 in Section 2.3.2 that switching on major loads can cause *inrush current*, which can be captured in the current measurements if the reporting rate of the sensor is sufficiently high. An example is shown in Figure (a) on the next slide. Here, the inrush current is denoted by $\Delta I_{\text{inrush}}$. The change in the steady-state current that is due to the increased load is also denoted by $\Delta I_{\text{steady}}$. Figure (b) on the next slide shows the scatter plot of $\Delta I_{\text{inrush}}$ versus $\Delta I_{\text{steady}}$ based on all the inrush current cases that were observed at a commercial facility during one day. One immediate observation is related to the outlier case that is marked as $\text{①}$. More importantly, we can see that the majority of the points in this scatter plot are *clustered into two groups*, which are marked as $\text{②}$ and $\text{③}$. Note that the points in $\text{③}$ experience much larger inrush current than the points in $\text{②}$ when compared at the same amount of change in the steady-state current.
• **Example 2.11 (Cont.):** Accordingly, the scatter plot helps us identify two load clusters at ① and ② and also a major outlier at ③.

• **Note:** We will discuss clustering and classification further in Section 3.7 in Chapter 3, Section 4.3 in Chapter 4, and Section 5.4 in Chapter 5.
Table of Contents

2.1. Instrument Transformers
2.2. Non-Contact Voltage and Current Sensors
2.3. Sampling Rate, Reporting Rate, and Accuracy
2.4. RMS Voltage and Current Profiles
2.5. RMS Voltage and Current Transient Responses
2.6. RMS Voltage and Current Oscillations
2.7. Events in RMS Voltage and Current Measurements
2.8. Three-Phase Voltage and Current Measurements
2.9. Measuring Frequency
• In Example 2.4, load switching caused a short transient response in the power system, in form of an inrush current.

• Such load-induced transient responses comprise the majority of the (rather minor) transient responses that one can see in voltage and current measurements on an ongoing basis.

• However, there are also many other, often more major and more important, causes of transient responses, such as faults and equipment actuations. Analysis of transient responses can be useful in many power system applications, such as in control, protection, stability analysis, and reliability assessment.

• The time resolution (i.e., the sensor’s reporting rate) that is needed depends on the time scale of the transient event being monitored.
2.5.1. Transient Responses Caused by Faults:

• Example 2.12: An animal-caused fault can cause voltage and current transient responses at a distribution feeder. The fault occurred on a lateral not too far from the substation. Measurements are done on the secondary side of a load transformer at a commercial load location on another lateral. The circuit is equipped with fuses and a circuit breaker.

![Diagram of a distribution feeder with fault location and measurement points.](www.citizentribune.com)
• **Example 2.12 (Cont.):** Three stages are marked. Stage ① shows the momentary drops in voltage and current during the initial animal-caused short-circuit fault. Stage ② shows the momentary power outage (at the location of the sensor) due to the operation of the circuit breaker. Sometime during stages ① and ②, the fuse on the faulted lateral burns, isolating the fault. Stage ③ shows the transient in voltage and current after the circuit breaker recloses to restore service.
• **Impact at Other Locations:** The fault in Example 2.12 also caused transient responses in *other* locations on the power grid, *even outside the faulted feeder*. The voltage measurements in Figure (a) are taken at the distribution substation at the same distribution feeder. The voltage measurements in Figure (b) are taken at another substation that is *several miles away* from the faulted location. As we move away from the faulted location, the transient response becomes less significant.
• **Impact at Other Locations (Cont.):** While the animal-induced fault in Example 2.12 caused transient responses in a region that may span *tens of miles*; there are also other events that have “*system-wide*” transient impacts that may span *hundreds of miles*. Some of such cases may include system-wide oscillations. We will discuss system-wide transient responses in Sections 2.6.1 and 2.9.2, and also in Chapter 6.
2.5.2. Transient Responses Caused by Equipment Actuations:

• **Example 2.13**: A PV farm is interconnected to a power distribution system. Figure (a) shows the RMS voltage that is measured at the point of inter-connection. Only one phase is shown here. A major step change is visible, which is caused by a step-up transformer tap change event in the power system. It is not caused by the PV farm.
• **Example 2.13 (Cont.):** This voltage event resulted in a transient response in the RMS current of the PV farm, as shown in Figure (b). The *pattern* of the *changes* in the RMS current reveals how the inverters at the PV farm respond to the step-up disturbance in voltage.

![Inverter's Dynamic Response](image)

The transient response takes less than 1 second. It includes *undershoots* and overshoots. The transient signature is for the most part a reflection of the *dynamics* of the inverter control system. A step-down disturbance may too cause transient responses [93].
Monitoring the transient responses that are caused by equipment operation can inform us about the *state of the health* of the equipment and the possible signs of incipient faults (i.e., early-stage faults) that could become problematic in the future, as well as the overall response of the system under potential contingencies.

The reporting rate (the resolution of the data) is a key factor to allow capturing sufficient information to reveal potential issues.

We will discuss *incipient faults* in details in Section 4.3 in Chapter 4.
# Table of Contents

2.1. Instrument Transformers  
2.2. Non-Contact Voltage and Current Sensors  
2.3. Sampling Rate, Reporting Rate, and Accuracy  
2.4. RMS Voltage and Current Profiles  
2.5. RMS Voltage and Current Transient Responses  
2.6. RMS Voltage and Current Oscillations  
2.7. Events in RMS Voltage and Current Measurements  
2.8. Three-Phase Voltage and Current Measurements  
2.9. Measuring Frequency
• Many problems in the power grid begin with or are manifested through oscillations; which can be captured in our measurements.

• At the transmission level, system-wide oscillations are often associated with the electromechanical dynamics of the power system that are excited by a major disturbance, such as losing a major power generation unit or losing a transmission line.

• At the distribution level, local oscillations are often somewhat minor and they are often associated with equipment responses to various disturbance, circuit resonance, or certain load operations.
• Oscillations in power systems are often characterized based on their *oscillatory modes*. If all the oscillatory modes are *stable*, then the oscillations decay and diminish over time. However, if one or more oscillatory modes are *unstable*, then the oscillations grow in magnitude until corrective actions are taken or the system fails.

• In this section (Section 2.6), we discuss examples of system-wide and local oscillations that can be seen in voltage and current measurements. We also discuss how to obtain the oscillatory modes of the system from voltage and current measurements.
2.6. RMS Voltage and Current Oscillations

2.6.1. Wide-Area Oscillations in Power Transmission Systems

• System-wide oscillations, also known as *wide-area oscillations*, are common phenomena in power transmission systems.

• They are caused by the electromechanical oscillations of rotational generators in response to faults, transmission line switching, sudden changes in the output of generators, or sudden changes in major loads.

• Wide-area oscillations may affect the magnitude (as we will see in this section), *phase angle* (see Section 3.4.3), and *frequency* (see Section 2.9.2) of voltage across the entire interconnected power system.
Some of the common classes of wide-area oscillations include:

- **Control mode oscillations**: one generator swings against the rest of the power system. *Typical Frequency Range*: 0.01 Hz to 0.15 Hz.

- **Inter-area mode oscillations**: two coherent groups of generators swing against each other, causing excessive power transfers across the network. *Typical Frequency Range*: 0.15 Hz to 1.0 Hz.

- **Local plant mode oscillations**: some generators suffer from poorly tuned exciters, governors, or other generator controllers. *Typical Frequency Range*: 1.0 Hz to 2.0 Hz.

*Note 1*: The above frequency ranges are approximate.

*Note 2*: These are the frequencies of the oscillations.
Example 2.14: Here are the transient inter-area oscillations that were observed in voltage measurements at a 500 kV transmission line after a disturbance, which was caused by losing a major generation unit.

The disturbance resulted in an initial voltage sag followed by ring-down oscillations, i.e., decaying oscillations that ultimately disappear in ambient noise after a few seconds. The dominant mode of oscillations had a frequency of 0.7 Hz, which is within the range of 0.15 Hz to 1.0 Hz.
• **Inter-Area Oscillations**: Among different types of wide-area oscillations, understanding inter-area oscillations is particularly important due to their impact on power system stability and the fact that they involve many parts of the power system in *distant regions* (see the example on the next slide) with complex dynamic behavior.

• Inter-area oscillations are used in the analysis of power system dynamics because of their relation to small-signal stability [94].

• Inter-area oscillations can arise for a variety of reasons, such as excessive power transfers, inefficient damping controls at some generators, or unfavorable load characteristics.
Here are some of the primary modes of inter-area oscillations that are observed in the *Western Interconnection* in the United States.

Each mode involves two geographical regions. For example, the North-South mode involves generators in Canada and the Pacific Northwest region in the U.S. oscillating against generators in the Desert Southwest and Southern California, causing excessive power swings between Northern California and Oregon as well as between Northern and Southern California.
2.6.2. Local Transient Oscillations in Power Distribution Systems

• Some oscillations in power systems are caused by a relatively minor *local issue* in the power distribution network, such as due to resonance between a capacitor and an inductor.

• They often affect only the same feeder where the issue occurs.

• Also, they are likely to be visible only in current measurements, compared to the voltage measurements.

• Also, some local transient oscillations may have very high frequencies at the order of *kHz* and may last for only a fraction of an AC power system cycle (few milliseconds).

• We will discuss local transient oscillations in Chapter 4.
2.6.3. Modal Analysis of Oscillations

- An oscillatory signal $x(t)$ can be mathematically modeled as

$$x(t) = \sum_{i=1}^{m} A_i e^{\sigma_i t} \cos(w_i t + \varphi_i),$$

where $m$ is the number of oscillatory modes. For each mode $i$, we have:

- $A_i$: amplitude
- $w_i$: angular frequency
- $\sigma_i$: damping factor
- $\varphi_i$: phase angle

- If signal $x(t)$ includes a DC term, then the DC term can be modeled as a mode with zero frequency, zero damping factor, and zero phase angle.

- $x(t)$ can be any *time series*, such as the RMS voltage or current data.
Example 2.15: Oscillatory signal $x_1(t)$ has $m = 2$ modes, where

$$A_1 = 3.5, \varphi_1 = \pi/4, w_1 = 2\pi \times 0.25, \sigma_1 = -0.1,$$

$$A_2 = 1.75, \varphi_2 = \pi/6, w_2 = 2\pi \times 0.8, \sigma_2 = -0.5.$$

Both modes are stable and decay over time.
**Example 2.15 (Cont.):** Oscillatory signal $x_2(t)$ has $m = 3$ modes. The first and the second modes are stable and identical to those of signal $x_1(t)$. The parameters of the third mode are

- $A_3 = 0.15$, $\varphi_3 = 0$, $\omega_3 = 2\pi \times 2$, $\sigma_3 = 0.1$.

The third mode is *not* stable, because $\sigma_3 > 0$. As a result, the oscillations corresponding to this mode quickly grow in magnitude.
• The model on Slide 74 can be represented also in *discrete-time* domain by applying the Z-Transform from digital signal processing. Let $\Delta t$ denote the reporting interval of the sensor. At each discrete time $k$, we can model the measurement signal as

$$x(t = \Delta t \times k) = \sum_{i=1}^{m} z_i^k R_i,$$

where for each mode $i = 1, \ldots, m$, we define:

$$z_i = e^{(\sigma_i + j\omega_i) \Delta t} \quad \text{and} \quad R_i = A_i e^{j\phi_i}.$$

• Since our focus is on digital sensors, for the rest of this section, we use the above discrete-time model. Also, for notational simplicity, we use $x(k)$ to denote the measurement at discrete time $k$, instead of the more precise but cumbersome notation $x(t = \Delta t \times k)$. 
2.6. RMS Voltage and Current Oscillations

- **Prony Method**: This is a popular “modal analysis” technique to estimate all four parameters, \( \omega_i, \sigma_i, A_i, \) and \( \phi_i \), for a given measurement signal \( x(t) \). This is done in three steps.

- **Step 1**: Suppose a total of \( n \) measurement points are available, where \( n \gg m \). A discrete linear auto-regressive (AR) predictor model can be used to capture the dynamics of \( x(t) \):

  \[
  x(m) = a_1 x(m - 1) + a_2 x(m - 2) + \ldots + a_m x(0).
  \]

  Here we approximate the measurement at discrete time \( m \) (on the left hand side) as a linear combination (a linear regression) of all the previous measurements at discrete times \( m - 1, m - 2, \ldots, 0 \).

- We can similarly approximate \( x(m + 1), x(m + 2), \ldots, x(n - 1) \).
Step 1 (Cont.): We can accordingly obtain:

\[
\begin{bmatrix}
  x(m) \\
  x(m+1) \\
  x(m+2) \\
  \vdots \\
  x(n-1)
\end{bmatrix}
= 
\begin{bmatrix}
  x(m-1) & \cdots & x(0) \\
  x(m) & \cdots & x(1) \\
  x(m+1) & \cdots & x(2) \\
  \vdots & & \vdots \\
  x(n-2) & \cdots & x(n-m-1)
\end{bmatrix}
\begin{bmatrix}
a_1 \\
\vdots \\
a_m
\end{bmatrix}
\]

\[
\Psi = (n-m) \times 1 \\
\times = (n-m) \times m \\
a = m \times 1
\]

Here, \( a \) is the vector of *unknowns* in this equation.
2.6. RMS Voltage and Current Oscillations

- **Step 1 (Cont.):** Since \( n \gg m \), we can formulate the following least-squares (LS) optimization problem based on the relationship on the previous slide in order to obtain the unknown coefficients:

\[
\min_{\mathbf{a}} \| \Psi - \mathbf{Xa} \|_2.
\]

- We can solve this LS problem by using an LS solver; such as `lsqlin` in MATLAB [102]. Or we can obtain the solution in closed-form as:

\[
\mathbf{a} = (\mathbf{X}^T \mathbf{X})^{-1} \mathbf{X}^T \Psi.
\]

- So far, we have completed obtaining the AR predictor model.
• **Step 2**: Given the coefficients $a_1, \ldots, a_m$ from Step 1, we need to solve the following *discrete-time characteristics polynomial* of degree $m$:

\[
1 - \sum_{i=1}^{m} a_i z^{-i} = 0. \quad \rightarrow \quad z^m - \sum_{i=1}^{m} a_i z^{m-i} = [1 - a^T] \begin{bmatrix} z^m \\ z^{m-1} \\ \vdots \\ z \\ 1 \end{bmatrix} = 0.
\]

• We can solve the above equation over unknown $z$ by using a solver such as **roots** in MATLAB [104] to obtain the **poles** (in discrete-time domain) corresponding to the oscillatory modes of the measured signal.

• The poles in *continuous-time domain* are obtained as:

\[
\sigma_i + j\omega_i = \ln(z_i)/\Delta t, \quad i = 1, \ldots, m.
\]
2.6. RMS Voltage and Current Oscillations

• **Step 3:** Once $z_i$ is known for all $i = 1, \ldots, m$, we can use the model on Slide 77 to construct the following relationship:

\[
\begin{bmatrix}
    x(0) \\
    x(1) \\
    x(2) \\
    \vdots \\
    x(n-1)
\end{bmatrix} =
\begin{bmatrix}
    1 & \cdots & 1 \\
    z_1 & \cdots & z_m \\
    z_1^2 & \cdots & z_m^2 \\
    \vdots & \ddots & \vdots \\
    z_1^{n-1} & \cdots & z_m^{n-1}
\end{bmatrix}
\begin{bmatrix}
    R_1 \\
    \vdots \\
    R_m
\end{bmatrix}.
\]

\[
\Phi 
\begin{bmatrix}
    n \times 1
\end{bmatrix}
\quad
Z 
\begin{bmatrix}
    n \times m
\end{bmatrix}
\quad
R 
\begin{bmatrix}
    m \times 1
\end{bmatrix}
\]

• Here, $\mathbf{R}$ is the vector of *unknowns* in this equation.

\[
\mathbf{R} = (\mathbf{Z}^T \mathbf{Z})^{-1} \mathbf{Z}^T \mathbf{\Phi} \quad \rightarrow \quad A_i = |R_i|, \quad \varphi_i = \angle R_i, \quad i = 1, \ldots, m.
\]
• **Note 1:** Once all the $m$ oscillatory modes are calculated, one may still need to do some post-processing before the results can be used. For example, it is known that the Prony method usually results in aliased modes. Any oscillatory mode with a frequency that is higher than half the reporting rate of the sensor is prone to aliasing. Such oscillatory modes are not reliable and should be *discarded* from the results.

• **Note 2:** Accuracy of the Prony estimation may increase by increasing parameter $m$; see *Exercise 2.17*. However, in general, you may have to try different choices for parameter $m$ to find the proper selection.

• **Note 3:** Adjustments, improvements, extensions, and alternative methods have been proposed in the literature, such as in [106–113].
2.6. RMS Voltage and Current Oscillations

- **Example 2.16**: Again, consider the ring-down oscillations in the voltage measurements in Example 2.14. The reporting interval is $\Delta t = 0.1$ sec. By choosing $m = 101$, the Prony method gives 101 oscillatory modes. A total of 86 modes have frequencies higher than $0.5/0.1 = 5$ Hz. Thus, they are prone to aliasing and are *discarded*. The remaining 15 modes include one DC mode with an amplitude of 524.19 kV and 7 pairs of complex conjugate oscillatory modes, as listed below:

<table>
<thead>
<tr>
<th>Mode</th>
<th>Frequency (Hz)</th>
<th>Damping (Hz)</th>
<th>Amplitude (kV)</th>
<th>Phase Angle (°)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>± 1.6056</td>
<td>−0.9081</td>
<td>5.0083</td>
<td>±45.419</td>
</tr>
<tr>
<td>2</td>
<td>± 1.5002</td>
<td>−0.2515</td>
<td>1.1597</td>
<td>±74.344</td>
</tr>
<tr>
<td>3</td>
<td>± 1.2395</td>
<td>−0.3032</td>
<td>2.2171</td>
<td>±55.979</td>
</tr>
<tr>
<td>4</td>
<td>± 1.0872</td>
<td>−0.4831</td>
<td>2.2250</td>
<td>±45.605</td>
</tr>
<tr>
<td>5</td>
<td>± 0.7241</td>
<td>−0.0767</td>
<td>2.3696</td>
<td>±30.464</td>
</tr>
<tr>
<td>6</td>
<td>± 0.6540</td>
<td>−0.1128</td>
<td>3.0296</td>
<td>±105.92</td>
</tr>
<tr>
<td>7</td>
<td>± 0.2890</td>
<td>−0.3598</td>
<td>4.6563</td>
<td>±162.87</td>
</tr>
</tbody>
</table>
• Example 2.16 (Cont.): All the obtained oscillatory modes:
• **Example 2.16 (Cont.):** Once we combine, i.e., *add together*, all the $1 + 2 \times 7 = 15$ obtained oscillatory modes, we can reconstruct an estimation (i.e., a Prony estimation) of the original measurements.

The Prony estimation versus the original measurements:

We can see that the Prony estimation is particularly accurate during the first few seconds and then gradually drifts from the measurements.
Fourier Method: While Prony analysis forms a sum of damped sinusoidal terms, Fourier analysis forms a sum of sustained sinusoidal terms, where the damping factor is zero at all frequencies.

In this regard, we can still use the Fourier Transform (FT) or Fast Fourier Transform (FFT) as a quick method to estimate only the frequency, not the damping factor, of the oscillatory modes in the time-series of the measurements. The FFT of the measurements can be obtained by using the command fft in MATLAB [114].

Since FFT is computationally less complex than the Prony analysis, it can be used to detect oscillations in the measurements; e.g., see Example 2.24 in Section 2.9.2. Once detected, the oscillation can be further characterized by using the Prony method.
The *frequency spectrum* of the voltage measurements in Example 2.16 is shown below. The frequencies that are marked on this figure are comparable with those in the table on Slide 84. In particular, the frequency modes at 0.30 Hz, 0.63 Hz, 0.73 Hz, 1.23 Hz, and 1.63 Hz in the figure below more or less match the frequency modes in rows 7, 6, 5, 3, and 1 in the table on Slide 84. These are the primary modes of oscillations that an FFT analysis can identify *relatively quickly*. 
2.1. Instrument Transformers
2.2. Non-Contact Voltage and Current Sensors
2.3. Sampling Rate, Reporting Rate, and Accuracy
2.4. RMS Voltage and Current Profiles
2.5. RMS Voltage and Current Transient Responses
2.6. RMS Voltage and Current Oscillations
2.7. Events in RMS Voltage and Current Measurements
2.8. Three-Phase Voltage and Current Measurements
2.9. Measuring Frequency
• Events in power systems can be defined broadly as *any change* in any component in the system that is “worth studying”.

• Events may include major *faults* and *equipment failures* that can affect stability and reliability of the system. We saw instances of such events in Examples 2.12 and 2.14.

• Events may also include *load switching* or *transformer tap changing* that are *benign yet informative* about the power system and its elements. We saw instances of such events in Examples 2.11 and 2.13.

• Of course, whether or not an event is “worth studying” depends on what we are trying to achieve in our study. An event could be of great importance for one smart grid monitoring purpose but useless for another smart grid monitoring purpose!
Dependence to the Purpose of Analysis:

- For example, when we study inter-area oscillations in transmission networks, the oscillations in the frequency of the voltage measurements are a key event to study (see Section 2.9.2); but an animal-caused fault at a distribution line, such as what we saw in Example 2.12, is just a small noise in the analysis of system-wide oscillations.

- In contrast, when we study the operation of the protection system at a distribution line, the animal-caused fault is a key event; but a change in the voltage frequency due to inter-area oscillations are of no interest.
2.7. Events in RMS Voltage and Current Measurements

Dependence to the Type of Sensors:

• The definition of events in a study may also depend on the type of sensors that are used and their characteristics.

• For example, some events could last for only a very short period of time; therefore, they *may not even show up* in our measurements, depending on the sampling rate and the reporting rate of the sensor.

• Furthermore, some events may not manifest in the typical RMS voltage and RMS current measurements, but they do show up if we have access to the phase angle measurements of the voltage and current phasors (see Chapter 3), or the actual waveform measurements of voltage and current (see Chapter 4).
2.7.1. Transient Events versus Sustained Events

• The basis for this classification is shown below:

![Graph showing transient and sustained changes in measurements over time.]

• If the event causes a major transient change in the measurements, then it is a transient event, as the transient change is likely worth studying. If the event causes a major sustained change, then the event is a sustained event, as the sustained change is likely worth studying.

• It is possible that an event is both transient and sustained.
2.7.2. Event Detection Methods

• Most methods follow these three general steps:

1) A window of measurements is considered;
2) One or more indexes are calculated based on window of measurements;
3) An event is detected if the index(es) exceed certain threshold(s).
• **Min-Max (Detection Index)**: A simple detection index is the difference between the maximum and the minimum values within the window of measurements:

\[
\text{Min-Max Index} = \max\{x_w(t)\} - \min\{x_w(t)\}.
\]

• An event is detected if the following holds for a given threshold:

\[
\text{Min-Max Index} > \text{Threshold}.
\]
• **Example 2.17**: Consider the daily voltage profile in Example 2.9. Recall that the measurements are reported on a *minutely* basis. If we set the *detection window size* to 20 minutes, then each window includes 20 measurement points. The day is divided into $24 \times \frac{60}{20} = 72$ windows. In this example, we assume that the measurement windows do *not* overlap. Figure below shows the Min-Max Index that for all the 72 windows. The *detection threshold* is set to 70 kV (dashed line).
• **Example 2.17 (Cont.):** Since we set the detection threshold to 70 kV, a window of measurements is deemed to contain an event if the difference between the maximum reported voltage and the minimum reported voltage within the window is at least 70 kV.

This results in detecting *eight events*. Five of them are among the events that we had previously marked in Example 2.9, labeled as 3, 4, 5, 8, and 9. They are positioned above the threshold curve.

Importantly, four of the events that we had previously marked in Example 2.9 through a manual selection are *not* detected here, labeled as 1, 2, 6, and 7. They are positioned *below* the threshold curve.
2.7. Events in RMS Voltage and Current Measurements

- **Median Absolute Deviation (Detection Index):** Detection indexes also can be defined based on principles in statistics. Here, we discuss one such index. Let us define the *median absolute deviation* as

\[
\text{MAD} = \gamma \text{Median}\{|x_w(t) - \text{Median}\{x_w(t)\}|\}.
\]

- A typical value for coefficient \(\gamma\) is 1.4826 [115].

- An event is detected in the window of measurements \(x_w(t)\) if any measurement within this window is *outside* the following range:

\[
[\text{Median}\{x_w(t)\} - \zeta\text{MAD}, \text{Median}\{x_w(t)\} + \zeta\text{MAD}].
\]

A typical value for \(\zeta\) is 3.
2.7. Events in RMS Voltage and Current Measurements

- An example for a window of measurements during which some measurements *drop below* this range, i.e., below $\text{Median}\{x_w(t)\} - \zeta \text{MAD}$ is shown in Figure (a). An example for a window of measurements during which some measurements *exceed above* this range, i.e., above $\text{Median}\{x_w(t)\} + \zeta \text{MAD}$ is shown in Figure (b).
• We can express the range \([\text{Median}\{x_w(t)\} - \zeta \text{MAD}, \text{Median}\{x_w(t)\} + \zeta \text{MAD}\] also in form of *two event detection indexes*, as follows:

\[
\text{MAD-Low Index} = \min\{x_w(t)\} - (\text{Median}\{x_w(t)\} - \zeta \text{MAD})
\]

\[
\text{MAD-High Index} = \max\{x_w(t)\} - (\text{Median}\{x_w(t)\} + \zeta \text{MAD})
\]

• An event is detected in a window of measurements if the MAD-Low Index is *less than zero* or the MAD-High Index is *greater than zero*. 
Example 2.18: Again, consider the daily voltage profile in Example 2.9. The window size is 20. Figures below show the MAD-Low Index and the MAD-High Index calculated for each of the 72 windows, respectively.
• **Example 2.18 (Cont.):** Seven of the events that we had previously marked in Example 2.9 are detected, labeled as 1, 2, 3, 4, 5, 6, and 9.

Event 8 is not detected; however, this event can be detected once we use *partially overlapping windows*, as we will discuss later in this section.

Event 7 cannot be detected, even with partially overlapping windows, unless we reduce parameter $\zeta$, which may result in detecting too many cases as events. To understand why the situation with Event 7 is different, recall from Example 2.9 that Event 7 is likely caused by a local issue in the distribution network. Therefore, the voltage measurements may *not* be the best indicator (regardless of the choice of the detection index), if our goal is to detect local events such as Event 7. In such cases, it is better also to check the current measurements instead of the voltage measurements, in order to search for potential local events.
2.7. Events in RMS Voltage and Current Measurements

• **Overlapping Windows**: It is very common in event detection to use windows that have some overlap with each other. Such overlap can help avoid missing the events that may fall on the borderline of two adjacent but non-overlapping windows.

• For instance, in Example 2.18, we did not detect Event 8 because it appeared partially in one window and partially in another window. However, once we use a sequence of windows with 50% overlap, i.e., each window overlaps with half of the previous window and also with half of the next window, we are able to detect this event.
• **Dynamic Windows:** Sometimes it may help to also use dynamic windows. For example, if no event is detected in a window of measurements, then we can try *increasing* or *decreasing* the size of the detection window before we conclude that there is indeed no event in the considered window of measurements.

• Using dynamic windows might be a necessity if we expect the events to have different lengths in time; e.g., see [127].
• **Frequency Spectral Methods**: When it comes to transient events that involve oscillations, it might be better to use frequency spectral methods to detect such events. A common such method is FFT.

• In this method, FFT is applied to each window of voltage or current measurements in order to obtain the frequency spectrum associated with the measurements in that window.

• An event is detected if the amplitude of any frequency component in the frequency spectrum exceeds a predetermined threshold.
• Figure (a) shows the voltage measurements during the oscillatory event that we previously saw in Example 2.14. Figure (b) shows the corresponding frequency spectrum that is obtained by applying FFT to the window of measurements in Figure (a). The event detection threshold is 1.5 kV. The oscillatory event is detected since the frequency spectrum exceeds the threshold around 0.63 Hz–0.73 Hz.
• There exist several other event detection methods that can be used to detect events in voltage and current measurements.

• Some of these methods can be seen as variations or extensions of the methods that we discussed above. For example, one can use mean instead of median in the MAD method.

• There are also many options among the frequency spectral event detection methods, such as the Yule-Walker method [113] and the Wavelet method [116, 117], to name a few.
• Some recent methods also use *machine learning*.

• In *supervised* event detection, *prior knowledge* is used to identify and characterize the common patterns for the kind of events that we are interested in detecting. The event detection algorithm then looks for those specific patterns in order to detect the intended events whose characteristics are learned; see the methods in [118–122].

• In *unsupervised* event detection, *no prior knowledge* is used. Instead, the goal is to learn the normal trends in measurements and then detect an event whenever there is an *abnormality*, i.e., a major deviation from the identified normal trends; see the methods in [123–126].
2.7.3. Events in Other Types of Measurements

• The focus in this Chapter has been on events in RMS voltage measurements and RMS current measurements. However, events can be of interest in any type of smart grid measurements.

• Events in *voltage and current phasor measurements* will be discussed in Section 3.7 in Chapter 3.

• Events in *voltage and current waveform measurements* will be discussed in Section 4.4 in Chapter 4.

• Events in *active and reactive power measurements* and *power factor measurements* will be discussed in Section 5.2.1 in Chapter 5.
2.1. Instrument Transformers
2.2. Non-Contact Voltage and Current Sensors
2.3. Sampling Rate, Reporting Rate, and Accuracy
2.4. RMS Voltage and Current Profiles
2.5. RMS Voltage and Current Transient Responses
2.6. RMS Voltage and Current Oscillations
2.7. Events in RMS Voltage and Current Measurements
2.8. Three-Phase Voltage and Current Measurements
2.9. Measuring Frequency
• The power grid is a three-phase power system.

• Since the three phases are often balanced at transmission systems, it might be sufficient to measure voltage and current on only one phase when we monitor the operation of the power transmission system.

• However, when it comes to monitoring power distribution systems, the three phases are often unbalanced due to the distribution of loads, or even due to an unbalanced power distribution network topology.

• Therefore, it is often necessary to monitor all three phases in power distribution networks, including at load locations.
• Recall that phase unbalance can be in different forms:

Balanced

Unbalanced

(a) Balanced

(b) Unbalanced
2.8. Three-Phase Voltage and Current Measurements

2.8.1. Three-Phase RMS Profiles

- Figures below show the RMS voltage and RMS current profiles at each phase of a three-phase load. The RMS voltage and current profiles are not identical across the three phases. However, the RMS profiles do generally resemble each other in this example.
• We may also measure the *line-to-line* voltage profile:

![Graph showing line-to-line voltage profile](image)

• The angle $\theta$ can be obtained as

$$\cos(\theta) = \frac{V_A^2 + V_B^2 - V_{AB}^2}{2 V_A V_B}.$$
• **Example 2.19**: Suppose RMS voltage at Phases A and B is measured as

\[ V_A = 286.63 \, V \]
\[ V_B = 287.26 \, V \]

And the RMS line-to-line voltage across Phases A and B is measured as

\[ V_{AB} = 497.70 \, V. \]

We obtain \( \cos(\theta) = -0.5042 \). Therefore, we have \( \theta = -120.28^\circ \).

• **Note**: We will discuss measuring phase angle in Chapter 3 as part of the broader discussion on voltage and current phasor measurements.
2.8.2. Measuring Phase Unbalance

• If the voltages in a three-phase power system are not balanced, then some equipment, such as induction motors, can perform poorly. Thus, we may need to assess the extent of unbalance in voltage measurements.

• For instance, the National Equipment Manufacturers Association (NEMA) has introduced *Percentage Unbalance* (PU) as a metric:

\[
PU = \frac{1}{\Gamma} \max \left\{ |V_{AB} - \Gamma|, |V_{BC} - \Gamma|, |V_{CA} - \Gamma| \right\} \times 100%,
\]

Where

\[
\Gamma = \frac{1}{3} (V_{AB} + V_{BC} + V_{CA})
\]

Most motors allow a 1% voltage unbalance; but higher PU should be avoided. The ANSI C84.1 standard is to limit voltage unbalance to 3%.
• **Example 2.20**: The line-to-line voltages are measured at a load as:

\[
\begin{align*}
V_{AB} &= 497.70 \, V \\
V_{BC} &= 494.87 \, V \\
V_{CA} &= 496.98 \, V
\end{align*}
\]

We can obtain:

\[
PU = \frac{1.6467}{496.52} \times 100\% = 0.33\%.
\]

Since \( PU < 1\% \), such voltage unbalance does not affect the motor loads.
2.8.3. Phase Identification

• Utilities often do not have reliable records about how the three phases of each feeder are connected to the loads.

• The phase identification problem is the problem of identifying the correct phase connection configuration at each load.
Wrong phase labeling is a major source of error in the analysis of power distribution systems. Therefore, it is critical to find ways to correctly identify the phase labeling for the loads on each phase.

Q: Can we use sensor measurements to solve this problem?

The method to be used depends on the type of measurements.

We will discuss some methods here.

We will discuss other methods in Chapters 3 and 4 as well.
• **Example 2.21**: The voltage measurements at a single-phase load as shown in (a). The phase connection is *unknown* at this load. The voltage measurements on three phases at the substation are shown in (b). They are labeled as Phases A, B, and C. **Q**: What is the phase of the load?

All the measurements are reported at 30 readings per second.
The phase identification method in Example 2.21 can also be discussed in the context of the event analysis in Section 2.7. That is, we identified the phase connection by detecting an event on the single-phase measurements in Figure (a) and comparing it with an event that we detected at the same window on Phase C of the three-phase measurements in Figure (b), while considering the fact that no similar event was detected at this window on Phase A and Phase B.

Q: What if we do not have sensors with such high reporting rates?
One statistical approach is based on obtaining the correlation between the voltage measurements at the unknown phase and the voltage measurements at each of the three known reference phases. For example, the correlation coefficient between $V_1$ and $V_A$ is obtained as

$$\text{Corr} (V_1, V_A) = \frac{\text{Cov} (V_1, V_A)}{\sqrt{\text{Var} (V_1) \text{Var} (V_A)}},$$

We can similarly obtain $\text{Corr} (V_1, V_B)$ and $\text{Corr} (V_1, V_C)$.

A higher correlation coefficient indicates stronger correlation between the two voltage profiles, suggesting that the measurements are done at the same phase of the three-phase circuit.

Q: How does the correlation analysis help?
Example 2.22: Let us again consider the same example. However, this time suppose all the voltage measurements are reported at the rate of one reading every two seconds. This reporting rate is 60 times slower than the reporting rate in the previous Example. We obtain the correlation coefficients over 15 minutes of measurements:

Since \( \text{Corr}(V_1, V_A) = 0.9289 \), \( \text{Corr}(V_1, V_B) = 0.9286 \), \( \text{Corr}(V_1, V_C) = 0.9751 \). we conclude that the single-phase load is connected to Phase C.
# Table of Contents

2.1. Instrument Transformers  
2.2. Non-Contact Voltage and Current Sensors  
2.3. Sampling Rate, Reporting Rate, and Accuracy  
2.4. RMS Voltage and Current Profiles  
2.5. RMS Voltage and Current Transient Responses  
2.6. RMS Voltage and Current Oscillations  
2.7. Events in RMS Voltage and Current Measurements  
2.8. Three-Phase Voltage and Current Measurements  
2.9. Measuring Frequency
2.9. Measuring Frequency

- A fundamental characteristic of voltage and current waveforms in AC power systems is their frequency (see Section 1.2.1 in Chapter 1).

\[ f = \frac{1}{T}. \]

- Q: What is the **nominal** frequency in the U.S.?

- Q: Is the **nominal** frequency the same in different countries?
2.9. Measuring Frequency

• The instrument to measure frequency is *frequency meter*. However, most *digital* voltage and current sensors also measure frequency. Therefore, it is no longer common to have a separate frequency sensor.

• A frequency measurement is more accurate if the signal is *sinusoidal* with no or little distortion. Therefore, frequency is often measured based on voltage, as opposed to current that are prone to *distortion* due to nonlinear loads, etc. We will discuss the distortions and harmonics in voltage and current waveform in Chapter 4.
• An example for frequency measurements during normal grid operating conditions is shown below. The frequency measurements in this figure are made at a load location. In North America, frequency is typically maintained at 60 ± 0.036 Hz [132]. The frequency measurements in the figure below are generally within this range.
2.9.1. Generation-Load Imbalance

• Managing the operation of the electric grid includes a constant effort to balance electric power generation with electric power consumption.

• Unbalance between electric power generation and electric power consumption can affect the frequency of the power system:

\[ \text{Frequency} \approx (P_{\text{Generation}} - P_{\text{Consumption}}) \]

• The impact of unbalance between electric power generation and electric power consumption can be explained based on the electromechanical operation of turbine generators (next slide).
2.9. Measuring Frequency

- If we lose a large generator, then frequency suddenly **decreases**.
- If we lose a large load, then frequency suddenly **increases**.
- Since each inter-connection is one very large circuit, a disturbance can affect the frequency all across the **same inter-connection**.

- For example, losing a generator in California can cause a drop in Frequency all across the Western Interconnection in the United States.
2.9. Measuring Frequency

• If electric power consumption exceeds electric power generation, then the turbine generators *slow down slightly*, converting some of their mechanical kinetic energy (*inertia*) into extra electric power to help meet the increased load. Since the frequency of the generated power is proportional to the turbine’s rotor speed, increasing electric power consumption results in a drop in the system frequency.

• If electric power consumption falls below electric power generation, then the turbine generators *speed up slightly*, increasing the frequency.

• It takes a few seconds for a turbine generator to increase or decrease its mechanical torque to increase or decrease its rotor speed. Several generators may have to adjust their generation level for the system frequency to be ultimately brought back to its normal level.
• **Example 2.23**: Figure below shows the impact of a generator tripping in the Western Interconnection that caused a sudden drop in frequency.

![Frequency Response Graph](image)

- The frequency drops to as low as 59.91 Hz. The rate of frequency drop decays within three to four seconds because of the *inertial response* of the system. For example, when frequency drops, motor loads slow down, which results in less generation-load mismatch.
• **Example 2.23 (Cont.):** Next, the governor control of generators starts to arrest and then halt the frequency decline within 8–10 seconds. This procedure is known as *primary frequency response* (governor response).

• Finally, the secondary frequency response kicks in by Automatic Generation Control (AGC), which deploys *regulating reserves*, i.e., the fast-responding generators, to bring frequency back to around 60 Hz.
The exact amount of the change in the frequency may vary based on the time of day and the season. Nevertheless, one can use the *frequency response characteristic* (FRC) to estimate change in frequency as a result of a change in load-generation imbalance.

FRC is calculated using historical frequency measurements during generation and load loss events. The current estimates of FRC for all three North American interconnections are [134]:

- Eastern Interconnection: $-2760 \text{ MW} / 0.1 \text{ Hz}$
- Western Interconnection: $-1482 \text{ MW} / 0.1 \text{ Hz}$
- Texas Interconnection: $-650 \text{ MW} / 0.1 \text{ Hz}$
• The negative sign in FRC means there is an inverse relationship between generation loss and frequency change.

• For example, on average, a 1000 MW generation loss in the Western Interconnection causes a frequency change in the order of

\[ -1000 \times \frac{0.1}{1482} = -0.067 \text{ Hz}. \]

• As another example, on average, a 1000 MW load loss in the Eastern Interconnection causes a frequency change in the order of

\[ 1000 \times \frac{0.1}{2760} = 0.036 \text{ Hz}. \]

• Conversely, FRC can be used to estimate the amount of generator loss from frequency measurements. For example, the size of the lost generator in Example 2.23 is about \(-0.085 \times -1482/0.1 = 1260 \text{ MW}.\)
• **Inertia of Renewable Energy Resources:** Other applications of frequency measurements include: estimating the inertia in the power system [135], examining the impact of renewable generation on frequency response [136], and evaluating the performance of an interconnection to bring frequency back to normal within a five minute period after a generation loss or a load loss event [133].

• **Note:** Recently, synchronized phasor measurements are also used to measure power system frequency, in particular in a *synchronized fashion* across the same interconnection; see Section 3.3 in Chapter 3.
2.9. Measuring Frequency

2.9.2. Frequency Oscillations

• Recall from Section 2.6.1 that wide-area oscillations can affect the magnitude, phase angle, and frequency of voltage. We already saw the impact of wide-area oscillations on voltage magnitude in Example 2.14.

• Next, we will see the impact of wide-area oscillations on voltage frequency. We will see the impact of wide-area oscillations on voltage phase angle in Section 3.4.3 in Chapter 3.
• **Example 2.24**: Figure (a) shows a *damped oscillation in frequency measurements*. The duration of the oscillation is about six seconds. The largest peak-to-peak amplitude during the event is 10 mHz. The frequency of the most dominant oscillation mode is 1.23 Hz. This oscillation event could be detected using FFT analysis, as in Figure (b). The FFT is applied to the differential of the frequency to remove its DC offset. Event detection is triggered due to an increase in the FFT magnitude beyond a threshold. The peak of FFT magnitude is at 1.23 Hz.