Distribution Grid Reliability Analysis Considering Regulation Down Load Resources Via Micro-PMU Data

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***Abstract*—There is a growing interest by power system operators to encourage load resources to offer frequency regulation. There are several studies that evaluate the system-wide benefits of such load resource participation. However, the current literature often overlooks the potential adverse impact on power distribution feeders. This paper seems to address this open problem. We focus on a scenario where load resources offer *regulation down* service. We start by developing a novel *data-driven* approach to use distribution-level μPMU data to analyze *transient* load behaviours. Subsequently, we model the aggregate load transient profile, in form of an aggregate three-phase surge current profile, that is induced on a distribution feeder once a group of loads responds to a regulation down event. The impact of delay, e.g., due to sensing, communications, and load response, is taken into consideration. Distribution grid reliability is then analyzed based on different characteristics of typical over-current protection relay devices. Both *momentary* and *permanent* reliability indices are calculated. Case studies suggest that it is possible to jeopardize power distribution grid reliability if several regulation down load resources are on the same feeder. The probability of failure depends on the distribution grid protection system and the amount and distribution of delay in the regulation aggregation system.**

NOMENCLATURE

τ	Delay time: sensing, communications, etc.
t_{op}, t_{trip}	Operation time and tripping time
I, I_P	Input current and peak-up current
θ, θ_{max}	Disk travel and maximum disk travel
K_I	Constant that relates torque to current
m	Moment inertia of disk
K_d	Drag magnet damping factor
τ_F	Spring torque at maximum travel
τ_s	Inertia spring torque
K, α	Constant relay parameters
Φ	Set of annual line contingencies
Ψ	Set of annual regulation down contingencies
$\lambda^{p/m}$	Permanent/momentary interruption frequency
n	Number of interrupted customers
r	Interruption duration
N	Total number of customers
ρ	Probability of contingency by surge current

I. INTRODUCTION

Frequency regulation is a required ancillary service in power system operation in order to maintain the grid nominal frequency, e.g., at 60 Hz in North America, by balancing power generation and power consumption on a second-by-second basis. Regulation up service is concerned with increasing generation or decreasing consumption when frequency drops below a certain threshold. Regulation down service is concerned with decreasing generation or increasing consumption when frequency exceeds above a certain threshold, c.f. [1].

Traditionally, frequency regulation is offered by generator resources. However, it is now a growing trend in practice to encourage and facilitate offering frequency regulation by energy storage systems and controllable loads. Examples of controllable loads that are considered to offer frequency regulation include air-conditioning units [2], [3] and electric vehicles [4].

While the system-wide benefits of using load resources for frequency regulation are well-studied, e.g., see [3], the current literature often overlooks the impact on power distribution feeders. Note that, based on the hierarchical structure of the power grid, any system-wide service that is offered by a load resource is physically mediated by distribution feeders. Therefore, it is of crucial importance to examine whether and to what extent the use of frequency responsive loads could have any adverse effect on the operation of distribution systems.

Addressing the above open problem is the focus of this paper. We seek to understand the potential impact of offering *regulation down* service by load resources on the distribution grid *reliability*. We show that there is, in fact, the risk of jeopardizing power distribution grid reliability if the impact on distribution grid protection systems is not carefully taken into consideration when one designs market programs to utilize load resources to offer system-wide regulation down service. The contributions in this paper can be summarized as follows:

- A novel *data-driven* approach is developed to use experimental distribution-level μPMU data, i.e., voltage and current measurements, on three phases, at a sampling rate that is hundred thousand times faster than that of a typical smart meter, to analyze *transient* load behaviors.
- A new method is developed to model the aggregate load transient profile, in form of an aggregate three-phase surge current profile, that is induced on a distribution feeder once a group of load resources responds to a regulation down event. This is done by carefully analyzing several individual load surge current signatures as well as by taking into account factors such as sensing delay, communications delay, and load response delay.
- A distribution grid reliability analysis is conducted in presence of regulation down load resources by taking

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into account the models and different characteristics of different typical over-current distribution-level protection relay devices. Based on whether or not a recloser device is used in the distribution grid protection system, both *momentary* and *permanent* reliability indices are analyzed.
- The results in this paper suggest that it is indeed likely to jeopardize power distribution grid reliability if several regulation down load resources are located on one feeder. The actual likelihood depends on the underlying distribution grid protection system and the amount and distribution of delay in the regulation aggregation system.

Besides the literature on load-assisted frequency regulation, this paper is related also to papers that discuss the tradeoff between using more resources for energy and ancillary services versus the power grid stability and reliability. The focus so far has been mostly on the system-wide and transmission-level stability and reliability issues [5]. In contrast, in this paper, we are concerned with distribution-level reliability.

This paper is also related to the broad literature on distribution system reliability. While the majority of studies in this field addressed contingencies that are triggered by non-electric causes, such as a downed power line [6], our analysis is more comparable to the smaller group of papers, e.g., in [7, pp. 40-45], that addressed contingencies that are triggered by electric causes. In our case, the contingency is due to the aggregate surge current induced by a regulation down event.

II. Regulation down Frequency Responsive Loads

Frequency regulation is the mechanism of balancing power generation and power consumption in real time. It can be provided by on-line generation, storage, or load resources. Regulation helps maintaining the stability and reliability of the power gird [1]. Regulation services are either *regulation down* or *regulation up*. If generation is greater than demand, then frequency increases and regulation down service is needed. It is provided by a generator/load if it decreases/increases its generation/consumption. If generation is less than consumption, then frequency drops and regulation up service is needed.

The focus in this paper is on regulation down service. As an example, consider the frequency measurements in Fig. 1. Suppose we seek to keep frequency below 60.05 Hz. In that case, one *regulation down event* occurs, as marked by the arrow.

In practice, a regulation down event could be detected locally, i.e., by the regulation resource, or globally, i.e., by the regional grid operator. In case of the latter, a regulation down command is sent to the load or to the regulation down aggregator that represents the load in the wholesale regulation market.

Many independent system operators (ISOs) have already adopted mechanisms to allow load resources to offer regulation services. For example, the California ISO now has a program for Non-Generator Resources (NGRs) with Regulation Energy Management (REM) to enable resources with limited energy capacities to competitively bid in the regulation market [8]. The PJM inter-connection has also introduced the RegD and RegA regulation signals to encourage fast responding loads, generators, and storage units to provide regulation services [9].

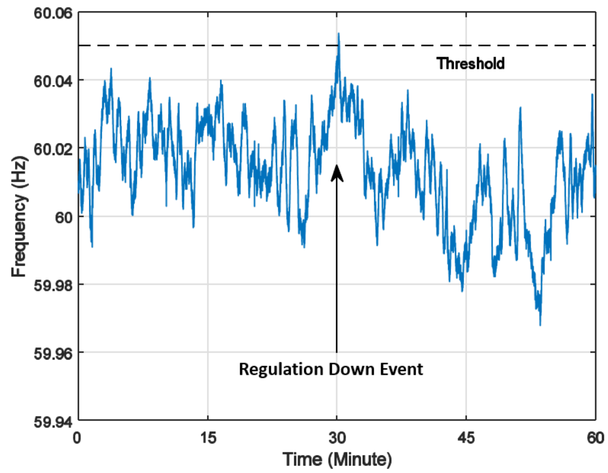

Fig. 1. A regulation down event occurs when frequency exceeds a threshold.

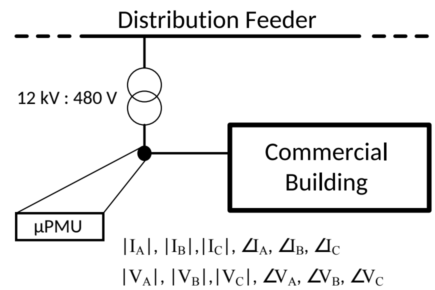

Fig. 2. Test system with a μPMU and its corresponding measurements.

III. Data-Driven Load Transient Models

The first step in analyzing reliability in power distribution systems with load resources providing regulation down service is to model the *transient* behavior of such loads at the moment that they are called upon, i.e., when a regulation down event occurs, as in Fig. 1. One option to obtain such model is to derive mathematical dynamic models for each load resource. This would require accurate knowledge of each load. Alternatively, in this section, we use a *data-driven* approach using data from a distribution-level phasor measurement unit, a.k.a a μPMU.

A. Individual Load Transient Signatures

μPMU is a new sensor device that is installed on distribution feeders, often at load or substation transformers, to provide precise time-stamped GPS-synchronized reading of voltage and current phasors, enabling thorough monitoring of the distribution grid conditions [10], [11]. These devices are gradually becoming commercially available [12]. In this paper, we use the data from a μPMU that is installed at the secondary side of a 12 kV to 480 V transformer at a commercial building in Riverside, CA. The sampling rate is 120 Hz, i.e., one sample every 8.33 msec. This is *hundred thousand times* faster than the once per 15 minutes reading rate of a typical smart meter.

As shown in Fig. 2, our μPMU reports four fundamental measurements on three phases (12 channels): voltage magnitude, voltage phase angle, current magnitude, and current phase angle. In this paper, we use *voltage phase angle* to calculate the

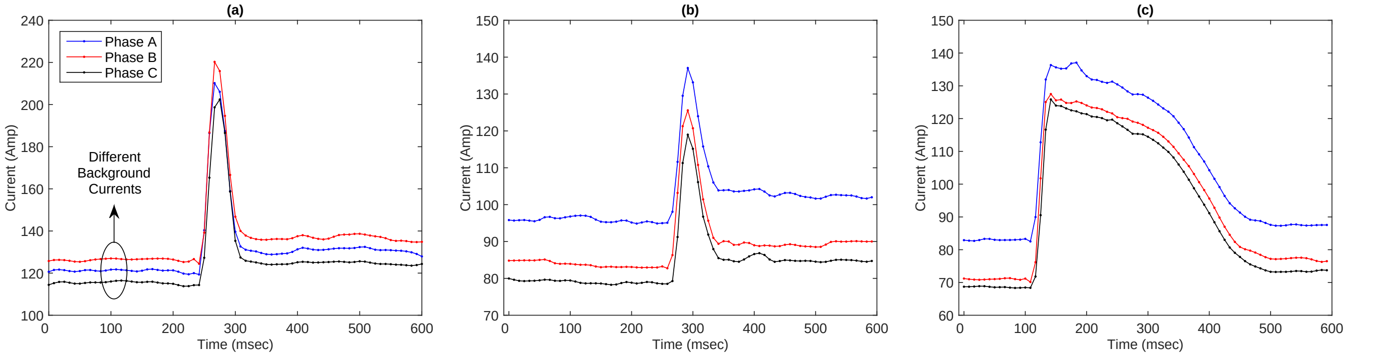

Fig. 3. The three *current surge signatures* that represent the three load types with significant current surges that are identified through analysis of μPMU data.

instantaneous voltage frequency. But more importantly, we use the *current magnitude* to understand the dynamics of the current surge when a major load device, such as an air-conditioner, is hypothetically called upon at a regulation down event. As we will explain in Section IV, we are particularly interested in analyzing current magnitude data, because most practical distribution-level protection relays are sensitive to the current.

The central idea in our data-driven load transient modelling approach is to examine the current magnitude data during one entire day, March 28, 2016, to identify and analyze all major *current surge signatures*. Note that, we had to analyze $120 \times 60 \times 60 \times 24 = 10,368,000$ samples of current data per each phase. In practice, each current surge may take from only a few milliseconds to only a few hundred milliseconds.

In total, our analysis identified 169 current surge signatures with at least 40% momentary increase in the current magnitude across all three phases. However, a closer look at the collected data revealed that these 169 current surge signatures can be classified into *three* groups, where the current surge signatures in each group can be seen as repetitions or slight variations of each other. In other words, in the end, we identified *three loads* that would be responsible for any major current surge in the commercial building of interest. They are shown in Fig. 3. All three signatures are believed to belong to the building Heating Ventilation and Air-Conditioning (HVAC) system. Accordingly, their corresponding three load units are good candidates to be recruited as frequency regulation load resources, see [2].

B. Aggregate Load Transient Profiles

Suppose there are N regulation down frequency responsive loads of $M < N$ different load types across a distribution feeder. In a typical distribution feeder in today's power systems, where the penetration of distribution generation resources is still relatively small, the instantaneous current that goes through the protection relay at the feeder head is the summation of the instantaneous current that is drawn by each load unit. Accordingly, in this paper, we use the model in Fig. 4 to calculate the transient current that goes through the protection relay at and a few milliseconds after a regulation down event would occur. In this figure, without loss of generality, we assume that $N > 3$ and the load resources are of the $M = 3$ load types in Figs. 3(a), (b), (c) in Section III-A.

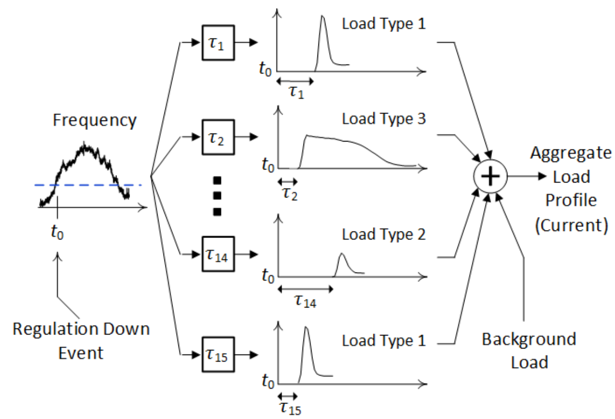

Fig. 4. The method of calculating the aggregate load current profile at a regulation down event under different load types and different delay values.

For the model in Fig. 4, the aggregate current load profile is decomposed into two components. The first component is the *feeder background load* that is the summation of the current that is drawn by all loads that do *not* offer regulation down service. At the millisecond time resolution, the feeder background load can be considered as a constant at and around the moment that a regulation down event occurs. The second component is a *aggregate current profile* of the N load units that *do* offer regulation down service. This second component is the one that generates the transient response at and around the moment that a regulation down event occurs. Note that, here, we are essentially modeling the loads at and around the moment that a regulation down event occurs as constant current, c.f. [13].

In practice, there would be a slight *lagging*, i.e., a very small delay (typically bounded by 4 seconds due to regulation market requirements), between the moment that the regulation down event occurs and the moment that the current surge signature appears for each regulation down frequency responsive load. For each load unit $i = 1, \ldots, N$ such delay is modeled as

$$\begin{aligned} \text{Delay } \tau_i = \; & \text{Sensing Delay +} \\ & \text{Communications Delay +} \\ & \text{Load Response Delay.} \end{aligned} \tag{1}$$

If frequency is sensed locally[1] by each loads, then commu-

[1]Local sensing could also introduce sensing error affecting the response [14].

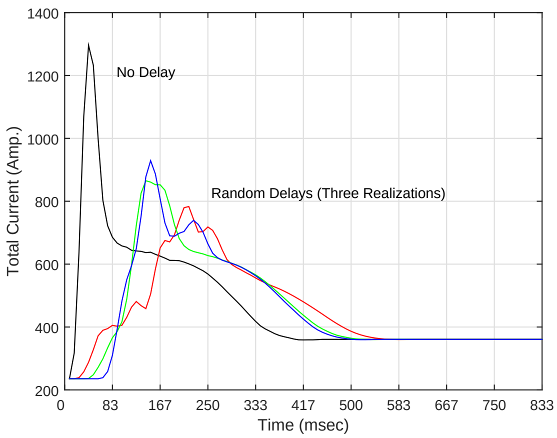

Fig. 5. Examples of the aggregate load transient profile under different delay scenarios. The regulation down event is assumed to occur at time zero.

nications delay is not a factor, but the sensing delay could be different at different loads. If regulation down commands are dispatched by a central entity, then communications delay could have significant impact. Regardless of the method of sensing/communicating, load response delay could be different for different loads based on their internal control mechanisms.

Suppose $N = 15$, where there are five regulation down load resources of $M = 3$ types. If $\tau_1 = \ldots = \tau_N = 0$, i.e., if there is absolutely no delay, then the aggregate load transient profile is obtained as marked in Fig. 5. This figure also shows three different realizations of the case where the delays are random with a Gaussian distribution and parameters $\mu = 100$ msec and $\sigma = 50$ msec. We can see that random delays can result in different aggregate load transient profiles.

IV. Analysis of Distribution Grid Reliability

A. Protection Over-Current Relay Models

In power system protection, relays provide the intelligence for identifying contingencies in order to control the operation of circuit breaker. A relay could be overcurrent, overvoltage, differential, etc. However, the most common relays at distribution level are overcurrent with and without reclosing capability [15]. The overcurrent relay operates, i.e., picks up, when the feeder current exceeds a predetermined threshold, a.k.a., the *pick-up current*. The overcurrent relays are categorized based upon their time-current characteristics (TCC), including instantaneous overcurrent relays, definite time over current relays, inverse time over current relays, and directional overcurrent relays. Unlike the first three types, the directional overcurrent relays provide adequate protection in weakly meshed distribution networks as well as in bi-directional network. However, in the context of typical radial or directional distribution systems, the inverse time relay is the most common one, which is also known as the inverse definite minimum time (IDMT) relay.

In IDMT relays, which are of interest in this paper, the TCC inversely depends on the current. Therefore, these relays are sub-categorized based on the inverseness, including short time inverse, moderately inverse, long time inverse, very inverse, and extremely inverse. The TCC of relays in pick-up/reset modes can be derived from the dynamic equation of the relay's induction disk rotation with respect to contingency current [16]:

$$K_I I^2 = m\frac{\mathrm{d}^2\theta}{\mathrm{d}t^2} + K_d\frac{\mathrm{d}\theta}{\mathrm{d}t} + \frac{\tau_F - \tau_s}{\theta_{max}}\theta + \tau_s, \tag{2}$$

The above equation is often approximated by disregarding the second derivation and the linear terms, as follows:

$$K_I I^2 \approx K_d\frac{\mathrm{d}\theta}{\mathrm{d}t} + \tau_s. \tag{3}$$

Using (3) to model the induction disk rotation around the pick-up current, we can now define the maximum disk rotation with respect to operation time of the relay in tripping mode as

$$\theta_{max} = \int_0^{t_{op}} \frac{\tau_s}{K_d}(I^2/I_P^2 - 1)dt. \tag{4}$$

The tripping time for each disk rotation is calculated as

$$\begin{aligned} t_{trip} &= K_d\theta_{max}/\tau_s\left(I^2/I_P^2 - 1\right) \\ &= K_{trip}/\left(I^2/I_P^2 - 1\right). \end{aligned} \tag{5}$$

Finally, the tripping criteria for each disk rotation becomes:

$$\int_0^{t_{op}} (1/t_{trip})dt = 1. \tag{6}$$

Note that, the operating time of relay t_{op} can be moved up/down by using time multiplier setting (TMS) and by imposing intentional delay time to provide different TCC for different relay technologies. Consequently, the following equation provides a modification of (5) with respect to the desired exponent of plug setting multiplier, TMS, and the intentional delay time:

$$t_{op} = TMS\left(\frac{K_{op}}{(I/I_P)^{\alpha_{op}} - 1} + L\right). \tag{7}$$

The above procedure can be repeated to obtain the reset time for $I \leq I_P$ via the reset time multiplier setting (RTMS).

The final result is as follows:

$$t_{reset} = RTMS\left(\frac{K_{reset}}{1 - (I/I_P)^{\alpha_{reset}}}\right). \tag{8}$$

The constant coefficients K_{op} and K_{reset} depend on the type of the relay and the standard being considered. For instance, IEEE, ANSI, and IEC suggest different coefficients for the very inverse curve [16], [17]. In particular, the intentional delay time is assumed to be zero under the IEC standard, while it is non-zero under the ANSI and IEEE standards. The TCC coefficients of tripping and resetting for most common relays are provided in Table I [17], and the corresponding inverse time characteristic curves (ITCCs) are depicted in Fig. 6.

B. Reliability Evaluation Considering regulation down

If the feeder is reinforced by a recloser, then the momentary average interruption frequency index (MAIFI) would increase. If there is no recloser, then the system average interruption frequency index (SAIFI) and the system average interruption duration index (SAIDI) would increase [6], [18]. A momentary

TABLE I
TCC COEFFICIENTS OF DIFFERENT IDTM RELAYS

Case	Inverse Time	Operation			Reset	
		K_{op}	α_{op}	L	K_r	α_r
I	Short†	0.02394	0.02	0.01694	2.261	2
II	Moderate‡	0.05150	0.02	0.11400	4.850	2
III	Long§	5.9500	2.00	0.18000	5.950	2
IV	Very Long‡	19.610	2.00	0.49100	21.60	2
V	Extreme‡	28.200	2.00	0.12170	29.10	2

† C02 Standard, ‡ ANSI/IEEE Standard, § C08 Standard

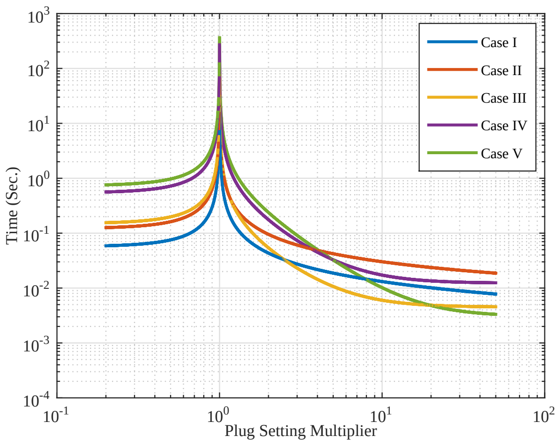

Fig. 6. The ITCCs of the different protection relays in Table I.

interruption is one that lasts between less than one to five minutes. SAIFI, SAIDI, and MAIFI indices are calculated as

$$\text{SAIFI} = \left(\sum_{i \in \Phi} \lambda_i^p n_i + \sum_{i \in \Psi} \rho_i n_i \right) / N, \tag{9}$$

$$\text{SAIDI} = \left(\sum_{i \in \Phi} \lambda_i^p r_i n_i + \sum_{i \in \Psi} \rho_i r_i n_i \right) / N, \tag{10}$$

$$\text{MAIFI} = \left(\sum_{i \in \Phi} \lambda_i^m n_i + \sum_{i \in \Psi} \rho_i n_i \right) / N. \tag{11}$$

The first term in the numerator expresses the interruption frequency/duration due to fault occurrence in the main feeder and laterals. The second term defines the imposed interruption frequency/duration due to regulation down surge current.

V. CASE STUDIES

The dynamic response of the five standard relays in Table I are examined based on the parameters of a typical 12.47 kV distribution feeder. The feeder is considered under 7 MVA load and serves 400 customers. The main feeder is four miles long. The feeder also has additional four miles of laterals [19].

Permanent and momentary failures could occur in lines, transformers, or busbars. However, in this study, the reliability evaluations are limited to interruptions corresponding to the fault in lines and unnatural failures due to the current surges only. To such aim, the probability of annual permanent and annual momentary failures are set to 0.065 and 0.06 faults per km, respectively [18]. The fault detection process is considered to be 30 minutes while customers experience interruption due to surge currents. Line repair time is three hours [18]. To calculate the permanent and momentary reliability indices, it is assumed that the under-study feeder is reinforced by one circuit breaker and one recloser. The number of regulation down load resources for all three types are set to be equal as 27.33 percent of loads.

A. *Impact of Surge Current on Different Relay Types*

In this section, we evaluate the reliability of the under-study network reinforced by the mentioned relays. The relays and their settings are presented in Table II, where the RTMS is 0.025. In order to reflect randomness in the aggregated surge current, the problem is solved 10,000 times and the probability of tripping for each relay type is calculated under a Gaussian distribution with $\mu = 100$ and $\sigma = 50$ msec. For each random scenario, the problem is solved for two different cases: with and without recloser. Recall that, if the feeder is reinforced by recloser, then the current surge may affect momentary reliability indices, otherwise the permanent indices are affected.

Fig. 7 shows the disk position for a single surge current, corresponding to a hypothetical single regulation down event. The feeder current starts from a background current, which is 72.66% of the nominal current. The total current then reaches the feeder nominal loading once the current surge is damped. There is a surge current at $t = 108$ msec. In this case, no relay responds to the surge since the surge magnitude is less than the pick-up currents. Hence, the disk does not rotate at the tripping/resetting direction. The disk can rotate at resetting direction, if it detects a fault current before the present sampling. At $t = 130$ to 140 msec, the current is larger than the pick-up current. Thus, the disk starts to rotate with different speed, based on its TCC curve. The disk speed (slope of disk position) depends on the sampling current. The relays in cases I, II, and III trip at 208, 275, and 341 msec, respectively. The relays in cases IV and V do not trip in this test.

Next, we conduct a probabilistic annual reliability evaluation, assuming one regulation down event per day. The results are shown in Table II. Here, the reliability indices associated to under-study cases are calculated through (9)-(11). As an example, consider the case where we use the short time inverse relay. In this case, on one hand, the regulation down events would increase SAIFI by about 7.3 faults per year. As a result, the customers would experience 3.65 hours additional interruption due to the potential regulation down induced surge currents. On the other hand, if the short time inverse relay is reinforced by recloser, then the customer would experience only 7.3 momentary interruptions, lasting less than one minute.

B. *Impact of Delay Randomness on Tripping Probability*

In this section, we examine how the tripping probability is affected due to changes in the delay distribution parameters. The results are shown in Fig. 8, where $\mu = 4\sigma$ varies from zero to 250 msec. All relays trip when the mean and variance of delay are small, i.e., $\mu = 4\sigma \leq 50$ msec. However, the tripping probability reduces as the mean and variance of delay increases. The rate of reduction depends on the type of relay. For example, *Case I-III* are more sensitive to current changes

TABLE II

DIFFERENT RELAY CASES AND SETUPS AND THEIR CORRESPONDING RELIABILITY RESULTS UNDER REGULATION DOWN EVENTS

Case / Inverse Time		Relay Setting		Tripping Probability	Reliability Indices Deviaiton			Annual Reliability Indices		
					Without Recloser		With Recloser	Without Recloser		With Recloser
		TMS	Pick-up %		ΔSAIFI	ΔSAIDI	ΔMAIFI	SAIFI	SAIDI	MAIFI
I	Short	0.050	160	0.0200	7.3000	3.6500	7.300	8.1369	6.1606	8.0725
II	Moderate	0.025	160	0.0190	6.9350	3.4675	6.9350	7.7719	5.9781	7.7075
III	Long	0.030	160	0.0120	4.3800	2.1900	4.3800	5.2169	4.7006	5.1525
IV	Very Long	0.025	125	0.0090	3.2850	1.6425	3.2850	4.1219	4.1531	4.0575
V	Extreme	0.025	120	0.0010	0.3650	0.1825	0.3650	1.2019	2.6931	1.1375

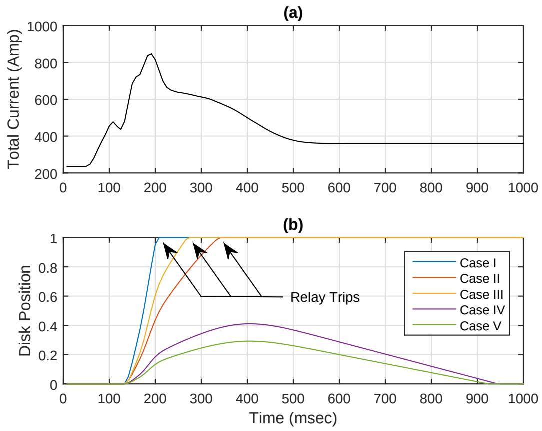

Fig. 7. Changes in relay disk position in response to a regulation down event induced aggregate surge current: (a) total surge current; (b) disk positions.

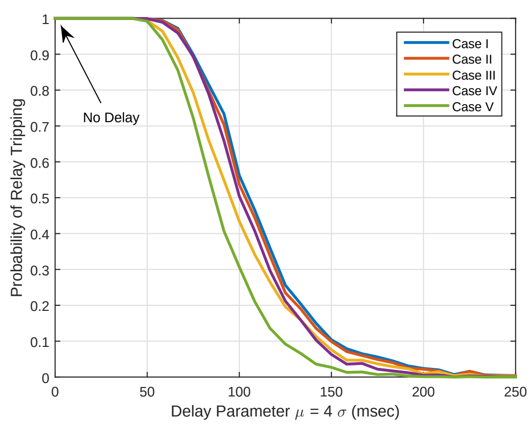

Fig. 8. Impact of delay distribution parameters on tripping probability.

than *Case IV* and *V*, since the K_{op}, K_r, and intentional delays of the two latter cases are larger than the first three types.

VI. CONCLUSIONS

This paper takes the first steps in analyzing the reliability of power distribution systems in presence of regulation down load resources. Using experimental μPMU data and by taking into account the exact current surge signatures of practical regulation-eligible load types, the characteristics of practical distribution-level protection relay devices, and the impact of delay, e.g., due to sensing, communications, and load response, we showed that it is possible to jeopardize distribution grid reliability if several regulation down load resources are on the same feeder. Both momentary and permanent reliability indices are then calculated. The results in this paper shall raise flags to the power system operators and utilities to carefully take into consideration the potential risks and reliability concerns at the distribution level when they design programs to utilize load resources to offer system-wide regulation down services.

We plan to extend this analysis by investigating the reliability consequences of offering regulation down service by distributed energy resources (DERs) such as energy storage systems.

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