Distributed Multi-Interface Multi-Channel Random Access Using Convex Optimization

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Abstract—The aggregate capacity of wireless ad-hoc networks can be increased substantially if each node is equipped with multiple network interface cards (NICs) and each NIC operates on a distinct frequency channel. Most of the recently proposed channel assignment algorithms are based on *combinatorial* techniques. Combinatorial channel assignment schemes may sometimes result in computationally complicated algorithms as well as inefficient utilization of the available frequency spectrum. In this paper, we analytically model channel and interface assignment problems as tractable *continuous* optimization problems within the framework of *network utility maximization* (NUM). In particular, the link data rate models for both single-channel reception and multi-channel reception scenarios are derived. The assignment of both non-overlapped and partially-overlapped channels are also considered. We then propose two distributed multi-interface multi-channel random access (DMMRA) algorithms for *single-channel reception* and *multi-channel reception* scenarios. The DMMRA algorithms are fast, distributed, and easy to implement. Each algorithm solves the formulated NUM problem for each scenario. DMMRA requires each node to only iteratively solve a *local*, *myopic*, and *convex* optimization problem. Convergence and optimality properties of our algorithms are studied analytically. Simulation results show that our proposed algorithms significantly outperform utility-optimal combinatorial channel assignment algorithms in terms of both achieved network utility and throughput.

Index Terms—Multi-interface multi-channel wireless ad-hoc networks, random access, persistent probabilities, network utility maximization, convex optimization, single-channel reception, multi-channel reception, partially overlapped frequency channels.

1 Introduction

Multi-interface multi-channel wireless ad-hoc networks have recently received increasing attention especially under the context of wireless mesh networks (cf. [1]–[11]). Various applications include community and neighborhood networking, enterprise networking, and metropolitan area networking [12]. The IEEE 802.11s standardization project also includes using multiple frequency channels for wireless mesh networking [13].

In a multi-interface multi-channel wireless network, each node is equipped with multiple (usually 2 or 3) network interface cards (NICs). Each NIC operates on a distinct orthogonal frequency channel. The number of channels varies from 3 (as in IEEE 802.11b/g) to 12 (as in IEEE 802.11a). Using orthogonal channels can reduce the interference among simultaneous transmissions and substantially increase the network capacity. It has recently been shown in [14] that the performance can further improve if not only the orthogonal channels, but also the *partially overlapped* frequency channels are used.

There exists a wide range of related work aiming to design efficient channel and interface assignment algorithms. Some of them (e.g., in [8], [9]) focus on *heuristic* schemes, while the others take the analytical approach and formulate channel and interface assignment as *optimization problems* with various objectives such as throughput maximization [4], [5], [7], fair resource allocation [2], and network utility maximization [1].

A common approach in optimization-based channel assignment is to formulate channel and interface assignment as *combinatorial* problems: each NIC is assumed to operate over exactly *one* channel, either permanently or for a long period of time. Examples include the formulation of *integer optimization problems* [1], [2], *mixed-integer optimization problems* [3], and *graph coloring problems* [6], [7]. Knowing that most combinatorial problems are NP-hard [11, Appendix A], finding the optimal solution may require examining *all* the possible combinations within the search space. Thus, combinatorial channel assignment is usually computationally complicated. Their distributed implementation is also very challenging due to issues such as the *ripple-effect* problem [7], [11].

In this paper, we improve optimization-based channel assignment in *complexity* and *distributed implementation* aspects. We focus on *random access* systems which are appropriate for distributed deployments. Our proposed schemes to find optimal link and node persistent probabilities can be applied to the existing medium access protocols by using the mapping between persistent probabilities and contention window sizes [15].

The key contributions of this paper are as follows:

- We formulate optimization-based channel and interface assignments in random access systems, where the optimization variables are *transmission* and *listening* probabilities in each wireless node. This includes elaborate mathematical modeling of the achievable data rate on each wireless link.

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- We study channel and interface assignment not only in NICs with single-channel reception, but also in NICs with *multi-channel reception*. Although multi-channel reception is not widely used in practice, its deployment is feasible, making it a promising candidate for future ad-hoc networking architectures.
- Unlike most of the existing channel assignment schemes, here we address assigning not only orthogonal channels, but also partially overleaped channels. This better utilizes the frequency spectrum and further improves the network performance.
- We propose a fast, distributed, and easy to implement algorithm, called *distributed multi-interface multi-channel random access* (DMMRA), to solve a *network utility maximization* problem in multi-channel random access systems. We prove that DMMRA outperforms combinatorial channel assignment. We also analytically study the optimality and convergence properties of the DMMRA algorithm.
- *Performance Improvement:* Simulation results show that the DMMRA algorithm with *single-channel reception* leads to 36% and 23% higher network utility and aggregate throughput compared to the *utility-optimal combinatorial interface assignment and channel allocation* algorithm in [1]. When *multi-channel reception* model is implemented, the utility and throughput further increase by 57% and 71%, respectively.

This paper differs from the existing related work in the literature in several aspects. The algorithms in [2]–[6] are *centralized*, while we propose a channel and interface assignment algorithm which can be implemented in a *distributed* fashion as long as the wireless nodes can exchange some control messages with their two-hop neighbors. On the other hand, our focus is on random access models where each node *independently* selects its own transmission and listening probabilities; however, the algorithms in [4], [5] focus on *scheduling-based* medium access control. Similar to [3]–[5], we formulate channel assignment as a *continuous* optimization problem. However, the optimization problems in [3]–[5] are first formulated as discrete (i.e., combinatorial) problems, then simply converted to continuous problem by simply *relaxing* integer constraints, leading to major optimality losses. Our results are also related to the multi-channel random access models in [8]–[10]; however, the proposed schemes in [8]–[10] are *heuristic* (such as the multi-channel extension of the IEEE 802.11 distributed coordination function in [8]), while here we consider analytical modeling of random access in various multi-channel scenarios. Finally, none of the prior work in [1]–[11] address partially overlapped channel assignment and multi-channel reception.

The rest of this paper is organized as follows. The data rates for various scenarios are modeled in Section 2. The NUM problems are formulated in Section 3. The DMMRA algorithms are proposed in Section 4. Simulation results are presented in Section 5. Conclusions are given in Section 6. All proofs are given in the Appendices. A list of key notations is given in Table 1.

TABLE 1
List of Key Notations

$\mathcal{N}$, N	Set of all nodes in the network and its cardinality
$\mathcal{L}$, L	Set of all links in the network and its cardinality
$\mathcal{L}_n^{\text{in}}$, $\mathcal{L}_n^{\text{out}}$	Set of incoming and outgoing links of node n
$\mathcal{N}_n^{\text{in}}$, $\mathcal{N}_n^{\text{out}}$	Set of incoming and outgoing nodes of node n
$\mathcal{C}$, C	Set of available frequency channels and its cardinality
$\mathcal{I}_n$, I_n	Set of available NICs in node n and its cardinality
$P_n^{(i)(c)}$	Persistent probability for node n on NIC i, channel c
$Q_n^{(i)(c)}$	Listening probability for node n on NIC i, channel c
$p_{nm}^{(i)(c)}$	Persistent probability for link (n,m) on NIC i, channel c
$\gamma_{nm}^{(c)}$	Fixed peak data rate for link (n,m) on channel c
r_{nm}	Average data rate for link (n,m)
$u(\cdot)$	Utility function
Φ, Ψ	Feasible sets in problems (NUM-S) and (NUM-M)

2 System Model

Consider a multi-interface multi-channel wireless ad-hoc network with $\mathcal{N} = \{1, \ldots, N\}$ as the set of wireless nodes and $\mathcal{L} = \{1, \ldots, L\}$ as the set of *unidirectional* wireless links[1]. For each node $n \in \mathcal{N}$, we denote the set of *incoming* links by $\mathcal{L}_n^{\text{in}} \subset \mathcal{L}$, with size $L_n^{\text{in}} = |\mathcal{L}_n^{\text{in}}|$, and the set of outgoing links by $\mathcal{L}_n^{\text{out}} \subset \mathcal{L}$, with size $L_n^{\text{out}} = |\mathcal{L}_n^{\text{out}}|$. We also define $\mathcal{N}_n^{\text{in}} = \{m : (m,n) \in \mathcal{L}_n^{\text{in}}\}$ as the set of *in-neighbors* and $\mathcal{N}_n^{\text{out}} = \{m : (n,m) \in \mathcal{L}_n^{\text{out}}\}$ as the set of *out-neighbors* of node n, respectively. The set of available frequency channels is denoted by $\mathcal{C} = \{1, \ldots, C\}$. The set of NICs for each node $n \in \mathcal{N}$ is denoted by $\mathcal{I}_n$, with size $I_n = |\mathcal{I}_n|$. Each node $n \in \mathcal{N}$ has L_n^{out} separate queues, where each queue stores the packets for one of the outgoing links of node n (see Fig. 1). Time is divided into equal-length slots. Similar to *slotted Aloha* systems (cf. [16, Section 4.2]), at each time slot, node n may choose to transmit packets to each of its out-neighbors $m \in \mathcal{N}_n^{\text{out}}$ using its NIC $i \in \mathcal{I}_n$ over channel $c \in \mathcal{C}$ with a *link persistent probability* $p_{nm}^{(i)(c)}$. For the network in Fig. 1, node n has $I_n = 2$ NICs and $L_n = 2$ outgoing links, where $\mathcal{I}_n = \{i, j\}$ and $\mathcal{N}_n^{\text{out}} = \{m, s\}$. We also have: $\mathcal{C} = \{1, 2, 3\}$. In node n, those packets which are destined to node m are enqueued in queue $[n, m]$. Similarly, the packets which are destined to node s are enqueued in queue $[n, s]$. At each time slot, a packet from queue $[n, m]$ is sent to node m (i.e., through link (n, m)) using NIC i over channels 1, 2, or 3, with probabilities $p_{nm}^{(i)(1)}$, $p_{nm}^{(i)(2)}$, and $p_{nm}^{(i)(3)}$, respectively. Each NIC may only transmit one packet at a time. Furthermore, each NIC makes *independent* decisions on selecting its transmission and listening probabilities. Next, we obtain the average data rate model for each link in various scenarios.

2.1 NICs with Single-Channel Reception

In this section, we consider the case where each NIC can decode the received packets over only *one* channel

1. Clearly, bidirectional traffic can be modeled as two unidirectional links, one in each direction. That is, if the link between two nodes $n, m \in \mathcal{N}$ is bidirectional, then we have $(n, m), (m, n) \in \mathcal{L}$.

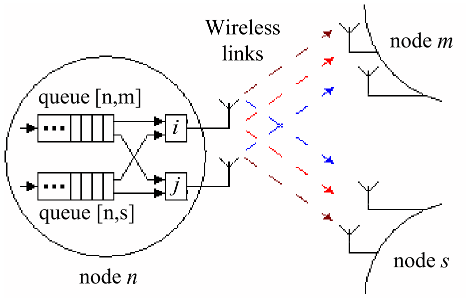


Fig. 1. An ad-hoc network with $\mathcal{N} = \{n, m, s\}$ as the set of nodes. Each node has two NICs and there are $C = 3$ channels available, denoted by 3 colors. Nodes m and s are the out-neighbors of node n. We have: $\mathcal{I}_n = \{i, j\}$.

at a time. We assume that all available channels are orthogonal. For each node $n \in \mathcal{N}$, let $Q_n^{(i)(c)}$ denote the probability that node n *listens* to channel $c \in \mathcal{C}$ using its NIC $i \in \mathcal{I}_n$. To be able to listen to channel c, NIC i on node n needs to be in the *receive mode* (i.e., does not transmit any packet) and *operates* over channel c. The key feature in single-channel reception model is that if node n is in the receive mode, and operates over channel $d \neq c$, then it *cannot* decode the signals transmitted over channel c. In this case, for each node $n \in \mathcal{N}$, we have[2]

$$\sum_{c\in\mathcal{C}} \left(P_n^{(i)(c)} + Q_n^{(i)(c)}\right) = 1, \quad \forall i \in \mathcal{I}_n, \tag{1}$$

where $P_n^{(i)(c)}$ denotes the probability that node n transmits some data from NIC $i \in \mathcal{I}_n$ over channel $c \in \mathcal{C}$ to *one* of its out-neighbors. We call $P_n^{(i)(c)}$ the *node persistent probability* for NIC i of node n over channel c. We have

$$P_n^{(i)(c)} = \sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)}. \tag{2}$$

For each link $(n, m) \in \mathcal{L}$, we first consider the case where there is *no interference* in the network (i.e., there are only two nodes). Let $\tilde{A}_m^{(c)}$ denote the action set for all cases where at least one NIC $j \in \mathcal{I}_m$ transmits packets over channel c. The probability of this happening is

$$\mathbb{P}\left(\tilde{A}_m^{(c)}\right) = 1 - \prod_{j\in\mathcal{I}_m} \left(1 - P_m^{(j)(c)}\right). \tag{3}$$

Let $\hat{A}_m^{(c)}$ denote the action set for all cases where no NIC on node m transmits packets over channel c, and no NIC listens to channel c either. This happens with probability

$$\mathbb{P}\left(\hat{A}_m^{(c)}\right) = \prod_{j\in\mathcal{I}_m} \left(1 - P_m^{(j)(c)} - Q_m^{(j)(c)}\right). \tag{4}$$

Since the sets $\tilde{A}_m^{(c)}$ and $\hat{A}_m^{(c)}$ are two *disjoint* sets (i.e., $\tilde{A}_m^{(c)} \cap \hat{A}_m^{(c)}$ is an empty set), we have

$$\mathbb{P}\left(\tilde{A}_m^{(c)} \cup \hat{A}_m^{(c)}\right) = \mathbb{P}\left(\tilde{A}_m^{(c)}\right) + \mathbb{P}\left(\hat{A}_m^{(c)}\right). \tag{5}$$

The transmission from NIC $i \in \mathcal{I}_n$ on sending node $n \in \mathcal{N}$ over channel $c \in \mathcal{C}$ can be received *correctly* by the receiving node $m \in \mathcal{N}_n^{\text{out}}$ only if *at least* one NIC $j \in \mathcal{I}_m$ is *listening* to channel c and *none* of the other NICs on node m are transmitting packets over frequency channel c. From (3)-(5), this happens with probability

$$\begin{aligned} 1 - \mathbb{P}\left(\tilde{A}_m^{(c)} \cup \hat{A}_m^{(c)}\right) = \prod_{j\in\mathcal{I}_m} \left(1 - P_m^{(j)(c)}\right) \\ - \prod_{j\in\mathcal{I}_m} \left(1 - P_m^{(j)(c)} - Q_m^{(j)(c)}\right). \end{aligned} \tag{6}$$

Next, we model the effect of interference in a network with N nodes. For each pair of nodes $s, m \in \mathcal{N}$, we define $\delta_{sm} = 1$ if node s is within the *interference range*[3] of node m, and $\delta_{sm} = 0$ otherwise. Since the interference range is *at least* as large as the communication range, $\delta_{sm} = 1$ if $s \in \mathcal{N}_m^{\text{in}}$. A transmission from NIC i on node n to node m via link $(n, m) \in \mathcal{L}$ over channel c *does not encounter collision* if there is no simultaneous transmission over channel c from any NIC $j \in \mathcal{I}_n \backslash \{i\}$ on node n, or any NIC $k \in \mathcal{I}_s$ on node s with $\delta_{sm} = 1$. This happens with probability

$$\begin{aligned} \left(\prod_{j\in\mathcal{I}_n\backslash\{i\}} \left(1 - P_n^{(j)(c)}\right)\right) \\ \times \left(\prod_{s\in\mathcal{N}\backslash\{n,m\}} \prod_{k\in\mathcal{I}_s} \left(1 - \delta_{sm} P_s^{(k)(c)}\right)\right). \end{aligned} \tag{7}$$

For each wireless link $(n, m) \in \mathcal{L}$, let r_{nm} denote the average data rate, which is a function of the following persistent and listening probability vectors:

$$\boldsymbol{p} = \left(p_{nm}^{(i)(c)}, \forall n \in \mathcal{N}, m \in \mathcal{N}_n^{\text{out}}, i \in \mathcal{I}_n, c \in \mathcal{C}\right), \tag{8}$$

$$\boldsymbol{Q} = \left(Q_n^{(i)(c)}, \forall n \in \mathcal{N}, i \in \mathcal{I}_n, c \in \mathcal{C}\right). \tag{9}$$

From (2), (6) and (7), we have [16][4]

$$\begin{aligned} & r_{nm}(\boldsymbol{p}, \boldsymbol{Q}) = \\ & \sum_{i\in\mathcal{I}_n} \sum_{c\in\mathcal{C}} \gamma_{nm}^{(c)} p_{nm}^{(i)(c)} \left(\prod_{j\in\mathcal{I}_n\backslash\{i\}} \left(1 - P_n^{(j)(c)}\right)\right) \\ & \left(\prod_{s\in\mathcal{N}\backslash\{n,m\}} \prod_{k\in\mathcal{I}_s} \left(1 - \delta_{sm} P_s^{(k)(c)}\right)\right) \\ & \left(\prod_{j\in\mathcal{I}_m} \left(1 - P_m^{(j)(c)}\right) - \prod_{j\in\mathcal{I}_m} \left(1 - P_m^{(j)(c)} - Q_m^{(j)(c)}\right)\right), \end{aligned} \tag{10}$$

where $\gamma_{nm}^{(c)}$ denotes the fixed peak data rate[5] for link (n, m) over frequency channel c (i.e., the data rate achieved by link (n, m) over channel c if there is no other transmission in the network at the same time). The data rate model in (10) sums up all the average data rates that can be achieved by transmitting packets from each NIC $i \in \mathcal{I}_n$ and over each frequency channel $c \in \mathcal{C}$.

2.2 NICs with Multi-Channel Reception

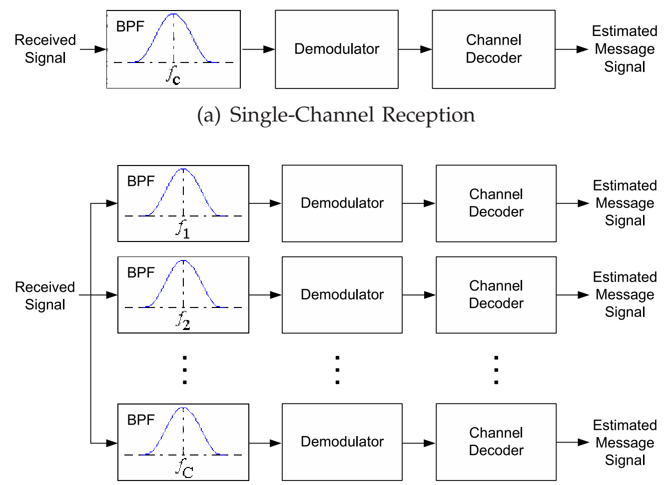
Next, consider the case where each NIC can decode *multiple* simultaneously received packets as long as they are transmitted over different orthogonal channels. That is, each NIC listens to *all* frequency channels at its receive

2. Here we assume that each NIC operates either in *transmit* or *receive* mode. If an NIC also operates in *idle* mode, then the equality in (1) is replaced with non-strict inequality "$\leq$" and NIC $i \in \mathcal{I}_n$ would be operating in *idle* mode with probability $1 - \sum_{c\in\mathcal{C}}(P_n^{(i)(c)} + Q_n^{(i)(c)})$.

3. In this paper, we assume that the transmission powers and consequently the interference ranges are fixed for all nodes. Joint channel assignment and power control is studied e.g., in [17], [18].

4. We notice that *node* persistent probability vector $\boldsymbol{P} = (P_n^{(i)(c)}, \forall n \in \mathcal{N}, i \in \mathcal{I}_n, c \in \mathcal{C})$ can be constructed from *link* persistent probability vector $\boldsymbol{p}$ using (2). Thus, for each wireless link $(n, m) \in \mathcal{L}$, we can denote the average data rate for the single-channel reception scenario as $r_{nm}(\boldsymbol{p}, \boldsymbol{Q})$, rather than $r_{nm}(\boldsymbol{p}, \boldsymbol{P}, \boldsymbol{Q})$, to avoid redundancy.

5. In general, we may have $\gamma_{nm}^{(c)} \neq \gamma_{nm}^{(d)}$ for any $c \neq d$ as the channel properties are usually different at different frequency bands.



(a) Single-Channel Reception

(b) Multi-Channel Reception

Fig. 2. Building blocks of the receiver unit in single-channel reception and multi-channel reception models. BPF stands for band-pass filter. For each channel $c \in \mathcal{C}$, the central frequency of the channel filter is shown by f_c.

mode and applies the band-pass channel filters to *all* the received signals. The output of each filter is then processed separately (i.e., in parallel) in C distinct *radio frequency (RF) chains* to distinguish transmissions over different channels [19]. Figs. 2 (a) and (b) show the basic building blocks of the receiver device when the single-channel and multi-channel reception models are used, respectively. Most of the existing commercial NICs do *not* yet implement multi-channel reception. However, we will show in Section 5 that it can significantly improve the network performance. Thus, it is an attractive and promising candidate for future deployments. Note that for the setting in Fig. 2(b) to be implemented in practice, various issues such as synchronization and coordination among the RF chains need to be addressed carefully.

As in Section 2.1, we first consider the no interference case. Let $\breve{A}_m^{(-c)}$ denote the action set where all NICs on node m transmit packets on some channels *other than* channel c, and no NIC is in the receive mode. We have

$$\mathbb{P}\left(\breve{A}_m^{(-c)}\right) = \prod_{j\in\mathcal{I}_m} \sum_{d\in\mathcal{C}\setminus\{c\}} P_m^{(j)(d)}. \tag{11}$$

Since the sets $\tilde{A}_m^{(c)}$ (defined in Section 2.1) and $\breve{A}_m^{(-c)}$ are *disjoint* (i.e., $\tilde{A}_m^{(c)} \cap \breve{A}_m^{(-c)}$ is an empty set), we have

$$\mathbb{P}\left(\tilde{A}_m^{(c)} \cup \breve{A}_m^{(-c)}\right) = \mathbb{P}\left(\tilde{A}_m^{(c)}\right) + \mathbb{P}\left(\breve{A}_m^{(-c)}\right). \tag{12}$$

In the multi-channel reception model, for any link $(n,m) \in \mathcal{L}$, the transmission from NIC $i \in \mathcal{I}_n$ on node $n \in \mathcal{N}$ over channel $c \in \mathcal{C}$ can be received correctly by node $m \in \mathcal{N}_n^{\text{out}}$ if *at least* one NIC $j \in \mathcal{I}_m$ is in the *receive mode* and *none* of the other NICs on node m are transmitting packets over channel c. From (3), (11), and (12), this happens with probability

$$\begin{aligned} &1-\mathbb{P}\left(\tilde{A}_m^{(c)} \cup \breve{A}_m^{(-c)}\right) = \\ &\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right) - \prod_{j\in\mathcal{I}_m}\sum_{d\in\mathcal{C}\setminus\{c\}} P_m^{(j)(d)}. \end{aligned} \tag{13}$$

When the interference is taken into account, from (2), (7), and (13), for each link $(n,m)\in\mathcal{L}$, we have

$$\begin{aligned} &r_{nm}(\boldsymbol{p}) = \\ &\sum_{i\in\mathcal{I}_n}\sum_{c\in\mathcal{C}} \gamma_{nm}^{(c)} p_{nm}^{(i)(c)} \left(\prod_{j\in\mathcal{I}_n\setminus\{i\}}\left(1-P_n^{(j)(c)}\right)\right) \\ &\left(\prod_{s\in\mathcal{N}\setminus\{n,m\}}\prod_{k\in\mathcal{I}_s}\left(1-\delta_{sm}P_s^{(k)(c)}\right)\right) \\ &\left(\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right) - \prod_{j\in\mathcal{I}_m}\sum_{d\in\mathcal{C}\setminus\{c\}} P_m^{(j)(d)}\right). \end{aligned} \tag{14}$$

Note that since a node can listen to *all* channels when it is in the receive mode, the data rate model in (14) does *not* depend on the listening probability vector $\boldsymbol{Q}$.

2.3 Partially Overlapped Frequency Channels

In this section, we extend the data rate models in (10) and (14) to the general cases, where both orthogonal (i.e., non-overlapped) and partially overlapped channels are used. Unlike orthogonal channels that can only cause *co-channel interference*, partially overlapping channels may also lead to *adjacent channel interference*. Thus, we borrow the concept of *multiple interference ranges* from our recent work in [14, Section II-D]. For each pair of nodes $s,m \in \mathcal{N}$, we define $\delta_{sm}^{(cd)} = 1$ if node s is within the interference range of node m, while node s is operating over channel c and node m is operating over channel d; otherwise, $\delta_{sm}^{(cd)} = 0$. In general, *the smaller the frequency spectrum overlapping between two channels c and d, the shorter the corresponding interference range* (cf. [14, Fig. 3]). In fact, as two wireless links use lower overlapped channels, the less is the interference power that they cause on each other's transmissions. Thus, the interfering transmissions need to be in shorter distance to block each other's packets.

We first assume that NICs use single-channel reception model and there is no interference in the network. For each link $(n,m)\in\mathcal{L}$, let $\tilde{B}_m^{(c)}$ denote the action set for all cases where at least one NIC $j \in \mathcal{I}_m$ transmits packets on some channel $d \in \mathcal{C}$ such that $\delta_{mm}^{(cd)} = 1$. Also, let $\hat{B}_m^{(c)}$ denote the action set for all cases where no NIC on node m transmits packets on any channel $d\in\mathcal{C}$ such that $\delta_{mm}^{(cd)}=1$, and no NIC listens to channel c either. We have

$$\mathbb{P}\left(\tilde{B}_m^{(c)}\right) = 1-\prod_{j\in\mathcal{I}_m}\left(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}\right), \tag{15}$$

$$\mathbb{P}\left(\hat{B}_m^{(c)}\right) = \prod_{j\in\mathcal{I}_m}\left(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}-Q_m^{(j)(c)}\right). \tag{16}$$

Since the sets $\tilde{B}_m^{(c)}$ and $\hat{B}_m^{(c)}$ are disjoint sets, we have

$$\begin{aligned} 1-\mathbb{P}\left(\tilde{B}_m^{(c)}\cup\hat{B}_m^{(c)}\right) &= \prod_{j\in\mathcal{I}_m}\left(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}\right) \\ &- \prod_{j\in\mathcal{I}_m}\left(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}-Q_m^{(j)(c)}\right). \end{aligned} \tag{17}$$

To model the interference, we can modify the collision avoidance probability model in (7) as

$$\begin{aligned} &\left(\prod_{j\in\mathcal{I}_n\setminus\{i\}}\left(1-\sum_{d\in\mathcal{C}}\delta_{nm}^{(cd)}P_n^{(j)(d)}\right)\right) \\ &\times\left(\prod_{s\in\mathcal{N}\setminus\{n,m\}}\prod_{k\in\mathcal{I}_s}\left(1-\sum_{d\in\mathcal{C}}\delta_{sm}^{(cd)}P_s^{(k)(d)}\right)\right). \end{aligned} \tag{18}$$

From (17) and (18), when the NICs implement single-channel reception and all partially overlapped channels are available, the average data rate of link $(n,m) \in \mathcal{L}$ is

$$
\begin{aligned}
&r_{nm}(\boldsymbol{p},\boldsymbol{Q}) = \\
&\quad \textstyle\sum_{i\in\mathcal{I}_n}\sum_{c\in\mathcal{C}}\gamma_{nm}^{(c)} p_{nm}^{(i)(c)}\Big(\prod_{j\in\mathcal{I}_n\setminus\{i\}}\Big(1-\sum_{d\in\mathcal{C}}\delta_{nm}^{(cd)}P_n^{(j)(d)}\Big)\Big) \\
&\quad \Big(\textstyle\prod_{s\in\mathcal{N}\setminus\{n,m\}}\prod_{k\in\mathcal{I}_s}\Big(1-\sum_{d\in\mathcal{C}}\delta_{sm}^{(cd)}P_s^{(k)(d)}\Big)\Big) \\
&\quad \Big(\textstyle\prod_{j\in\mathcal{I}_m}\Big(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}\Big) \\
&\quad -\textstyle\prod_{j\in\mathcal{I}_m}\Big(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}-Q_m^{(j)(c)}\Big)\Big).
\end{aligned} \tag{19}
$$

We now consider the multi-channel reception scenario. For any link $(n,m)\in\mathcal{L}$, let $\breve{B}_m^{(-c)}$ denote the set of actions for all cases where no NIC on node m transmits packets over channel c or any other channel $d \in \mathcal{C}\setminus\{c\}$ such that $\delta_{mm}^{(cd)} = 1$, and no NIC is silent either. In other words, all NICs on node m transmit packets on some channels *other than* those channels that have (full or partial) overlapping with channel c. We have

$$
\mathbb{P}\left(\breve{B}_m^{(-c)}\right) = \textstyle\prod_{j\in\mathcal{I}_m}\sum_{d\in\mathcal{C}}\left(1-\delta_{mm}^{(cd)}\right) P_m^{(j)(d)}. \tag{20}
$$

Notice that for any node $m\in\mathcal{N}$, if frequency channels $c,d \in \mathcal{C}$ are either fully or partially overlapped, then $1-\delta_{mm}^{(cd)}=0$. From (15), (18), and (20), when the NICs use multi-channel reception and all non-overlapped as well as partially overlapped channels are being available, the average data rate of link $(n,m)\in\mathcal{L}$ becomes

$$
\begin{aligned}
&r_{nm}(\boldsymbol{p}) = \\
&\quad \textstyle\sum_{i\in\mathcal{I}_n}\sum_{c\in\mathcal{C}}\gamma_{nm}^{(c)} p_{nm}^{(i)(c)} \\
&\quad \times\Big(\textstyle\prod_{j\in\mathcal{I}_n\setminus\{i\}}\Big(1-\sum_{d\in\mathcal{C}}\delta_{nm}^{(cd)}P_n^{(j)(d)}\Big)\Big) \\
&\quad \Big(\textstyle\prod_{s\in\mathcal{N}\setminus\{n,m\}}\prod_{k\in\mathcal{I}_s}\Big(1-\sum_{d\in\mathcal{C}}\delta_{sm}^{(cd)}P_s^{(k)(d)}\Big)\Big) \\
&\quad \Big(\textstyle\prod_{j\in\mathcal{I}_m}\Big(1-\sum_{d\in\mathcal{C}}\delta_{mm}^{(cd)}P_m^{(j)(d)}\Big) \\
&\quad -\textstyle\prod_{j\in\mathcal{I}_m}\sum_{d\in\mathcal{C}}\Big(1-\delta_{mm}^{(cd)}\Big)P_m^{(j)(d)}\Big).
\end{aligned} \tag{21}
$$

It can be verified that, if all the channels are orthogonal (i.e., $\delta_{sm}^{cd}=0$ for all $s,m\in\mathcal{N}$ and any $c\neq d$), then the rates in (19) and (21) reduce to (10) and (14), respectively.

3 Network Utility Maximization

Expressions in (1)-(21) model the average link data rates for different multi-interface multi-channel random access scenarios. In this section, we formulate the random access problems for each scenario in a unified framework.

3.1 Problem Formulation

Within the NUM framework, the resource allocation problem can be formulated either at *link layer* [20], [21] or at *transport layer* [22]. Here, for the ease of exposition, we limit our study to the link-layer NUM[6]. In this regard, each link $(n,m)\in\mathcal{L}$ is assumed to maintain a *utility* $u(r_{nm})$, which is an increasing and concave function of its rate r_{nm} and indicates the degree of satisfaction of link (n,m) on its data rate. The utility of link (n,m) is also a function of all the persistent and listening probabilities $\boldsymbol{p}$ and $\boldsymbol{Q}$. Assuming that the *single*-channel reception model is used, we are interested in finding the optimal solution of the following NUM problem:

$$
\underset{\langle \boldsymbol{p},\boldsymbol{Q}\rangle\in\Phi}{\text{maximize}} \quad \textstyle\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}_n^{\text{out}}} u(r_{nm}(\boldsymbol{p},\boldsymbol{Q})), \tag{NUM-S}
$$

where the data rates are as in (10) if only the orthogonal channels are used, and as in (19) if both orthogonal and partially overlapped channels are used. We also have

$$
\begin{aligned}
\Phi = \{\, &\langle \boldsymbol{p},\boldsymbol{Q}\rangle : p_{nm}^{(i)(c)}, P_n^{(i)(c)}, Q_n^{(i)(c)} \in [0,1], \\
&P_n^{(i)(c)} = \textstyle\sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)},\ \sum_{d\in\mathcal{C}}\Big(P_n^{(i)(d)}+Q_n^{(i)(d)}\Big)=1, \\
&\forall n\in\mathcal{N},\ m\in\mathcal{N}_n^{\text{out}},\ i\in\mathcal{I}_n,\ c\in\mathcal{C}\,\}.
\end{aligned}
$$

On the other hand, if *multi*-channel reception is used, then the following NUM problem is being solved:

$$
\underset{\boldsymbol{p}\,\in\,\Psi}{\text{maximize}} \quad \textstyle\sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}_n^{\text{out}}} u(r_{nm}(\boldsymbol{p})), \tag{NUM-M}
$$

where the data rates are as in (14) if only the orthogonal channels are used, and as in (21) if both orthogonal and partially overlapped channels are used. We have

$$
\begin{aligned}
\Psi = \{\, &\boldsymbol{p} : p_{nm}^{(i)(c)}, P_n^{(i)(c)} \in [0,1],\ P_n^{(i)(c)} = \textstyle\sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)}, \\
&\textstyle\sum_{d\in\mathcal{C}} P_n^{(i)(d)} \le 1,\ \forall n\in\mathcal{N},\ m\in\mathcal{N}_n^{\text{out}},\ i\in\mathcal{I}_n,\ c\in\mathcal{C}\,\}.
\end{aligned}
$$

Sets Φ and Ψ include only linear constraints. Therefore, they are *convex* sets [24]. Various concave utility functions can also be considered to achieve different design objectives. A popular class of utility functions are α-fair utilities [25], where for each link $(n,m)\in\mathcal{L}$, we have

$$
u(r_{nm}) = \begin{cases} (1-\alpha)^{-1} r_{nm}^{1-\alpha}, & \text{if } \alpha\in(0,1)\cup(1,\infty), \\ \log r_{nm}, & \text{if } \alpha = 1. \end{cases} \tag{22}
$$

Using (22), a wide range of efficient and fair resource allocations among the link-layer flows can be modeled. In particular, optimization problems (NUM-S) and (NUM-M) reduce to throughput maximization with $\alpha\to 0$, to proportional fair allocation with $\alpha=1$, to harmonic mean fairness with $\alpha=2$, and to max-min fairness with $\alpha\to\infty$.

Unlike most of the previously proposed optimization-based channel assignment models, where the formulated optimization problems are combinatorial and *discrete-valued* (cf. [1]–[3], [6], [7]), problems (NUM-S) and (NUM-M) are *continuous-valued*. They make the analysis of our models substantially easier. Notice that, within the NUM framework at the link-layer, the combinatorial channel assignment problem can be formulated as the

6. We can extend the model to a transport-layer NUM similar to the joint congestion control and medium access control design in [23].

following *mixed-integer* optimization problem [1]:

$$\underset{\substack{\langle \boldsymbol{p}, \boldsymbol{Q}\rangle \in \Phi \\ \boldsymbol{x} \in \Upsilon}}{\text{maximize}} \;\; \sum_{n\in\mathcal{N}} \sum_{m\in\mathcal{N}_n^{\text{out}}} u(r_{nm}(\boldsymbol{p}, \boldsymbol{Q})),$$

$$\begin{aligned} \text{subject to } & \textstyle\sum_{c\in\mathcal{C}} x_n^{(i)(c)}=1,\ \forall n\in\mathcal{N},\, i\in\mathcal{I}_n, \\ & P_n^{(i)(c)} \leq x_n^{(i)(c)},\ \ \forall n\in\mathcal{N},\, i\in\mathcal{I}_n,\, c\in\mathcal{C}, \\ & Q_n^{(i)(c)} \leq x_n^{(i)(c)},\ \ \forall n\in\mathcal{N},\, i\in\mathcal{I}_n,\, c\in\mathcal{C}, \end{aligned} \quad \text{(NUM-C)}$$

where for each node $n \in \mathcal{N}$, any NIC $i \in \mathcal{I}_n$, and each channel $c \in \mathcal{C}$, the *integer* variable $x_n^{(i)(c)}$ is defined as

$$x_n^{(i)(c)} = \begin{cases} 1, & \text{if NIC } i \text{ operates over channel } c, \\ 0, & \text{otherwise.} \end{cases} \quad (23)$$

We also have $\boldsymbol{x} = (x_n^{(i)(c)},\ \forall n\in\mathcal{N},\ i\in\mathcal{I}_n\ c\in\mathcal{C})$ and

$$\Upsilon = \left\{\boldsymbol{x} : x_n^{(i)(c)} \in \{0, 1\},\ \forall n\in\mathcal{N},\ i\in\mathcal{I}_n\ c\in\mathcal{C}\right\}.$$

From the first constraint in (NUM-C), each NIC can be assigned to exactly one channel. In the context of combinatorial channel assignment, the selection of the *operating channel* for each NIC is called *interface-to-channel binding* [7]. From the second and the third constraints, NIC i cannot transmit over or listen to channel $c \in \mathcal{C}$ if it is *not* operating on channel c. By solving the mixed-integer problem (NUM-C), we can select not only the operating channel, but also the persistent and listening probabilities corresponding to the operating channel of each NIC to achieve utility-optimal performance within the combinatorial channel and interface assignment framework.

Theorem 1: Let $U^\star_{SCR}$, $U^\star_{MCR}$, and $U^\star_{Comb}$ denote the optimal solution of problems (NUM-S), (NUM-M), and (NUM-C), respectively. We can show the following:
(a) Random access with multi-channel reception outperforms random access with single-channel reception:

$$U^\star_{SCR} \leq U^\star_{MCR}. \quad (24)$$

(b) Random access with single-channel reception outperforms NUM-based combinatorial channel assignment:

$$U^\star_{Comb} \leq U^\star_{SCR}. \quad (25)$$

The proof of Theorem 1 is given in Appendix A.

3.2 Examples

First, consider a *unidirectional ring topology* with $N = 3$ nodes, $L = 3$ links, and $C = 3$ channels. The utilities are logarithmic (i.e., $\alpha = 1$). Each node has one NIC. We have: $\mathcal{N} = \{n, m, s\}$, $\mathcal{L} = \{(n, m), (m, s), (s, n)\}$, and $\mathcal{C} = \{1, 2, 3\}$. For any $c \in \mathcal{C}$, $\gamma^{(c)}_{nm} = \gamma^{(c)}_{ns} = \gamma^{(c)}_{sn} = 11$ Mbps. In this scenario, combinatorial channel assignment strategies can only assign the *same* channel to all NICs in the network. Otherwise, at least two nodes *cannot* communicate with each other. Thus, we have: $U^\star_{Comb} = 3\log(11 \times \frac{1}{3} \times (1-\frac{1}{3}) \times (1-\frac{1}{3})) = 1.465$, where each link is optimally active with probability $\frac{1}{3}$. On the other hand, $U^\star_{SCR} = 3\log(11\times\frac{1}{2}\times(1-0)\times(1-\frac{1}{2})) = 3.035$, where $P_n^{(1)(1)} = Q_n^{(1)(3)} = 0.5$, $P_n^{(1)(2)} = P_n^{(1)(3)} = Q_n^{(1)(1)} = Q_n^{(1)(2)} = 0$, $P_m^{(1)(2)} = Q_m^{(1)(1)} = 0.5$, $P_m^{(1)(1)} = P_m^{(1)(3)} = Q_m^{(1)(2)} = Q_m^{(1)(3)} = 0$, $P_s^{(1)(3)} = Q_s^{(1)(2)} = 0.5$, and $P_s^{(1)(1)} = P_s^{(1)(2)} = Q_s^{(1)(1)} = Q_s^{(1)(3)} = 0$. That is, on average, each node transmits to its out-neighbor on one channel during half of the time slots and listens to its in-neighbor on a *different* channel during the other half of the time slots. As a result, each of the 3 links in the network is active on a *distinct* channel and the available frequency spectrum is fully utilized. The performance gain is $\frac{3.035}{1.465} = 2.1$. In this example, we have: $U^\star_{Comb} < U^\star_{SCR} = U^\star_{MCR}$.

Next, consider a *bidirectional ring topology*, where $\mathcal{N} = \{n,\ m, s\}$ and $\mathcal{L} = \{(n, m), (m, n), (m, s), (s, m), (s, n), (n, s)\}$. The rest of the parameters are the same as the previous example. Again, any combinatorial channel assignment assigns the same channel to all NICs. We have: $U^\star_{Comb} = 6\log(11 \times \frac{1}{6} \times (1 - \frac{1}{3}) \times (1 - \frac{1}{3})) = -1.229$, where each link is optimally active with probability $\frac{1}{6}$. Each node is also optimally active with probability $2\times\frac{1}{6} = \frac{1}{3}$ as it has two outgoing links. On the other hand, $U^\star_{SCR} = 6\log(11 \times 0.2113 \times (1 - 0.2113) \times 0.5774) = 0.341$, where each node listens to one distinct channel with probability 0.5774. Each node also transmits to its out-neighbors using two different channels, other than the channel that it listens to. Finally, $U^\star_{MCR} = 6\log(11 \times \frac{1}{4} \times (1-0) \times (1-\frac{1}{2})) = 1.9107$, where each node transmits to both of its out-neighbors using one of the three channels with probability $\frac{1}{4}$. Each NIC is silent with probability $\frac{1}{2}$. In this example, $U^\star_{Comb} < U^\star_{SCR} < U^\star_{MCR}$.

Theorem 1 and the above examples show that our proposed random access models can outperform combinatorial channel and interface assignment. We will investigate this issue further in Section 5.2.

4 DMMRA Algorithms

Although the objective functions in problems (NUM-S) and (NUM-M) are concave in link rates $\boldsymbol{r} = (r_{nm}, \forall (n, m) \in \mathcal{L})$, they are *not* concave in persistent and listening probabilities $\boldsymbol{p}$ and $\boldsymbol{Q}$ due to the *product forms* in (10), (14), (19), and (21). Thus, finding the optimal solutions of these *non-convex* optimization problems are not easy in general. In this section, we discuss some of the features of problems (NUM-S) and (NUM-M) which will help us to develop our distributed multi-interface multi-channel random access (DMMRA) algorithms.

4.1 Local NUM Problems

For each node $n\in\mathcal{N}$ and any NIC $i \in \mathcal{I}_n$, we define

$$\boldsymbol{p}_n^{(i)} = \left(p_{nm}^{(i)(c)},\ \forall m \in \mathcal{N}_n^{\text{out}},\ c \in \mathcal{C}\right), \quad (26)$$

$$\boldsymbol{Q}_n^{(i)} = \left(Q_n^{(i)(c)},\ \forall c \in \mathcal{C}\right), \quad (27)$$

to be the persistent and listening probabilities of NIC i, respectively. Consider the following *local* and *myopic* optimization problem in NIC $i \in \mathcal{I}_n$, when the single-channel reception model is being used:

$$\underset{\langle \boldsymbol{p}_n^{(i)}, \boldsymbol{Q}_n^{(i)}\rangle \in \Phi_n^{(i)}}{\text{maximize}} \;\; \sum_{m\in\mathcal{N}} \sum_{s\in\mathcal{N}_m^{\text{out}}} u(r_{ms}(\boldsymbol{p}, \boldsymbol{Q})). \quad \text{(Local-NUM-S)}$$

Here, the average data rates are as in (10) and we have

$$\begin{aligned}\Phi_n^{(i)} = \{ & \langle \boldsymbol{p}_n^{(i)}, \boldsymbol{Q}_n^{(i)} \rangle : p_{nm}^{(i)(c)}, P_n^{(i)(c)}, Q_n^{(i)(c)} \in [0,1], \\ & P_n^{(i)(c)} = \textstyle\sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)}, \\ & \textstyle\sum_{d \in \mathcal{C}} \left(P_n^{(i)(d)} + Q_n^{(i)(d)} \right) = 1,\ \forall m \in \mathcal{N}_n^{\text{out}},\ c \in \mathcal{C} \}.\end{aligned} \tag{28}$$

Similarly, consider the following local and myopic problem when the multi-channel reception model is used:

$$\underset{\boldsymbol{p}_n^{(i)} \in \Psi_n^{(i)}}{\text{maximize}} \ \textstyle\sum_{m \in \mathcal{N}} \sum_{s \in \mathcal{N}_m^{\text{out}}} u(r_{ms}(\boldsymbol{p})). \quad \text{(Local-NUM-M)}$$

Here, the average data rates are as in (14) and we have:

$$\begin{aligned}\Psi_n^{(i)} = \{ & \boldsymbol{p}^{(i)} : p_{nm}^{(i)(c)}, P_n^{(i)(c)} \in [0,1],\ \textstyle\sum_{d \in \mathcal{C}} P_n^{(i)(d)} \le 1, \\ & P_n^{(i)(c)} = \textstyle\sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)},\ \forall m \in \mathcal{N}_n^{\text{out}},\ c \in \mathcal{C} \}.\end{aligned} \tag{29}$$

The objective functions in (Local-NUM-S) and (Local-NUM-M) are the same as those in (NUM-S) and (NUM-M), respectively. However, the variables in (Local-NUM-S) and (Local-NUM-M) are local to NIC i in node n.

For the case with single-channel reception, we define

$$\boldsymbol{p}_n^{(-i)} = \left(\boldsymbol{p}_n^{(j)}, \forall j \in \mathcal{I}_n \backslash \{i\},\ \boldsymbol{p}_m^{(k)}, \forall k \in \mathcal{I}_m, m \in \mathcal{N} \backslash \{n\} \right), \tag{30}$$

$$\boldsymbol{Q}_n^{(-i)} = \left(\boldsymbol{Q}_n^{(j)}, \forall j \in \mathcal{I}_n \backslash \{i\},\ \boldsymbol{Q}_m^{(k)}, \forall k \in \mathcal{I}_m, m \in \mathcal{N} \backslash \{n\} \right). \tag{31}$$

The above are the persistent and listening probabilities corresponding to all NICs in the network *other than* NIC i in node n. By solving problem (Local-NUM-S), we can select $\boldsymbol{p}_n^{(i)}$ and $\boldsymbol{Q}_n^{(i)}$ such that the total utility is maximized *assuming* that $\boldsymbol{p}_n^{(-i)}$ and $\boldsymbol{Q}_n^{(-i)}$ are *fixed*.

Theorem 2: Problems (Local-NUM-S) and (Local-NUM-M) are *convex* optimization problems.

The proof of Theorem 2 is given in Appendix B. From Theorem 2, we can use various *convex programming* techniques (cf. [24]) to solve problems (Local-NUM-S) and (Local-NUM-M). The optimal solutions of problems (Local-NUM-S) and (Local-NUM-M) can also be obtained in *closed-form* for some *simple* scenarios:

Theorem 3: For a two-node single-interface multi-channel network (i.e., when $N = 2$, $C > 1$, and $I_n = 1$ for all $n \in \mathcal{N}$) with α-fair utilities, let $\boldsymbol{M}(\boldsymbol{p}_n^{(-i)})$ denote a $C \times C$ matrix that the entry in row c and column d is

$$\boldsymbol{M}^{(cd)}\left(\boldsymbol{p}_n^{(-i)}\right) = 1 + \frac{\gamma_{nm}^{(d)} / \gamma_{nm}^{(c)}}{\sqrt[\alpha]{\left(\frac{1 - \sum_{e \in \mathcal{C}} p_{mn}^{(j)(e)}}{\sum_{e \in \mathcal{C}} \gamma_{mn}^{(e)} p_{mn}^{(j)(e)}} \gamma_{nm}^{(c)} \right)^{\alpha - 1}}}. \tag{32}$$

Here j denotes the (only) NIC of node m. If $\boldsymbol{M}(\boldsymbol{p}_n^{(-i)})$ is an invertible matrix, then the optimal solution of problem (Local-NUM-M) can be obtained as

$$\boldsymbol{p}_n^{(i)^*} = \boldsymbol{M}(\boldsymbol{p}_n^{(-i)})^{-1}\, \mathbf{1}, \qquad \forall n \in \mathcal{N},\ i \in \mathcal{I}_n, \tag{33}$$

where $\mathbf{1}$ is a $C \times 1$ unit vector.

The proof of Theorem 3 is given in Appendix C. We note that in certain cases, problem (Local-NUM-M) may have non-unique solutions. For example, if $\alpha = 1$ and peak rates are the same for all channels, any vector $\boldsymbol{p}_n^{(i)}$

Algorithm 1 - DMMRA-S: Executed by each node $n \in \mathcal{N}$ when the NICs implement *single*-channel reception.

```
Allocate memory for p and Q.
Randomly choose p and Q such that ⟨p, Q⟩ ∈ Φ.
repeat
    for each NIC i ∈ I_n do
        Either transmit to node m ∈ N_n^out on channel c ∈ C
        or listen to channel c ∈ C with probabilities p_nm^(i)(c)
        and Q_n^(i)(c), respectively.
    end for
    if t ∈ T_n^(i) for some i ∈ I_n then
        Solve problem (Local-NUM-S) using IPM [24].
        Update p_n^(i) and Q_n^(i) according to the solution.
        Inform p_n^(i) and Q_n^(i) to nodes in 2R^max distance.
    end if
    if a message is received from another node then
        Update p and Q accordingly.
    end if
until node n leaves the network.
```

such that $\sum_{c \in \mathcal{C}} p_{nm}^{(i)(c)} = \frac{1}{2}$ is optimal. When the network has a large number of nodes we can solve the convex problems (Local-NUM-S) and (Local-NUM-M) using the *interior-point method* (IPM) [24] via *local* iterations.

For each node $n \in \mathcal{N}$ and any NIC $i \in \mathcal{I}_n$, we rewrite the objective function of problem (Local-NUM-S) to be

$$\textstyle\sum_{s \in \mathcal{N}, \delta_{ns} = 1} \sum_{m \in \mathcal{N}_s^{\text{in}}} u(r_{ms}(\boldsymbol{p}, \boldsymbol{Q})) + \Gamma(\boldsymbol{p}_n^{(-i)}, \boldsymbol{Q}_n^{(-i)}), \tag{34}$$

where Γ depends only on $\boldsymbol{p}_n^{(-i)}$ and $\boldsymbol{Q}_n^{(-i)}$, but not $\boldsymbol{p}_n^{(i)}$ and $\boldsymbol{Q}^{(i)}$. Thus, the value of Γ needs *not* be known to solve problem (Local-NUM-S). On the other hand, each node n can determine $\sum_{s \in \mathcal{N}, \delta_{sn} = 1} \sum_{m \in \mathcal{N}_s^{\text{in}}} u(r_{ms}(\boldsymbol{p}, \boldsymbol{Q}))$ if it is given the persistent and listening probabilities of all nodes within its $2R^{\max}$ distance, where $R^{\max} = \max_{m \in \mathcal{N}} R_m$ and R_m is the interference range of node m. To see this, note that any node $s \in \mathcal{N}$ can calculate the data rate r_{ms} for any of its incoming links (m, s), where $m \in \mathcal{N}_n^{\text{in}}$, if it knows the persistent and listening probabilities of all nodes which are located within its $R^{\max}$ distance. Moreover, any node $s \in \mathcal{N}$ such that $\delta_{ns} = 1$ is located within the $R^{\max}$ distance of node n. Therefore, problem (Local-NUM-S) can be solved *locally* if all nodes within $2R^{\max}$ distance of node n can inform their persistent and listening probabilities to node n. A similar statement is true for problem (Local-NUM-M).

4.2 Algorithms

Our proposed DMMRA algorithms, when the *single*-channel reception and *multi*-channel reception models are being used, are shown in Algorithm 1 (DMMRA-S) and Algorithm 2 (DMMRA-M), respectively. For each node $n \in \mathcal{N}$ and any of its NICs $i \in \mathcal{I}_n$, let $\mathcal{T}_n^{(i)}$ be an unbounded set of time slots at which node n updates NIC i's persistent and listening probabilities. We assume that the updates are *asynchronous* across the

Algorithm 2 - DMMRA-M: Executed by each node $n \in \mathcal{N}$ when the NICs implement *multi*-channel reception.

```
Allocate memory for p.
Randomly choose p such that p ∈ Ψ.
repeat
    for each NIC i ∈ I_n do
        Transmit to node m ∈ N_n^out on channel c ∈ C with
          probability p_nm^(i)(c).
    end for
    if t ∈ T_n^(i) for some i ∈ I_n then
        Solve problem (Local-NUM-M) using IPM.
        Update p_n^(i) according to the solution.
        Inform p_n^(i) to nodes within 2R^max distance.
    end if
    if a message is received from another node then
        Update p accordingly.
    end if
until node n leaves the network.
```

network. That is, $\mathcal{T}_n^{(i)} \cap \mathcal{T}_n^{(j)} = \varnothing$ for all $j \in \mathcal{I}_n \backslash \{i\}$ and $\mathcal{T}_n^{(i)} \cap \mathcal{T}_m^{(k)} = \varnothing$ for all $m \in \mathcal{N} \backslash \{n\}$ and any $k \in \mathcal{I}_m$. In line 2 of Algorithm 1, node n randomly initiates all of its persistent and listening probabilities. Lines 4 to 14 are then executed repeatedly at every time slot until node n leaves the network or switches off. In lines 4 to 6, node n either transmits or receives packets according to its persistent and listening probabilities. On the other hand, lines 8 to 10 are executed only if there exists an NIC $i \in \mathcal{I}_n$ such that $t \in \mathcal{T}_n^{(i)}$. That is, the *current* time slot is a time slot at which the persistent and listening probabilities of NIC i need to be updated by solving problem (Local-NUM-S). Recall from Theorem 2 that problem (Local-NUM-S) is convex. Thus, it can easily be solved using IPM. In line 10, node n announces its updated persistent and listening probabilities to all nodes within its $2R^{\max}$ distance, where $R^{\max}$ is defined in Section 4.1. This can be done using *limited-scope message flooding* [26, pp. 408]. Upon reception of the new probability values from other nodes, in line 13, node n updates its local memory accordingly. Finally, for distributed implementation of Algorithm 1, we need to slightly modify the feasible sets in (28) such that for each $n \in \mathcal{N}$, any NIC $i \in \mathcal{I}_n$, and each $m \in \mathcal{N}_n^{\text{out}}$, we have: $p_{nm}^{(i)(c)}, Q_n^{(i)(c)} \in [\epsilon, 1-\epsilon]$, where $0 < \epsilon \ll \frac{1}{2}$ is a *small* design parameter (e.g., $\epsilon = 10^{-6}$). This requires all NICs to listen to all channels and transmit over all channels with small *non-zero* probabilities. Similar assumptions are made to avoid node starvation in the single-channel random access algorithms in [20] and [21].

Algorithm 2 works similarly. The persistent probabilities are adjusted by solving the convex optimization problem in (Local-NUM-M). Note that, in general, Algorithm 2 is less complex compared to Algorithm 1 as problem (Local-NUM-M) has fewer variables and constraints compared to (Local-NUM-S). Both Algorithms 1 and 2 are distributed and allow each node to adjust its operation based on a few simple local tasks and some limited-scope message exchange with other nodes.

4.3 Optimality, Convergence, and Complexity

In this section, we investigate the optimality, convergence, and complexity of DMMRA algorithms. For each node $n \in \mathcal{N}$ and any NIC $i \in \mathcal{I}_n$, we define

$$\begin{aligned} &\boldsymbol{f}_{n,SCR}^{(i)}\left(\boldsymbol{p}_n^{(-i)}, \boldsymbol{Q}_n^{(-i)}\right) \\ &\quad = \underset{\langle \boldsymbol{p}_n^{(i)}, \boldsymbol{Q}_n^{(i)} \rangle \in \Phi_n^{(i)}}{\arg\max} \sum_{m \in \mathcal{N}} \sum_{s \in \mathcal{N}_m^{\text{out}}} u(r_{ms}(\boldsymbol{p}, \boldsymbol{Q})), \end{aligned}$$

and

$$\boldsymbol{f}_{n,MCR}^{(i)}\left(\boldsymbol{p}_n^{(-i)}\right) = \underset{\boldsymbol{p}_n^{(i)} \in \Psi_n^{(i)}}{\arg\max} \sum_{m \in \mathcal{N}} \sum_{s \in \mathcal{N}_m^{\text{out}}} u(r_{ms}(\boldsymbol{p})).$$

We also define $\boldsymbol{f}_{SCR} = (\boldsymbol{f}_{n,SCR}^{(i)},\ \forall\, i \in \mathcal{I}_n, n \in \mathcal{N})$ and $\boldsymbol{f}_{MCR} = (\boldsymbol{f}_{n,MCR}^{(i)},\ \forall\, n \in \mathcal{N}, i \in \mathcal{I}_n)$ as the *mapping functions* corresponding to Algorithms 1 and 2, respectively. Let $\mathcal{F}_{SCR}$ and $\mathcal{F}_{MCR}$ denote the set of *fixed points* (cf. [27, pp. 181]) of mappings $\boldsymbol{f}_{SCR}$ and $\boldsymbol{f}_{MCR}$, respectively. That is, if $\langle \boldsymbol{p}^*, \boldsymbol{Q}^* \rangle \in \mathcal{F}_{SCR}$, then $\boldsymbol{f}_{SCR}(\boldsymbol{p}, \boldsymbol{Q}) = \langle \boldsymbol{p}^*, \boldsymbol{Q}^* \rangle$. Similarly, if $\boldsymbol{p}^* \in \mathcal{F}_{MCR}$, then $\boldsymbol{f}_{MCR}(\boldsymbol{p}) = \boldsymbol{p}^*$. Also let $\mathcal{S}_{SCR}$ and $\mathcal{S}_{MCR}$ denote the set of all *stationary points* (cf. [28, pp. 194]) of problems (NUM-S) and (NUM-M), respectively. All local/global optimal solutions of problems (NUM-S) and (NUM-M) belong to sets $\mathcal{S}_{SCR}$ and $\mathcal{S}_{MCR}$, respectively. We can show the following:

Theorem 4: $\mathcal{F}_{SCR} = \mathcal{S}_{SCR}$ and $\mathcal{F}_{MCR} = \mathcal{S}_{MCR}$.

The proof of Theorem 4 is given in Appendix D. From Theorem 4, any fixed point of Algorithm 1 is indeed a stationary point of optimization problem (NUM-S) and vice versa. The same statement is true for Algorithm 2 and optimization problem (NUM-M).

Theorem 5: (a) For any choice of system parameters (i.e., the topology parameters, number of channels, number of NICs, links' peak data rates, and choice of utility functions), the fixed point of Algorithm 1 (Algorithm 2) always exists. (b) If the number of channels $C = 1$ and $\alpha \geq 1$, then Algorithm 1 (Algorithm 2) has a unique fixed point. (c) If $C > 1$, then Algorithm 1 (Algorithm 2) may have more than one (i.e., non-unique) fixed points.

The proof of Theorem 5 is given in Appendix E. From Theorems 4 and 5(b), if the number of channels $C = 1$, then the unique fixed point of Algorithm 1 (Algorithm 2) is the global optimal solution of problem (NUM-S) (problem (NUM-M)). On the other hand, from Theorem 4 and Theorem 5(c), if $C > 1$, then any fixed point of Algorithm 1 (Algorithm 2) is at least a local maximum of problem (NUM-S) (problem (NUM-M)).

Next, we discuss convergence. Let $U_{SCR}(t)$ and $U_{MCR}(t)$ denote the aggregate network utilities at time slot t, while running Algorithms 1 and 2, respectively.

Theorem 6: For any choice of system parameters, (a) At each time slot t, the instantaneous aggregate network utilities $U_{SCR}(t)$ and $U_{MCR}(t)$ are upper bounded:

$$U_{SCR}(t), U_{MCR}(t) \leq L\, u(C\, I^{\max} \gamma^{\max}),\ \forall (n,m) \in \mathcal{L}, \quad (35)$$

where $I^{\max} = \max_{n\in\mathcal{N}} I_n$ and $\gamma^{\max} = \max_{(n,m)\in\mathcal{L},c\in\mathcal{C}} \gamma_{nm}^{(c)}$.
(b) The instantaneous network utilities $U_{SCR}(t)$ and $U_{MCR}(t)$ are *non-decreasing*. That is, for any $T \geq 2$,

$$U_{SCR}(1) \leq \quad U_{SCR}(2) \quad \leq \cdots \leq U_{SCR}(T), \quad (36)$$
$$U_{MCR}(1) \leq \quad U_{MCR}(2) \quad \leq \cdots \leq U_{MCR}(T). \quad (37)$$

(c) Starting from any initial point $\langle \boldsymbol{p}, \boldsymbol{Q}\rangle$ and $\boldsymbol{p}$, Algorithms 1 and 2 converge to one of their fixed points, respectively, i.e., there exist U^*_{SCR} and U^*_{MCR} such that

$$U^*_{SCR} = \lim_{t\to\infty} U_{SCR}(t), \quad U^*_{MCR} = \lim_{t\to\infty} U_{MCR}(t). \quad (38)$$

The proof of Theorem 6 is given in Appendix F. From Theorem 4, U^*_{SCR} and U^*_{MCR} are the local maxima for problems (NUM-S) and (NUM-M), respectively. In many cases, the achieved fixed points are not only locally optimal, but also *globally* optimal. For example, we can verify that for the sample topologies in Section 3.2, we have: $U^*_{SCR} = U^\star_{SCR}$ and $U^*_{MCR} = U^\star_{MCR}$. We also note that the assumption of turn-taking is not a requirement for our algorithms and only helps us to construct some *sufficient* conditions to prove the convergence.

For the complexity of Algorithm 1, we notice that at each update interval (i.e., at each time slot $t \in \mathcal{T}_n^{(i)}$ for any node $n \in \mathcal{N}$ and each NIC $i \in \mathcal{I}_n$), we need to solve problem (Local-NUM-S) using IPM. It is known that IPM has *polynomial* (P) complexity [24]. Therefore, the complexity it takes to execute line 8 in Algorithm 1 is *upper-bounded* by a polynomial function of the *problem size*. Since problem (Local-NUM-S) is a *local* problem for each NIC with only a few variables and constraints, the problem size is small. Thus, Algorithm 1 is a tractable algorithm and can be implemented easily in practical random access systems. In particular, its complexity is significantly less compared to combinatorial interface and channel assignment algorithms (e.g., [1]–[3], [6], [7], [11]). Recall from Section 1 that combinatorial schemes have *non-polynomial* (NP) complexity. Similar statements are true for the complexity of Algorithm 2.

5 Performance Evaluation

In this section, we evaluate the performance of the DMMRA algorithms. We study the convergence, robustness, and optimality properties, evaluate the performance gain of assigning partially overlapped channels, and measure the signalling overhead. We also compare DMMRA with *utility-optimal combinatorial interface assignment and channel allocation* (UO-CIACA) [1] and *multi-channel medium access control* (MMAC) [8] algorithms.

In our simulation, we consider ten random topologies. Unless otherwise is stated, each topology includes $N = 10$ nodes, randomly located in a 500 m $\times$ 500 m square field. Communication and interference ranges are 150 m and 250 m, respectively. The peak data rates (i.e., $\gamma_{nm}^{(c)}$ for all $(n,m) \in \mathcal{L}$ and $c \in \mathcal{C}$) are selected randomly between 6 and 54 Mbps, as in the IEEE 802.11a standard. Except in Section 5.3, the utility functions are logarithmic. Each time slot is 1 sec. The simulation time is 1000 time slots.

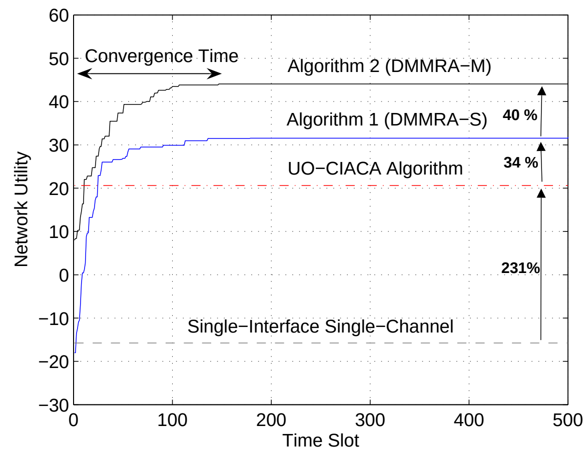


Fig. 3. Trend of network utility versus time for Algorithms 1 and 2 for the first simulated topology.

5.1 Convergence

In this experiment, we set the number of channels $C = 6$ and the number of NICs $I_n = 2$ for all $n \in \mathcal{N}$. The trends of the network utilities for the first simulated topology (i.e., topology number 1) when the DMMRA algorithms are being used are shown in Fig. 3. We can see that both Algorithms 1 and 2 converge to their fixed-points very fast, i.e., within 152 and 146 time slots, respectively. We can also observe that the utility values are *non-decreasing* and *bounded*, which confirm the results in Theorem 6(b). At the steady state, Algorithm 1 results in 34% higher utility, compared to UO-CIACA. Using Algorithm 2, the utility is further increased by 40%. Thus, equations (36) and (37) hold as *strict* inequalities in this case. Similar results are observed for other topologies.

5.2 Comparison with a Combinatorial Scheme

Next, we compare DMMRA with UO-CIACA [1] in terms of both utility and throughput. Simulation setting is the same as in Section 5.1. We notice that the UO-CIACA algorithm is designed to solve problem (NUM-C). In other words, DMMRA and UO-CIACA algorithms have the *same* design objective: *maximizing the network aggregate utility*. However, the formulation for the UO-CIACA scheme is a mixed-integer (i.e., combinatorial) problem, while the one for the DMMRA scheme is continuous. Therefore, implementing the UO-CIACA algorithm is NP-hard and is more complex than the DMMRA algorithm. We also notice that unlike the DMMRA algorithm which is distributed and requires information only from two-hop neighbors in each node, the UO-CIACA algorithm is centralized and is designed based on the availability of all global information in the network.

Results are shown in Fig. 4, where each point is the average of the measurements for all ten simulated topologies. We can see that both utility and throughput increase as there are more channels available. Algorithm 1 results in 36% and 23% higher utility and throughput, compared

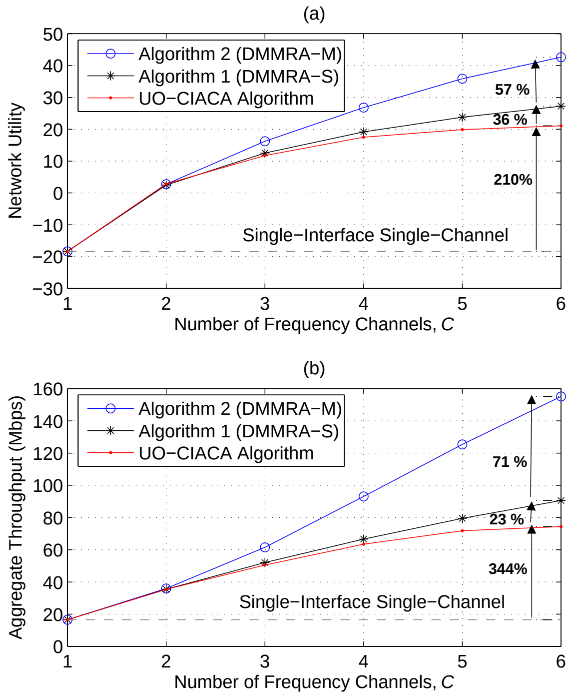


Fig. 4. Comparison between Algorithms 1 and 2 with (UO-CIACA) algorithm [1]. The number of available channels varies from 1 to 6. (a) Network utility, (b) Throughput.

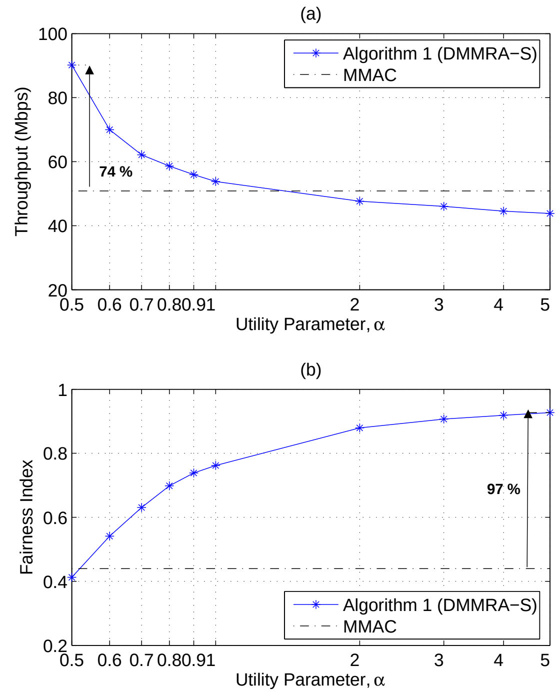


Fig. 5. Comparison between Algorithm 1 and MMAC [8]. Each node has one NIC and there are 6 channels available. For Algorithm 1, the parameter α is varied from 0.5 to 5. (a) Throughput, (b) Fairness index.

to UO-CIACA, respectively (see Theorem 1(b)). Using Algorithm 2 leads to further 57% and 71% increase in utility and throughout, respectively (see Theorem 1(a)).

5.3 Impact of Utility Parameter α

Recall from Section 3.1 that by changing the utility parameter α, various design objectives can be modeled. In particular, α can act as a *knob* in Algorithms 1 and 2 to control the trade-off between efficiency and fairness. In this section, we compare DMMRA-S algorithm with MMAC [8], which is the multi-channel extension of the IEEE 802.11 distributed coordination function. For each NIC, MMAC assigns the channel which has the *least scheduled transmission* within the neighborhood. Since the MMAC algorithm is a *heuristic*, we cannot match it to an exact optimization problem. However, as pointed out in [8], MMAC aims to increase network throughput. We also notice that the MMAC algorithm is fully distributed and does *not* require message passing (other than RTS/CTS/ACK); thus, it is less complex than the DMMRA algorithm. MMAC is designed for *single*-interface *multi*-channel networks. It also assumes that each NIC can listen to only one channel at a time. Thus, MMAC is suitable for comparison with Algorithm 1, where $I_n = 1$ for all $n \in \mathcal{N}$. We set the number of channels $C = 6$. Running both Algorithm 1 and MMAC for all ten topologies, the network throughput and fairness index, when α varies between 0.5 and 5, are shown in Fig. 5 (a) and (b), respectively. The fairness index is calculated among the data rates of all links as in [29]. We can see that, by increasing α, we can make the system more fair, but less efficient (and vice versa). If $\alpha = 0.5$, then Algorithm 1 results in 74% higher throughput, compared to MMAC. If $\alpha = 5$, Algorithm 1 results in 97% higher fairness index. Thus, Algorithm 1 can be tuned for better efficiency or better fairness.

5.4 Signalling Overhead

Both Algorithms 1 and 2 require message exchange among the neighboring nodes. In this section, we measure the signalling overhead for each algorithm and compare it with the network throughput. We assume that each probability value occupies two bytes. Thus, for each node $n \in \mathcal{N}$, the message size is $2C(L_n^{\text{out}} + 2)$ bytes and $2C(L_n^{\text{out}} + 1)$ bytes for Algorithms 1 and 2, respectively. For each NIC $i \in \mathcal{I}_n$, we assume the use of *limited-scope message flooding* [26, pp. 408] to distribute $\langle \boldsymbol{p}_n^{(i)}, \boldsymbol{Q}_n^{(i)} \rangle$ (for single-channel reception scenario) and $\boldsymbol{p}_n^{(i)}$ (for multi-channel reception scenario) to all nodes within $2R^{\max}$ distance. In limited-scope flooding, each control message has a *hop-count* field in its header. Every time that a node forwards a message, it decrements the hop-count by one. Thus, the control message is no longer forwarded as soon as the hop-count reaches zero. In this regard, if a node sets the hop-count to 2, then only the nodes within two hops will receive the control message.

The signalling overhead (in Kbps) and throughput (in Mbps), when the number of nodes N varies from 10 to 50, are shown in Fig. 6 (a) and (b), respectively. The

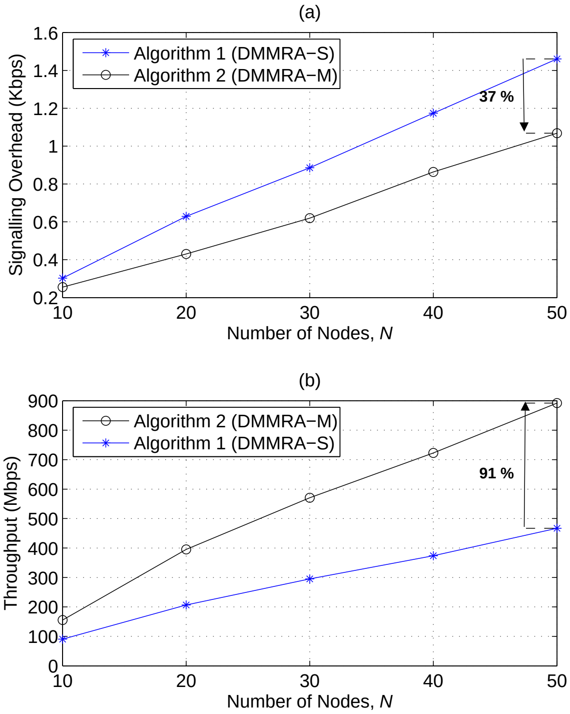


Fig. 6. Signalling overhead and aggregate network throughput for Algorithms 1 and 2 when the number of nodes N varies from 10 to 50. Each node is equipped with 2 NICs and there are 6 orthogonal channels available.

signalling overhead increases as the number of nodes increases. However, it is always negligible compared to the throughput (i.e., less than 0.002%). We notice that Algorithm 2 always incurs lower overhead as it has smaller messages and converges faster. When $N = 50$, Algorithm 2 results in 37% lower signalling overhead and 91% higher throughput, compared to Algorithm 1.

5.5 Partially Overlapped Channel Assignment

Given the data rate models in (19) and (21), Algorithms 1 and 2 can be used to assign not only the non-overlapped channels, but also the partially overlapped channels. This is particularly important when the number of orthogonal channels is limited; e.g., as in IEEE 802.11b standard, where only 3 out of 11 channels are non-overlapped. The throughput, when Algorithm 2 is used, is shown in Fig. 7. The results for Algorithm 1 are similar. The channel filters are assumed to be *raised cosine* with *roll-off factors* equal to 1 (cf. [19]). The dashed lines correspond to the measured throughput when either single channel (i.e., channel 1), two non-overlapped channels (i.e., channels 1 and 6), or three non-overlapped channels (i.e., channels 1, 6, and 11) are used. We see that, using Algorithm 2, assigning the partially overlapped channels 1, . . ., 6, instead of assigning only non-overlapped channels 1 and 6, results in 11% higher throughput. By assigning all partially overlapped channels 1, . . ., 11, instead of assigning only the non-overlapped channels 1, 6, and 11, the throughput is increased by 13%. Here,

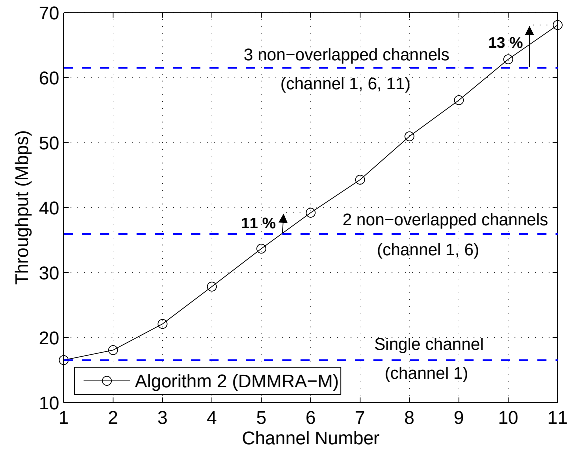


Fig. 7. Performance improvement when both orthogonal and partially overlapped channels are being used. The number of available channels is varied from 1 to 11.

the improvements are achieved with no extra resources. Thus, the available spectrum is utilized more efficiently.

5.6 Impact of Delayed and Outdated Information

In some practical scenarios, the nodes may receive *outdated* information about the persistent and listening probabilities of other nodes. This can be due to *communication delay* (e.g., queueing or propagation delay) or *message loss*. The latter can occur due to *channel imperfections* (e.g., fading) or *packet collision*. In this section, we study the effect of outdated information exchange on the performance of DMMRA algorithms. In particular, we consider the case where the communication medium imposes random delay on the exchanged messages. The trend of the network utility for the first simulated topology, when Algorithm 1 is being used and the messages experience delay up to 10 time slots (i.e., 10 seconds), is shown in Fig. 8. We can see that Algorithm 1 still converges to its fixed-point, even though the information used by the nodes is outdated. However, communication delay may cause *utility fluctuation* (compare Fig. 8 and Fig. 3). That is, at some time instances, since the node, which executes DMMRA, may not have accurate information about the persistent and listing probabilities of its neighboring nodes, the value of the utility may be decreased. Nevertheless, Algorithm 1 still (asynchronously) converges to its fixed point, but with lower convergence speed. Similar results are obtained for Algorithm 2.

5.7 Optimality

From Theorem 4, every fixed point of DMMRA algorithms is at least a *local* maximum for NUM problems. However, the fixed-points may not always be *globally* maximum. In this section, we investigate the optimality of Algorithms 1 and 2. Results are shown in Fig. 9. In this figure, the *average* utility is compared with the *optimal* utility for each topology. To obtain the results,

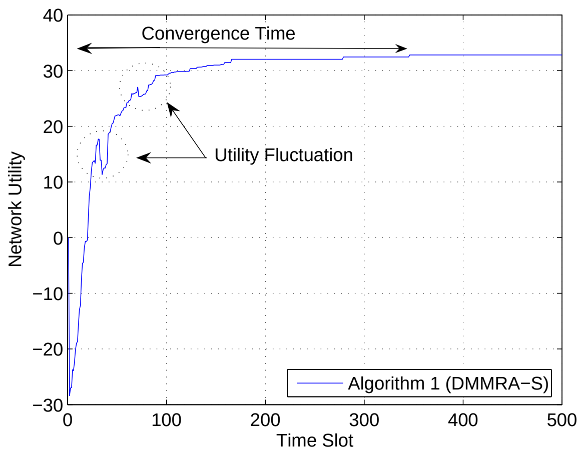


Fig. 8. Trend of network utility versus time for Algorithm 1 for the first simulated topology in the presence of delay.

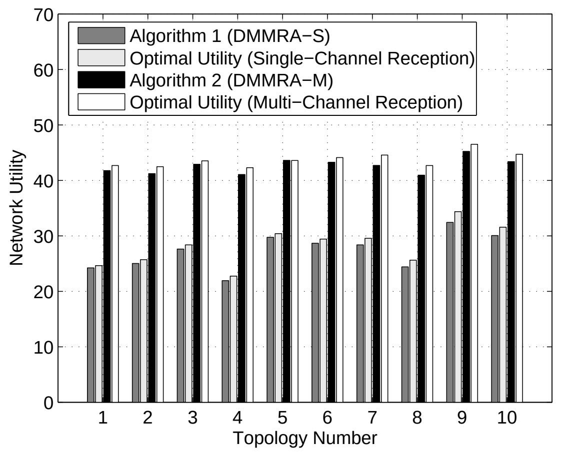


Fig. 9. Comparison between achieved aggregate network utility and its optimal value for each proposed algorithm and each simulated topology.

we calculate the utility at *all* fixed-points. This is done by partitioning the search space and running DMMRA with the *initial points* varying among the partitions. Specifically, for each topology, we obtain both *mean utility value* and *maximum utility value* among all fixed-points. The former indicates the *average* performance of our proposed DMMRA algorithms, while the latter indicates the *optimal* performance. From the results in Fig. 9, DMMRA achieves *near-optimal* solutions for all ten simulated topologies. On average, Algorithms 1 and 2 result in 96.5% and 97.4% optimality, respectively. In fact, although the fixed-points are not always globally optimal, they lead to near-optimal performance.

6 Conclusion

In this paper, we formulated a novel multi-interface multi-channel random access model under the framework of network utility maximization (NUM). First, we obtained the link data rate models as functions of the persistent and listening probabilities of each wireless node. We addressed both single-channel reception and multi-channel reception as well as both non-overlapped and partially overlapped channel assignment scenarios. Given the data rate models, the NUM problems are formulated accordingly. We then proposed two versions of a fast, distributed, and easy to implement multi-interface multi-channel random access algorithm, called DMMRA, to solve the NUM problems for each scenario. DMMRA requires each node to iteratively solve a *local*, *myopic*, and *convex* optimization problem, while it exchanges some control messages with its two-hop neighbors. We proved the convergence and optimality properties of the algorithms. Simulation results show that our algorithms have better performance compared to UO-CIACA and MMAC algorithms.

Appendix A
Proof of Theorem 1

Part (a): For each wireless node $m \in \mathcal{N}$, any of its NICs $j \in \mathcal{I}_m$, and each frequency channel $c \in \mathcal{C}$, we have:

$$1 - P_m^{(j)(c)} - Q_m^{(j)(c)} \overset{\text{by (1)}}{=} \sum_{d \in \mathcal{C}\setminus\{c\}} P_m^{(j)(d)} + \sum_{d \in \mathcal{C}\setminus\{c\}} Q_m^{(j)(d)} \geq \sum_{d \in \mathcal{C}\setminus\{c\}} P_m^{(j)(d)}.$$

Replacing the above inequality in (4) and (11), we have $\mathbb{P}(\hat{A}_m^{(c)}) \geq \mathbb{P}(\breve{A}_m^{(-c)})$, which implies that $(1 - \mathbb{P}(\tilde{A}_m^{(c)} \cup \hat{A}_m^{(c)})) \leq (1 - \mathbb{P}(\tilde{A}_m^{(c)} \cup \breve{A}_m^{(-c)}))$. Hence r_{nm} in (14) is always greater than or equal to the one in (10). Since the utility function $u(r_{nm})$ is an increasing function of r_{nm}, the inequality (24) is resulted when we sum up the utilities of all links.

Part (b): Let Λ denote the feasible set of problem (NUM-C). It is clear that, $\Lambda \subseteq \Phi$. Thus, any $\langle \bar{\boldsymbol{p}}, \bar{\boldsymbol{Q}} \rangle \in \Lambda$ is also a feasible solution of problem (NUM-S). Hence, *no* $\langle \bar{\boldsymbol{p}}, \bar{\boldsymbol{Q}} \rangle \in \Lambda$ can lead to an aggregate network utility which is greater than the *optimal* utility $U_{SCR}^{\star}$ over the set Φ. Therefore, the inequality in (25) always holds. ■

Appendix B
Proof of Theorem 2

For each node $n \in \mathcal{N}$ and any NIC $i \in \mathcal{I}_n$, the objective function of problem (Local-NUM-S) can be written as:

$$\sum_{m \in \mathcal{N}_n^{\text{out}}} u\left(\sum_{c \in \mathcal{C}} \left(\xi_{n,m}^{(i)(c)} p_{nm}^{(i)(c)} + \zeta_{n,m}^{(i)(c)} \left(1 - P_n^{(i)(c)}\right)\right)\right)$$
$$+ \sum_{m \in \mathcal{N}_n^{\text{in}}} u\left(\sum_{c \in \mathcal{C}} \left(\left(\theta_{n,m}^{(i)(c)} - \vartheta_{n,m}^{(i)(c)}\right)\left(1 - P_n^{(i)(c)}\right) + \vartheta_{n,m}^{(i)(c)} Q_n^{(i)(c)}\right)\right)$$
$$+ \sum_{m \in \mathcal{N}\setminus\{n\}} \sum_{s \in \mathcal{N}_m^{\text{out}}\setminus\{n\}} u\left(\sum_{c \in \mathcal{C}} \beta_{n,ms}^{(i)(c)} \left(1 - \delta_{ns} P_n^{(i)(c)}\right)\right),$$

where for each node $m \in \mathcal{N}_n^{\text{out}}$ and any channel $c \in \mathcal{C}$,

$$\xi_{n,m}^{(i)(c)} = \gamma_{nm}^{(c)} \left(\prod_{j \in \mathcal{I}_n \setminus\{i\}} \left(1 - P_n^{(j)(c)}\right)\right) \left(\prod_{s \in \mathcal{N}\setminus\{n,m\}} \prod_{k \in \mathcal{I}_s} \left(1 - \delta_{sm} P_s^{(k)(c)}\right)\right) \left(\prod_{l \in \mathcal{I}_m} \left(1 - P_m^{(l)(c)}\right) - \prod_{l \in \mathcal{I}_m} \left(1 - P_m^{(l)(c)} - Q_m^{(l)(c)}\right)\right),$$

$$\zeta_{n,m}^{(i)(c)} = \sum_{j \in \mathcal{I}_n \setminus\{i\}} \gamma_{nm}^{(c)} p_{nm}^{(j)(c)} \left(\prod_{k \in \mathcal{I}_n \setminus\{i,j\}} \left(1 - P_n^{(k)(c)}\right)\right) \left(\prod_{s \in \mathcal{N}\setminus\{n,m\}} \prod_{k \in \mathcal{I}_s} \left(1 - \delta_{sm} P_s^{(k)(c)}\right)\right) \left(\prod_{j \in \mathcal{I}_m} \left(1 - P_m^{(j)(c)}\right) - \prod_{j \in \mathcal{I}_m} \left(1 - P_m^{(j)(c)} - Q_m^{(j)(c)}\right)\right).$$

Similarly, we can obtain $\theta^{(i)(c)}_{n,m}$ and $\vartheta^{(i)(c)}_{n,m}$ for any node $m\in\mathcal{N}^{\text{in}}_n$ and any channel $c\in\mathcal{C}$; and also $\beta^{(i)(c)}_{n,ms}$ for any link $(m,n)\in\mathcal{L}\backslash(\mathcal{L}^{\text{in}}_n\cup\mathcal{L}^{\text{out}}_n)$ and each $c\in\mathcal{C}$. Notice that $\xi^{(i)(c)}_{n,m}$, $\zeta^{(i)(c)}_{n,m}$, $\theta^{(i)(c)}_{n,m}$, $\vartheta^{(i)(c)}_{n,m}$, and $\beta^{(i)(c)}_{n,ms}$ only depend on $\boldsymbol{p}^{(-i)}_n$ and $\boldsymbol{Q}^{(-i)}_n$. In fact, they can be treated as *constants* as far as problem (Local-NUM-S) is concerned. Since the utility functions are concave and $\Phi^{(i)}_n$ is a convex set, problem (Local-NUM-S) is a convex optimization problem. The proof for (Local-NUM-M) is similar. ■

Appendix C
Proof of Theorem 3

Let $\mathcal{N}=\{n,m\}$, $\mathcal{L}_n=\{i\}$, and $\mathcal{L}_m=\{j\}$. For wireless link $(n,m)\in\mathcal{L}$, we have:

$$r_{nm}(\boldsymbol{p})=\textstyle\sum_{c\in\mathcal{C}}\gamma^{(c)}_{nm}p^{(i)(c)}_{nm}\left(1-\sum_{d\in\mathcal{C}}p^{(j)(d)}_{mn}\right), \tag{39}$$

The data rate $r_{mn}(\boldsymbol{p})$ can also be obtained similarly. We can re-write optimization problem (Local-NUM-M) as:

$$\begin{aligned}\underset{\boldsymbol{p}^{(i)}_n\in\Psi^{(i)}_n}{\text{maximize}}\ & u\Big(\Big(1-\textstyle\sum_{e\in\mathcal{C}}p^{(j)(e)}_{mn}\Big)\Big(\sum_{c\in\mathcal{C}}\gamma^{(c)}_{nm}p^{(i)(c)}_{nm}\Big)\Big)\\ &+u\Big(\Big(\textstyle\sum_{e\in\mathcal{C}}\gamma^{(e)}_{mn}p^{(j)(e)}_{mn}\Big)\Big(1-\sum_{c\in\mathcal{C}}p^{(i)(c)}_{nm}\Big)\Big).\end{aligned} \tag{40}$$

From Theorem 2, problem (40) is a convex optimization problem. By solving the Karush-Kuhn-Tucker (KKT) *sufficient* and *necessary* optimality conditions (cf. [24, pp. 244]) and rearrangement the terms, we can show that for any fairness index α and for any channel $c\in\mathcal{C}$,

$$\left(\frac{\gamma^{(c)}_{nm}\left(1-\sum_{e\in\mathcal{C}}p^{(j)(e)^*}_{mn}\right)}{\left(\sum_{e\in\mathcal{C}}\gamma^{(e)}_{mn}p^{(j)(e)^*}_{mn}\right)}\right)^{1-\alpha}=\left(\frac{\sum_{d\in\mathcal{C}}(\gamma^{(d)}_{nm}/\gamma^{(c)}_{nm})p^{(i)(d)^*}_{nm}}{1-\sum_{d\in\mathcal{C}}p^{(i)(d)^*}_{nm}}\right)^{\alpha}. \tag{41}$$

From (32) and (41), the optimal solution of convex optimization problem (40) can be obtained by solving the following *system of linear equations*:

$$\textstyle\sum_{d\in\mathcal{C}}\boldsymbol{M}^{(cd)}\left(\boldsymbol{p}^{(-i)}_n\right)p^{(i)(d)^*}_{nm}=1,\qquad\forall c\in\mathcal{C}. \tag{42}$$

In vector representation, (42) is equivalent to:

$$\boldsymbol{M}\left(\boldsymbol{p}^{(-i)}_n\right)\boldsymbol{p}^{(i)^*}_n=\boldsymbol{1}. \tag{43}$$

The solution of the system of linear equations in (43) is obtained as in (33). Note that, any optimal solution of problem (40) should satisfy (43). Since problem (40) is always *feasible* (cf. [28, pp. 9]), the solution in (33) always exists, regardless of the choice of parameters. ■

Appendix D
Proof of Theorem 4

Consider the case where the NICs implement single-channel reception. For any fixed point $\langle\boldsymbol{p}^*,\boldsymbol{Q}^*\rangle\in\mathcal{F}_{SCR}$, the tuple $\langle\boldsymbol{p}^{(i)^*}_n,\boldsymbol{Q}^{(i)^*}_n\rangle$ is an optimal solution of the convex optimization problem in (Local-NUM-S) for any $i\in\mathcal{I}_n$ for all $n\in\mathcal{N}$. Since problem (Local-NUM-S) is *convex*, $\langle\boldsymbol{p}^*,\boldsymbol{Q}^*\rangle\in\mathcal{F}_{SCR}$ should satisfy the KKT conditions for (Local-NUM-S). By definition, each stationary point [28, pp. 194] of *non-convex* problem (NUM-S) also satisfies all the KKT conditions for problem (NUM-S). Since the objective functions in (NUM-S) and (Local-NUM-S) are the same and the set of constraints in (NUM-S) is the *union* of the set of constraints in (Local-NUM-S) for all $n\in\mathcal{N}$ and $i\in\mathcal{I}_n$, the KKT conditions for (NUM-S) are equal to the *union* of the KKT conditions for all $n\in\mathcal{N}$ and $i\in\mathcal{I}_n$. Thus, since $\langle\boldsymbol{p}^*,\boldsymbol{Q}^*\rangle\in\mathcal{F}_{SCR}$ satisfies the KKT conditions of (Local-NUM-S) for all nodes and all NICs, it also satisfies the KKT conditions for (NUM-S) and is indeed a local optimal solution for problem (NUM-S). This implies that, $\mathcal{F}_{SCR}\subseteq\mathcal{S}_{SCR}$. Following a similar argument, we can show that, $\mathcal{S}_{SCR}\subseteq\mathcal{F}_{SCR}$. Since $\mathcal{F}_{SCR}\subseteq\mathcal{S}_{SCR}$ and $\mathcal{S}_{SCR}\subseteq\mathcal{F}_{SCR}$, we have: $\mathcal{F}_{SCR}=\mathcal{S}_{SCR}$. The proof when the NUMs implement multi-channel reception model is similar. ■

Appendix E
Proof of Theorem 5

Part (a): It is easy to verify that for any node $n\in\mathcal{N}$ and any NIC $i\in\mathcal{I}_n$, we can select $p^{(i)(c)}_{nm}=\frac{1}{2CL_n}$ and $P^{(i)(c)}_n=Q^{(i)(c)}_n=\frac{1}{2C}$ for all $c\in\mathcal{C}$ and any $m\in\mathcal{L}^{\text{out}}_n$ as a *feasible* (not necessarily optimal) solution for problem (NUM-S). Similarly, we can select $p^{(i)(c)}_{nm}=\frac{1}{2CL_n}$ and $P^{(i)(c)}_n\frac{1}{2C}$ for all $c\in\mathcal{C}$ and $m\in\mathcal{L}^{\text{out}}_n$ as a *feasible* solution for problem (NUM-M). Thus, $|\Phi|\geq 1$ and $|\Psi|\geq 1$. Since both problems (NUM-S) and (NUM-M) are feasible problems, they have at least one stationary point [28, pp. 194]. From this, together with Theorem 4, we have: $|\mathcal{F}_{SCR}|=|\mathcal{S}_{SCR}|\geq 1$ and $|\mathcal{F}_{MCR}|=|\mathcal{S}_{MCR}|\geq 1$.

Part (b): From [20, Lemma 2], in a single-channel network, problems (NUM-S) and (NUM-M) can be transformed to *equivalent* convex optimization problems by introducing $\bar{p}^{(i)(c)}_{nm}\triangleq\log\,p^{(i)(c)}_{nm}$, $\bar{P}^{(i)(c)}_n\triangleq\log\,P^{(i)(c)}_n$, and $\bar{r}_{nm}\triangleq\log\,r_{nm}$ and showing that the new problem has positive definite Hessian with respect to the new variables when $\alpha\geq 1$. Since a convex problem has a unique stationary point, we have: $|\mathcal{S}_{SCR}|=|\mathcal{S}_{MCR}|=1$. From this, together with Theorem 4, $|\mathcal{F}_{SCR}|=|\mathcal{F}_{MCR}|=1$.

Part (c): Assume that $\langle\boldsymbol{p},\boldsymbol{Q}\rangle$ is a fixed point for Algorithm 1. From Theorem 4, it is also a stationary point of problem (NUM-S). Now consider another point $\langle\bar{\boldsymbol{p}},\bar{\boldsymbol{Q}}\rangle$, where for all nodes $n\in\mathcal{N}$, any node $m\in\mathcal{L}^{\text{out}}_n$, we have

$$\bar{p}^{(i)(c)}_{nm}\;=\;p^{(i)(C-c+1)}_{nm},\qquad\forall i\in\mathcal{I}_n,\ \forall c\in\mathcal{C}, \tag{44}$$

$$\bar{Q}^{(i)(c)}_n\;=\;Q^{(i)(C-c+1)}_n,\qquad\forall i\in\mathcal{I}_n,\ \forall c\in\mathcal{C}. \tag{45}$$

Clearly, if $p^{(i)(c)}_{nm}=p^{(i)(C-c+1)}_{nm}$ and $Q^{(i)(c)}_n=Q^{(i)(C-c+1)}_n$ for all $n\in\mathcal{N}$, $m\in\mathcal{L}^{\text{out}}_n$, $i\in\mathcal{I}_n$, and $c\in\mathcal{C}$, then $\langle\bar{\boldsymbol{p}},\bar{\boldsymbol{Q}}\rangle=\langle\boldsymbol{p},\boldsymbol{Q}\rangle$. Otherwise, $\langle\bar{\boldsymbol{p}},\bar{\boldsymbol{Q}}\rangle$ and $\langle\boldsymbol{p},\boldsymbol{Q}\rangle$ are different. From (10) and (19), it is easy to verify that: $\boldsymbol{r}(\boldsymbol{p},\boldsymbol{Q})=\boldsymbol{r}(\bar{\boldsymbol{p}},\bar{\boldsymbol{Q}})$. Thus, $\langle\bar{\boldsymbol{p}},\bar{\boldsymbol{Q}}\rangle$ is a stationary point of problem (NUM-S). From Theorem 4, it is also another fixed point of Algorithm 1. The proof for Algorithm 2 is similar. ■

APPENDIX F
PROOF OF THEOREM 6

Part (a): From (10), (14), (19), and (21), for each $(n,m)\in\mathcal{L}$, $r_{nm} \leq \sum_{i\in\mathcal{I}_n}\sum_{c\in\mathcal{C}}\gamma_{nm}^{c} \leq \sum_{c\in\mathcal{C}} I^{\max}\gamma^{\max} = CI^{\max}\gamma^{\max}$. Thus, the utility of each wireless link $(n,m)\in\mathcal{L}$ is upper bounded by $u(C\,I^{\max}\,\gamma^{\max})$ and the aggregate network utility is upper bounded by $L\,u(C\,I^{\max}\,\gamma^{\max})$.

Part (b): We prove by contradiction. Consider the case where the NICs implement multiple-channel reception and assume that at some time slot $t\in[2,T]$, $U_{SCR}(t{-}1) > U_{SCR}(t)$. In that case, there exist a node n and an NIC $i\in\mathcal{I}_n$ such that $t\in\mathcal{T}_n^{(i)}$ and solving problem (Local-NUM-S) at NIC i *reduces* the network utility (i.e., the objective in problem (NUM-S)). However, this is impossible as the objective functions in problems (Local-NUM-S) and (NUM-S) are the same. Thus, $U_{SCR}(t{-}1)\leq U_{SCR}(t)$ and (36) holds. The proof for (37) is similar.

Part (c): The existence of the limits in (38) result from parts (a) and (b). Note that any bounded non-decreasing real sequence always converges to a fixed point. ■

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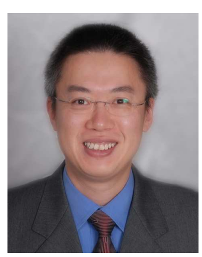

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