Optimal Market Participation of Distributed Load Resources Under Distribution Network Operational Limits and Renewable Generation Uncertainties

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***Abstract*—Distributed load resources are encouraged nowadays to actively participate in the energy market. As a part of the distribution system, they affect the power flow pattern of the network and interact with intermittent renewable generation in the distribution system. In this regard, one fundamental challenge, not yet addressed, is to derive an optimal market participation model, under the demand bidding paradigm, that *systemically* accounts for the *operational limits of a physical distribution grid* considering *uncertainty* associated with both the *electricity market* and *distribution network system*. Accordingly, this paper addresses the optimal demand biding under uncertain market and distribution system data and network operational limits. Assuming a price-taker distribution utility with renewable energy, inflexible and deferrable loads and a two-settlement market model, we develop a two-stage robust stochastic bidding formulation solved using a decomposition algorithm. We derive optimal bid curves that minimize energy procurement cost and fully comply with the operational standards of the distribution network. Moreover, novel indexes are proposed to help the utility evaluate the operational performance of its network with regard to deferrable loads and renewable resources. Finally, we illustrate the advantage of the proposed model from a set of numerical experiments on an example system and the 33-bus system.**

***Keywords*: Demand bidding, deferrable load, renewable resource, stochastic programming, two stage robust optimization.**

NOMENCLATURE

$\Omega_p(n)$	Set of the precedent nodes connected to n
$\Omega_d(n)$	Set of the decedent nodes connected to n
$\mathcal{N}_b$	Set of branches
$\mathcal{N}$	Set of buses
Ψ_r	Uncertainty set of renewable generation
N_r	Number of renewable resource buses
k	Index for scenario
K	Number of scenarios
n, m	Index for distribution nodes
nm, jn	Index for branches
$0, t, i$	Indexes for the substation, time step and the deferrable loads
Δt	Duration of time steps t in hour.
π_k	The probability of scenario k
$\underline{P}^R, \overline{P}^R$	the lower and upper limits of power traded in the real-time market at the substation
$\lambda^{D/R}$	Market price in day-ahead/ real-time market
d_n, q_n	Real and Reactive demand at node n
γ	Equals $tg(cos^{-1}(\text{power factor}))$
α, β	The start and end times of deferrable load
e	Total energy procurement of deferrable load
$r_{jn},\ x_{jn}$	Resistance and reactance of line jn
$\overline{U}_n, \underline{U}_n$	Upper and lower bounds of square voltage magnitude at bus n
$\overline{p}, \overline{q}$	Feeder's active and reactive capacity limits
$\Gamma^{r/R}$	Uncertainty budget related the renewable generations/ real-time market prices
$\bar{g}^r_n, \hat{g}^r_n$	Upper bound and deviation from it for renewable generation at node n
M, ϵ	Sufficient large and small numbers
$P^{D/R}$	Power component bid in the day-ahead/real-time market at the substation
Q	Reactive power at the substation
d^d_n	Active power of deferrable load at bus n
g^r_n	Real power of renewable resource at node n
p_{nm}, q_{nm}	Active and reactive power flow of line nm
v_n	Voltage magnitude of bus n
U_n	Squared voltage magnitude of bus n

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I. INTRODUCTION

Deferrable loads and distributed energy resources (DERs) can help electric utilities reduce their cost in energy trading and when participating in the electricity market. The benefits of using deferrable loads and DERs have been the subject of many recent studies, e.g., [1]–[10]. The idea is to formulate the bidding optimization problem of a utility, treated as an energy purchaser, to minimize the cost of energy procurement and to derive the optimal demand bid.

One fundamental challenge is to derive a utility market participation model that accounts for the operational constrains of distribution system considering uncertainty associated with the electricity market and distribution network system. Recall that beside cost management, a utility is also concerned with maintaining its network security and serving customers at satisfactory operating standards.

Ignoring operational constraints of the distribution system in the bidding problem can lead to costly or technically infeasible results not realizable in practice. One such situation can occur when the peak load hour of a distribution feeder does not coincide with the peak hour of market price. An example is shown in Fig. 1, for the case of a distribution feeder in Riverside, California, along with the price of the day-ahead market at the node connecting this feeder to the electricity market. We can see that if the utility shifts its deferrable loads to low-price hours in an attempt to reduce its costs, then it may inevitably breaches safety bands of voltage and/or current along the feeder. The problem gets even more complicated if the distribution network also serves several renewable generation units with intermittent generation outputs. This illustrative example shows only the tip of the iceberg about the potential issues that may arise at distribution level, if DERs are scheduled considering only market conditions.

Inspired from the above example, this paper aims to take a new look at the demand bidding problem. In this view, we extend the conventional utility optimal bidding problem in the literature to propose a stochastic robust network-constrained bidding approach in which the uncertainty of the operating conditions, market price, and the effect of deploying DERs on distribution feeders are considered simultaneously. To the best of our knowledge, there is no prior study to analyze the role of the distribution network constraints and derive optimal bid curves considering uncertainty in both distribution networks and electricity markets. A two-stage optimization problem is developed, where decision making is carried out in two steps, *before* and *after* uncertainty revelation. For modeling uncertainty, we use a hybrid stochastic-robust scheme as in [11], [12] so as to take the advantage of both methods.

A. Summary of Contributions

Given the variability of system loads, renewable energy, and market price, a deterministic demand bidding model may lead to undesired results if the randomness of the input parameters is not considered. Moreover, the role of network operating conditions and deferrable loads are to be captured in deriving bidding strategy of the utility. Ensuring the solution robustness and considering enough modeling details are thus important to obtain practically feasible and optimal results. This paper aims to address this specific problem; it presents a two-stage robust stochastic optimization model for the optimal bidding of a distribution utility with DERs, inflexible and deferrable loads, that trades energy in the day-ahead and real-time markets. The model includes network constraints and uncertainties of different resources mentioned above. The optimization problem is thus derived and solved using a decomposition method.
In view of the discussion above, our major contributions are:

1) On the modeling aspect, we develop the first robust stochastic demand bidding model for a distribution utility that considers uncertainty and captures the operational limits of the distribution network. From this standpoint, our model contributes to the literature by adding *realism* to the bidding process to ensure that market participation will *not* jeopardize the quality of service in the distribution network.
2) We then propose novel indexes that can help the utility evaluate the operational performance of its distribution feeders when deferrable loads and intermittent DERs are deployed. These indexes provide a good measure on how these new resources affect the operational constraints of the distribution network while the utility actively participates in the electricity market. Moreover, those indexes can show how *good* a feeder can follow the electricity market signals using its DERs *without* adverse effects on the distribution network constraints.

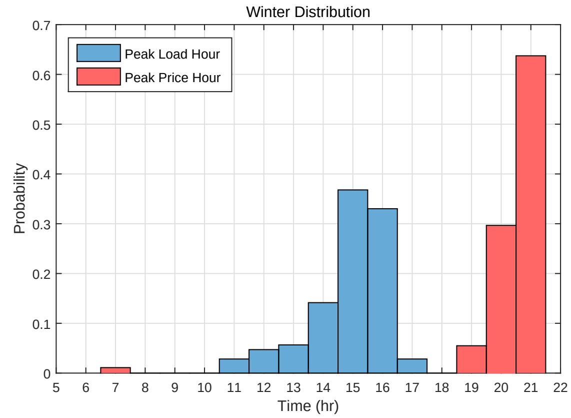

Fig. 1. The distribution of the peak load of a real-world feeder in Riverside, CA and the day-ahead market peak price at its market node in winter 2015.

3) Insightful case studies are presented. First, we show that ignoring network constraints may result in technically infeasible solutions or costly bidding strategies in an uncertain environment. We then analyze the impact of deferrable load locations and their penetration levels on the optimal bidding performance.

B. Related Work

In a number of papers published recently on the bidding problem of a utility (or a virtual power plant), information gap theory [8], stochastic programming [13], [14], and robust optimization [15]–[17] have been used. However, previous studies neglect the potential impact of the DER operation on the distribution feeder. That is, they did not consider the fact that demands and DERs are located in the distribution network and have direct impact on the operational conditions of such network. Moreover, unlike in [15], [16], where static robust optimization is applied, we develop a two-stage stochastic robust program to make our model more flexible and less conservative. Finally, robust optimization is used in [17] to hedge against uncertainty and to mitigate financial risks in the bidding process while the proposed model is concerned with removing the risk of distribution network operational limit violations.

In [18]–[20] the distribution network constraints are represented but they are based on game-theoretic method without considering uncertainties. In [21], robust optimization is used for operation of distribution girds.

Finally, although a number of papers have addressed the issue of randomness in the optimal demand bidding problem [7], [17], [22], no prior report has studied the network constrained counterpart considering deferrable loads and uncertain inflexible loads and renewable resources in the bidding strategy.

It is worthwhile to mention that no prior work in the demand bidding problem, to the best of our knowledge, proposed new indexes or extended the new indexes to evaluate the feeder performance in response to market conditions under the uncertain situation and in presence of the deferrable loads and renewable generations.

II. Problem Formulation

Consider a utility that is responsible for reliable and cost-efficient operation of a power distribution system and serves a number of deferrable and a number of inflexible loads as well as a number of distributed renewable generation resources. The utility procures energy from a wholesale electricity market by submitting demand bids. The utility is in charge of scheduling deferrable loads through a demand response program.

The utility must simultaneously address two challenges. On one hand, it must participate strategically in the wholesale electricity market to effectively procure energy at low cost. On the other hand, it must maintain the reliable operation of the distribution system. Accordingly, the goal of the analysis in this paper is to derive the optimal bidding strategy of the utility in the day-ahead market, together with the optimal schedules of the deferrable loads, all by considering the loads and renewable generation uncertainties and network constraints.

A. Electricity Market Model

Consider a two-settlement electricity market, such as California ISO, consisting of day-ahead and real-time markets. The demand bid at each market interval, such as each hour, has two components, namely, *power quantity* and *price*. The bid is submitted at closest point of participation at the nodal electricity market, which is substation. We assume a price-taker market participation scenario, where the price component of the demand bid is sufficiently large to assure that the bid is cleared. Accordingly, the cleared nodal market prices at the substation are taken as external parameters by the utility to manage its energy procurement cost. Note that, the price-taker assumption is valid in practice. For example, over 90% of the current demand bids in the California ISO market are of type self-schedule, i.e., they are price-taker [23]. In other words, once the demand MWh bid is determined in the day-ahead market using the optimization problem, then the utility submits it to the day-ahead market with a large price component no less than the predicted price.

Day-ahead market prices can often be forecasted with high accuracy in practice, c.f. [24]; therefore, they are considered deterministic in this paper. In contrast, the prices in the real-time market are uncertain and volatile; therefore, they are considered stochastic.

The power component of a demand bid submitted to the *day-ahead* market is by definition a non-negative quantity, i.e., $P^D[t] \geq 0$. However, given the volatile nature of the real-time market prices, and due to the network operational constraints, the power component of the bid submitted to the *real-time* market can be both positive or negative, where a negative bid indicates the supply offer for selling energy which arises from the excessive power generation or excessive power purchase from the day-ahead market. Accordingly, as a typical assumption in this context, e.g., see [22], we assume the power component of the bid submitted to the real-time market at time interval t has an upper bound and a lower bound in order to relieve the stress from the utility:

$$\underline{P^R}[t] \leq P^R[t] \leq \overline{P^R}[t], \ \ \forall t, \tag{1}$$

We assume in this paper the utility can only purchase energy from the day-ahead market. However, by increasing the renewable generation penetration, some utilities may also sell energy to the market. In such case, the constraint $P^D \geq 0$ changes to $P^D \geq \underline{P}$ where a negative $\underline{P}$ indicates the upper bound of the supply bid in the day-ahead market [16].

B. Deferrable Load Model

In addition to deciding on its bids, the utility has an extra degree of flexibility to schedule its deferrable loads to manage its network condition and the energy cost. Each deferrable load i is characterized with three parameters α_i, β_i and e_i. Parameters α_i and β_i denote the beginning and the end of the acceptable operating time intervals, where $\alpha_i < \beta_i$. Parameter $e_i \geq 0$ denotes the amount of energy that load i needs to consume during operation period. It is required that

$$\sum_{t=\alpha_i}^{\beta_i} d_i^d[t] \times \Delta t = e_i, \qquad d_i^d[t] \geq 0, \ \ \forall i, t, \tag{2}$$

where Δt denotes the length of the market interval. Some examples of deferrable loads include: charging electric vehicles, intelligent pools, and certain industrial equipment [7]. Note that, if only a portion of the load is deferrable, then we can conceptually divide the load to deferrable and inflexible.

C. Distribution Network Model

Given the feeder topology and the location of renewable generators and deferrable loads, the utility must satisfy power flow constraints. There are various well-known linearized DistFlows in the literature such as [25], [26] which could be used for our purpose. Without loss of generality, we use the linearized DistFlow equations drom [26], which are widely used in the literatures, e.g. [27]–[29], represented by

$$\sum_{m \in \Omega_d(n)} p_{nm}[t] = \sum_{j \in \Omega_p(n)} p_{jn}[t] - d_n^d[t] - d_n[t] + g_n^r[t] \qquad \forall n, t, \tag{3}$$

$$\sum_{m \in \Omega_d(n)} q_{nm}[t] = \sum_{j \in \Omega_p(n)} q_{jn}[t] - \gamma_n^d d_n^d[t] - q_n[t] + \gamma_n^r g_n^r[t] \ \forall n, t, \tag{4}$$

$$U_n[t] = U_j[t] - 2(r_{jn} p_{jn}[t] + x_{jn} q_{jn}[t]), \ \forall n, j \in \Omega_p(n), t, \tag{5}$$

where γ_n^d and γ_n^r are the coefficients converting the active power of the deferrable load and renewable generator at node n to their reactive power; and $U \triangleq v^2$ is the square of the voltage. The sets $\Omega_p(n)$ and $\Omega_d(n)$ represent the precedent and the decedent nodes connected to n, respectively, and p_{jn} shows the branch connecting node j to node n. Moreover, the squared voltage magnitude at each node and the flowing power through each line are restricted to upper and lower limits:

$$\underline{U}_n \leq U_n[t] \leq \overline{U}_n, \qquad \forall n, t, \tag{6}$$

$$\underline{p}_{nm} \leq p_{nm}[t] \leq \overline{p}_{nm}, \ \ \forall nm \in \mathcal{N}_b, t, \tag{7}$$

$$\underline{q}_{nm} \leq q_{nm}[t] \leq \overline{q}_{nm}, \ \ \forall nm \in \mathcal{N}_b, , t. \tag{8}$$

Note that, obtaining all characteristics of the voltage magnitude from the squared voltage magnitude is straightforward. Finally, the active and reactive power balance must hold at the substation where active power is purchased from the market:

$$P^D[t] + P^R[t] = \sum_{0m} p_{0m}[t], \ \forall t, \tag{9}$$

$$Q[t] = \sum_{0m} q_{0m}[t], \qquad \forall t. \tag{10}$$

D. Characterization of Uncertainty

In this paper, two different approaches are utilized to incorporate different sources of uncertainties, depending on how well they can be predicted and how their variations may affect power quality level of the distribution network. For example, the variation in real-time market price may not cause a problem to the power quality of distribution network, therefore, such price uncertainty can be treated as soft constraints. In contrast, erroneous prediction of the renewable generation level may affect quality of service and even unexpected service interruption, which must be treated as hard constraints.

We address the uncertainty pertaining to real-time market prices and inflexible loads through finite discrete random scenarios (in the context of *stochastic programming*), denoted by λ_k^r and d_{nk}, where $k = 1, ..., K$ is the scenario index.

We also model the randomness in renewable power generation g_{nk}^r through continuous uncertainty sets (in the context of *robust optimization*) as follows:

$$\Psi_r = \{g_{nk}^r = \bar{g}_n^r - w_{nk}^r \hat{g}_n^r, \ 0 \leq w_{nk}^r \leq 1, \\ \sum_n w_{nk}^r \leq \Gamma^r. \ \forall \ k, \forall \ n = 1, ..., N_r\}, \tag{11}$$

The last inequality in (11) is the budget constraint in robust optimization. Parameter $\Gamma^r \geq 0$ serves as a budget to control the maximum number of renewable resources that can deviate from their nominal values. If $\Gamma^r = 0$, then renewable power generation is assumed fixed at its nominal value and the problem is no longer a robust program. If $\Gamma^r = N_r$, then the system is protected against all the upper-bound deviations of the renewable resources. Between these two extreme cases, variable w_k^r is introduced to define the scaled deviation of the renewable resource from the estimated value $\bar{g}_{nk}^r$.

It is worth clarifying that, if *all* the uncertainties are modeled through robust optimization, then the model may be *too conservative* in some cases, as the robust program considers the worst case scenarios even for soft conditions. Similarly, if *all* the uncertainties are modeled through stochastic programming, then we may miss to plan adequately for those scenarios that may have serious adverse impact on the distribution network. Moreover, stochastic programming needs the information of the probabilistic distribution of data which may not be always available. Therefore, we propose to use a hybrid approach by considering both stochastic scenarios and robust formulation to add flexibility to the model.

E. Robust Stochastic Formulation

We formulate the distribution network-constrained optimal demand bidding problem in the presence of deferrable loads and uncertain renewable generation resources, as a two-stage optimization problem, where decision making is carried out in two subsequent steps, *before* and *after* uncertainty revelation. In the first-stage, the optimal day-ahead energy bid $P^D[t]$ is determined with the goal of minimizing the cost in the day-ahead market cost and the expected cost in the real-time market. Next, given the optimal bid in the day-ahead market, in the second stage, the real-time energy bid $P_k^r[t]$ is computed. However, as indicated in Subsection II-D, DER output is not known and can take any value in the set described in (11). To protect the utility against the worst case realization of the DER output, we present the following robust stochastic network constrained optimal bidding problem:

$$\mathop{\textbf{Min}}_{P^D \geq 0} \ \sum_{t=1}^{t=T} \Bigg(\lambda^D[t] P^D[t] \Delta t + \\ \mathop{\textbf{Max}}_{g^r \in \Psi_r} \mathop{\textbf{Min}}_{\mathcal{F}} \sum_{k=1}^{k=K} \pi_k[t] \lambda_k^R[t] P_k^R[t] \Delta t \Bigg), \tag{12}$$

where Ψ_r is defined in (11) and we have

$$\mathcal{F} = \{(1) - (10) \ \forall \ k = 1, ..., K\}. \tag{13}$$

The objective function in (12) is the total daily cost of energy procurement, i.e., the deterministic cost of energy procurement from the day-ahead market, plus the robust stochastic cost of energy procurement from the real-time market. Note that, the uncertainty of the day-ahead market price and its impact on the bidding problem are discussed in [30]. If needed, our model can capture uncertainty also in the day-ahead market prices. This would require slight modification in the objective function in (12, where different discrete scenarios for the day-ahead market prices λ^D should be considered. If needed, notation P^D can also change to P_k^D. The inner most minimization problem, which is the second stage decision problem in robust optimization, treats $P^D[t]$ from the first stage as well as the revealed uncertainty as constant; it accordingly minimizes the expected cost of energy purchased (sold) in real-time market. The limit on the day-ahead demand bid is determined in the first stage, which is $P^D[t] \geq 0$. Sets Ψ_r and $\mathcal{F}$ are defined in relationship with the uncertainty set of the renewable resources and with the distribution network constraints, respectively.

Note that, all constraints in problem (12)-(13) are indexed by k, meaning they must hold for every scenario. The decision variable pertaining to the day-ahead market includes $P^D[t]$ and those related to the real-time market are $P_k^R[t]$, $Q_k[t]$, $p_{mnk}[t]$, $q_{mnk}[t]$, $v_{nk}[t]$, and $d_{ik}^d[t]$. Variables g_k^r and w_k^r are used for modeling the worst case uncertainty scenario. The formulation in (12) incorporates both stochastic programming and robust optimization methods that we discussed in Subsection II-D. In the former, bidding decisions are made to minimize the expected cost for the utility. In the latter, the worst-case realizations of renewable production is anticipated. Note that, variables g_k^r and w_k^r are used for modeling the worst case scenario. It is relevant to note that the proposed two stage robust method is preferred when the utility needs to hedge against the adverse impacts of the randomness of the DER output and other network operating conditions [31].

III. Solution Method

The multi-stage optimization problem (12) is difficult to solve [32]. Thus, this section is dedicated to solve problem (12). First, in order to have a clear presentation, we recast optimization program (12) in the following compact form:

$$\underset{y\geq 0}{\textbf{Min}}\; b^T y + \underset{u\in\Psi_r}{\textbf{Max}}\; \underset{x}{\textbf{Min}} \sum_k a_k^T x_k, \tag{14a}$$

$$Ay + B_k x_k = 0 \qquad \forall k, \tag{14b}$$

$$C_k x_k = u_k \qquad \forall k, \tag{14c}$$

$$D_k x_k \leq f_k \qquad \forall k. \tag{14d}$$

Note that, y denotes the vector of all the first stage variables; and u_k and x_k denote the recourse variables. For each k, u_k includes the summation of a parameter and variable w_k. Constraints (14b)-(14d) stand for the ones in set $\mathcal{F}$ in (13).

In this paper, we propose to solve the min-max-min problem in (14) using the *column and constraint generation* (CCG) method [32]. The idea is to implement the solution procedure in a *master* and a *subproblem* framework. The master problem is a relaxed version of the original problem in (14) and the subproblem is the middle-level and bottom-level problems.

A. Subproblem: Obtaining Upper Bound

Given a solution for the top-level variables $y^\star$ at iteration l, we solve the following problem in each scenario k:

$$\underset{u_k\in\Psi_r}{\textbf{Max}}\; \underset{x_k}{\textbf{Min}}\; a_k^T x_k, \tag{15a}$$

$$\textbf{s.t. } Ay^\star + B_k x_k = 0 : \mu_{1k}, \tag{15b}$$

$$C_k x_k = u_k \quad : \mu_{2k}, \tag{15c}$$

$$D_k x_k \leq f_k \quad : \mu_{3k}. \tag{15d}$$

Dual variables are indicated after each constraints. Since the feasible set in (15b)-(15d) is a polyhedral and $a_k^T x_k$ is linear, the inner minimization problem in (15) is a linear program, for which *strong duality* holds. Accordingly, we can replace the inner minimization problem with its equivalent dual optimization problem [33, pp 224-227]. Hence, the $\max - \min$ structure in problem (15) can be converted to a $\max - \max$, or simply a $\max$, optimization problem. As a result, for each scenario k, we can reformulate problem (15) as:

$$\underset{u,w,\mu}{\textbf{Max}}\; (-Ay^\star)^T \mu_{1k} + (u_k)^T \mu_{2k} - f_k^T \mu_{3k}, \tag{16a}$$

$$\textbf{s.t. } - D_k^T \mu_3 + C_k^T \mu_2 + B_k^T \mu_1 = a_k, \tag{16b}$$

$$\mu_{3k} \geq 0, \tag{16c}$$

$$u_k = \bar{u} - w_k \hat{u_k}, \tag{16d}$$

$$w_k \in [0,1]^{N_r}, \sum_n w_{nk} \leq \Gamma^r. \tag{16e}$$

Algorithm 1: solving problem (14)

Initialization: $LB \leftarrow -\infty$, $UB \leftarrow \infty$ and $l \leftarrow 0$
repeat
    Solve master problem (17) to obtain $(x^{j\star}, \xi^\star, y^\star)$.
    Update $LB \leftarrow \max\{LB, O_{\text{master}}\}$.
    Solve subproblem (16) to obtain $(u^\star, w^\star, \mu^{j\star})$.
    Update $UB \leftarrow \min\{UB, b^T y^\star + \sum_k O_{\text{sub},k}\}$.
    $l \leftarrow l + 1$
until $UB - LB \leq \epsilon$
return UB *and* $(x^\star = x^{(j=l)\star}, u^\star = u^{(j=l)\star}, y^\star)$

Here, we have also explicitly stated the constraints for each scenario k in uncertainty set Ψ_r as (16d)-(16e). Problem in (16) is a bi-linear program, where bi-linearity is due to the term $(u_k)^T \mu_{2k}$ in the objective function. Since the uncertainty set Ψ_r is a budget set which is polyhedral, the optimal solution of problem (16) occurs at the *extreme points of its uncertainty set*. Thus, the bi-linear program in (16) can be transformed into an equivalent mixed integer linear program, c.f., [34], [35] which can be solved effectively for each scenario k.

For use in the next subsection, let $O_{\text{sub},k}$ denote the optimal objective value of the subproblem for scenario k. Using the summation of $O_{\text{sub},k}$ over all scenarios k, we can obtain an *upper bound* for the global solution in problem (14).

B. Master Problem: Obtaining Lower Bound

Suppose we are at iteration l. Let $u_k^{j\star}$ denote the optimal solution of u_k that is obtained from subproblem (16) at a prior iteration $j-1$, for all $j \in 1, ..., l$. We formulate the master problem in problem (14) as:

$$\underset{y\geq 0,\xi,x^j}{\textbf{Min}}\; b^T y + \xi, \tag{17a}$$

$$\textbf{s.t. } \xi \geq \sum_k a_k^T x_k^j \qquad \forall j \leq l, \tag{17b}$$

$$Ay + B_k x_k^j = 0 \quad \forall j \leq l, \forall k, \tag{17c}$$

$$C_k x_k^j = (u_k^j)^\star \quad \forall j \leq l, \forall k, \tag{17d}$$

$$D_k x_k^j \leq f_k \quad \forall j \leq l, \forall k, \tag{17e}$$

where x_k^j is the vector variables x_k corresponding to u_k^j. Note that, $u_k^{j\star}$ for all $j <= l$ and for all k are obtained from the previous iterations $1, ..., l-1$.

If all extreme points are included in the formulation, then the master problem is equivalent to problem (14), see [32] for details. Therefore, at each iteration, the optimal objective value of the master problem, denoted by O_{master}, provides a *lower bound* on the optimal objective value of problem (14).

At the beginning, when $l = 0$, there does not exist any constraint as part of the expressions in (17b)-(17e). However, as the iterations evolve, the expressions in (17b)-(17e) generate and add new constraints to the master problem.

Algorithm 1 shows how the optimal solution of problem (14) is obtained. Note that, UB and LB indicate the best upper bound and the best lower bound obtained by running subproblem (16) and master problem (17); and ϵ is the optimality tolerance.

IV. Extensions and Remarks

So far, the uncertainty of the real-time price was modeled using a discrete scenario-based method. Nevertheless, as discussed in [7], the utility may choose to model the uncertainty of the real-time price also using a set-based method. In that case, the proposed formulation can be modified accordingly to incorporate such model, as we describe next.

Suppose the real-time prices vary as follows:

$$\lambda_k^R[t] = \bar{\lambda}^R[t] - w_k^R[t]\hat{\lambda}^R[t], \tag{18a}$$
$$0 \leq w_k^R[t] \leq 1. \tag{18b}$$

Similar to (11), we take Γ^R as control parameter, where

$$\sum w_k^R[t] \leq \Gamma^R. \tag{19}$$

The constraints in (18) and (19) can now be added to optimization problems (14), (15), and (16). In this case, a represents the uncertain parameter λ^R whose optimal value should be determined. In this setting, a two stage robust program is formed in which uncertainty appears *both* in the objective function and in the constraints.

Using a similar approach outlined in Section III, the new problem formulation based on set-based real-time price uncertainty modeling can be solved using the CCG algorithm. Observe that, in this situation a worst case scenario is generated for *both* renewable generation and real-time price. Since the real-time price appears in the constraints of the dual problem in (16), i.e., in (16b), and the renewable generation outputs only appear in the objective function, the optimal value of the renewable generation still happens at the extreme points of its uncertainty set and we can solve subproblem (16) using the same procedure outlined in Section III. Simulation results show that the proposed algorithm converges quickly in only a few iterations, see Section VI-B.7.

V. Resource Flexibility Indexes

A. Market Condition Index

With the growing penetration of DERs, ISOs are willing to take advantage of the flexibilities that these resources offer, to enhance quality of service and the electricity market efficiency. However, these resources often are *not* dispatched by ISOs, and in many cases are *not* directly visible to ISOs.

California ISO, which is our focus in this section, has recently introduced a new index to help better manage the energy consumption of flexible and responsive loads according to the market conditions. This index is known as the Grid State Indicator (GSI), which is a number between 0 and 10 reflecting the market conditions into the distribution network [36]. GSI is calculated based on historical locational marginal prices (LMPs). A low GSI indicates a good condition for customers to use energy for purposes such as electric vehicle charging. A high GSI shows that market conditions have deviated more from average conditions, and reduction in the use of price-responsive end-user is recommended.

GSI is calculated based on historical locational marginal prices (LMPs) and it thus varies significantly from one location to another and from one time interval to another. However, the GSI index only reflects the market condition and suggests the deployment of the flexible loads according to the market condition.

The main limitation of GSI is as follows: it ignores the fact that flexible resources are located at distribution networks and their market participation may have adverse effect on distribution network constraints. In this regard, there are still two main questions that need to be answered:

1) What is the risk of the deployment of deferrable loads on violating the distribution network constraints in the presence of uncertain on-site renewable resources?
2) What feeders are least affected, in terms of increasing their cost of power procurement, by mitigating the risk of constraint violation by not operating deferrable loads?

Here, we aim to answer both questions. We consider not only the market conditions as in GSI, but also the impact of the flexible resources on the distribution network constraints. Specifically, we introduce two new indexes that can help the utility look beyond the GSI market conditions and identify feeders with better performance for the operation of deferrable loads considering system uncertainties. It is also important for the ISOs that the DERs can actively participate in the electricity market and could follow the market economic signals. The two indexes in this paper aim to show how good a feeder responds to the market signals without having negative effect on the distribution network.

B. Proposed Local Condition Indexes

In the context of power distribution systems, utilities are obliged to meet customers load demand within the standard power quality requirements. Accordingly, the operational constraints governing distribution network must be satisfied for any operating point. Limitations on voltage and line flow are commonly assumed as operational constraints, see (6)-(8). To such aim, several *power quality indexes* are developed in the literature, mostly structured on two rather broad categories: *violation from constraints* and *deviation from nominal values*. In this paper, a Power Quality Constraint Violation Index (PQCVI) is formulated based on the former category, to quantify possible risks in neglecting such constraints in the optimal bidding problem. Mathematically, we have:

$$\text{PQCVI}(\%) = \sum_k \pi_k \times \mathbb{I}_k(\text{Network Constraint Violation}) \times 100\,, \tag{20}$$

where, $\mathbb{I}_k$ (Network Constraint Violation) is an indicator function which is 1 if the network constraints are violated and 0 otherwise. If PQCVI is low, then the feeder is likely in a good condition to take advantage of its flexible resources with a low risk of violating distribution network constraints.

Index PQCVI is useful to assess the risk of violating the technical constraints while operating the flexible resources as the market, if such constraints are ignored. It answers the first question in Section V-A. But, it does not give any indication on the impact on financial aspects, i.e., cost savings. Those aspects can be taken into consideration by using another index, *Feeder Cost Inflexibility Index* (FCII), which is defined as:

$$\text{FCII}(\%) = \left(1 - \frac{\text{Optimal cost } \textit{without} \text{ constraints}}{\text{Optimal cost } \textit{with} \text{ constraints}}\right) \times 100. \tag{21}$$

This index shows how much additional cost is incurred to mitigate the likelihood of violations compared to the cost under ideal distribution network conditions, where there is no risk of violating network constraints. A lower FCII, such as 2%, means that the difference between the optimal objective values under constrained and unconstrained bidding problems is small; thus, one can mitigate technical constraints violations without significantly affecting the cost savings in optimal market participation. A higher FCII, such as 50%, means that it is likely to lose significantly on cost savings under optimal market participation of the load resources on a feeder in order to maintain the power quality of that particular feeder. In other words, FCII shows if the feeder is appropriate for deploying flexible loads, see the second question in Section V-A.

Note that, while GSI solely depends on the market prices, FCII depends on not only the market prices, but also on the distribution grid conditions and load resource parameters.

Both PQCVI and FCII are important indexes and should be considered simultaneously. PQCVI is related to the *extend of the risk in violating the constraints*, while FCII means *how cost-effective it is to remove the risk*. If these two indexes are low (high), then the feeder is more (less) appropriate for deploying its resources.

VI. Case Studies

A. Small Illustrative Example

A simple 4-node distribution network, shown in Fig. 2, is used to illustrate the importance of our analysis. We assume the day-ahead and real-time markets have two time slots. The day-ahead market prices of each time slot are 15 and 70 \$/MWh. The real-time market prices are 16 and 100 \$/MWh. One inflexible demand is located at bus 3 with active and reactive demand of 3 and 1 MWh and 1 and 0.4 MVarh in each time slot, respectively. A wind turbine is installed at bus 2. The deferrable load is located at bus 4 with a total consumptions of 4 MWh. The power factor is 0.9 for both the wind generation unit and the deferrable load. We assume that the utility can only purchase energy in both markets. Three cases are analyzed as follows:

(a) Case 1: Network is not modeled.

(b) Case 2: Network is modeled and wind generation in time slots 1 and 2 are 0.5 MW and 1.5 MW, respectively.

(c) Case 3: Similar to Case 2, but with wind generation being equal to 1 MW at each time slot.

The differences between Cases 1 and 2 would highlight the importance of considering the network constraints. Also, the differences between Cases 2 and 3 would highlight the importance of considering the uncertainties.

The operating conditions are shown in Fig. 2 and the optimal bids are shown in Fig. 3. First, note that different bid curves are achieved. The utility objective values in these cases are \$90,

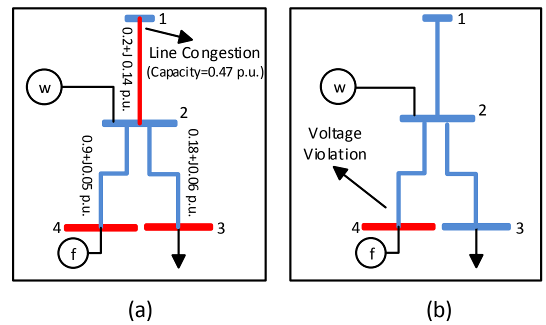

Fig. 2. Network constraints, when the utility bids according to a) Case 1; b) Case 2 with wind power reduced to that in Case 3. Overloaded feeders and voltage violations are shown in red. The wind power resource is denoted by "w" and the deferrable load is denoted by "f" .

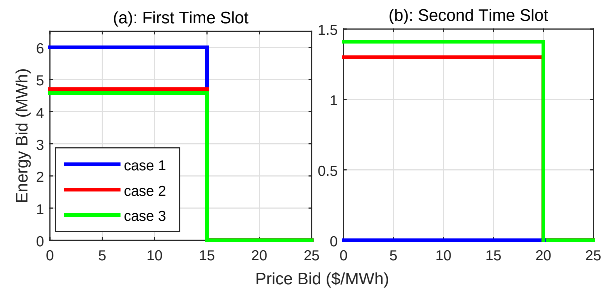

Fig. 3. Optimal bid curves for illustrative example in Section VI-A.

\$161.5, and \$163.46. The first case yields the lowest objective value but causes overload and voltage violation. Indeed, in Case 1, the model suggests to supply all deferrable loads in time slot 1 because the day-ahead market price is lower. However, the peak of the inflexible load occurs in time slot 1 and shifting all the deferrable loads to that time slot, to reduce cost, will overload feeder 1, as shown in Fig. 2(a). If the results of Case 1 are implemented, then the utility has to perform corrective actions, probably with high costs, in order to mitigate or remove violations. For instance, one such corrective action is to shift some demands to time slot 2 and purchase from the real-time market at a higher price of 100 \$/MWh. Fig. 2(b) also portraits the network conditions when the utility bids according to Case 2 but wind generation is changed to Case 3. We observe that voltage violation at bus 4 occurs when wind output changes.

These results clearly indicate that ignoring network constraints and/or the randomness of renewable generation lead to technically infeasible or expensive outcomes.

Finally, if we only consider cases 1 and 2, we see that PQCVI is 1 and FCII is 44.27%, meaning that, if the utility only follows the market prices and does *not* consider its network constraints, they will be violated. The FCII also says that the utility may not fully benefit from its deferrable loads and its cost saving is largely affected by the adverse impact of deploying deferrable loads on its distribution network. This feeder may not be a good choice for developing deferrable loads as the utility cannot take full advantage of them.

B. 33-bus Distribution System

Next, we apply the proposed method to the 33-bus distribution system. The data is adopted from [37] with some

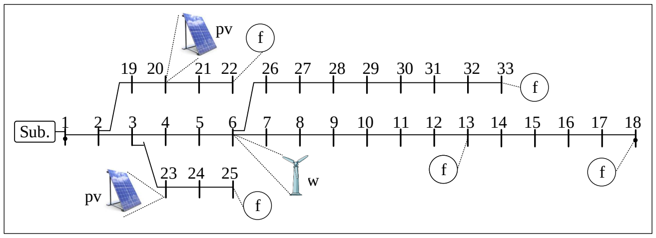

Fig. 4. The 33-bus distribution system with renewable resources and deferrable loads. “w” and “f” refer to wind power resource and deferrable load.

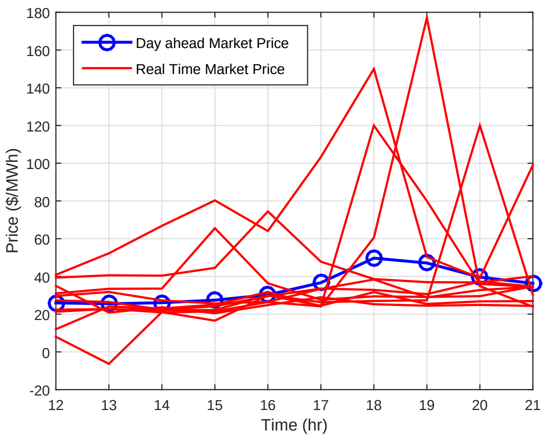

Fig. 5. The day-ahead market clearing price daily profile and the real-time market clearing price daily profile under different scenarios.

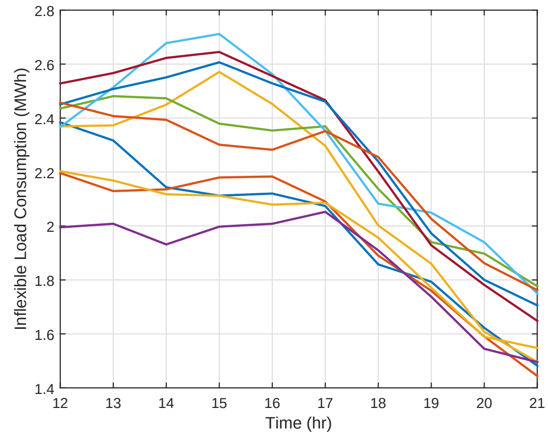

Fig. 6. The total inflexible load consumption in different scenarios

TABLE I
EXPECTED COSTS OF UNCONSTRAINED AND CONSTRAINED CASES

Γ^r	Cost ($)					
	Unconstrained			Constrained		
	Day Ahead Market	Real Time Market	Total	Day Ahead Market	Real Time Market	Total
0	528	175	703	581	139	720
1	551	175	726	604	139	743
2	552	177	729	606	139	745
3	555	175	730	607	140	747

modifications. The power and the voltage bases are 100 MVA and 12.66 Kv. One wind turbine and two PVs were placed at buses 6, 20 and 23 and five deferrable loads at buses 13, 18, 22, 25, 33, see Fig. 4. The deferrable loads can consume energy from hour 12 (noon) to hour 21, i.e., $T = 10$. The total energy consumption of the deferrable load is 0.0813 pu, equally divided amongst the five deferrable loads. If a load is inflexible, then its total consumption is divided equally within each time slot, i.e., 0.00813 pu in each hour. The maximum hourly wind generation is 0.0024, 0.0028, 0.0030, 0.0032, 0.0033, 0.0034, 0.0052, 0.0053, 0.0052, 0.0069 pu. The maximum hourly generation for each PV is 0.0025, 0.0025, 0.0016, 0.0008, 0.0002, 0, 0, 0, 0, 0 pu. We allowed up to 15% and 10% random deviations at each hour for wind generation and each PV, respectively. We set the maximum amount that the utility can sell in real-time to 0.0010 pu. Unless stated otherwise, the uncertainty budget is $\Gamma = 2$ and the substation capacity is 0.055 pu. We created 10 scenarios with equal probabilities for inflexible loads and real-time market prices as shown in Figs. 5 and 6. The scenarios are based on the data in November 2015 for a real-world feeder and the California ISO market nodal price for a real-world distribution substation where that feeder is connected to. Without loss of generality, we consider the price component of the day-ahead market bid to be equal to the market price, as we consider it deterministic.

In the following, we examine the impacts of several factors in the bidding problem *with* and *without* network model.

1) Different Budget of Uncertainties: Optimization results for different Γ^r values are presented in Table I for the constrained (with network model) and unconstrained (without network model) cases. Observe that as the uncertainty budget increases, the expected cost increases as well, given the fact that the utility is protected from more severe operating limit violations. It is seen that the day-ahead cost is higher than the real-time one in all cases. This is because the energy price in the real-time market is volatile and can drastically vary. The utility thus has to keep balance between these scenarios in order to minimize its expected cost by choosing its bid in the day-ahead market. Moreover, the utility can benefit from energy sale in the real-time market in some scenarios.

The day-ahead bids in sample hours 13, 14, 15 and 16 and for $\Gamma^r = 3$ are depicted in Fig. 7 for the unconstrained and constrained cases. It is seen that in the constrained case the utility is willing to purchase more from the day-ahead market. Detailed results show that the (energy) bid strategy for other Γ^r values is similar to that obtained for $\Gamma^r = 3$.

It is worth mentioning that if the linearized Distflow model in [25] is used, then the expected costs would become \$719, \$742, \$744 and \$746 for $\Gamma = 0, 1, 2, 3$, respectively. These numbers are only about \$1 different than the numbers in Table I. Furthermore, the bidding pattern does *not* change when the linearized Distflow model in [25] is used. These results confirm that the main findings and conclusions of this paper do not depend on the exact choice of the linearized Distflow model being used.

2) Deferrable Loads Location: Two scenarios are described as follows. The first scenario is similar to that explained in the previous section in which five deferrable loads are placed in the system, but the active and reactive limits of the feeder connecting node 2 to 19 are less than 1 MW and 0.2 MVar.

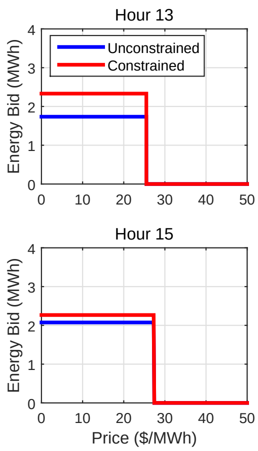

Fig. 7. Day-ahead energy bids for $\Gamma^r = 3$ for sample hours

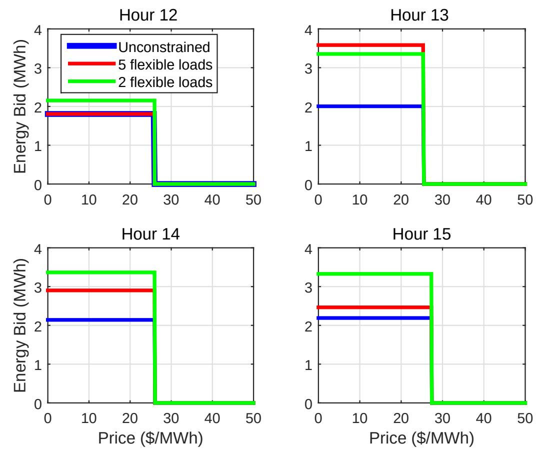

Fig. 8. Day-ahead bids with different number of deferrable loads

TABLE II
EXPECTED COSTS WITH DIFFERENT NUMBER OF DEFERRABLE LOADS AND SIMILAR TOTAL DEMAND

# of Flexible Loads	Cost ($)					
	Unconstrained			Constrained		
	Day Ahead Market	Real Time Market	Total	Day Ahead Market	Real Time Market	Total
2	552	177	729	665	102	767
5	552	177	729	605	140	745

The second scenario considers two deferrable loads located at buses 5 and 33. The total consumption of these loads remains as in the first scenario, i.e. 0.0813 pu

The optimization results for the unconstrained and constrained cases are shown in Table II. As seen, the expected costs of the unconstrained cases are the same in both scenarios. However, in the constrained case, the expected cost is higher when two deferrable loads are considered. Specifically, the energy bids in the day-ahead market are higher for the scenario with two deferrable loads in hours 12, 13, 14 and 15 as shown in Fig. 8. The higher cost here is due to the network constraints. That is, the utility has to shift more deferrable loads to the hours of higher prices as shown in Fig. 9. With five deferrable loads, the utility has better choices to manage consumption and cost, as seen from Table II. FCII for five and two deferrable loads are 2.14% and 4.95% highlighting that the utility can better benefit from its deferrable loads when they are distributed.

3) Renewable Generation Location: In this subsection, we analyze the impact of size and site of renewable generations on the expected cost of the utility. The first scenario is similar to that explained in the previous section in which one wind turbine and two PV units were placed in the system. The second scenario considers one wind turbine located at bus 2 with the same generation output of the wind turbine in scenario 1 and one PV unit is located at bus 3. The generation output of the PV unit is equal to the total generation of PV units in scenario 1. Again, the unconstrained model cannot capture the location of the renewable generation and thus the expected cost is the same for both scenarios. The expected cost for this case is $729. The expected cost of scenario 1 is $745 while it

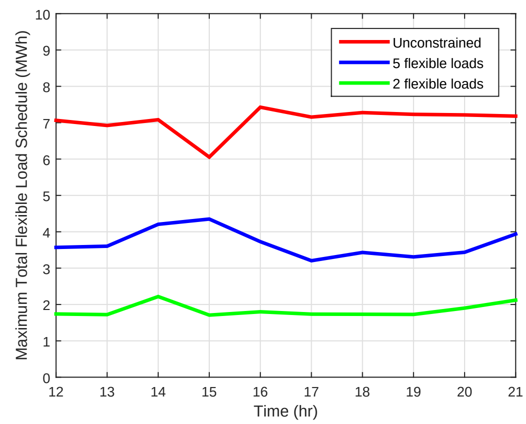

Fig. 9. Total maximum deferrable load among the scenarios in each hour

increases to $748 in the second scenario. That is, the location of renewable generation directly affects the cost of the utility in the demand bidding problem.

4) Deferrable Load Penetration: The impact of increasing the number of deferrable loads was studied in this section. From the previous discussion, the total deferrable load was 0.0813 pu We constructed a scenario in which the deferrable load portion is equal to $\rho \times 0.0813$ pu, and the remaining $(1-\rho) \times 0.0813$ pu is inflexible with consumptions are being equally divided among all the hours. We increased ρ from 0 to 1. Fig. 10 presents the results with and without grid constraints for $\Gamma^r = 2$. We can see that increasing the penetration level of the deferrable loads reduces the cost. As seen, for low penetration level, network constraints have slight impacts on the bidding cost while for higher value of this parameter, network model impacts become significant and can lead to operating limit violations. For example, for $\rho = 1$, in Fig. 10(b), the feeder connected to the substation, operates outside its allowable limits. Moreover, the worst voltage deviation among the nodes in all scenarios and time slots is harsh.

5) Proposed Indexes: Table III provides the values of the indexes related to Section V for different ρ in Section VI-B.4. It is observed that when the deferrable load penetration increases, PQCVI increases. Notice that FCII is low. That is, the expected cost between considering versus ignoring network constraints is *not* significant. Therefore, the utility, by

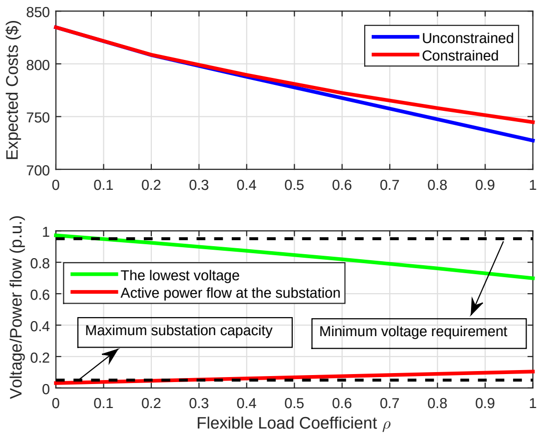

Fig. 10. Impact of penetration level of deferrable loads.(a): The expected costs, b) the power flow at the substation and the lowest voltage for the optimal solution of the unconstrained case

TABLE III
PROPOSED LOCAL INDEXES FOR DIFFERENT LEVEL OF DEFERRABLE LOAD PENETRATION

Index	Flexible Load Coefficient ρ					
	0	0.2	0.4	0.6	0.8	1
PQCVI (%)	0	40	70	100	100	100
FCII (%)	0	0.04	0.22	0.62	1.39	2.34

applying the proposed model, can prevent from violating its constraints at the expense of a slight increase in the expected cost compared to using the conventional model which may not be feasible in practice. Note that, in this example, PQCVI is high for $\rho \geq 0.6$. This means that if the utility ignores the network constraints in the bidding problem, there is a high probability that these constraints are violated in practice and the utility incurs high costs. However, if the model considers the operational constraints, then the model acts proactively and prevents from the network constraint violation in reality. The indexes were calculated for one day. The average value can be obtained in the long term and used for planning purposes. Interestingly, both FCII and PQCVI can be a good measure to develop deferrable load penetration in distribution network under uncertainties. These two indexes need to be considered simultaneously; because if only one index is considered, the utility may not see the big picture in his decision making process. If both indexes are high in the long term, then the feeder in question may not be a good choice for recruiting deferrable loads.

6) Comparison Based on Actual Outcome: To better evaluate our method, several experiments based on the actual output is done. To such aim, we performed the following steps:

- We obtained the day-ahead demand bid using our model *with* and *without* constraints. To highlight the importance of our model, we limit the substation capacity to 4MW. We also limit the bid in the real-time market to be less than 1MW to relieve the utility form the stress of the real-time market [22].
- After obtaining the day-ahead demand bid, we solved a cost minimization problem considering the network constraints and fixed values of the renewable generations, inflexible loads, real-time energy prices and day-ahead demand bids.
- As for the renewable generations, two different values are considered. In the first one, the renewable generations are equal to the worst case scenario obtained from our optimization problem and in the other case, their values are equal to their predicted values $\overline{g}^r$.
- As for the real-time prices and the inflexible loads, we consider their values in each scenario.

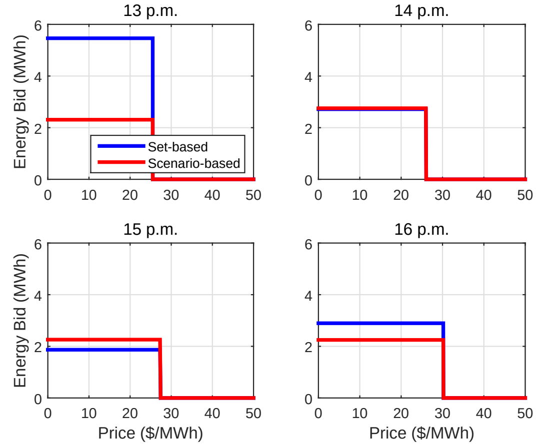

Fig. 11. Optimal bid curves when the real-time prices are modeled through a discrete scenario-based approach versus a continuous set-based approach.

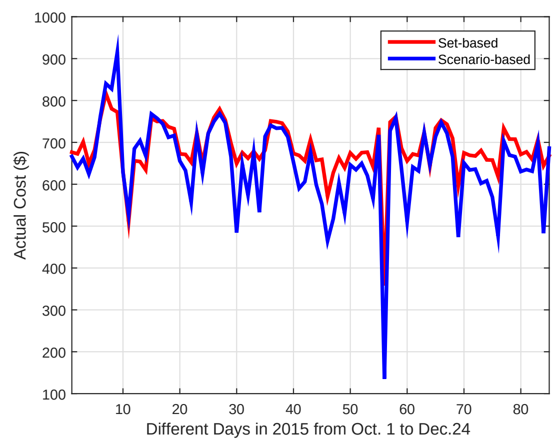

Fig. 12. Actual cost when the real-time prices are modeled through a discrete scenario-based approach versus a continuous set-based approach.

Table IV shows the actual cost and the nodes with voltage violations for different values of the renewable generations, real-time market prices and inflexible loads. We consider 20 different realizations for actual values. We can see that the proposed model attains a feasible solution, while the unconstrained model failed to reach any feasible solution in all scenarios. That is, in this case study, the utility cannot correct the network constraints problems due bidding in real-time market. It must take other actions. The cost is lower when the renewable generations operate at their nominal values.

7) Actual Outcome for Different Modeling of Real-Time Prices : In this section, we examine the case where the uncertainty in real-time prices is modeled as continuous uncertainty sets as opposed to discrete random scenarios. Consider the network-constrained model with the assumptions described in

TABLE IV
THE ACTUAL COSTS AND VOLTAGE VIOLATIONS FOR DIFFERENT ACTUAL OUTPUTS.

Model			Scenarios									
			1	2	3	4	5	6	7	8	9	10
Constrained	Worst Case	Cost $	696	707	809	685	739	795	809	814	789	717
	Nominal Case	Cost $	679	690	763	660	722	778	776	792	768	697
Unconstrained	Worst Case	Cost $	Infeasible									
		Violated Node Voltage	9-18, 29-33	9-18, 29-33	9-18, 30-33	9-18, 29-33	8-18, 28-33	8-18, 28-33	9-18, 29-33	9-18, 29-33	9-18, 30-33	9-18, 30-33
	Nominal Case	Cost $	Infeasible									
		Violated Node Voltage	9-18, 29-33	9-18, 29-33	9-18, 30-33	9-18, 29-33	8-18, 29-33	8-18, 29-33	9-18, 29-33	9-18, 29-33	9-18, 28-33	9-18, 29-33

Section VI-B.1. The uncertainty budget is fixed at 10 and the real-time prices are assumed to vary within the minimum and maximum values in each hour, as shown in Fig. 5. Fig. 11 shows the bidding patterns in both cases. The expected cost increases from $745 to $811 for the set-based model. Next, we carry out experiments based on the actual outputs using real-time prices from October 1 to December 24 2015 as follows:

- We obtained the day-ahead demand bid using the proposed models for continuous and discrete uncertainty modeling approaches for real-time prices.
- After obtaining the day-ahead demand bids, we solved a cost minimization problem considering network constraints and fixed values of the renewable generations, inflexible loads, real-time energy prices and day-ahead demand bids.
- Output levels of the renewable resources were set equal to the predicted values.
- Real-time prices and inflexible loads were set in each scenario based on the actual real-time prices from October 1 2015 to December 24 2015.

Fig. 12 shows the actual daily cost of the utility across three months using the two proposed methods. It is seen that, on most days, the model with discrete scenarios attains a lower cost solution compared to the model where the real-time prices are modeled by the uncertainty sets. There are few days/hours that the uncertainty set approach leads to lower costs. For example, on October 9, the cost of the robust model is $780 while it increases to $950 if the model with discrete scenarios for real-time prices is used.

In future, the indexes proposed in this paper can be combined with field measurements at power distribution feeders, e.g., from smart meters or phasor measurement units [38], to examine in real-time the impact of the aggregated participation of DERs in electricity markets and transmission-level services on distribution system reliability.

VII. CONCLUSIONS

In this paper, we proposed the optimal bidding problem of a utility with intermittent renewable generations and deferrable loads that participates in a two-settlement market. We considered the uncertainty of the real-time market price, inflexible demand, and DERs generation and developed a two-stage robust stochastic optimization problem which to the best of our knowledge is the first bidding model that simultaneously considers uncertainty and network model. A two stage robust stochastic optimization model was thus derived which was solved using a decomposition algorithm.

We observed that the bidding strategy of the utility drastically changes when network constraints are considered, compared to the simplified unconstrained case that can lead to infeasible or costly outcomes. The effect of the deferrable load locations in the distribution system was also evaluated and it was shown that a large number of deferrable loads, compared to small numbers but with large capacity of such loads, can better reduce the bidding cost in the presence of network constraints. Novel indexes were also proposed to help the utility apprise the distribution network performance. The possibility of modeling the uncertainty of the real-time prices with a set-based uncertainty method was discussed and illustrated with numerical results.

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