

# Optimal Bidding in Performance-based Regulation Markets: An MPEC Analysis with System Dynamics

Ashkan Sadeghi-Mobarakeh, *Student Member, IEEE*, Hamed Mohsenian-Rad, *Senior Member, IEEE*

**Abstract**—In this paper, we address the problem of optimal bidding in performance-based regulation markets for a large price-maker regulation resource. Focusing on the case of the California Independent System Operator (ISO), detailed market components are considered, such as regulation capacity payment, regulation mileage payment, performance accuracy adjustment, automatic generation control (AGC) dispatch, and participation factor. Our analysis also incorporates system dynamics of the regulation resource, for different resource types and technologies. In principle, our problem formulation is a mathematical program with equilibrium constraints (MPEC). However, our fundamentally new formulations introduce several new challenges in solving the MPEC problem in the context of performance-based regulation markets that are not perviously addressed. In fact, global optimization techniques fail to solve the original non-linear program due to its complexity. Therefore, we undergo several innovative steps to transform the problem into a mixed-integer linear program which is solved with accuracy, reliability, and computational efficiency. Insightful case studies are presented using data from a California ISO regulation market project.

**Keywords:** Performance-based regulation market, system dynamics, price-maker, optimal bidding, MPEC, California ISO.

## NOMENCLATURE

$i, j$	Index for generators
$k$	Index for random scenarios
$(t)$	Continuous time
$(s)$	Complex frequency
$[\tau]$	Discrete regulation time slot
$C$	Superscript for regulation capacity
$M$	Superscript for regulation mileage
$MCP$	Market clearing price
$r$	Regulation capacity/mileage allocation
$\mu, \xi, \eta, \gamma$	Dual variables in market optimization problem
$U$	Resource maximum regulation capacity
$O$	Regulation price offer
$y$	Mechanical output of generator
$z$	Auxiliary binary variables
$\theta$	Auxiliary real variables
$K$	Number of random scenarios
$R$	Regulation capacity/mileage requirement
$m$	Resource-specific mileage multiplier
$AGC$	Automatic generation control signal
$L, \epsilon$	Some large and small numbers
$T$	Zero and pole parameters in transfer functions

## I. INTRODUCTION

In October 2011, the United States Federal Energy Regulatory Commission (FERC) set forth Order 755 to remedy undue

discrimination in the procurement of frequency regulation in wholesale electricity markets and to ensure that providers of frequency regulation receive just and reasonable rates [1]. This ruling requires regional transmission operators (RTOs) and independent service operators (ISOs) to compensate frequency regulation resources based on *the actual service provided*. The compensation must include a capacity payment that considers the unit's opportunity marginal cost and a mileage payment for the performance that reflects the quality of regulation service provided by the resource when following the dispatch signal. Many ISOs, including the California ISO (CAISO), have already adopted this new market mechanism.

In this paper, we study *optimal market participation* in the CAISO performance-based regulation market [2], [3]. We develop a new mathematical foundation to optimize the capacity and mileage bids. Our design is not limited to a particular resource type or technology. Instead, we try to understand the key concepts in this new market paradigm. We seek to answer the following fundamental questions: *Given the specific setup, rules, and requirements of a practical performance-based regulation market such as the one that exists in California, what is the best bidding strategy for a regulation resource? How should the system dynamics of each regulation resource technology affect optimal bidding? Given the complexity of the optimal bidding problem, how can we obtain solutions that are accurate and computationally efficient?*

The analysis in this paper can be compared with three groups of work in the literature. First, there are studies that evaluate or improve the existing performance-based regulation markets, from the *viewpoint of ISOs*, such as PJM, MISO, or CAISO [2]–[8]. In contrast, our focus is on optimal bidding in performance-based regulation markets from the *viewpoint of market participants* who seek to maximize their own profit.

The second group seeks profitable operation of regulation resources in non-performance-based [9], [10] or performance-based [11]–[13] regulation markets. Our work is different from the existing studies in this group in at least three key aspects. First, the prior studies are limited to small *price-taker* regulation resources, i.e., those who cannot affect the regulation market prices. In contrast, our focus here is on large *price-maker* regulation resources<sup>1</sup>. Second, we take into account various details about the underlying regulation market rules and dispatch mechanisms. For example, unlike in [12], where the AGC signal is assumed to be fixed, here, we model the AGC dispatch mechanism using *participation factors*, which are directly linked to how the regulation bids are cleared for each regulation resource. Finally, none of the prior studies,

The authors are with the Department of Electrical Engineering, University of California, Riverside, CA, USA, e-mails: {asade004, hamed}@ee.ucr.edu. This work is supported by NSF Grants ECCS 1253516 and ECCS 1307756.

<sup>1</sup>A recent study has shown that as few as six resources provided over 78% of the total regulation service to CAISO during May and June 2014 [14].

including those in [11]–[13], investigates optimal bidding under the CAISO performance-based regulation market model.

Since the focus in this paper is on price-maker operation of regulation resources, our problem formulation naturally belongs to *mathematical programs with equilibrium constraints* (MPEC), which are widely studied in the literature on electricity markets, e.g., see [15], [16]. However, to the best of our knowledge, this is the first work to study an MPEC problem in a performance-based regulation market. The analysis in this paper is fundamentally different from the prior studies, e.g., due to the need to address system dynamics. The new formulations in this paper introduce several new challenges in solving MPEC problems that are not perviously addressed. The contributions in this paper are summarized as follows:

- 1) A new analytical framework is proposed to optimize the bids of a large and price-maker generation firm in a performance-based regulation market to maximize its profit. The system dynamics of the regulation resource- for different resource types and technologies- are taken into account. Both deterministic and stochastic optimization scenarios are considered. The formulated problem is an *MPEC with system dynamics*, which to the best of our knowledge, has not been studied before.
- 2) Two different formulations are addressed in this paper. The first one is a *non-linear program* (NLP) for the novel MPEC problem in this new context. By construction, it has fundamental difficulties, to the extent that when the standard NLP software tools are applied, the results are very sensitive to the choice of initial points, and in some cases no feasible solution can be found. The second problem is based on making some practical assumptions so as to tackle the nonlinearities in the original NLP and transform it into a series of tractable *mixed-integer linear programs* (MILPs). Both solution accuracy and computation time are significantly improved.
- 3) Insightful case studies are presented and discussed in details. The impact of high, medium and low market mileage requirements on the optimal bidding of the generation firm are analyzed. The role of the system performance accuracy, and its impact on the strategic behaviour of the generation firm is studied.

It is worth adding that the new optimization-based regulation market participation approach in this paper is general and *not* restricted by the regulation resource technology. For example, large energy storage units or electric vehicle aggregators may also benefit from the analysis in this paper.

Compared to its conference version in [17], in this journal version, Section III on solution method is entirely new, Section IV on case studies is also entirely new, and the detailed models on system dynamics in Section II are more comprehensive.

## II. PROBLEM FORMULATION

Consider a generation firm that bids in a performance-based regulation market. In this paper, our focus is on the CAISO market in California, where each bid has two components: 1) a regulation capacity offer in MW/interval and its associated price in \$/MW/interval; and 2) a mileage price offer in \$/MW.

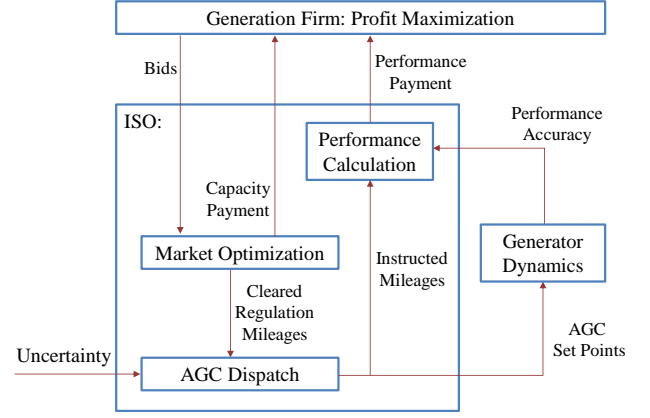


Fig. 1. The main components in our analysis and their interactions.

Each market interval takes 15 minutes [2, pp. 4-6]. Here, we only formulate the regulation market and neglect the markets for energy and all ancillary services, other than regulation. Our model is what we think a market participant may use to mimic the operation of the CAISO regulation market, such that it can accordingly select its bids. Also, we assume that the resources bid their maximum regulation capacities to the market. That is, the regulation capacity offers  $U$  are considered as parameters.

In each market interval, the ISO clears the *regulation capacities* and *regulation mileages* of all market participants by solving an optimization problem that seeks to minimize the total cost of regulation, and by taking into account its estimation of the AGC signal during the upcoming market interval. In the case of CAISO, the resolution of the AGC signal is four seconds; thus, each regulation market interval includes  $15 \times 60/4 = 225$  AGC set points [2, p. 11]. Once each market interval is implemented and the actual AGC signal and the metered contribution of each regulation resource is realized, CAISO calculates the payment to each resource based on its performance in following the AGC signal.

The main components in our analysis and the interactions among those components are shown in Fig. 1. Next, we will explain each component and its related mathematical models.

### A. Profit Model

Suppose the generation firm of interest has  $i = 1, \dots, I$  generators. Randomness is modeled by  $k = 1, \dots, K$  scenarios [18]. Let  $AGC_{i,k}[\tau]$  denote the realization of the AGC set point that is sent to generator  $i$  at time slot  $\tau$  of length four seconds under random scenario  $k$ . The total *expected profit* for the generation firm when it offers only regulation up service<sup>2</sup> during each 15 minutes market interval is obtained as

$$\sum_i MCP^C r_i^C + \frac{1}{K} MCP^M \times \sum_{i,k} \left[ \left( \sum_{\tau} AGC_{i,k}[\tau] - |AGC_{i,k}[\tau] - y_{i,k}[\tau]| \right) / \right]$$

<sup>2</sup>The problem formulation under regulation down service is very similar. Therefore, we only focus on regulation up service in order to avoid having long equations that do not add to the technical value of the discussion.

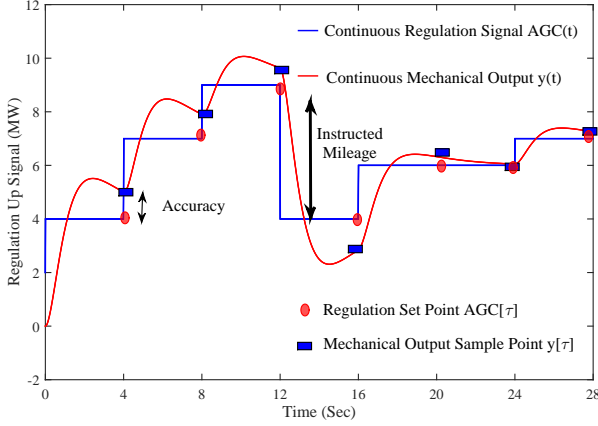


Fig. 2. An example on calculating accuracy and instructed mileage.

$$\left( \sum_{\tau} AGC_{i,k}[\tau] \right) \times \left[ \sum_{\tau} |AGC_{i,k}[\tau] - AGC_{i,k}[\tau - 1]| \right], \quad (1)$$

where the first term is the *regulation capacity payment* and the second term is the *regulation mileage payment*. The former is obtained by multiplying the cleared regulation up capacity price  $MCP^C$  to the awarded regulation capacity  $r_i^C$  for each generator  $i$ . As for the second term, different ISOs may calculate it in different ways. CAISO calculates this term by multiplying the cleared regulation up mileage price  $MCP^M$  by a summation over the product of two terms, namely the *performance accuracy adjustment*, i.e., the first bracket, and the *instructed mileage up*, i.e., the second bracket [3]. These two terms are shown in Fig. 2. Next, we explain each term.

The performance accuracy adjustment is associated with the level of accuracy in following the AGC signal. It is calculated as the sum of all AGC set points minus the sum of deviations from each AGC set point. The result is then divided by the sum of all AGC set points. Note that, CAISO treats positive and negative deviations *equally* [19, p. 14]. Here,  $y_{i,k}[\tau]$  is the mechanical output of generator  $i$  in response to set point  $AGC_{i,k}[\tau]$ , at time slot  $\tau$  and under scenario  $k$ . If generator  $i$  follows the AGC signal exactly, i.e., if  $y_{i,k}[\tau] = AGC_{i,k}[\tau]$  at all time slots, then the performance accuracy adjustment is *one*. Otherwise, it can be as low as *zero*. A mathematical expression for  $y_{i,k}[\tau]$  will be derived in Section II-B.

The instructed mileage up is associated with the shape of the AGC signal. It is the absolute change in the AGC regulation up set points that take new values every four seconds, represented by  $|AGC_{i,k}[\tau] - AGC_{i,k}[\tau - 1]|$  in (1). This term indicates what mileage the ISO would *ideally* expect from generator  $i$  in each four second set point interval [19, pp. 8].

### B. System Dynamics

The ability of a generator in following the AGC signal depends on its technology and physical characteristics. Without loss of generality, we consider a governor-turbine control model for each generator, where a speed governor senses the changes in its power command set points, i.e., the AGC

TABLE I  
TRANSFER FUNCTIONS OF GENERATORS [20], [21]

Generation Technology	Transfer Function
Non-Reheat Steam Unit	$\frac{1}{(1 + T_1 s)}$
Reheat Steam Unit	$\frac{1 + T_2 s}{(1 + T_3 s)(1 + T_4 s)(1 + T_5 s)}$
Hydraulic Unit	$\frac{(1 - T_6 s)(1 + T_7 s)}{(1 + 0.5T_6 s)(1 + T_8 s)}$

set points, and converts them into valve actions. A turbine then converts the changes in valve positions into changes in mechanical power output, i.e., generation signal  $y_{i,k}(t)$ .

The governor-turbine control is often modeled as a two-state dynamic system: one state corresponds to the *speed governor* and one state corresponds to the *turbine valve position* [20]. Other states may also be considered depending on the generator technology. System dynamics in *frequency domain* are then modeled using appropriate *transfer functions*:

$$Y_{i,k}(s) = G_i(s)AGC_{i,k}(s), \quad \forall i, k, s. \quad (2)$$

Here,  $s$  is the complex frequency variable and  $G_i(s)$  is the transfer function of generator  $i$ . Also,  $Y_{i,k}(s)$  and  $AGC_{i,k}(s)$  denote the Laplace transformations of time-domain continuous signals  $y_{i,k}(t)$  and  $AGC_{i,k}(t)$ , respectively. Several examples of generator transfer functions are listed in Table I for different technologies, where  $T_1, T_2, \dots, T_8$  are known parameters.

The time-domain continuous signal  $AGC_{i,k}(t)$  can be represented in form of a weighted summation of several *unit step functions*:  $u(t) = 0$  for  $t < 0$  and  $u(t) = 1$  for  $t \geq 0$  [22, p. 22]. The weights are the AGC set points  $AGC_{i,k}[\tau]$  at each time slot  $\tau$ , see Fig. 2 for an example. Therefore, we can write

$$AGC_{i,k}(t) = \sum_{\tau=1}^{225} \left[ (AGC_{i,k}[\tau] - AGC_{i,k}[\tau - 1]) u(t - 4(\tau - 1)) \right]. \quad (3)$$

By taking Laplace transform from (3), we obtain

$$AGC_{i,k}(s) = \frac{1}{s} \sum_{\tau=1}^{225} e^{-4(\tau-1)s} (AGC_{i,k}[\tau] - AGC_{i,k}[\tau - 1]). \quad (4)$$

Next, suppose the system dynamics of generator  $i$  is modeled by the first transfer function in Table I. This transfer function has one pole and no zero. From (2) and (4), we have:

$$Y_{i,k}(s) = \frac{y_{i,k}(0^-)}{s + 1/T_1} + \frac{\sum_{\tau=1}^{225} e^{-4(\tau-1)s} (AGC_{i,k}[\tau] - AGC_{i,k}[\tau - 1])}{s(1 + T_1 s)}, \quad (5)$$

where the first term is the zero input condition response and the second term is the zero initial condition response [22,

pp. 767-774]. By applying the inverse Laplace transform, the mechanical output of generator  $i$  in time domain becomes

$$y_{i,k}(t) = y_{i,k}(0^-)e^{-t/T_1} + \sum_{\kappa=1}^{225} \left[ \left( AGC_{i,k}[\kappa] - AGC_{i,k}[\kappa-1] \right) \times \left( 1 - e^{-(t-4(\kappa-1))/T_1} \right) u(t-4(\kappa-1)) \right]. \quad (6)$$

The above expression is a continuous signal. The mechanical output of generator  $i$  at time slot  $\tau$  is obtained as:

$$y_{i,k}[\tau] = y_{i,k}(t = 4\tau). \quad (7)$$

Once we replace (7) in (1), we can express regulation mileage payment, i.e., the second term in (1), based on the AGC set points and the transfer function parameters of generator  $i$ .

The other transfer functions in Table I can be analyzed similarly. Specifically, we can apply *partial fraction expansion* to these other transfer functions and calculate the corresponding  $y_{i,k}(t)$  and  $y_{i,k}[\tau]$  accordingly, see [22, pp. 767-774].

### C. Regulation Market Model

1) *Market Optimization Problem*: Once CAISO collects all regulation bids, it solves the following optimization problem:

$$\min \sum_j O_j^C r_j^C + \sum_j r_j^M O_j^M \quad (8)$$

$$\text{s.t.} \quad \sum_j r_j^C \geq R^C \quad : \quad MCP^C, \quad (9)$$

$$\sum_j r_j^M \geq R^M \quad : \quad MCP^M, \quad (10)$$

$$r_j^C \geq 0 \quad : \quad \mu_j, \quad \forall j, \quad (11)$$

$$r_j^C \leq U_j \quad : \quad \xi_j, \quad \forall j, \quad (12)$$

$$r_j^M \geq r_j^C \quad : \quad \eta_j, \quad \forall j, \quad (13)$$

$$r_j^M \leq m_j r_j^C \quad : \quad \gamma_j, \quad \forall j, \quad (14)$$

where  $j$  is the index for all regulation market participants. For each constraint, the *dual variable* is shown on the right side of the colon symbol. The resource-specific mileage multiplier  $m_j$  for each generator  $j$  is a parameter that is determined by CAISO using the historical data of generator  $j$ .

The objective function in (8) is the total bid-in cost of regulation up capacity and mileage. Constraints (9) and (10) indicate the market regulation up capacity requirement and the market regulation up mileage requirement, respectively. The dual variables corresponding to these two constraints indicate the regulation up capacity price and the regulation up mileage price, respectively. Constraints (11) and (12) assure operating all regulation resources within their operational limits. Note that, the multiplication  $U_j$  is in MW / interval and it is limited by the ramp rate of generator  $j$ . Constraints (13) and (14) are intended to link the rewarded regulation capacity and the rewarded regulation mileage for each resource. Specifically, from (13), the mileage reward of a resource cannot be less than its capacity reward. From (14), it also cannot be greater than the product of its regulation up capacity reward and

resource-specific mileage multiplier. This is consistent with the principle of establishing a uniform clearing price for mileage that considers the expected resource performance [23].

Note that, the mileage payments that are obtained as the solutions of problem (8)-(14) are *not* financially binding because the actual mileage payments are calculated once the AGC signals are realized and the accuracy of each resource in following its corresponding AGC signal is measured.

2) *AGC Dispatch Method*: Let  $AGC_k[\tau]$  denote the overall regulation set point at time slot  $\tau$  and under scenario  $k$  that is calculated by the ISO based on the overall imbalance between supply and demand in the power system. In this section, we discuss how  $AGC_k[\tau]$  is divided into several *resource-specific* AGC set points. Note that, for the generation firm of interest, the resource-specific AGC set points are denoted by  $AGC_{i,k}[\tau]$  for  $i = 1, \dots, I$ , see Section II-A. AGC dispatch is done using the concept of *participation factor*, which indicates the portion of  $AGC_{i,k}[\tau]$  out of  $AGC_k[\tau]$  [20, p. 617].

The choice of participation factor may differ among ISOs. However, it is often in some way related to the regulation market outcome. That is, it depends on certain solutions of the market optimization problem in (8)-(14). In this paper, we use a variation of the method in [23, p. 12] and assume that

$$AGC_{i,k}[\tau] = \min \left\{ r_i^C, \frac{r_i^M}{\sum_i r_i^M + \sum_{j \neq i} r_j^M} AGC_k[\tau] \right\}, \quad (15)$$

where the participation factor of generator  $i$  is a fraction of its mileage over the total cleared mileage. As regulation mileage and regulation capacity are determined separately in the CAISO market optimization procedure, the min function is needed in (15) to make sure that AGC set points do not exceed the cleared regulation mileage capacity  $r_i^C$ .

We shall point out that even though the above mechanism closely matches the regulation market optimization problem that is used by CAISO, it may not guarantee that the aggregation of the regulation services that are allocated to all market participants always matches the total AGC signal. Therefore, CAISO, and other ISOs, usually use a post-optimization corrective action to make sure that the intended AGC level is met. This is done iteratively and by first exploiting the low-price regulation resources, whose bids are below the market clearing price and therefore act as price-taker [24, p. 128]. Accordingly, the impact of such post-optimization heuristic corrective action is likely to be minimal on the AGC signal that is allocated to the large and price-maker regulation resources. Therefore, for the rest of this paper, we do not consider any such post-optimization heuristic corrective action in our analysis.

### D. Optimal Bidding Problem

In summary, if a generation firm tends to maximize its own profit in (1), it must solve the following optimization problem:

$$\begin{aligned} & \text{maximize} \quad (1) \\ & \text{subject to} \quad (6), (7), (8) - (14), (15). \end{aligned} \quad (16)$$

The above problem incorporates all the components in Fig. 1. This problem is a *bi-level* program, where the market optimization problem in (8)-(14) forms the lower-level problem.

### III. SOLUTION METHOD

#### A. Non-linear Programming Formulation

As a standard procedure in bi-level programming, one can replace the lower-level problem (8)-(14) with its *equivalent* set of Karush-Kuhn-Tucker (KKT) conditions [25, p. 224]:

$$(9) - (14) \quad (17)$$

$$O_j^C - MCP^C - \mu_j + \xi_j + \gamma_j - m_j \eta_j = 0 \quad \forall j, \quad (18)$$

$$O_j^M - MCP^M - \gamma_j + \eta_j = 0 \quad \forall j, \quad (19)$$

$$\left( R^C - \sum_j r_j^C \right) MCP^C = 0, \quad (20)$$

$$\left( R^M - \sum_j r_j^M \right) MCP^M = 0, \quad (21)$$

$$(U_j - r_j^C) \xi_j = 0 \quad \forall j, \quad (22)$$

$$r_j^C \mu_j = 0 \quad \forall j, \quad (23)$$

$$(m_j r_j^C - r_j^M) \eta_j = 0 \quad \forall j, \quad (24)$$

$$(r_j^M - r_j^C) \gamma_j = 0 \quad \forall j, \quad (25)$$

$$MCP^C, MCP^M \geq 0, \mu_j, \xi_j, \gamma_j, \eta_j \geq 0 \quad \forall j, \quad (26)$$

where (18) and (19) are the gradient equilibrium conditions and (20)-(25) enforce *complementarity slackness*. By replacing the lower-level linear program in (8)-(14) with the constraints in (17)-(26), problem (16) takes the standard form *mathematical program with equilibrium constraint* (MPEC) as follows:

$$\begin{aligned} & \text{maximize} && (1) \\ & \text{subject to} && (6), (7), (15), (17) - (26). \end{aligned} \quad (27)$$

The above optimization problem is *non-linear* and *non-smooth* and hard to solve. The difficulty is not only due to its MPEC structure, but more importantly also due to the intricate formulations of the objective function and the AGC dispatch mechanism. One may attempt solving problem (27) using global optimization techniques [16, pp. 415-430], [26]. However, as we will see in Section IV-B, these techniques either fail to find any solution, or their solutions are not accurate. Therefore, next, we seek to convert problem (27) into an equivalent, or slightly different, but more tractable optimization problem. Our goal is to solve problem (27) by solving a series of mixed integer linear programs (MILP). Note that, MILPs can be solved accurately using commercial software, such as CPLEX ([www.ibm.com](http://www.ibm.com)) and MOSEK ([www.mosek.com](http://www.mosek.com)).

#### B. Mixed-Integer Linear Program Formulation

1) *Reformulation of the AGC Dispatch Model*: The AGC dispatch model in (15) is difficult to handle due to the fractional multiplier and the minimization function. First, consider the fractional multiplier, which can be tackled as follows.

**Theorem 1.** *We can replace constraint (15) in problem (27) with the following constraint without affecting the solution:*

$$AGC_{i,k}[\tau] = \min \left\{ r_i^C, r_i^M AGC_k[\tau] / R^M \right\}. \quad (28)$$

The proof of Theorem 1 is given in the Appendix.

Next, we need to find a way to tackle the nonlinearity in the minimization function in (28). One option is to introduce some auxiliary binary variables so as to replace the nonlinear minimization function in (28) with a set of MILP constraints. This approach is explained in details, e.g., in Section III-A in [27]. However, such approach has curse of dimensionality because it requires adding  $225 \times I \times K$  new binary variables to the optimization problem. Therefore, alternatively, we assume that the AGC set points are always upper-bounded by the cleared regulation capacity. Hence, we replace (28) with

$$AGC_{i,k}[\tau] = r_i^M AGC_k[\tau] / R^M \quad \forall i, k, \tau, \quad (29)$$

$$AGC_{i,k}[\tau] \leq r_i^C \quad \forall i, k, \tau. \quad (30)$$

Of course, the above constraints may *not always* be equivalent to (28); but they provide a practical approximation for the AGC dispatch model. The approximation becomes exact if the following inequality holds across the system parameters:

$$m_i AGC_k[\tau] \leq R^M, \quad \forall k. \quad (31)$$

To see why, we note that, from (31), we have

$$r_i^M AGC_k[\tau] / R^M \leq r_i^M / m_i \leq r_i^C, \quad (32)$$

where the second inequality is due to (14). Thus, if (31) holds, then (28) reduces to (29), making it an accurate model. Interestingly, this condition often holds in practice. For example, for the six major CAISO regulation resources in Footnote 1, the average resource-specific mileage multipliers for regulation up service are 2.7, 2.24, 4.55, 2.3, 2.5, and 3.6 [14], which are small enough to often satisfy the inequality in (31).

2) *Reformulation of the Profit Model*: To tackle the nonlinearities in the objective function in (16), we start by replacing the AGC signal by its model in (29). After reordering the terms, the objective function becomes:

$$\begin{aligned} & \sum_i MCP^C r_i^C + \frac{1}{K R^M} MCP^M \times \sum_{i,k} \left[ r_i^M - \right. \\ & \quad \left( \sum_{\tau} | r_i^M AGC_k[\tau] - R^M y_{i,k}[\tau] | \right) / \\ & \quad \left( \sum_{\tau} AGC_k[\tau] \right) \times \\ & \quad \left. \left[ \sum_{\tau} | AGC_k[\tau] - AGC_k[\tau - 1] | \right] \right]. \end{aligned} \quad (33)$$

We can see that the denominator in the fractional multiplier, i.e.,  $\sum_{\tau} AGC_k[\tau]$ , is now a constant. The summation over absolute values, i.e.,  $\sum_{\tau} | AGC_k[\tau] - AGC_k[\tau - 1] |$ , is also a constant. These constant terms no longer add to the complexity of the problem, resolving some of the initial nonlinearities.

Next, we combine (29) with (6) and (7), and rewrite the subtraction inside the first absolute value term in (33) as

$$r_i^M AGC_k[\tau] - R^M y_{i,k}[\tau] = \phi_{i,k}[\tau] + r_i^M \psi_{i,k}[\tau], \quad (34)$$

where

$$\begin{aligned}\phi_{i,k}[\tau] &\triangleq -R^M y_{i,k}(0^-) e^{-4\tau/T_1}, \\ \psi_{i,k}[\tau] &\triangleq \sum_{l=1}^{\tau} \left[ \left( AGC_k[l] - AGC_k[l-1] \right) e^{-4(\tau-1+l)/T_1} \right].\end{aligned}$$

Here,  $\phi_{i,k}[\tau]$  and  $\psi_{i,k}[\tau]$  are constant, i.e., they do *not* depend on optimization variables. The term  $\phi_{i,k}[\tau]$  includes a *diminishing exponential term* which approaches zero as  $\tau$  increases. Hence, for any small  $\epsilon > 0$ , there exists a threshold

$$\tau_{th} \triangleq \lceil -T_1 \log(\epsilon)/4 \rceil \quad (35)$$

such that for any time slot  $\tau \geq \tau_{th}$ , we have  $\exp(-4\tau/T_1) \leq \epsilon$ . Therefore, if  $\epsilon$  is sufficiently small, then we can approximate  $\phi_{i,k}[\tau] \approx 0$  for any  $\tau \geq \tau_{th}$  and any  $i$  and  $k$ .

To tackle the nonlinearity in  $|\phi_{i,k}[\tau] + r_i^M \psi_{i,k}[\tau]|$ , we can replace it with a new auxiliary variable  $\theta_{i,k}[\tau]$  for any  $\tau < \tau_{th}$ . Accordingly, we can rewrite the absolute-value summation as

$$\sum_{\tau < \tau_{th}} \theta_{i,k}[\tau] + \sum_{\tau \geq \tau_{th}} r_i^M |\psi_{i,k}[\tau]|. \quad (36)$$

The following linear constraints are also needed:

$$-\theta_{i,k}[\tau] \leq \phi_{i,k}[\tau] + r_i^M \psi_{i,k}[\tau] \leq \theta_{i,k}[\tau], \quad \forall \tau < \tau_{th}. \quad (37)$$

Since we intend to maximize the profit expression in (36), and because of the negative sign before the second term inside the brackets, exactly one of the two inequalities in (37) holds as equality once the optimization problem is solved.

Next, consider the optimization problem in (8)-(14), i.e., the so-called *lower-level* problem. This problem is a linear program and accordingly carries *zero duality gap*, i.e., *strong duality* holds for this optimization problem [25, pp. 215-236]. Therefore, by setting the primal objective function equal to the dual objective function in problem (8)-(14), and after reordering the terms, we can write the objective function as

$$\begin{aligned} & R^C MCP^C + R^M MCP^M - \sum_i r_i^M MCP^M - \\ & \sum_{j \neq i} \xi_j U_j - \sum_{j \neq i} (O_j^C r_j^C + O_j^M r_j^M) + \frac{1}{KR^M} MCP^M \times \\ & \sum_{i,k} \left[ r_i^M - \left( \sum_{\tau < \tau_{th}} \theta_{i,k}[\tau] + \sum_{\tau \geq \tau_{th}} r_i^M |\psi_{i,k}[\tau]| \right) / \right. \\ & \quad \left. \left( \sum_{\tau} AGC_k[\tau] \right) \right] \times \\ & \quad \left[ \sum_{\tau} |AGC_k[\tau] - AGC_k[\tau-1]| \right]. \end{aligned} \quad (38)$$

The key property of the above objective function is that if  $MCP^M$  is taken as a constant, then the expression in (38) is *linear* with respect to the rest of optimization variables.

3) *Reformulation of the Market Optimization Problem:* The *complimentary-slackness* constraints in (20)-(25) are non-

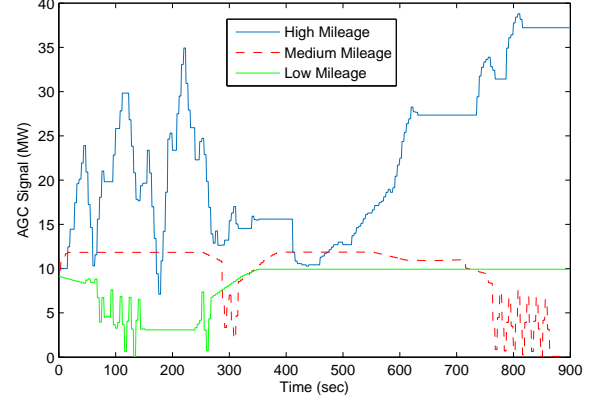


Fig. 3. The AGC signals for the illustrative example in Section IV-A-1.

linear [25, p.18]; however, one can use the techniques in [15], [26] and rewrite them in form of equivalent MILP constraints:

$$\sum_j r_j^C - R^C \leq Lz^C \quad (39)$$

$$MCP^C \leq L(1 - z^C) \quad (40)$$

$$\sum_j r_j^M - R^M \leq Lz^M \quad (41)$$

$$MCP^M \leq L(1 - z^M) \quad (42)$$

$$r_j^C \leq Lz_j^\mu \quad \forall j, \quad (43)$$

$$\mu_j \leq L(1 - z_j^\mu) \quad \forall j, \quad (44)$$

$$U_j - r_j^C \leq Lz_j^\xi \quad \forall j, \quad (45)$$

$$\xi_j \leq L(1 - z_j^\xi) \quad \forall j, \quad (46)$$

$$r_j^M - r_j^C \leq Lz_j^\gamma \quad \forall j, \quad (47)$$

$$\gamma_j \leq L(1 - z_j^\gamma) \quad \forall j, \quad (48)$$

$$-r_j^M + m_j r_j^C \leq Lz_j^\eta \quad \forall j, \quad (49)$$

$$\eta_j \leq L(1 - z_j^\eta) \quad \forall j, \quad (50)$$

where  $z^C, z^M, z_j^\mu, z_j^\xi, z_j^\gamma$ , and  $z_j^\eta$  are auxiliary binary variables corresponding to the complimentary-slackness constraints. For example, from (9), (39), (40), and (26), if  $z^C = 0$ , then  $\sum_j r_j^C = R^C$ ; and if  $z^C = 1$ , then  $MCP^C = 0$ . Please refer to [15], [26] for more details about the above reformulation. Note that, solving the system of MILP equalities and inequalities in (17)-(19), (26) and (39)-(50) is equivalent to solving the regulation market optimization problem in (8)-(14).

4) *Resulted Mixed-Integer Linear Program:* From the results in Sections III-B-1, III-B-2, and III-B-3, we now propose to reformulate problem (27) as follows:

**maximize** (38)

**subject to** (17) – (19), (26), (29), (30), (37), (39) – (50), (51)

where the optimization variables comprise of three different groups: first, the cleared market prices  $MCP^C$  and  $MCP^M$ ; second,  $r_j^C, r_j^M, \mu_j, \xi_j, \eta_j, \gamma_j, z_j^C, z_j^M, z_j^\mu, z_j^\xi, z_j^\gamma, z_j^\eta$  where  $j$  is the index for *all* regulation resources; third,  $\theta_{i,k}[\tau], O_i^C, O_i^M$ , where  $i$  is the index for all regulation resources that belong to the generation firm of interest. In contrast,  $O_j^C$  and



$O_j^M$  for any  $j \neq i$ , i.e., the price bids of the other regulation resources, serve as parameters.

The optimization problem in (51) is an MILP as long as  $MCP^M$  is taken as a constant. Therefore, one can solve problem (51) by combining an *one dimensional* exhaustive search over single variable  $MCP^M$  with standard MILP solution techniques. Solving problem (51) using this hybrid approach has several advantages over solving problem (27) using global optimization techniques, see Section IV-B-1.

It is worth noting that, the practicality of an exhaustive search depends on the dimension and size of the search space. In particular, the *one-dimensional* exhaustive search that we proposed above has a very low computational complexity. Another issue that plays to our advantage is the fact that the range of regulation mileage prices in the CAISO regulation market is relatively small [14]; therefore, the one-dimensional exhaustive search is over a relatively small set. That being said, we believe that there might exist even better solution approaches to solve the original NLP problem in (27) that could be explored as an extension of this paper in the future.

#### IV. CASE STUDIES

##### A. Case I: Small Illustrative Example

Consider a small performance-based regulation market with three generators. Generator 1 is of interest to submit optimal bids. The dynamics of this generator are modeled by the first transfer function in Table I, where  $T_1 = 7.5$  sec and  $y_1(0^-) = AGC_{1,k}[1]$ . We have  $U_1 = 40$  MW and  $m_1 = 4$ . For other generators,  $U_2 = 40$  MW,  $U_3 = 50$  MW,  $m_2 = 3$ ,  $m_3 = 3$ ,  $O_2^C = 8$  \$/MW,  $O_3^C = 10$  \$/MW,  $O_2^M = 3$  \$/MW, and  $O_3^M = 2$  \$/MW. Other parameters are  $L = 1000$ ,  $K = 1$ , and  $\epsilon \rightarrow 0$ .

1) *High, Medium and Low Mileage Scenarios*: The realization of the AGC signal under these three mileage scenarios are shown in Fig. 3. They are derived from the data in [28], [29]. The corresponding total mileage, i.e., the summation of the absolute value differences between every two consecutive AGC set points, are obtained as 197.41 MW, 118.47 MW, and 72.41 MW, respectively. The market regulation capacity requirement is 80 MW for all three scenarios. The market mileage requirements are 200 MW, 120 MW, and 80 MW, respectively. The market outcome under the proposed optimal bidding mechanism is shown in Table II. When the mileage requirement is high, Generator 1 seeks to maximize its profits by obtaining mileage revenue, where  $r_1^M$  is high. As the market mileage requirement decreases, Generator 1 may earn less revenue from mileage. Hence, it seeks to increase the regulation capacity revenue. Accordingly, it uses its price-making ability to increase  $MCP^C$  while decreasing  $MCP^M$ . In contrast, when the mileage requirement is low, Generator 1 has very low incentive to earn mileage revenue. Therefore, it offers a *very high* regulation capacity price bid  $O_i^C$  and a *very low* regulation mileage price bid  $O_i^M$ . Nevertheless, the ISO still clears Generator 1's bids because the cleared mileage and the cleared capacity are *linked together*, through the market optimization problem in (8)-(14). To elaborate, from (13)-(14), we have  $r_i^C \leq r_i^M \leq m_i r_i^C$ , which means that for the low mileage bid price  $O_i^M$ , the ISO is willing to clear resource

$i$  in the amount of  $r_i^M$ . However,  $r_i^M$  should be at least equal to  $r_i^C$ , meaning that the ISO will clear the regulation capacity  $r_i^C$ , even for a high price bid regulation capacity  $O_i^C$ . Therefore, in this case, Generator 1 can obtain profit without having concerns about its regulation performance in the market. Unfortunately, the fact that such higher profit comes at the expense of decreasing the clearing mileage prices suggests that Generator 1's market power in this example results in discouraging the other regulation resources from offering high performance regulation service in a performance-based regulation market, which is clearly not desirable.

Another point to highlight here is that the cleared regulation mileage, i.e.,  $r^M$ , is *not* financially binding, and a regulation resource will be compensated for the regulation capacity service it provides, i.e.,  $r^C$ , as well as its actual performance. Calculating the actual performance constitutes the product of the instructed mileage and the accuracy, see Fig. 1.

TABLE II  
MARKET OUTCOMES FOR ILLUSTRATIVE EXAMPLE

Mileage	Generator 1					Generators 2, 3		Market	
	$r_1^C$	$r_1^M$	Instructed Mileage	Accuracy	Revenue	$r_2^C, r_3^C$	$r_2^M, r_3^M$	$MCP^C$	$MCP^M$
High	40	160	157.87	96.03	719	40 0	40 0	8.5	2.5
Medium	40	80	78.11	96.66	551	40 0	40 0	10	2
Low	40	40	36.21	98.16	480	40 0	40 0	12	0

2) *Fast and Slow System Dynamics*: Consider the high mileage scenario and suppose the system dynamics of Generator 1 is modeled based on the second transfer function in Table I with three poles and one zero. All other parameters are the same as in Section IV-A-1, except that Generator 1 starts from zero condition. The parameters of the transfer function are  $T_2 = 1$ ,  $T_3 = .5$ ,  $T_4 = 10\alpha$ , and  $T_5 = 25\alpha$ , where  $\alpha$  is a number between 0.1 and 25. As we increase  $\alpha$ , we move two poles to the left, creating a generator with slower dynamics. Two examples are shown in Fig. 4, where  $\alpha = 1$  for the Fast generator and  $\alpha = 5$  for the slow dynamic. The market outcomes when we change  $\alpha$  from 0.1 to 25 are shown in Fig. 5. Here, the impact of increasing  $\alpha$  is represented in terms of reducing the performance accuracy from 100% to 10%.

As expected, faster responding generators can collect more mileage revenue due to high accuracy in following the AGC signal. When accuracy is almost perfect, Generator 1 offers a relatively high price for mileage and a relatively low price for capacity. As the generator's ability to follow the AGC signal degrades, it tries to use its price-making capability to decrease  $MCP^M$  and rather increase  $MCP^C$  to continue making profit, but of course at an inevitability lower rate. The thresholds to switch to new offers are at 76.1% and 25.2% performance accuracies, as marked in Figs. 5 (a) and (b).

3) *Price Taker versus Price Maker Bidding*: Since the focus of this paper is on price-making market participation, where the regulation firm of interest can change the price to make

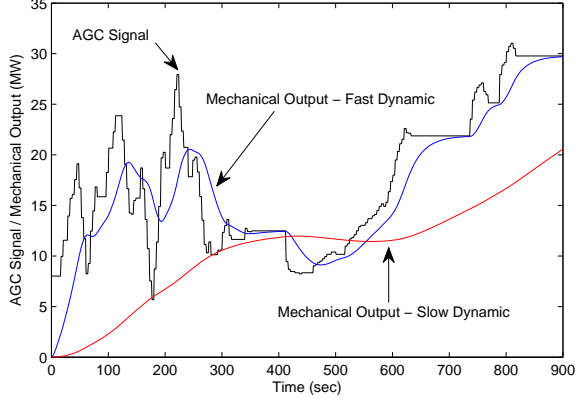


Fig. 4. The AGC signal versus the output of Fast and Slow generators.

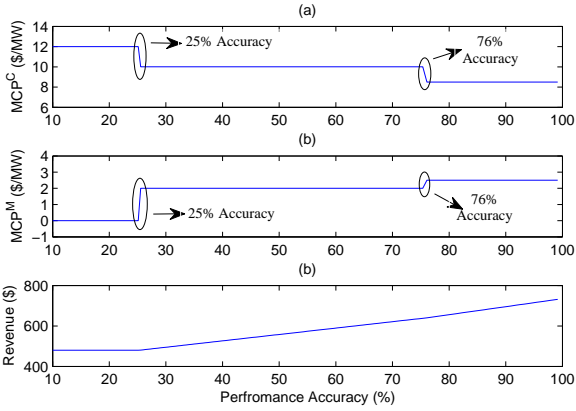


Fig. 5. The impact of changing performance accuracy: (a) cleared regulation capacity price, b) cleared regulation mileage price, c) total revenue.

more profit, next we compare the maximum profit of the firm for the price-taker and the price-maker design cases. The results are shown in Fig. 6. The high mileage scenario is considered for the results in this figure. As expected, the two designs are similar when the resource capacity is small. However, as the resource capacity increases, the regulation firm that exploits its price-making capability can increase its profit. It is interesting to note that, the profit under the price taker design soon experiences *saturation* because increasing the regulation capacity does no longer help in this case. In contrast, under the price-maker design, the revenue increases noticeably, making more incentives for the regulation firm.

### B. Case II: California ISO Regulation Market Data

Next, we increase the size and complexity of the case study using the data from the *CAISO Regulation Energy Management Project-Phase 1* [29]. The AGC data of all participating resources are used at 4 second intervals during six days in April 2012. These AGC signals are summed up at 10 representative high mileage and low mileage requirement times, to generate  $K = 10$  scenarios, where the average mileage is 629.83 MW. In some scenarios, the mileage is as low as 0 MW and in some other scenarios it is as high as 1372.9 MW. The peak AGC signal among all scenarios is 103 MW. We consider 19 participants where  $I = 7$  of

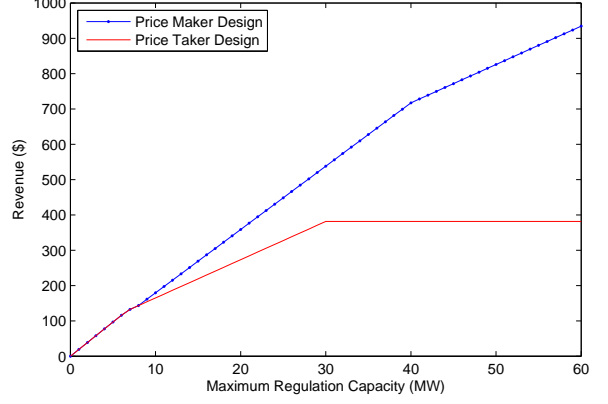


Fig. 6. Comparing price-taker and price-maker bidding. The need for a price-maker design is more evident as the capacity of the resource increases.

TABLE III  
MARKET REQUIREMENTS AND PRICES RESULTS FOR  
THE LARGE MARKET EXAMPLE

Market	Parameters		Results	
	$R^C$	$R^M$	$MCP^C$	$MCP^M$
	400	800	13	1.5

them belong to the firm of interest. We assumed all the regulation resources of the firm of interest have the same dynamic characteristics as Generator 1 in Section IV-A-1. For generator  $i = 7$  of the firm of interest, we have  $m_7 = 15$  which is twice  $R^M / \max_{k,\tau} \{AGC_k[\tau]\} = 800/103 = 7.8$ . It means our assumption in (29) does not hold for this generator. This assumption *does* hold for all other generators of the firm of interest. All other parameters for generator offers, regulation capacities, resource-specific mileage multipliers, and market requirements are from [14], [30]. The parameters and the results are shown in Tables III, IV, and V. The cleared prices under optimal bidding are  $MCP^C = 13$  MW and  $MCP^M = 1.5$  MW, and the total revenue of the firm is \$2418.

1) *Optimality and Computational Complexity*: We examine two different ways to obtain the optimal bids. First, we solve the NLP in (27) in GAMS ([www.gams.com](http://www.gams.com)), using the following Global Optimization solvers: CONOPT, SNOPT and IPOPT. Second, we solve the reformulated MILP in (51) in MATLAB using *intlinprog* command. No difference was observed in the results for the illustrative example in Section IV-A. However, the results were drastically different for the large test case in this Section. Specifically, two solvers, namely SNOPT and IPOPT, failed to reach any solution for the NLP in (27). As for CONOPT, the results were *very sensitive* to the choice of *initial point*, as it is shown in Fig. 7. Here, the iterations along the x-axis are only for the case of the NLP. Note that, the MILP final solution is also shown by a constant line. We can see that the number of iterations as well as the final results are significantly different for different choices of the initial point. In one case, the final objective value was infeasible. In contrast, the solution from MILP is unique and it outperforms all the solutions from NLP. We can conclude that our proposed solution method in Section III is necessary in order to obtain an optimal bidding solution.



TABLE IV  
MARKET PARAMETERS AND RESULTS FOR ALL RESOURCES  
 $j \neq i$  IN THE LARGE MARKET EXAMPLE

Generators	Parameters				Results	
	$m$	$U$	$O^C$	$O^M$	$r^C$	$r^M$
1	3	30	11	3.5	22	22
2	3	40	12	2.6	0	0
3	5	25	9	0.5	25	125
4	12	40	8.78	2.8	40	40
5	5	40	10.5	4.5	0	0
6	3	45	14	4.02	0	0
7	4	50	12	3.8	0	0
8	5	70	11.2	2.9	70	70
9	2	10	10.3	0.9	10	20
10	3	30	9.75	1.5	30	30
11	4	33	9.5	2	33	33
12	4	20	12	2.4	20	20

TABLE V  
MARKET PARAMETERS AND RESULTS FOR ALL RESOURCES  $i$  OF THE  
FIRM OF INTEREST IN THE LARGE MARKET EXAMPLE

Generators	Parameters		Results						
	$m$	$U$	$O^C$	$O^M$	$r^C$	$r^M$	Instructed Mileage	Accuracy	Revenue
1	4	15	1.5	13	15	15	11.82	90.1	211
2	3	25	1.5	13	25	25	19.7	90.1	352
3	2	15	1.5	13	15	30	23.64	90.1	227
4	4	15	1.5	13	15	15	11.82	90.1	211
5	2	15	1.5	13	15	30	23.64	90.1	227
6	5	25	1.5	13	25	25	19.7	90.1	352
7	15	40	1.5	13	40	300	236.4	90.1	839

Next, we show the results for solving the NLP in (27) and MILP in (51), where in both cases we conduct *exhaustive search* over  $MCP^M$ . The results are shown in Figs. 8 (a) and (b). We can report two results. First, exhaustive search does help solving the NLP for *certain initial points*. However, no feasible solution is found or the solution is not optimal for some other initial points. In one case, exhaustive search found the optimal  $MCP^M$ , yet GAMS was unable to find the optimal solutions for other variables, causing a major optimality gap. Second, the computational time for exhaustive search is 2072 seconds versus only 66 seconds when applied to NLP formulation versus the MILP formulation. Here, we have  $\epsilon = 2 \times 10^{-6}$  and  $\tau_{th} = 25$ . Therefore, the solution method in Section III is *accurate, reliable, and computationally tractable*.

## V. CONCLUSIONS

A new analytical framework was proposed for optimal bidding of large, price-maker regulation resources in the California ISO performance-based regulation market. Regulation resources in a firm seek to maximize their total profit, while taking into account the system dynamics of their physical resources as well as various details in the California ISO market. Accordingly, a novel MPEC analysis with system dynamics was conducted which introduced several new challenges. Specifically, we showed that global optimization tools fail to find a feasible solution or the solution that they find is inaccurate. Therefore, the original non-linear programming problem was carefully transformed into a mixed-integer linear program which is solved with accuracy, reliability, and with

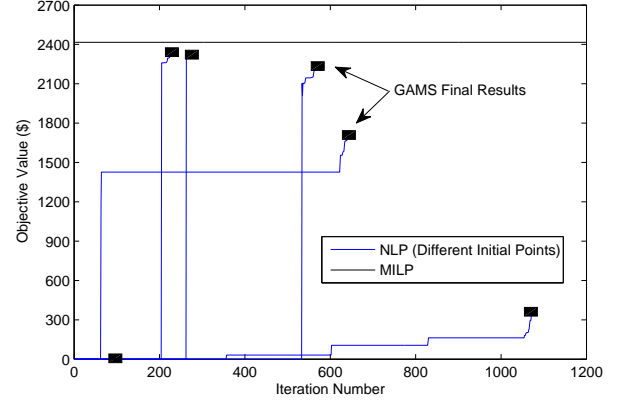


Fig. 7. The results on solving the NLP in (27) versus the MILP in (51). The former solutions are very sensitive to the choice of NLP initial point. Infeasible solutions are represented with zero objective values.

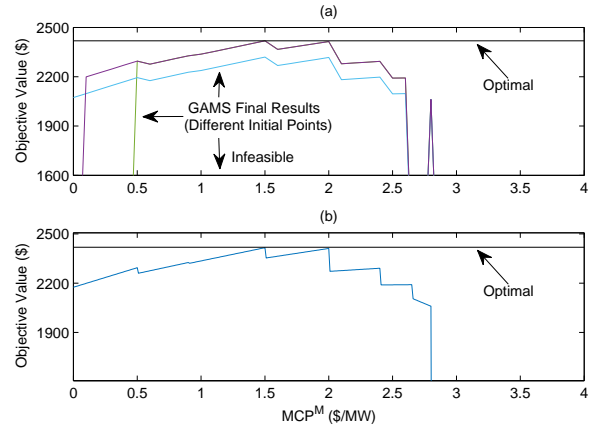


Fig. 8. Optimal objective value when we conduct exhaustive search over single variable  $MCP^M$ : (a) NLP formulation, b) MILP reformulation.

computational efficiency. Diverse case studies were discussed based on data from a California ISO regulation market project.

Of course, our proposed method also has some limitations. First, if the uncertainty is considered in the market optimization problem, then the size of the problem may grow noticeably in order to incorporate several random scenarios. Second, we assumed linear system dynamics for regulation resources. In practice, some regulation resources may have nonlinear dynamics. Third, the AGC dispatch mechanism in (15) is only one possible option in practice. More sophisticated AGC dispatch mechanisms can be considered.

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## APPENDIX: PROOF OF THEOREM 1

First, we note that, by substituting  $y_{i,k}[\tau]$  in (1) with its mathematical expressions that are given in (6) and (7), we can

rewrite (1), i.e., the objective function, as follows:

$$\begin{aligned}
 & \sum_i MCP^C r_i^C + \frac{1}{K} MCP^M \times \\
 & \sum_{i,k} \left[ \left( \sum_{\tau} AGC_{i,k}[\tau] - \left| AGC_{i,k}[\tau] - y_{i,k}(0^-) e^{-4\tau/T_1} \right. \right. \right. \\
 & \quad \left. \left. - \sum_{\kappa=1}^{225} \left[ \left( AGC_{i,k}[\kappa] - AGC_{i,k}[\kappa-1] \right) \times \right. \right. \right. \\
 & \quad \left. \left. \left. \left( 1 - e^{-4(\tau-\kappa+1)/T_1} \right) u(4(\tau-\kappa+1)) \right] \right| \right) \Bigg] / \\
 & \quad \left( \sum_{\tau} AGC_{i,k}[\tau] \right) \times \\
 & \quad \left[ \sum_{\tau} \left| AGC_{i,k}[\tau] - AGC_{i,k}[\tau-1] \right| \right], \tag{52}
 \end{aligned}$$

Accordingly, we can rewrite problem (27) as

$$\begin{aligned}
 & \text{maximize} \quad (52) \\
 & \text{subject to} \quad (15), (17) - (26). \tag{53}
 \end{aligned}$$

Next, consider the first constraint in (26), i.e.,  $MCP^M \geq 0$ . Let us examine two possible cases based on this constraint.

*Case I)* Suppose  $MCP^M > 0$ . From (21), we must have:

$$R^M - \sum_i r_i^M - \sum_{j \neq i} r_j^M = 0 \rightarrow \sum_i r_i^M + \sum_{j \neq i} r_j^M = R^M.$$

Substituting the above in (15), constraint (28) is directly obtained. In this case, the constraints (15) and (28) are exactly equivalent; thus, the optimal solution of problem (53) does not change after replacing constraint (15) with (28) under Case I.

*Case II)* Suppose  $MCP^M = 0$ . In this case, the objective function in optimization problem (53) reduces to:

$$\sum_i MCP^C r_i^C. \tag{54}$$

Accordingly, we can rewrite problem (53) as

$$\begin{aligned}
 & \text{maximize} \quad (54) \\
 & \text{subject to} \quad (15), (17) - (26). \tag{55}
 \end{aligned}$$

Next, suppose we replace constraint (15) with constraint (28). The revised optimization problem becomes:

$$\begin{aligned}
 & \text{maximize} \quad (54) \\
 & \text{subject to} \quad (28), (17) - (26). \tag{56}
 \end{aligned}$$

Finally, suppose we simply drop constraint (15) from (55) and drop constraint (28) from (56). In both cases, the following relaxed optimization problem is resulted:

$$\begin{aligned}
 & \text{maximize} \quad (54) \\
 & \text{subject to} \quad (17) - (26). \tag{57}
 \end{aligned}$$

Note that, unlike problems (55) and (56), problem (57) does not include  $AGC_{i,k}[\tau]$  as a variable for any  $i, k$  and  $\tau$ . Now, let us denote the optimal solution of problem (57) as:

$$MCP^{M*} = 0$$

$$r_j^{C*}, r_j^{M*}, O_i^{C*}, O_i^{M*}, \mu_j^*, \xi_j^*, \eta_j^*, \gamma_j^*, MCP^{C*}, \quad \forall i, j. \tag{58}$$

Since the only constraint in (55) that includes  $AGC_{i,k}[\tau]$  is (15), where  $AGC_{i,k}[\tau]$  is defined in terms of variables  $r_i^C$  and  $r_i^M$  for any  $i$  and parameter  $AGC_k[\tau]$  for any  $k$  and  $\tau$ , the following is a *feasible* solution for problem (55):

$$\begin{aligned} MCP^{M^*} &= 0 \\ r_j^{C^*}, r_j^{M^*}, O_i^{C^*}, O_i^{M^*}, \mu_j^*, \xi_j^*, \eta_j^*, \gamma_j^*, MCP^{C^*}, &\quad \forall i, j, \\ AGC_{i,k}[\tau] &= \min \left\{ r_i^{C^*}, \frac{r_i^{M^*} AGC_k[\tau]}{\sum_i r_i^{M^*} + \sum_{j \neq i} r_j^{M^*}} \right\} \quad \forall i, k, \tau. \end{aligned} \quad (59)$$

Similarly, the only constraint in (56) that includes  $AGC_{i,k}[\tau]$  is (28), where  $AGC_{i,k}[\tau]$  is defined in terms of variables  $r_i^C$  and  $r_i^M$  for any  $i$  and parameters  $R^M$  and  $AGC_k[\tau]$  for any  $k, \tau$ , the following is a *feasible* solution for problem (56):

$$\begin{aligned} MCP^{M^*} &= 0 \\ r_j^{C^*}, r_j^{M^*}, O_i^{C^*}, O_i^{M^*}, \mu_j^*, \xi_j^*, \eta_j^*, \gamma_j^*, MCP^{C^*}, &\quad \forall i, j, \\ AGC_{i,k}[\tau] &= \min \left\{ r_i^{C^*}, r_i^{M^*} AGC_k[\tau] / R^M \right\}, \quad \forall i, k, \tau. \end{aligned} \quad (60)$$

From (58), (59), and (60), the *optimal* solution of problem (57) readily forms a *feasible* solution for problem (55) and a *feasible* solution for problem (56). Therefore, since the objective functions are the same in (55), (56), and (57), and because  $AGC_{i,k}[\tau]$  does *not* appear anywhere in this objective function, we can conclude that the optimal objective value in (57) is a *lower bound* for the optimal objective values in (55) and (56). However, since, by construction, problem (57) is a relaxation of problems (55) and (56), its optimal objective value is also an *upper bound* for the optimal objective values of problems (55) and (56). Therefore, we can conclude that the three optimization problems in (55), (56) and (57) have *equal optimal objective values*. This means that, under Case II, both (15) and (28) are irrelevant constraints that can be *swapped* or even *removed* without affecting the optimal bids, also see [31, p. 305]. It is worth clarifying that  $AGC_{i,k}[\tau]$  may still take different values in (59) and (60); however, any such difference would *not* have any impact on the optimal bidding solutions in (58) or the optimal objective value in (54). That is, under Case II,  $AGC_{i,k}[\tau]$  is nothing but a slack variable.

To summarize, either (15) and (28) are both relevant constraints with impact on choosing the bids, which in that case they are equivalent as shown in Case I, or they are both irrelevant constraints with no impact on choosing the bids, which in that case we can swap them, as in Case II. ■