Coordinated Price-Maker Operation of Large Energy Storage Units in Nodal Energy Markets

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***Abstract*—In this paper, a new optimization framework is proposed to coordinate the operation of *large*, *price-maker*, and *geographically dispersed* energy storage / battery systems in a nodal transmission-constrained energy market. The energy storage units are assumed to be *investor-owned* and *independently-operated*, seeking to maximize their total profit. Various design factors are taken into consideration such as the location, size, efficiency, and charge and discharge rates of the energy storage units as well as the joint impact of the energy storage operations on the locational marginal prices. While the formulated optimization problem is originally nonlinear and hard to solve, nonlinearities are tackled both in the objective function and in the constraints and the problem is transformed into a tractable mixed-integer linear program, for which the *global optimal* solutions are found for the charge and discharge schedules of each energy storage unit. Both deterministic and stochastic design scenarios are addressed. Various case studies are presented. It is observed that transmission line congestion is often, but not always, desirable for the coordinated storage systems. Locational diversity, practice of arbitrage, robust design, self-scheduling versus economic bidding, and the overall power system performance are also investigated.**

***Keywords*: Energy storage, coordinated charge and discharge schedules, transmission-constrained market, locational marginal price, price-maker, arbitrage, mixed-integer linear programming.**

NOMENCLATURE

$\mathcal{N}$	Set of all buses
$\mathcal{G}$	Set of generator buses
$\mathcal{D}$	Set of load buses
$\mathcal{S}$	Set of storage buses
$\mathcal{L}$	Set of transmission lines
x	Storage discharge variable
y	Storage charge variable
m	Storage charge and discharge indicator
z	Storage charge level
i, j	Subscript indicating a bus
l	Subscript indicating a transmission line
G	Superscript indicating generator power
D	Superscript indicating load power
S	Superscript indicating storage power
SD	Superscript indicating storage discharge power
SC	Superscript indicating storage charge power
H	Imaginary part of Y-bus matrix
C	Capacity of transmission line
T	Hourly market horizon
t	Hourly time-slot index
K	Number of random scenarios
k	Random scenario index
λ	Locational marginal price
θ	Bus voltage phase angle
a, b, c	Price bids
ρ, ϕ, ψ	Lagrange multipliers
$\sigma, \delta, \zeta, \xi$	Lagrange multipliers
r, ω	Auxiliary optimization variables
α, β	Storage efficiency parameters
L	A fixed large number

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I. INTRODUCTION

The large-scale deployment of batteries and other energy storage systems is one of the *priority areas* to build a smart grid, as identified by the Department of Energy [1] and the National Institute of Standards and Technology [2]. The applications of grid-scale energy storage are diverse and include bulk energy support, synchronous reserve, non-synchronous reserve, voltage support, and frequency regulation [3]–[5].

While many of the existing major battery projects in the United States and elsewhere are fully or partially funded by the state and federal governments [6]–[8], it is necessary to also plan for a more *sustainable* option where the battery systems are *investor-owned* and *independently-operated*. Such investor-owned and independently-operated battery systems will have to participate in the existing electricity markets. For example, in California, the California Public Utilities Commission has adopted an energy storage procurement framework that requires the three large investor-owned utility companies in California to install 1,325 MW of energy storage by 2020 [9].

With investor-owned independently-operated energy storage systems being on the horizon, it is natural to ask the following questions: *What is the most profitable charge and discharge schedule for a single energy storage system or a group of geographically dispersed but coordinated energy storage systems in a wholesale electricity market? Is arbitrage an optimal option? What is the impact of the energy storage systems on the price of electricity when the energy storage systems are large and price-maker? In a transmission-constrained network, how does energy storage affect congestion?*

Answering the above questions is the focus of this paper. The contributions in this paper can be summarized as follows:

- An optimization-based framework is proposed to coordinate the operation of large, price-maker, and geographically dispersed energy storage / battery systems in a nodal transmission-constrained energy market. Both deterministic and stochastic optimization scenarios are considered. The objective is to maximize the total profit of the coordinated energy storage units. To the best of our knowledge, this problem has not been addressed before.

- The proposed framework takes into account design factors such as the location, size, efficiency, and charge and discharge rates of the energy storage units as well as the joint / coordinated impact of the energy storage systems operations on the locational marginal prices.
- While the formulated optimization problem is initially nonlinear and hard to solve, the nonlinearities are tackled both in the objective function and constraints and the formulated optimization problem is transformed into a tractable mixed-integer linear program. Accordingly, the global optimal solution is obtained for the coordinated price-maker operation of the energy storage systems.
- Several case studies are presented. The role of arbitrage, the impact of congestion on different transmission lines, capacity of the congested lines, locational diversity, robust design, self-scheduling, economic bidding, and energy storage efficiency are investigated. Although the analysis in this paper is from the viewpoint of the energy storage systems with focus on profitability, the impact of the coordinated price-maker operation of the energy storage systems on the overall power system performance is also studied in terms of the total generation cost.

The analysis and results in this paper can be compared with three groups of papers in the literature on operating batteries and other energy storage systems. First, there are papers that optimize the use of batteries to improve efficiency and reliability at distribution and microgrid level [10]–[12] or at transmission level [13]–[16], but their focus is not on the *profitability* and *market participation* aspects. In contrast, the focus here is on investor-owned energy storage systems that are primarily concerned with their own profit. Second, there are papers that combine and co-locate batteries with other energy resources, such as wind farms [17]–[19], solar farms [20], [21], or demand response aggregators [22]–[24]. Accordingly, in these papers, the operations of battery systems are often dependent on those of the other energy resources.

The third group, which includes [25]–[28], is particularly close to this paper. The focus is similarly on the profitable operation of investor-owned batteries and other energy storage systems in wholesale electricity markets. However, the results here are new and unique in at least two key aspects. First, the prior studies in this third group, including [16], [25]–[28] do not consider *large* and *price-maker* battery systems. Instead, the focus has mostly been on *relatively small* and *price-taker* battery and other energy storage systems. Second, the studies in [16], [25]–[28] operate either single energy storage units or a group of energy storage units that are *not* coordinated for higher profit. In contrast, here, the focus is on *coordinated operation* of geographically dispersed energy storage systems. Accordingly, this paper addresses some interesting aspects such as *locational diversity* and *practice of arbitrage*.

This paper is comparable also with the literature on energy market participation in contexts *other than coordinated energy storage systems*, e.g., see [29]–[32]. In particular, the problem formulation here is related to the general class of mathematical programming with equilibrium constraints (MPEC) problems [33]. However, to the best of our knowledge, no prior study has addressed the optimal coordinated scheduling and market participation of large, price-maker, geographically dispersed storage units that are investor-owned and independently-operated.

In this paper, the focus is only on *bulk* energy support and the use of storage units in *energy* markets. Other, non-bulk applications of storage units are beyond the scope of this paper.

II. Problem Formulation

Consider a power grid with $\mathcal{N}$ as the set of buses and $\mathcal{L}$ as the set of transmission lines. The transmission line between buses i and j is denoted by (i,j). Let $\mathcal{S} \subseteq \mathcal{N}$ denote the set of buses where the energy storage units are located. Since the focus in this paper is on coordinated charging and discharging of energy storage systems, the storage units at all buses are assumed to belong to the same firm. Suppose the day-ahead energy market is divided into $T = 24$ hourly time slots. At each time slot $t = 1, \ldots, T$, and for each storage unit $i \in \mathcal{S}$, let $m_i[t] \in \{0,1\}$ denote the type of the bid that is submitted to the wholesale market. If $m_i[t] = 0$, then unit i is *charged* during time slot t. Accordingly, it submits a *demand bid* to the energy market. If $m_i[t] = 1$, then unit i is *discharged* during time slot t. Accordingly, it submits a *supply bid* to the energy market. Let $x_i[t] \geq 0$ denote the supply energy bid in MWh that unit i submits to the wholesale market at time t. Also let $y_i[t] \geq 0$ denote the demand energy bid in MWh that unit i submits to the market at time t. The following constraints assure submitting one type of bid at each bus at a time:

$$0 \leq x_i[t] \leq m_i[t] x_i^{\max}, \tag{1}$$

$$0 \leq y_i[t] \leq (1 - m_i[t]) y_i^{\max}, \tag{2}$$

where $x_i^{\max}$ and $y_i^{\max}$ denote the maximum discharge rate and the maximum charge rate of the energy storage unit i, respectively. The price bid in \$/MWh that unit i submits to the wholesale market at time slot t is denoted by $c_i[t]$.

Let $z_i[t]$ denote the charge level of unit i at time slot t. The following constraints should hold at all times:

$$z_i^{\min} \leq z_i[t] \leq z_i^{\max}, \tag{3}$$

where $z_i^{\min} \geq 0$ and $z_i^{\max} \geq 0$ denote the minimum and the maximum charge levels for energy storage unit i. Next, suppose P_i^S denotes the amount of power that energy storage unit i injects to the power grid during time slot t. If unit i draws power from the grid, then P_i^S takes a negative value. Without loss of generality, the energy storage units are assumed to have ideal round-trip efficiencies:

$$z_i[t+1] = z_i[t] - P_i^S[t], \tag{4}$$

The case with non-ideal energy storage units, together with a few other extension scenarios, is explained in Section IV-A.

Next, optimization problem is formulated to coordinate charging and discharging of energy storage systems in a nodal electricity market. Let $\lambda_i[t]$ denote the *locational marginal price* (LMP) at bus i at time slot t. To maximize profit, the following optimization problem needs to be solved:

$$\begin{aligned} \textbf{max} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{S}} \lambda_i[t] P_i^S[t] \\ \textbf{s.t.} \quad & \text{Eqs. } (1)-(4), \end{aligned} \tag{5}$$

where the variables are $x_i[t]$, $y_i[t]$, $m_i[t]$, and $z_i[t]$. Here, $\lambda_i[t]$ and $P_i^S[t]$ depend on not only $x_i[t]$, $y_i[t]$, $m_i[t]$, and $z_i[t]$; but also the overall wholesale market conditions. Specifically, $\lambda_i[t]$ and $P_i^S[t]$ are obtained once the ISO solves the following economic dispatch problem across the power market:

$$
\begin{aligned}
\textbf{min} \quad & \sum_{i\in\mathcal{G}} a_i[t]P_i^G[t] - \sum_{j\in\mathcal{D}} b_i[t]P_i^D[t] + \sum_{i\in\mathcal{S}} c_i[t]P_i^S[t] \\
\textbf{s.t.} \quad & P_i^G[t] - P_i^D[t] + P_i^S[t] \\
& \quad = \sum_j H_{ij}(\theta_i[t]-\theta_j[t]), \qquad \forall\, i \in \mathcal{N}, \\
& P_i^{G,\min}[t] \le P_i^G[t] \le P_i^{G,\max}[t], \qquad \forall\, i\in\mathcal{G}, \\
& P_i^{D,\min}[t] \le P_i^D[t] \le P_i^{D,\max}[t], \qquad \forall\, i\in\mathcal{D}, \\
& -y_i[t] \le P_i^S[t] \le x_i[t], \qquad \forall\, i\in\mathcal{S}, \\
& -C_{ij} \le H_{ij}(\theta_i[t]-\theta_j[t]) \le C_{ij}, \qquad \forall\,(i,j)\in\mathcal{L},
\end{aligned} \tag{6}
$$

where the variables are the power injection $P_i^G[t]$ of each generator $i \in \mathcal{G}$, the power consumption $P_i^D[t]$ of each load $i \in \mathcal{D}$, the power injection / consumption $P_i^S[t]$ of each energy storage unit $i \in \mathcal{S}$, and the voltage phase angle $\theta_i[t]$ at each bus $i \in \mathcal{N}$. Notation $\mathcal{G} \subseteq \mathcal{N}$ is the set of generation buses and $\mathcal{D} \subseteq \mathcal{N}$ is the set of load buses. For notational simplicity, it is assumed that $P_i^S[t] = 0$, $P_i^G[t] = 0$, and $P_i^D[t] = 0$, for any $i \notin \mathcal{S}$, any $i \notin \mathcal{G}$, and any $i \notin \mathcal{D}$, respectively. For each generator i, $a_i[t]$ denotes the bidding price for selling energy, and $P_i^{G,\min}[t]$ and $P_i^{G,\max}[t]$ are the minimum and the maximum energy to be sold at time slot t. For each load i, $b_i[t]$ denotes the bidding price for buying energy, and $P_i^{D,\min}[t]$ and $P_i^{D,\max}[t]$ are the minimum and the maximum energy to be purchased at time slot t. For each line $(i,j) \in \mathcal{L}$, C_{ij} denotes the transmission capacity. Both the primal and the dual variables in (6) are of importance. Specifically, $P_i^S[t]$ is a primal variable and $\lambda_i[t]$ is a dual variable. The latter corresponds to the power balance constraint at bus i. Throughout this paper, parameter θ_1 is zero.

III. Solution Method

In this section, a method is presented to reformulate the coupled nonlinear optimization problem in (5)-(6) into a single mixed-integer linear program, whose global optimal solution can be found within a short amount of computational time. Here, *two sources of nonlinearity* need to be tackled. First, the inherent nonlinearity in the definition of LMP term $\lambda_s[t]$, which comes from the fact that problem (6) needs to be incorporated into (5) in form of a constraint. Second, the non-convex nonlinearity in the objective function in (5) which is due to the multiplication of LMP $\lambda_i[t]$ to the energy storage charging power injection / consumption variable $P_i^S[t]$.

A. Nonlinearity in Constraints

One can write the Karush-Kuhn-Tucker (KKT) conditions [34, p. 243] for the minimization problem in (6) as

$$
\begin{aligned}
& P_i^G[t] - P_i^D[t] + P_i^S[t] \\
& \quad = \sum_{j\neq i} H_{ij}(\theta_i[t]-\theta_j[t]), \qquad \forall\, i\in\mathcal{N},
\end{aligned} \tag{7}
$$

$$P_i^{G,\min}[t] \le P_i^G[t] \le P_i^{G,\max}[t], \qquad \forall\, i\in\mathcal{G}, \tag{8}$$

$$P_i^{D,\min}[t] \le P_i^D[t] \le P_i^{D,\max}[t], \qquad \forall\, i\in\mathcal{D}, \tag{9}$$

$$-y_i[t] \le P_i^S[t] \le x_i[t], \qquad \forall\, i\in\mathcal{S}, \tag{10}$$

$$-C_{ij} \le H_{ij}(\theta_i[t]-\theta_j[t]) \le C_{ij}, \qquad \forall\,(i,j)\in\mathcal{L}, \tag{11}$$

$$a_i[t] - \lambda_i[t] - \sigma_i[t] + \delta_i[t] = 0, \qquad \forall\, i\in\mathcal{G}, \tag{12}$$

$$-b_i[t] + \lambda_i[t] - \zeta_i[t] + \xi_i[t] = 0, \qquad \forall\, i\in\mathcal{D}, \tag{13}$$

$$c_i[t] - \lambda_i[t] - \rho_i[t] + \varrho_i[t] = 0, \qquad \forall\, i\in\mathcal{S}, \tag{14}$$

$$
\begin{aligned}
& -\sum_{j>i} H_{ij}(\phi_{ij}[t]-\psi_{ij}[t]) \\
& +\sum_{j<i} H_{ji}(\phi_{ji}[t]-\psi_{ji}[t]) \\
& +\sum_{j\neq i} H_{ij}\lambda_i[t] - \sum_{j\neq i} H_{ji}\lambda_j[t] = 0, \qquad \forall\, i\in\mathcal{N},
\end{aligned} \tag{15}
$$

$$\phi_{ij}[t](C_{ij} + H_{ij}(\theta_i[t]-\theta_j[t])) = 0, \qquad \forall\,(i,j)\in\mathcal{L}, \tag{16}$$

$$\psi_{ij}[t](H_{ij}(\theta_i[t]-\theta_j[t]) - C_{ij}) = 0, \qquad \forall\,(i,j)\in\mathcal{L}, \tag{17}$$

$$\sigma_i[t](P_i^{G,\min}[t] - P_i^G[t]) = 0, \qquad \forall\, i\in\mathcal{G}, \tag{18}$$

$$\delta_i[t](P_i^G[t] - P_i^{G,\max}[t]) = 0, \qquad \forall\, i\in\mathcal{G}, \tag{19}$$

$$\zeta_i[t](P_i^{D,\min}[t] - P_i^D[t]) = 0, \qquad \forall\, i\in\mathcal{D}, \tag{20}$$

$$\xi_i[t](P_i^D[t] - P_i^{D,\max}[t]) = 0, \qquad \forall\, i\in\mathcal{D}, \tag{21}$$

$$\rho_i[t](-y_i[t] - P_i^S[t]) = 0, \qquad \forall\, i\in\mathcal{S}, \tag{22}$$

$$\varrho_i[t](P_i^S[t] - x_i[t]) = 0, \qquad \forall\, i\in\mathcal{S}, \tag{23}$$

$$\sigma_i[t] \ge 0, \delta_i[t] \ge 0, \qquad \forall\, i\in\mathcal{G}, \tag{24}$$

$$\zeta_i[t] \ge 0, \xi_i[t] \ge 0, \qquad \forall\, i\in\mathcal{D}, \tag{25}$$

$$\rho_i[t] \ge 0, \varrho_i[t] \ge 0, \qquad \forall\, i\in\mathcal{S}, \tag{26}$$

$$\phi_{ij}[t] \ge 0, \psi_{ij}[t] \ge 0, \qquad \forall\,(i,j)\in\mathcal{L}. \tag{27}$$

Since problem (6) is a linear (hence convex) optimization problem and Slater's condition holds, c.f. [34, p. 226], solving problem (6) is equivalent to solving the *system of nonlinear equations* in (7)-(27). Note that, the nonlinearity is due to the complementary slackness constraints in (16)-(23).

For each transmission line $(i,j) \in \mathcal{L}$, the nonlinear equation in (16) holds if at least one of the following is true:

$$\phi_{ij}[t] = 0, \tag{28}$$

or

$$C_{ij} + H_{ij}(\theta_i[t]-\theta_j[t]) = 0. \tag{29}$$

Let us introduce a new binary variable $r_{ij}^{\phi}[t]$ such that $r_{ij}^{\phi}[t] = 1$ if (28) holds and $r_{ij}^{\phi}[t] = 0$ if (29) holds. The nonlinear equation in (18) can be replaced with the following constraints:

$$\phi_{ij}[t] \le \big(1 - r_{ij}^{\phi}[t]\big)L, \tag{30}$$

$$C_{ij} + H_{ij}(\theta_i[t]-\theta_j[t]) \le r_{ij}^{\phi}[t]L, \tag{31}$$

where L is a large number compared to the storage capacity. To verify the above statement, first, assume that $r_{ij}^{\phi}[t] = 1$. In that case, from (27) and (30), one can conclude (28), while (31) is not binding. Next, assume that $r_{ij}^{\phi}[t] = 0$. In that case, from (11) and (31), one can conclude (29), while (30) is not binding. Similarly, one can replace (17) and (18)-(23) with

$$\psi_{ij}[t] \le \big(1 - r_{ij}^{\psi}[t]\big)L, \tag{32}$$

$$C_{ij} - H_{ij}(\theta_i[t]-\theta_j[t]) \le r_{ij}^{\psi}[t]L, \tag{33}$$

and

$$\sigma_i[t] \leq \big(1 - r_i^{\sigma}[t]\big)L, \tag{34}$$
$$P_i^G[t] - P_i^{G,\min}[t] \leq r_i^{\sigma}[t]L, \tag{35}$$
$$\delta_i[t] \leq \big(1 - r_i^{\delta}[t]\big)L, \tag{36}$$
$$P_i^{G,\max}[t] - P_i^G[t] \leq r_i^{\delta}[t]L, \tag{37}$$
$$\zeta_j[t] \leq \big(1 - r_j^{\zeta}[t]\big)L, \tag{38}$$
$$P_j^D[t] - P_j^{D,\min}[t] \leq r_j^{\zeta}[t]L, \tag{39}$$
$$\xi_j[t] \leq \big(1 - r_j^{\xi}[t]\big)L, \tag{40}$$
$$P_j^{D,\max}[t] - P_j^D[t] \leq r_j^{\xi}[t]L, \tag{41}$$
$$\rho_i[t] \leq \big(1 - r_i^{\rho}[t]\big)L, \tag{42}$$
$$P_i^S[t] + y_i[t] \leq r_i^{\rho}[t]L, \tag{43}$$
$$\varrho_i[t] \leq \big(1 - r_i^{\varrho}[t]\big)L, \tag{44}$$
$$x_i[t] - P_i^S[t] \leq r_i^{\varrho}[t]L, \tag{45}$$

respectively. Note that, it is also needed to have

$$r_{ij}^{\phi}[t], r_{ij}^{\psi}[t], r_i^{\sigma}[t], r_i^{\delta}[t], r_i^{\zeta}[t], r_i^{\xi}[t], r_i^{\rho}[t], r_i^{\varrho}[t] \in \{0,1\}. \tag{46}$$

B. Nonlinearity in Objective Function

So far, optimization problem (6) has been reformulated as a set of linear equality and inequality constraints over continuous and binary variables. These constraints can be added into problem (5) to model $\lambda_i[t]$ and $P_i^S[t]$ as functions of the charging and discharging schedules and bidding choices of the energy storage units. However, the resulted optimization problem would still be nonlinear and hard to solve, due to the non-convex nonlinearity in the objective function in (5). In this section, it is explained how this nonlinearity can be tackled. The analysis here is an extension of the results in [33, Section 6.4.3.1] to the case of coordinated energy storage systems.

The first step is to multiply both sides in (14) by P_i^S. After reordering the terms, for each bus $i \in \mathcal{S}$, the result becomes

$$\lambda_i[t]P_i^S[t] = c_i[t]P_i^S[t] - \rho_i[t]P_i^S[t] + \varrho_i[t]P_i^S[t]. \tag{47}$$

Next, note that from (22) and (23), one can write

$$\rho_i[t]P_i^S[t] = -\rho_i[t]y_i[t], \varrho_i[t]P_i^S[t] = \varrho_i[t]x_i[t]. \tag{48}$$

After substituting the above in (47), it becomes

$$\lambda_i[t]P_i^S[t] = c_i[t]P_i^S[t] + \rho_i[t]y_i[t] + \varrho_i[t]x_i[t]. \tag{49}$$

Finally, the following equality is obtained if one sums up both sides in (50) over all storage buses in the system:

$$\begin{aligned}\sum_{i\in\mathcal{S}} \lambda_i[t]P_i^S[t] =& \sum_{i\in\mathcal{S}} c_i[t]P_i^S[t] + \sum_{i\in\mathcal{S}} \rho_i[t]y_i[t] \\ &+ \sum_{i\in\mathcal{S}} \varrho_i[t]x_i[t].\end{aligned} \tag{50}$$

Note that, the above expression still includes non-convex nonlinear terms on its right hand side. Therefore, more steps need to be taken in order to solve optimization problem (5).

Since problem (6) is a linear optimization problem and Slater's condition is satisfied, *strong duality* holds:

$$\begin{aligned}&\sum_{i\in\mathcal{G}} a_i[t]P_i^G[t] - \sum_{i\in\mathcal{D}} b_i[t]P_i^D[t] + \sum_{i\in\mathcal{S}} c_i[t]P_i^S[t] \\ &= \sum_{i\in\mathcal{G}} \sigma_i[t]P_i^{G,\min} - \sum_{i\in\mathcal{G}} \delta_i[t]P_i^{G,\max}[t] \\ &\quad + \sum_{i\in\mathcal{D}} \zeta_i[t]P_i^{D,\min} - \sum_{i\in\mathcal{D}} \xi_i[t]P_i^{D,\max}[t] \\ &\quad - \sum_{i\in\mathcal{S}} \rho_i[t]y_i[t] - \sum_{i\in\mathcal{S}} \varrho_i[t]x_i[t] \\ &\quad - \sum_{(i,j)\in\mathcal{L}} \phi_{ij}[t]C_{ij} - \sum_{(i,j)\in\mathcal{L}} \varphi_{ij}[t]C_{ij},\end{aligned} \tag{51}$$

where the left hand side is the primal optimal objective value and the right hand side is the dual optimal objective value for problem (6). From (47) and (51), and after reordering the terms, the objective function in problem (5) becomes

$$\begin{aligned}\sum_{t=1}^{T} \Bigg[&\sum_{i\in\mathcal{G}} \sigma_i[t]P_i^{G,\min} - \sum_{i\in\mathcal{G}} \delta_i[t]P_i^{G,\max}[t] \\ &+ \sum_{i\in\mathcal{D}} \zeta_i[t]P_i^{D,\min} - \sum_{i\in\mathcal{D}} \xi_i[t]P_i^{D,\max}[t] \\ &- \sum_{(i,j)\in\mathcal{L}} \phi_{ij}[t]C_{ij} - \sum_{(i,j)\in\mathcal{L}} \varphi_{ij}[t]C_{ij}, \\ &- \sum_{i\in\mathcal{G}} a_i[t]P_i^G[t] + \sum_{i\in\mathcal{D}} b_i[t]P_i^D[t] \Bigg].\end{aligned} \tag{52}$$

The above expression is a linear function of variables $\sigma_i[t]$, $\delta_i[t]$, $\zeta_i[t]$, $\xi_i[t]$, $\phi_{ij}[t]$, $\varphi_{ij}[t]$, $P_i^G[t]$, and $P_i^D[t]$. Therefore, the nonlinearity in the objective function in (5) is now tackled.

C. Resulted Mixed-Integer Linear Program

Problem (5)-(6) can now be reformulated as

$$\begin{aligned}\textbf{max} \quad & (52) \\ \textbf{s.t.} \quad & (1)-(4), (7)-(15), \\ & (24)-(27), (30)-(46),\end{aligned} \tag{53}$$

where the optimization variables are $m_i[t]$, $x_i[t]$, $y_i[t]$, $z_i[t]$, $c_i[t]$, $P_i^G[t]$, $P_i^D[t]$, $P_i^S[t]$, $\theta_i[t]$, $\lambda_i[t]$, $\sigma_i[t]$, $\delta_i[t]$, $\zeta_i[t]$, $\xi_i[t]$, $\rho_i[t]$, $\varrho_i[t]$, $\phi_{ij}[t]$, $\varphi_{ij}[t]$, $r_i^{\sigma}[t]$, $r_i^{\delta}[t]$, $r_i^{\zeta}[t]$, $r_i^{\xi}[t]$, $r_i^{\rho}[t]$, $r_i^{\varrho}[t]$, $r_{ij}^{\phi}[t]$, and $r_{ij}^{\varphi}[t]$. Problem (53) is a mixed-integer linear program (MILP). It can be solved efficiently using any MILP solver, such as CPLEX [35] or MOSEK [36].

IV. Remarks and Extensions

In this section, some pointers and remarks are discussed about a few directions to extend the analysis in Section III.

A. Charge and Discharge Efficiency Parameters

Recall that for the system model in Section II, the storage units are assumed to have ideal round-trip efficiencies. Suppose $\alpha_i \geq 1$ and $\beta_i \leq 1$ denote the efficiency of storage unit i during discharging and during charging, respectively. These parameters can be incorporated into the problem formulation by revising the equality constraint in (4) as follows:

$$z_i[t+1] = z_i[t] - \alpha_i P_i^{SD}[t] + \beta_i P_i^{SC}[t]. \tag{54}$$

Here, $P_i^{SD}[t]$ denotes the power that energy storage unit i injects to the power grid at time slot t if it is discharged; and $P_i^{SC}[t]$ denotes the power that energy storage unit i draws from the power grid at time slot t if it is charged. If the energy storage unit is ideal, then $\alpha_i = \beta_i = 1$ and (54) reduces to (4). The new variables $P_i^{SD}[t]$ and $P_i^{SC}[t]$ are modeled as

$$P_i^S = P_i^{SD} - P_i^{SC}, \tag{55}$$
$$0 \leq P_i^{SD} \leq m_i[t] x_i^{\max}, \tag{56}$$
$$0 \leq P_i^{SC} \leq (1 - m_i[t]) y_i^{\max}. \tag{57}$$

B. Stochastic Optimization

The deterministic analysis in this paper can be extended to the case where the energy storage system operator is uncertain about the supply and demand bids that are submitted to the wholesale electricity market. Under such alternative design framework, the energy storage system uses the historical bid data that are often available to public, e.g., see [37], and coordinates its price-maker operation by solving a *stochastic* version of the optimization problem in (5)-(6).

Suppose K denotes the number of *random* market scenarios. Let $\lambda_{i,k}[t]$ denote the LMP at bus i at time t under random scenario k, where $k = 1, \ldots, K$. The energy storage system seeks to coordinate its units such that it maximizes its *expected* total profit. Accordingly, problem (5) needs to be replaced by the following scenario-based stochastic optimization problem:

$$\begin{aligned} \textbf{max} \quad & \sum_{t=1}^{T} \sum_{i \in \mathcal{S}} \frac{1}{K} \left(\sum_{k=1}^{K} \lambda_{i,k}[t] P_{i,k}^S[t] \right) \\ \textbf{s.t.} \quad & \text{Eqs. } (1) - (4), \end{aligned} \tag{58}$$

where the variables are $x_i[t]$, $y_i[t]$, $m_i[t]$, and $z_{i,k}[t]$. Note that, since the charge levels of the storage units depend on the market bidding outcome, a separate charge level variable $z_{i,k}[t]$ is defined for each random scenario k. Accordingly, constraints (3) and (4) should hold under every scenario k.

As in Section II, $\lambda_{i,k}[t]$ and $P_{i,k}^S[t]$ must be obtained by solving an economic dispatch problem across the power market. However, the key difference here is that there are K different economic dispatch problems that need to be formulated, solved, and integrated into the problem in (58), each corresponding to one random scenario. Specifically, under each scenario k, variables $\lambda_{i,k}[t]$ and $P_{i,k}^S[t]$ are obtained by solving the following optimization problem:

$$\begin{aligned} \textbf{min} \quad & \sum_{i \in \mathcal{G}} a_{i,k}[t] P_{i,k}^G[t] - \sum_{j \in \mathcal{D}} b_{i,k}[t] P_{i,k}^D[t] \\ & + \sum_{i \in \mathcal{S}} c_i[t] P_{i,k}^S[t] \\ \textbf{s.t.} \quad & P_{i,k}^G[t] - P_{i,k}^D[t] + P_{i,k}^S[t] \\ & \quad = \sum_j H_{ij}(\theta_{i,k}[t] - \theta_{j,k}[t]), \qquad \forall\, i \in \mathcal{N}, \\ & P_{i,k}^{G,\min}[t] \leq P_{i,k}^G[t] \leq P_{i,k}^{G,\max}[t], \qquad \forall\, i \in \mathcal{G}, \\ & P_{i,k}^{D,\min}[t] \leq P_{i,k}^D[t] \leq P_{i,k}^{D,\max}[t], \qquad \forall\, i \in \mathcal{D}, \\ & -y_i[t] \leq P_{i,k}^S[t] \leq x_i[t], \qquad \forall\, i \in \mathcal{S}, \\ & -C_{ij} \leq H_{ij}(\theta_{i,k}[t] - \theta_{j,k}[t]) \leq C_{ij}, \quad \forall\, (i,j) \in \mathcal{L}, \end{aligned} \tag{59}$$

where the optimization variables are $P_{i,k}^G[t]$, $P_{i,k}^D[t]$, $P_{i,k}^S[t]$, and $\theta_{i,k}[t]$. Note that, the system parameters $a_{i,k}[t]$, $b_{i,k}[t]$, $P_{i,k}^{G,\min}[t]$, $P_{i,k}^{G,\max}[t]$, $P_{i,k}^{D,\min}[t]$, $P_{i,k}^{D,\max}[t]$ are now defined separately for each random scenario k. The rest of the analysis and the solution method are similar to those in Section III.

C. Self-Scheduling versus Economic Bidding

In practice, e.g., in the California Independent System Operator (ISO) energy market, the bids are classified into two types: *self-schedule bids* and *economic bids*. A self-schedule bid does *not* include a price component. It indicates that the buyer (seller) is willing to buy (sell) electricity regardless of the price. An economic bid *does* include a price component. It indicates that the buyer (seller) is willing to buy (sell) electricity as long as the cleared market price is less (more) than the submitted price bid. While the system model in this paper is based on economic bidding, it can be adjusted easily to allow self-schedule bidding. This can be done by adding the following constraint to the optimization problem:

$$L(1 - m_i) \leq c_i \leq L(1 - m_i). \tag{60}$$

If $m_i = 0$, then (60) reduces to $c_i = L$. That is, if the storage unit at bus i is charged, then the price component in its demand bid is *very large*, making sure that the bid is cleared under any market condition. If $m_i = 1$, then (60) reduces to $c_i = 0$. That is, if the storage unit at bus i is discharged, then the price component in its supply bid is *zero*, making sure that the bid is cleared under any market condition. Note that, under a deterministic problem formulation, the optimal performance of self-schedule bidding is as good as economic bidding. However, if the problem formulation is stochastic and the storage operator is uncertain about the market conditions, then economic bidding may outperform self-scheduling.

D. Robust Design and Risk Management

For the stochastic problem formulation in Section IV-B, the goal is to maximize the expected / average total profit. One can extend the analysis to also include robust design or risk management. As an alternative for the optimization problem

TABLE I
TRANSMISSION LINES

Line	Buses	Line	Buses	Line	Buses	Line	Buses
1	(1,2)	12	(6,9)	23	(12,14)	34	(23,24)
2	(1,3)	13	(6,10)	24	(12,15)	35	(24,25)
3	(2,4)	14	(6,28)	25	(12,16)	36	(25,26)
4	(2,5)	15	(8,28)	26	(14,15)	37	(25,27)
5	(2,6)	16	(9,11)	27	(15,18)	38	(27,28)
6	(3,4)	17	(9,10)	28	(15,23)	39	(27,29)
7	(4,6)	18	(10,20)	29	(16,17)	40	(27,30)
8	(4,12)	19	(10,17)	30	(18,19)	41	(29,30)
9	(5,7)	20	(10,21)	31	(19,20)	-	-
10	(6,7)	21	(10,22)	32	(21,22)	-	-
11	(6,8)	22	(12,13)	33	(22,24)	-	-

in (58), consider the following optimization problem:

$$\begin{aligned} \textbf{max} \quad & \underset{k=1,\ldots,K}{\textbf{minimum}} \left(\sum_{t=1}^{T} \sum_{i \in \mathcal{S}} \lambda_{i,k}[t] P^S_{i,k}[t] \right) \\ \textbf{s.t.} \quad & \text{Eqs. } (1)-(4). \end{aligned} \tag{61}$$

Here, the design goal is to maximize the *worst-case* total profit, i.e., the lowest total profit that may occur among the considered random scenarios. Compared to problem (58), problem (61) is intended for a *risk-averse* operator. Note that, problem (61) is nonlinear in its current form due to the use of max function in the objective. One can transform problem (61) into a linear mixed-integer program as follows:

$$\begin{aligned} \textbf{max} \quad & \omega \\ \textbf{s.t.} \quad & \text{Eqs. } (1)-(4), \\ & \omega \leq \sum_{t=1}^{T} \sum_{i \in \mathcal{S}} \lambda_{i,k}[t] P^S_{i,k}[t], \quad k = 1, \ldots, K. \end{aligned} \tag{62}$$

where ω is a continuous auxiliary variable. The rest of the analysis is as in Sections IV-B and III. Note that, risk management can be conducted also by incorporating a financial risk model, such as the conditional value-at-risk (CVaR) [38], into the problem formulation in (58), e.g., see [27].

V. CASE STUDIES

In this section, the performance of the proposed method is evaluated based on the IEEE 30bus test system [39], as shown in Fig. 1. Here, the network includes 30 buses and 41 transmission lines, where $\mathcal{G} = \{1, 5, 8, 13, 21, 23, 27, 28\}$, $\mathcal{D} = \{2, 3, 7, 10, 11, 12, 14, 16, 17, 18, 19, 20, 22, 26, 29, 30\}$, and $\mathcal{B} = \{4, 16, 24, 30\}$. The transmission lines are indexed as in Table I. The generation and load data are given in Tables II and III. Throughout this section, the power quantities are in GW, energy quantities are in GWh, and price quantities are in \$1000/GWh or \$/MWh. Each storage unit has 1 GWh energy storage capacity. Unless stated otherwise, the energy storage efficiency is assumed to be 100%. An entire day is studied, where $T = 24$ time slots. Without loss of generality, all generators are assumed to submit economic bids and all loads are assumed to submit self-schedule bids. The MILP problems are solved using CPLEX [35]. Parameter L is 1000.

If the transmission capacity constraints are relaxed, then, in the *absence* of the storage units, the cleared market prices of electricity are 41.4, 37.9, 35.7, 35.1, 36.2, 43.0, 52.5, 58.0, 50.5, 44.7, 42.5, 41.8, 41.1, 40.8, 41.9, 43.9, 47.6, 55.2, 63.0, 70.3, 66.0, 58.8, 52.6, and 46.1 \$/MWh, matching the average market profile in California energy market in March 2014 [40].

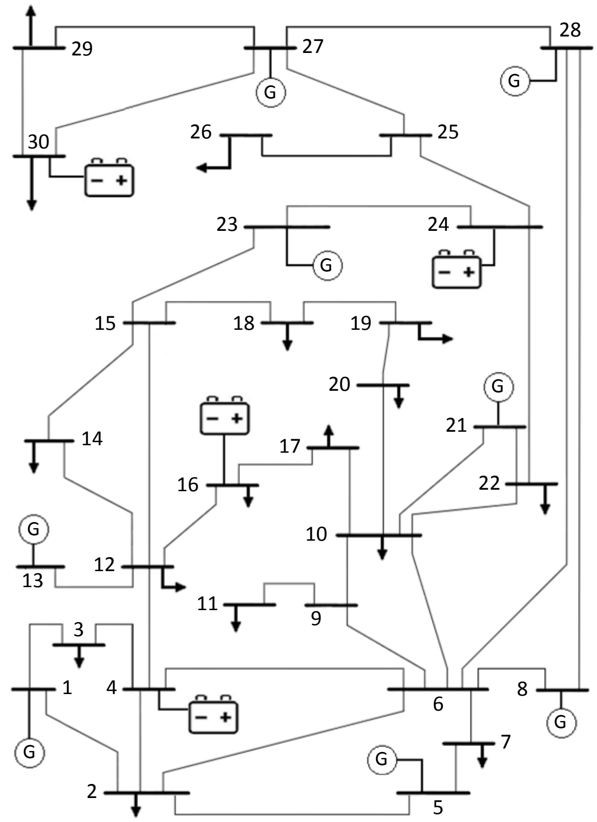

Fig. 1. The IEEE 30-bus test network with four energy storage systems.

A. Not Congested versus Congested Scenarios

First, consider the case where no transmission line is congested. The optimal charge and discharge schedules of the energy storage units are obtained as shown in Fig. 2. One can see that the energy storage units are charged during off-peak hours 3, 4, 5, 12, 13, 14, and 15, and discharged during peak hours 8 and 20. In presence of the energy storage units, the cleared market price of electricity change to 41.4, 37.9, 35.7, 35.6, 36.2, 43.0, 52.5, 57.0, 50.5, 44.7, 42.5, 41.8, 41.1, 41.3, 41.9, 43.9, 47.6, 55.2, 63.0, 68.9, 66.0, 58.8, 52.6, and 46.1 \$/MWh. The total profit that the coordinated energy storage units make through their market participation is \$194,696. Note that, since the network is not congested, the location of the energy storage systems does not matter in this case. Accordingly, one could switch the schedules at different storage buses and he/she would still see the same performance.

Next, consider the case where the transmission line between buses 2 and 4, i.e., line 3, is congested. Since this line is connected to an energy storage bus, its congestion is expected to have noticeable impact on the operation of the energy storage system. Suppose $C_{2,4} = 0.2$ GW $= 200$ MW. The optimal charge and discharge schedules of the storage units are obtained as shown in Fig. 3. One can see that the results

TABLE II
GENERATION DATA

Bus #	Hourly Energy Bids																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	1.9	1.4	1.9	2.6	1.6	1.0	1.9	1.6	2.2	2.1	1.8	1.9	1.8	0.8	1.3	2.1	1.9	2.3	2.2	1.8	2.4	1.4	2.2	1.0
5	1.4	1.4	1.3	1.4	1.8	2.1	2.2	2.2	2.1	1.9	1.1	1.6	1.3	2.1	1.7	1.4	1.3	2.2	2.4	1.3	2.2	2.1	1.9	1.3
8	1.0	0.8	1.3	1.9	1.4	1.0	2.1	2.2	1.4	1.0	1.6	1.8	1.8	1.9	2.1	2.3	1.4	2.2	1.9	2.3	1.6	2.2	2.4	2.2
13	2.2	1.6	2.2	1.1	2.4	2.6	1.7	2.2	1.1	1.1	2.1	2.1	2.4	1.1	2.3	1.8	1.7	1.8	1.9	1.1	1.8	1.9	2.1	1.1
21	1.3	2.1	2.1	1.8	1.4	1.1	1.4	1.4	1.6	1.4	1.9	1.6	1.9	1.3	1.0	0.9	2.2	1.4	1.7	2.6	2.1	2.1	1.1	1.9
23	1.8	1.4	1.1	1.0	1.3	1.4	1.3	1.8	1.4	1.3	1.4	1.4	1.4	2.6	1.4	1.9	1.9	2.0	1.8	1.4	2.1	1.6	1.7	1.6
27	1.9	1.6	1.6	2.4	2.2	1.9	2.7	1.3	2.2	2.1	1.3	2.2	1.3	1.7	1.9	2.1	2.1	1.8	1.4	2.4	1.4	1.9	1.3	1.0
28	1.1	1.8	1.8	1.4	1.3	1.6	1.3	1.3	1.5	2.2	2.6	1.8	1.1	1.4	1.1	1.6	1.7	1.6	2.2	1.7	1.9	1.1	1.6	1.8

Bus #	Hourly Price Bids																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
1	40.2	37.9	35.2	34.7	39.2	43.3	50.0	51.3	51.8	43.6	42.0	42.9	39.0	40.8	41.2	42.7	50.1	57.9	66.1	67.1	61.6	59.2	49.4	44.7
5	38.3	41.2	36.2	34.0	36.2	45.6	57.2	48.6	50.5	42.7	40.9	46.0	40.2	43.1	42.4	44.3	46.0	51.4	57.2	69.8	67.8	54.8	54.9	45.1
8	42.1	38.4	37.1	34.3	37.8	40.8	53.1	57.6	51.3	43.0	40.1	42.4	41.8	42.6	39.9	41.8	47.6	56.1	59.8	68.9	66.0	56.9	50.3	48.2
13	44.0	35.1	34.3	36.9	34.9	41.2	52.5	60.1	49.2	44.7	44.9	40.0	43.6	41.3	44.3	45.2	48.9	53.1	63.9	70.3	65.2	58.8	50.8	46.1
21	41.4	37.0	34.8	36.0	34.2	38.5	51.3	58.0	48.1	44.0	42.5	41.8	40.6	41.7	41.9	44.9	44.8	57.2	62.1	65.2	64.0	60.4	52.1	44.0
23	37.6	40.6	35.7	35.1	34.5	41.9	48.2	57.0	47.0	45.2	42.9	43.4	41.1	40.0	43.0	43.2	46.4	54.0	63.0	71.4	66.7	58.1	53.7	46.7
27	44.7	38.9	38.0	37.8	36.7	44.1	45.3	61.2	47.8	45.8	41.6	40.6	42.3	38.7	39.4	46.5	48.1	55.2	61.6	68.0	62.9	57.5	52.6	45.4
28	39.5	35.9	36.8	35.6	35.3	43.0	55.4	54.2	48.7	47.1	44.1	41.1	39.3	39.2	40.7	43.9	47.0	54.8	61.0	75.1	64.8	56.0	53.0	43.3

TABLE III
LOAD DATA

Bus #	Hourly Energy Bids																							
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24
2	0.53	0.41	0.59	0.48	0.42	0.55	0.65	0.27	0.83	0.44	0.64	0.47	0.20	0.48	0.40	0.33	0.32	0.78	0.33	0.98	0.83	0.81	0.59	0.26
3	0.42	0.46	0.42	0.25	0.26	0.47	0.48	0.75	0.31	0.25	0.25	0.18	0.29	0.19	0.65	0.59	0.26	0.36	0.92	0.88	0.57	0.77	0.31	0.64
7	0.51	0.29	0.21	0.17	0.78	0.72	0.73	0.42	0.23	0.49	0.59	0.79	0.23	0.27	0.33	0.28	0.54	0.42	0.45	0.60	0.79	0.50	0.34	0.21
10	0.65	0.40	0.29	0.56	0.54	0.54	0.51	0.53	0.56	0.34	0.31	0.32	0.29	0.26	0.56	0.34	0.45	0.29	0.58	0.83	0.82	0.39	0.47	0.48
11	0.43	0.65	0.32	0.39	0.41	0.63	0.40	0.61	0.78	0.52	0.48	0.43	0.37	0.47	0.41	0.23	0.46	0.83	0.25	0.89	0.60	0.48	0.69	0.34
12	0.24	0.24	0.47	0.28	0.62	0.43	0.59	0.89	0.62	0.49	0.18	0.57	0.41	0.55	0.19	0.49	0.57	0.63	0.82	0.71	0.27	0.91	0.74	0.56
14	0.63	0.43	0.23	0.58	0.28	0.42	0.22	0.52	0.33	0.34	0.39	0.36	0.38	0.32	0.26	0.54	0.64	0.53	0.72	0.56	0.52	0.23	0.71	0.49
16	0.35	0.28	0.51	0.34	0.52	0.64	0.88	0.92	0.44	0.38	0.73	0.54	0.58	0.50	0.63	0.48	0.31	0.11	0.82	0.40	0.59	0.57	0.41	0.26
17	0.41	0.41	0.21	0.45	0.19	0.23	0.32	0.44	0.50	0.51	0.26	0.27	0.41	0.47	0.41	0.41	0.57	0.57	0.94	0.76	0.48	0.25	0.55	0.65
18	0.54	0.48	0.47	0.56	0.21	0.26	0.28	0.72	0.82	0.32	0.42	0.64	0.61	0.16	0.25	0.53	0.43	0.64	0.97	0.49	0.41	0.38	0.72	0.71
19	0.23	0.51	0.40	0.38	0.44	0.28	0.45	0.58	0.31	0.63	0.39	0.43	0.47	0.41	0.34	0.29	0.68	0.75	0.39	0.31	0.90	0.87	0.28	0.18
20	0.38	0.37	0.53	0.43	0.22	0.40	0.34	0.61	0.74	0.71	0.22	0.12	0.62	0.34	0.47	0.67	0.51	0.68	0.35	1.10	0.59	0.35	0.30	0.65
22	0.61	0.32	0.36	0.34	0.61	0.54	0.72	0.72	0.61	0.57	0.54	0.26	0.28	0.58	0.26	0.52	0.43	0.55	0.76	0.47	0.95	0.68	0.64	0.45
26	0.12	0.67	0.59	0.27	0.59	0.63	0.43	0.65	0.44	0.39	0.42	0.35	0.48	0.16	0.54	0.66	0.54	0.86	0.31	0.64	0.57	0.94	0.53	0.72
29	0.68	0.19	0.48	0.62	0.53	0.22	0.62	0.36	0.37	0.54	0.53	0.46	0.30	0.73	0.39	0.28	0.47	0.36	0.63	0.85	0.86	0.45	0.33	0.16
30	0.47	0.61	0.32	0.14	0.26	0.40	0.86	0.29	0.43	0.28	0.53	0.37	0.48	0.35	0.63	0.56	0.50	0.28	0.68	0.57	0.65	0.86	0.87	0.44

are different from those in Fig. 2. While peak shaving is still conducted at $t = 8, 20$, the energy storage systems now practice *arbitrage* at $t = 4, 13, 14$. For example, at hour 4, the energy storage unit at bus 4 sells electricity to the market while the storage units at buses 16, 24 and 30 purchase electricity. The total profit that the coordinated storage system makes through energy market participation is \$201,831, i.e., 3.5% higher than the profit where no line is congested. Interestingly, this higher profit also comes with some benefits to the power system as a whole, as it will be discussed in Section V-B.

The deviations in locational marginal prices from their base values when transmission line 3 is not congested are shown in Fig. 4(a) and (b) for cases *without* and *with* energy storage units, respectively. One can see that congestion has caused locational price diversity from the base levels. In particular, the price is now lower than the base price at bus 2, which is on one side of the congested line. Also, the price is now higher than the base price at bus 4, which is on the other side of the congested transmission line. Note that, under congestion, the storage units conduct peak shaving not only *across time* but also to some extent *across location*. Specifically, while price deviation at peak hour $t = 8$ ranges from -17.40 \$/MWh to 6.31 \$/MWh when the energy storage units are *not* operational; it ranges from -16.45 \$/MWh to 4.77 \$/MWh when the energy storage units *are* operational.

The results for all 41 possible single-line congestion scenarios are shown in Fig. 5. One can make multiple observations. First, in 18 cases, i.e., the cases of congestion at lines 2, 3, 4, 7, 8, 11, 13, 14, 15, 17, 20, 23, 24, 29, 32, 35, 38, and 41, the total profit *increases* compared to the case where no line is congested. Some increases are significant. For example, when the line between buses 6 and 10, i.e., line 13, is congested, the total profit is \$585,877. This is three times higher than the profit where no line is congested. The coordinated storage units practice arbitrage in *all* 18 cases. Second, in 13 cases, i.e., the cases of congestion at lines 1, 5, 6, 9, 10, 21, 22, 26, 28,

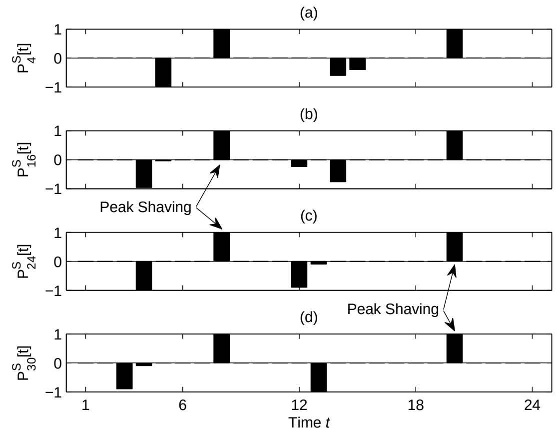

Fig. 2. The optimal charge and discharge schedules of the four energy storage units in Fig. 1 when there is no congested transmission line.

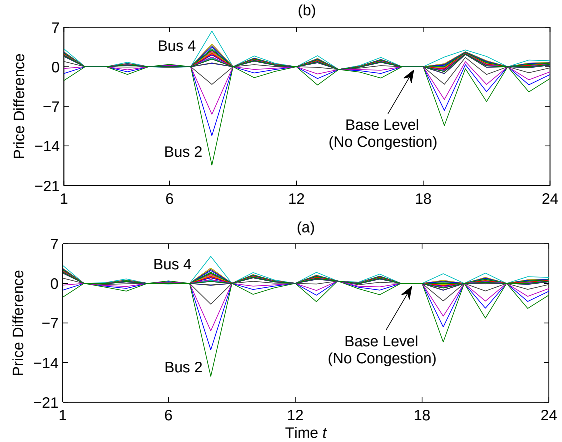

Fig. 4. Locational marginal prices at all 30 buses when line 3 is congested: (a) without energy storage operation; (b) with energy storage operation.

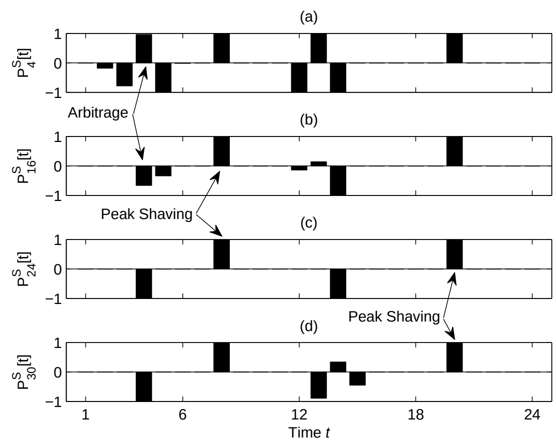

Fig. 3. The optimal charge and discharge schedules of the four energy storage units in Fig. 1 when transmission line 3 is congested.

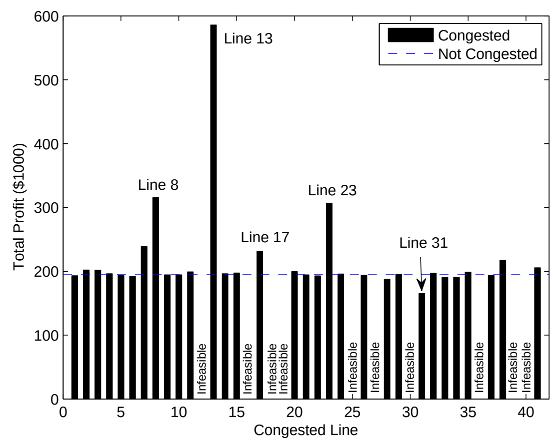

Fig. 5. The total profit of the coordinated storage systems for different congestion scenarios, and comparison with the case without congestion.

31, 33, 34, and 37, the profit *decreases* compared to the case where no transmission line is congested. While most decreases are minor, the highest drop in profit occurs at 15.1% when line 31 is congested. The noticeable drop in profit in this case is inevitable in order to keep the economic dispatch problem feasible. Note that, if no energy storage unit is available, then the economic dispatch problem becomes infeasible when line 31 is congested. Finally, in 10 cases, the cases of congestion at lines 12, 16, 18, 19, 25, 27, 30, 36, 39, and 40 network congestion results in an infeasible network operation.

The above results were obtained using a computer with a 2.90 GHz CPU and 8.00 GB RAM. The time to construct and solve the optimization problem without transmission constraints was 147 sec. As for the cases when a transmission line is congested, the computation time was 152 sec on average.

B. Total Generation Cost

While the design in this paper is from the viewpoint of the coordinated energy storage units to decide on how they must coordinate their price-maker operation in a nodal energy market to maximize their total profit, it is interesting to also assess the impact of coordinated energy storage units on the overall performance of the power system. Therefore, in this section, the total generation cost is assessed *with* and *without* the presence of coordinated energy storage units. The results are shown in Fig. 6. One can see that, in all cases, the presence of the coordinated energy storage units is in fact beneficial to the power system as a whole. First, note that without the energy storage units, the energy market is infeasible in 18 congestion scenarios. Adding the energy storage units to the market resolves infeasibility in 8 of these 18 cases. As for the 23 cases where the market is feasible even without energy storage units, adding the energy storage units to the market reduces the total generation cost in *all* 23 cases. On average, the total generation cost reduces by 2.23% from $8,910,387 to $8,710,465. The situation is similar even if the network is not congested. In that case, if the coordinated storage units join the energy market, then the total generation cost reduces by 2.24% from $8,874,464 to $8,675,742.

C. Locational Diversity

In this section, locational diversity is investigated by comparing the results in Section V-A with those where all the energy storage units are located at one bus, rather than being

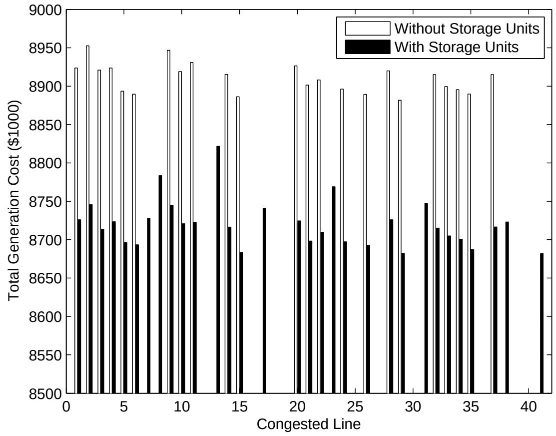

Fig. 6. The total power generation cost in the power system with and without the presence of storage systems and for different congestion scenarios.

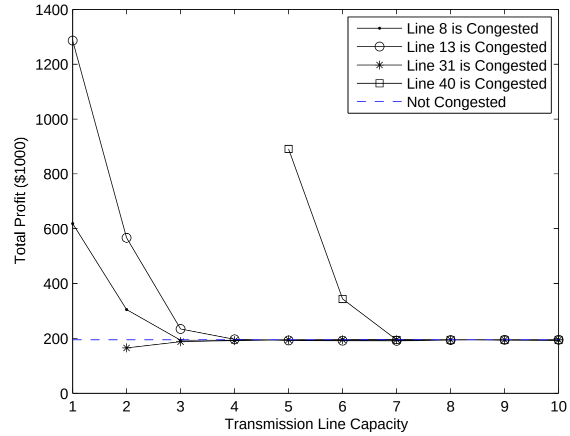

Fig. 8. The total profit of the coordinated energy storage systems for a few line congestion scenarios under different transmission line capacities.

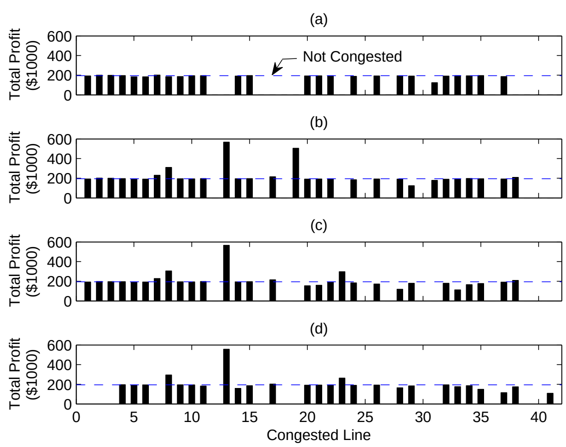

Fig. 7. The total profit of the coordinated energy storage units for different congestion scenarios when all energy storage units are located at the same bus: (a) all storage units are at bus 4, (b) all storage units are at bus 16, (c) all storage units are at bus 24, and (d) all storage units are at bus 30.

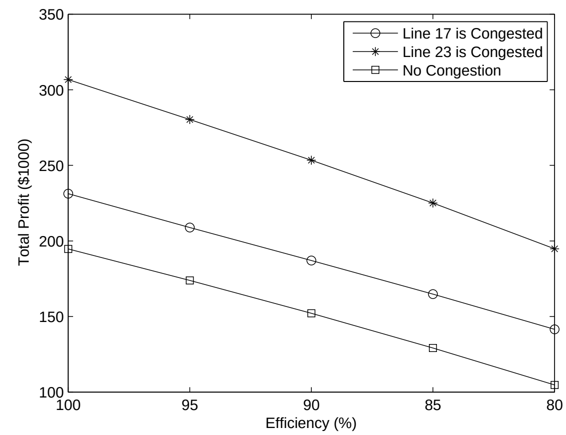

Fig. 9. The impact of energy storage efficiency on the total profit of the storage systems. The impact can differ across different congestion scenarios.

distributed at four different buses. The results are shown in Fig. 7. These results are comparable with those in Fig. 5. First, one can see that the number of infeasible market scenarios has significantly increased, because placing all storage units at one bus leads to limited opportunities to relieve congestion. Second, even among those scenarios where the market is feasible, the total profit of the storage units often decreases when they are all moved to the same bus. For example, the number of congestion scenarios at which the total profit drops below the profit under $194,696, i.e., the profit where no line is congested, is 19, 17, 18, and 21 for the cases where all storage units are at buses 4, 16, 24, and 30, respectively.

D. Transmission Line Capacity

For the case studies so far it has been assumed that a congested line has a transmission capacity of 0.2 GW. In this section, the cases are considered where the transmission capacity varies from 0.1 GW to 1 GW. The results are shown in Fig. 8 for a few example congestion scenarios. As expected, increasing the transmission line capacity results in the profit approaching $194,696, i.e., the profit where no line is congested. The results here confirm the earlier observations that congestion can often (but not always) help coordinated energy storage units make more profit in an energy market.

E. Storage Efficiency

In this section, the impact of energy storage efficiency is examined on the profitability of coordinated energy storage units. The total profit of the energy storage systems versus the energy storage efficiency for three example congestion scenarios are shown in Fig. 9. The efficiency at 95% is implemented by selecting $\alpha_i = 1.0256$ and $\beta_{s,u} = 0.9744$. The efficiency at 90% is implemented by selecting $\alpha_i = 1.0526$ and $\beta_i = 0.9474$. The efficiency at 85% is implemented by selecting $\alpha_i = 1.0811$ and $\beta_i = 0.9189$. Finally, the efficiency at 80% is implemented by selecting $\alpha_i = 1.1111$ and $\beta_i = 0.8889$. One can see that lower efficiency results in less but still considerable profit. The rate of profit degradation may or may not be the same for different congestion scenarios.

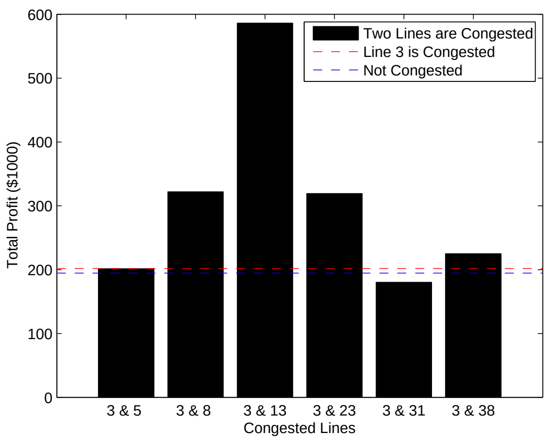

Fig. 10. The total profit of the coordinated storage systems when two transmission lines, i.e., line 3 plus another line, are congested simultaneously.

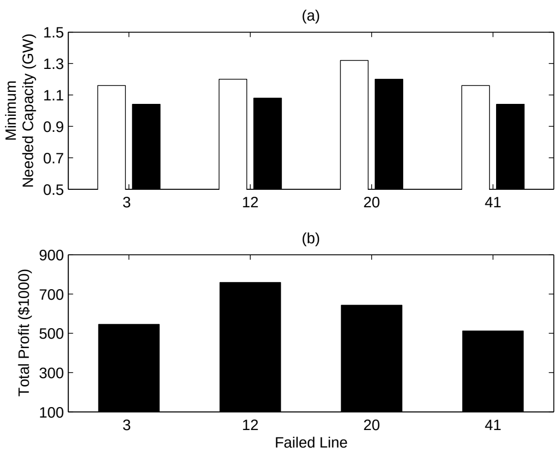

Fig. 11. The impact of energy storage on transmission failure contingencies: (a) the minimum required transmission line capacity to maintain feasible economic dispatch; (b) the total profit the coordinated energy storage systems.

F. Multiple Congested Transmission Lines

For all the case studies so far, only one transmission line is assumed to be congested at a time. Of course, the results may change if two lines are congested simultaneously. In this regard, consider the case where line 3 is congested and assume that a second line is congested as well. The results are shown in Fig. 10. Here, the transmission capacity of the two congested lines is assumed to be 0.2 GW. One can see that, in one case, i.e., when lines 3 and 5 are congested, the network total profit does not change compared to the case when only line 3 is congested. In two cases, i.e., when lines 3 and 8 or lines 3 and 13 or lines 3 and 23 or lines 3 and 38 are congested, the total profit of the energy storage units is *higher* than the total profit when only line 3 is congested. This means that dual line congestion is *advantageous* to the energy storage units in these cases. In contrast, there is one case, i.e., when lines 3 and 31 are congested, where the total profit of the storage units is *lower* than the total profit when only line 3 is congested. This means that dual line congestion is *disadvantageous* to the energy storage systems in these cases.

Next, consider a scenario where one transmission line fails, thus increasing power flow on the remaining transmission lines. The results are shown in Fig. 11. All 41 transmission lines are assumed to be capacity-constrained. Fig. 11(a) shows the minimum capacity that is needed at the remaining transmission lines when one line fails in order to keep economic dispatch feasible. For example, when the transmission line between buses 2 and 4, i.e., line 3, fails, the capacity of the remaining lines must be at least 1.16 GW and 1.04 GW to assure feasible dispatch *without* and *with* energy storage units, respectively. On average, the use of energy storage units allows handling line failure with 10% lower minimum (bottleneck) capacity across transmission lines. Interestingly, such benefit to the grid as a whole is accompanied with significant monetary advantages for the energy storage units, as shown in Fig. 11(b). Specifically, the total profit of the coordinated energy storage units is \$545,792, \$759,093, \$643,043, \$512,059 when transmission lines 3, 12, 20, and 41 fail, respectively.

G. Stochastic Optimization and Robust Design

In this section, the design extensions in Sections IV-B, IV-C, and IV-D are demonstrated. Suppose the storage units operator is not certain about the behavior of other market players. Yet, using historical bid data [37] and market simulation and forecasting methods [41], [42], it can construct a statistical model for the market parameters based on random scenarios. For the sake of demonstration, assume that $K = 3$. The first random scenario is based on the numbers in Tables II and III. The other two scenarios are generated based on *random deviations* from the parameters in Tables II and III. For example, under the second random scenario, the hourly price bids for the generator at bus 1 are 41.56 (+1.36), 38.66 (+0.76), 35.72 (+0.52), 36.50 (+1.80), 37.62 (-1.58), 39.07 (-4.23), 46.82 (-3.19), 56.00 (+4.70), 52.87 (+1.07), 46.68 (+3.08), 42.24 (+0.24), 39.06 (-3.84), 36.80 (-2.20), 41.94 (+1.14), 42.70 (+1.50), 38.68 (-4.02), 45.13 (-4.97), 57.66 (-0.24), 61.31 (-4.79), 72.67 (+5.57), 56.56 (-5.04), 63.51 (+4.31), 53.96 (+4.56), and 45.9 (+1.24). Note that, the numbers in parenthesis indicate the amount of deviation from the numbers in the first row in the second portion of Table II.

First, assume that the energy storage units are scheduled to maximize the total expected profit, i.e., based on the optimal solution of problem (58). Suppose no transmission line is congested. If the energy storage units submit self-schedule bids, then the expected total profit becomes \$190,104. If the energy storage units submit economic bids, then the expected total profit becomes \$192,951. One can see that, unlike in the deterministic case, there is noticeable difference between self-scheduling and economic bidding. This is because the price components of economic bids act as *filters* to avoid charging the energy storage units when the clearing market price is high and discharging them when the market price is low.

For the self-scheduling design in the previous paragraph, the total profit under random scenarios 1, 2, and 3 is \$190,691, \$184,947, \$194,675, respectively, where the average is \$190,104, as it was explained earlier. Accordingly, under the the worst-case scenario, the total profit can be as low as

$$\min\{\$190,691, \$184,947, \$194,675\} = \$184,947. \quad (63)$$

In order to increase profit under the worst-case scenario, one can schedule the energy storage units based on the robust design in Section IV-D, i.e., based on the optimal solution of problem (61). In that case, the total profit under random scenarios 1, 2, and 3 is \$187,183 \$187,405, \$187,674, respectively. The average of these three numbers is \$187,421. One can see that a robust design can considerably increase the total profit under the worst-case scenario; however, this comes at the cost of lowering the total profit on average.

VI. Conclusions

With the rise of investor-owned independently-operated energy storage systems on the horizon, in this paper, a group of large, price-maker, and geographically dispersed energy storage / battery systems was considered that seek to coordinate their charge and discharge schedules so as to maximize their total profit in a nodal transmission-constrained energy market. Such profit maximization problem was formulated as an optimization problem that takes into consideration the location, size, efficiency, and charge and discharge rates of the energy storage systems as well as the joint / coordinated impact of the energy storage operations on the locational marginal prices. Although the formulated optimization problem was initially nonlinear, non-convex, and hard to solve, the nonlinearities were tackled both in the objective function and constraints by transforming the problem into a tractable mixed-integer linear program, for which the global optimal solution was obtained.

The following conclusions can be highlighted based on the presented case studies. First, transmission line congestion is often, but not always, desirable for coordinated energy storage systems. Congestion may create more price differential across time slots, providing more opportunities for energy storage systems to make profit through their charge and discharge cycles. Second, while the design in this paper is from the viewpoint of energy storage systems with focus on profitability, the presence of energy storage systems in the energy market seems to also benefit the overall power system performance by reducing the total cost of generation. Third, arbitrage seems to be the optimal choice among the energy storage systems in all scenarios where congestion is beneficial to the coordinated energy storage units. Fourth, locational diversity is critical to assure profit for energy storage systems in transmission-constrained networks. That is, it is preferred if a firm distributes its energy storage units across the power network rather than installing them all in one location. Fifth, while the results are sensitive to the efficiency of the energy storage units, energy storage units with lower round trip efficiencies can still experience considerable profit. Finally, although there is no difference between self-scheduling and economic bidding market participation in a deterministic scenario, coordinated energy storage units do benefit from economic bidding when their operator faces electricity market uncertainty.

References

[1] Department of Energy, "The Smart Grid: An Introduction," [Online] http://energy.gov/oe/downloads/smart-grid-introduction-0, Jul. 2009.

[2] National Institute of Standards and Technology, "NIST Framework and Roadmap for Smart Grid Interoperability Standards," Jan. 2010.

[3] R. Byrne, V. Loose, M. Donnelly, and D. Trudnowski, "Methodology to determine the technical performance and value proposition for grid-scale energy storage systems," Sandia National Lab Report, Dec. 2012.

[4] K. C. Divya and J. Ostergaard, "Battery energy storage technology for power systemsan overview," *Electric Power Systems Research*, vol. 79, no. 4, pp. 511–520, Apr. 2009.

[5] B. P. Roberts and C. Sandberg, "The role of energy storage in development of smart grids," *Proceedings of the IEEE*, vol. 99, no. 6, pp. 1139–1144, Jun. 2011.

[6] N. Pinsky, "Southern California Edison Tehachapi Wind Energy Storage Project," [Online] https://www.smartgrid.gov, 2014.

[7] T. Wiley, "Aquion Energy Sodium-Ion Battery for Grid-level Applications Project," [Online] https://www.smartgrid.gov, 2014.

[8] J. S. Gates, "Duke Energy Notrees Wind Storage Demonstration Project," [Online] https://www.smartgrid.gov, 2014.

[9] California Public Utilities Commission, "Energy Storage Decision D.13-10-040," [Online] http://www.cpuc.ca.gov, Oct. 2013.

[10] M. Brenna, E. Tironi, and G. Ubezio, "Proposal of a local DC distribution network with distributed energy resources," in *Proc. of the IEEE International Conference on Harmonics and Quality of Power (ICHQP)*, Lake Placid, NY, 2004.

[11] S. X. Chen, H. B. Gooi, and M. Q. Wang, "Sizing of energy storage for microgrids," *IEEE Trans. on Smart Grid*, vol. 3, no. 1, pp. 142–151, Mar. 2012.

[12] J. H. Choi and J. C. Kim, "Advanced voltage regulation method of power distribution systems interconnected with dispersed storage and generation systems," *IEEE Trans. on Power Delivery*, vol. 16, no. 2, pp. 329–334, Apr. 2001.

[13] W. Dua, H. F. Wang, S. Cheng, J. Y. Wen, and R. Dunn, "Robustness of damping control implemented by energy storage systems installed in power systems," *International Journal of Electrical Power and Energy Systems*, vol. 33, no. 1, pp. 35–42, Jan. 2011.

[14] Z. Hu, F. Zhang, and B. Li, "Transmission expansion planning considering the deployment of energy storage systems," in *Proc. of IEEE PES General Meeting*, San Diego, CA, Jul. 2012.

[15] M. Parvania, M. Fotuhi-Firuzabad, and M. Shahidehpour, "Comparative hourly scheduling of centralized and distributed storage in day-ahead markets," *IEEE Trans. on Sustainable Energy*, vol. 5, no. 3, pp. 729–737, Jul. 2014.

[16] D. Pudjianto, M. Aunedi, P. Djapic, and G. Strbac, "Whole-systems assessment of the value of energy storage in low-carbon electricity systems," *IEEE Trans. on Smart Grid*, vol. 5, no. 2, pp. 1098–1109, Mar. 2014.

[17] D. L. Yao, S. S. Choi, K. J. Tseng, and T. T. Lie, "Determination of short-term power dispatch schedule for a wind farm incorporated with dual-battery energy storage scheme," *IEEE Trans. on Sustainable Energy*, vol. 3, no. 1, pp. 74–84, Jan. 2012.

[18] M. Dicorato, G. Forte, M. Pisani, and M. Trovato, "Planning and operating combined wind-storage system in electricity market," *IEEE Trans. on Sustainable Energy*, vol. 3, no. 2, pp. 209–217, Apr. 2012.

[19] D. Khatod, V. Pant, and J. Sharma, "Optimized daily scheduling of wind-pumped hydro plants for a day-ahead electricity market system," in *Proc. of the International Conference on Power Systems*, Kharagpur, India, Dec. 2009.

[20] C. A. Hill, M. C. Such, D. Chen, J. Gonzalez, and W. Grady, "Battery energy storage for enabling integration of distributed solar power generation," *IEEE Trans. on Smart Grid*, vol. 3, no. 2, Jun. 2012.

[21] Y. Zheng, Z. Y. Dong, F. J. Luo, K. Meng, J. Qiu, and K. P. Wong, "Optimal allocation of energy storage system for risk mitigation of discos with high renewable penetrations," *IEEE Trans. on Power Systems*, vol. 29, no. 1, Jan. 2014.

[22] C. Wu, H. Mohsenian-Rad, and J. Huang, "Vehicle-to-aggregator interaction game," *IEEE Trans. on Smart Grid*, vol. 3, no. 1, pp. 434–442, Mar. 2012.

[23] R. J. Bessa, M. A. Matos, F. J. Soares, and J. A. P. Lopes, "Optimized bidding of a ev aggregation agent in the electricity market," *IEEE Trans. on Smart Grid*, vol. 3, no. 1, pp. 443–452, Mar. 2012.

[24] Y. Xu, L. Xie, and C. Singh, "Optimal scheduling and operation of load aggregator with electric energy storage in power markets," in *Proc. of the IEEE North American Power Symposium*, Arlington, TX, 2010.

[25] H. Akhavan-Hejazi and H. Mohsenian-Rad, "Optimal operation of independent storage systems in energy and reserve markets with high wind penetration," *IEEE Trans. on Smart Grid*, vol. 5, no. 2, pp. 1088–1097, Mar. 2014.

[26] M. Kefayati and R. Baldick, "On optimal operation of storage devices under stochastic market prices," in *Proc. of IEEE Conference on Decision and Control*, Firenze, Italy, Dec. 2013.

[27] H. Mohsenian-Rad, “Optimal bidding, scheduling, and deployment of battery systems in california day-ahead energy market,” *IEEE Trans. on Power Systems (to appear)*, Jan. 2015.
[28] H. Akhavan-Hejazi and H. Mohsenian-Rad, “A stochastic programming framework for optimal storage bidding in energy and reserve markets,” in *Proc. of IEEE PES ISGT*, Washington, DC, Feb. 2013.
[29] E. G. Kardakos, C. K. Simoglou, and A. G. Bakirtzis, “Optimal bidding strategy in transmission-constrained electricity markets,” *Electric Power Systems Research*, vol. 109, pp. 141–149, Apr. 2014.
[30] G. Steeger, L. A. Barroso, and S. Rebennack, “Optimal bidding strategies for hydro-electric producers: A literature survey,” *IEEE Trans. on Power Systems*, vol. 29, no. 4, pp. 1758–1766, Jul. 2014.
[31] J. M. Arroyo and A. J. Conejo, “Optimal response of a thermal unit to an electricity spot market,” *IEEE Trans. on Power Systems*, vol. 15, no. 3, pp. 1098–1104, Aug. 2000.
[32] M. Kohansal and H. Mohsenian-Rad, “Price-maker economic bidding in two-settlement pool-based markets: The case of time-shiftable loads,” *IEEE Trans. on Power Systems (to appear)*, Feb. 2015.
[33] S. Gabriel, A. Conejo, J. Fuller, B. Hobbs, and C. Ruiz, *Complementarity Modeling in Energy Markets*. New York, NY: Springer, 2013.
[34] S. Boyd and L. Vandenberghe, *Convex Optimization*. New York, NY: Cambridge University Press, 2004.
[35] International Business Machines, “ILOG CPLEX Optimization Studio,” [Online] http://www.ibm.com/developerworks/downloads/ws/ilogcplex, Jul. 2014.
[36] E. Andersen and K. Andersen, “MOSEK Optimization Software,” [Online] http://www.mosek.com/products/mosek, Jul. 2014.
[37] http://oasis.caiso.com.
[38] R. T. Rockafellar and S. Uryasev, “Optimization of conditional value-at-risk,” *Journal of Risk*, vol. 2, pp. 21–41, 2000.
[39] http://www.ee.washington.edu/research/pstca.
[40] California Independent System Operator, “OASIS: Open Access Same-Time Information System,” [Online] oasis.caiso.com, Jul. 2014.
[41] G. B. Sheblé, *Computational auction mechanisms for restructured power industry operation*. Norwell, MA: Kluwer Academic Publishers, 1999.
[42] G. Aneiros, J. M. Vilar, R. Cao, and A. M. S. Roque, “Functional prediction for the residual demand in electricity spot markets,” *IEEE Trans. on Power Systems*, vol. 28, no. 2, pp. 4201–4208, Nov. 2013.

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