Distributed Multi-Interface Multi-Channel Random Access

A. Hamed Mohsenian Rad and Vincent W.S. Wong
Department of Electrical and Computer Engineering
The University of British Columbia, Vancouver, Canada
e-mail: {hamed, vincentw}@ece.ubc.ca

***Abstract*— The aggregate capacity of wireless ad-hoc networks can be substantially increased if each wireless node is equipped with multiple network interface cards (NICs) and each NIC operates over a distinct orthogonal frequency channel. Most of the recently proposed channel assignment algorithms are based on formulating *combinatorial* channel assignment problems. The key is to assign exactly one frequency channel to each NIC. However, combinatorial channel assignment models may result in computationally complicated algorithms as well as inefficient utilization of the available frequency spectrum. In this paper, we revisit channel assignment problem by formulating a novel *continuous* multi-interface multi-channel random access model. This includes elaborate modeling of the link data rates for various multi-interface multi-channel networking scenarios. We then propose a fast, fully distributed and easy to implement multi-interface multi-channel random access algorithm. Simulation results show that our proposed algorithm significantly outperforms combinatorial channel assignment algorithms in terms of achieved network utility and aggregate network throughput.**

I. Introduction

Multi-interface multi-channel wireless ad-hoc networks have recently received an increasing attention, especially under the context of wireless mesh networks [1]–[4]. In this scenario, each wireless node is equipped with multiple (usually 2 or 3) network interface cards (NICs). Each NIC operates over a distinct orthogonal frequency channel. The number of channels varies from 3 (as in IEEE 802.11b/g) to 12 (as in IEEE 802.11a). Using orthogonal channels can significantly reduce the interference among simultaneous transmissions and substantially increase the aggregate network capacity.

Most of the recently proposed channel assignment schemes formulate channel assignment as different kinds of *combinatorial* problems. In this regard, each NIC is assumed to be assigned to exactly *one* fixed channel. Examples include *graph coloring problems* [4], *integer optimization problems* [1], [2], and *mixed-integer optimization problems* [3]. It is known that the combinatorial problems are *NP*-hard [5]. That is, finding the optimal solutions may require examining *all* the possible combinations within the search space. Thus, the combinatorial channel assignment algorithms are usually computationally complicated. In addition, they display poor performance gain as the ratio between the number of channels and the number of NICs at each node increases [2], [3]. Finally, the distributed implementation of the combinatorial channel assignment algorithms is difficult because of several design challenges such as the ripple-effect problem [2].

In this paper, we overcome the performance bottlenecks of the previous combinatorial channel assignment algorithms in all the aforementioned aspects. In this regard, we formulate a novel *continuous* multi-interface multi-channel *random access* model. Unlike the combinatorial channel assignment models, where each NIC operates over exactly one channel, here we allow each NIC to switch among different frequency channels according to its *persistent probabilities* on each channel. Both single-channel reception and multi-channel reception scenarios are considered and the data rate models are obtained accordingly for each scenario. We then formulate the channel assignment problem within the framework of *network utility maximization* (NUM) (cf. [6]). Our formulated NUM problem is an extension of the recently proposed single-interface single-channel random access models in [7] and [8]. We then propose a fast, fully distributed, and easy to implement algorithm, called *distributed multi-interface multi-channel random access* (DMMRA), to solve the formulated multi-interface multi-channel NUM problem. Our algorithm only requires each node to iteratively solve a *local*, *myopic*, and *convex* optimization problem. Simulation results show that DMMRA algorithm with single-channel reception leads to 36% and 23% higher network utility and aggregate throughput compared to combinatorial channel assignment. If the multiple-channel reception is being used, then the network utility and the aggregate throughput further increase by 57% and 71%, respectively.

The rest of this paper is organized as follows. In Section II, we obtain the data rate models and formulate the NUM problems for both single-channel and multi-channel reception scenarios. The performance of the optimal solutions are analytically evaluated in Section III. Our DMMRA algorithm is proposed in Section IV. Simulation results are presented in Section V. The paper is concluded in Section VI.

II. Problem Formulation

Consider a multi-interface multi-channel wireless ad-hoc network with $\mathcal{N} = \{1, \ldots, N\}$ as the set of wireless nodes and $\mathcal{L} = \{1, \ldots, L\}$ as the set of unidirectional wireless links. For each node $n \in \mathcal{N}$, we denote the set of *incoming* links by $\mathcal{L}_n^{\text{in}} \subset \mathcal{L}$, with size $L_n^{\text{in}} = |\mathcal{L}_n^{\text{in}}|$, and the set of outgoing links by $\mathcal{L}_n^{\text{out}} \subset \mathcal{L}$, with size $L_n^{\text{out}} = |\mathcal{L}_n^{\text{out}}|$, respectively. We also define $\mathcal{N}_n^{\text{in}} = \{m : (m,n) \in \mathcal{L}_n^{\text{in}}\}$ as the set of *in-neighbors* and $\mathcal{N}_n^{\text{out}} = \{m : (n,m) \in \mathcal{L}_n^{\text{out}}\}$ as the set of *out-neighbors* of node n, respectively. The set of available channels is denoted by $\mathcal{C} = \{1, \ldots, C\}$. The set of NICs of each node $n \in \mathcal{N}$ is denoted by $\mathcal{I}_n$, with size $I_n = |\mathcal{I}_n|$. Each node $n \in \mathcal{N}$ has L_n^{out} separate queues, where each queue holds the packets for one of the outgoing links of node n (see Fig. 1). Time is divided into

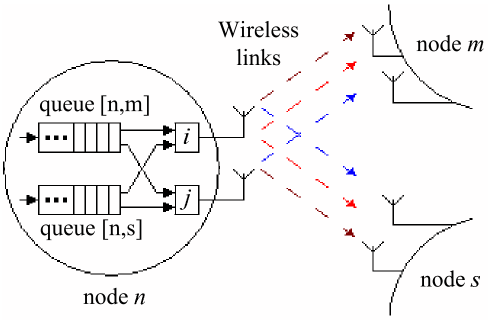

Fig. 1. An ad-hoc network with $\mathcal{N}=\{n,m,s\}$ as the set of wireless nodes. Each node has 3 NICs and there are $C=3$ channels available, denoted by 3 colors. Nodes m and s are the out-neighbors of node n. We have: $\mathcal{I}_n=\{i,j\}$.

equal-length slots. At each time slot, node n may choose to transmit packets to each of its out-neighbors $m\in\mathcal{N}_n^{\text{out}}$ using its NIC $i\in\mathcal{I}_n$ over channel $c\in\mathcal{C}$ with a *persistent probability* $p_{nm}^{(i)(c)}$. For the network in Fig. 1, node n has $I_n=2$ NICs and $L_n=2$ outgoing links, where $\mathcal{I}_n=\{i,j\}$ and $\mathcal{N}_n^{\text{out}}=\{m,s\}$. We also have $\mathcal{C}=\{1,2,3\}$. In node n, those packets which are destined to node m are enqueued in queue $[n,m]$. Similarly, the packets which are destined to node s are enqueued in queue $[n,s]$. At each time slot, a packet from queue $[n,m]$ is sent to node m (i.e., through wireless link (n,m)) using NIC i over channels 1, 2, or 3, with probabilities $p_{nm}^{(i)(1)}$, $p_{nm}^{(i)(2)}$, and $p_{nm}^{(i)(3)}$, respectively. Next, we obtain the average data rate model for each wireless link for two different multi-interference multi-channel wireless networking scenarios.

A. NICs with Single Channel Receptions

In this section, we consider the case where each NIC can decode the received packets over only *one* channel *at a time*. For each node $n\in\mathcal{N}$, let $Q_n^{(i)(c)}$ denote the probability that node n *listens* to channel $c\in\mathcal{C}$ using its NIC $i\in\mathcal{I}_n$. To be able to listen to channel c, NIC i on node n needs to be in the *receive mode* (i.e., does not transmit any packet) and also *operates* over channel c. The key feature of the single-channel reception model is that if node n is in the receive mode, and operates over channel $d\neq c$, then it *cannot* decode the signals transmitted over channel c. In this case, for each node $n\in\mathcal{N}$:

$$\sum_{c\in\mathcal{C}}\left(P_n^{(i)(c)}+Q_n^{(i)(c)}\right)=1,\qquad \forall i\in\mathcal{I}_n,\tag{1}$$

where $P_n^{(i)(c)}$ denotes the probability that node n transmits some data from NIC $i\in\mathcal{I}_n$ over channel $c\in\mathcal{C}$ to *one* of its out-neighbors. We call $P_n^{(i)(c)}$ the *node persistent probability* for NIC i of node n over channel c. We have:

$$P_n^{(i)(c)}=\sum_{m\in\mathcal{N}_n^{\text{out}}}p_{nm}^{(i)(c)}.\tag{2}$$

For each link $(n,m)\in\mathcal{L}$, we first consider the case where there is *no interference* in the network (i.e., assuming that there are only two nodes). Let $\tilde{A}_m^{(c)}$ denote the action set for all cases where at least one NIC $j\in\mathcal{I}_m$ transmits packets over channel c. The probability of this happening is:

$$\mathbb{P}\left(\tilde{A}_m^{(c)}\right)=1-\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right).\tag{3}$$

Let $\hat{A}_m^{(c)}$ denote the action set for all cases where no NIC on node m transmits packets over channel c, and no NIC listens to channel c either. The probability of this happening is:

$$\mathbb{P}\left(\hat{A}_m^{(c)}\right)=\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}-Q_m^{(j)(c)}\right).\tag{4}$$

Since $\tilde{A}_m^{(c)}\cap\hat{A}_m^{(c)}$ is an empty set, we have:

$$\mathbb{P}\left(\tilde{A}_m^{(c)}\cup\hat{A}_m^{(c)}\right)=\mathbb{P}\left(\tilde{A}_m^{(c)}\right)+\mathbb{P}\left(\hat{A}_m^{(c)}\right).\tag{5}$$

The transmission from NIC $i\in\mathcal{I}_n$ on sending node $n\in\mathcal{N}$ over channel $c\in\mathcal{C}$ can be received *correctly* by the receiving node $m\in\mathcal{N}_n^{\text{out}}$ only if *at least* one NIC $j\in\mathcal{I}_m$ is *listening* to channel c and *none* of the other NICs on node m are transmitting packets over frequency channel c. From (3)-(5), this happens with probability:

$$\begin{aligned}1-\mathbb{P}\left(\tilde{A}_m^{(c)}\cup\hat{A}_m^{(c)}\right)=&\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right)\\&-\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}-Q_m^{(j)(c)}\right).\end{aligned}\tag{6}$$

Next, we model the effect of interference in a network with N nodes. For each pair of nodes $s,m\in\mathcal{N}$, we define $\delta_{sm}=1$ if node s is within the *interference range* of node m, and $\delta_{sm}=0$ otherwise. Since the interference range is at least as large as the communication range, $\delta_{sm}=1$ if $s\in\mathcal{N}_m^{\text{in}}$. A transmission from NIC i on node n to node m via link $(n,m)\in\mathcal{L}$ over channel c *does not encounter collision* if there is no simultaneous transmission over channel c from any NIC $j\in\mathcal{I}_n\backslash\{i\}$ on node n, and any NIC $k\in\mathcal{I}_s$ on node s with $\delta_{sm}=1$. This happens with probability:

$$\begin{aligned}&\left(\prod_{j\in\mathcal{I}_n\backslash\{i\}}\left(1-P_n^{(j)(c)}\right)\right)\\&\quad\times\left(\prod_{s\in\mathcal{N}\backslash\{n,m\}}\prod_{k\in\mathcal{I}_s}\left(1-\delta_{sm}P_s^{(k)(c)}\right)\right).\end{aligned}\tag{7}$$

For each link $(n,m)\in\mathcal{L}$, let r_{nm} denote the average data rate, which is a function of the persistent and listening probability vectors: $\boldsymbol{p}=(p_{nm}^{(i)(c)},\ \forall\, n\in\mathcal{N},\ m\in\mathcal{N}_n^{\text{out}},\ i\in\mathcal{I}_n,\ c\in\mathcal{C})$, and $\boldsymbol{Q}=(Q_n^{(i)(c)},\ \forall\, n\in\mathcal{N},\ i\in\mathcal{I}_n,\ c\in\mathcal{C})$. From (2), (6) and (7), we have [9][1]:

$$\begin{aligned}&r_{nm}(\boldsymbol{p},\boldsymbol{Q})=\\&\sum_{i\in\mathcal{I}_n}\sum_{c\in\mathcal{C}}\gamma_{nm}^{(c)}p_{nm}^{(i)(c)}\left(\prod_{j\in\mathcal{I}_n\backslash\{i\}}\left(1-P_n^{(j)(c)}\right)\right)\\&\left(\prod_{s\in\mathcal{N}\backslash\{n,m\}}\prod_{k\in\mathcal{I}_s}\left(1-\delta_{sm}P_s^{(k)(c)}\right)\right)\\&\left(\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right)-\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}-Q_m^{(j)(c)}\right)\right),\end{aligned}\tag{8}$$

where $\gamma_{nm}^{(c)}$ denotes the fixed peak data rate for link (n,m) over frequency channel c. The data rate model in (8) sums up all the average data rates that can be achieved by transmitting packets from each NIC $i\in\mathcal{I}_n$ and over each channel $c\in\mathcal{C}$.

[1]The *node* persistent probability vector $\boldsymbol{P}=(P_n^{(i)(c)},\ \forall n\in\mathcal{N}, i\in\mathcal{I}_n, c\in\mathcal{C})$ can always be constructed from the *link* persistent probability vector $\boldsymbol{p}$ using the relationship in (2). Thus, for each wireless link $(n,m)\in\mathcal{L}$, we can denote the average data rate for the single-channel reception scenario as $r_{nm}(\boldsymbol{p},\boldsymbol{Q})$, rather than $r_{nm}(\boldsymbol{p},\boldsymbol{P},\boldsymbol{Q})$, to avoid redundancy.

B. NICs with Multiple Channel Reception

Next, consider the case where each NIC can decode *multiple* simultaneously received packets as long as they are transmitted over different orthogonal channels. That is, each NIC does not listen to only one channel while it is in the receive mode. Instead, it listens to *all* frequency channels and applies the band-pass channel filters to *all* the received signals. The output of each filter is then processed separately (i.e., in parallel) to distinguish transmissions over different channels [10]. We notice that, most of the existing commercial NICs do not yet implement multi-channel reception model. However, we will show in Section V that it can significantly improve the network performance. Thus, it is an attractive and promising candidate for future deployment. As in Section II-A, we first assume that there is no interference in the network. Let $\breve{A}_m^{(-c)}$ denote the action set where all NICs on node m transmit packets on some channels *other than* channel c, and no NIC is in the receive mode. We have:

$$\mathbb{P}\left(\breve{A}_m^{(-c)}\right) = \prod_{j\in\mathcal{I}_m} \sum_{d\in\mathcal{C}\setminus\{c\}} P_m^{(j)(d)}. \tag{9}$$

Since the sets $\tilde{A}_m^{(c)}$ (defined in Section II-A) and $\breve{A}_m^{(-c)}$ are *disjoint* (i.e., $\tilde{A}_m^{(c)} \cap \breve{A}_m^{(-c)}$ is an empty set), we have:

$$\mathbb{P}\left(\tilde{A}_m^{(c)} \cup \breve{A}_m^{(-c)}\right) = \mathbb{P}\left(\tilde{A}_m^{(c)}\right) + \mathbb{P}\left(\breve{A}_m^{(-c)}\right). \tag{10}$$

In the multi-channel reception model, for any link $(n,m)\in\mathcal{L}$, the transmission from NIC $i\in\mathcal{I}_n$ on node $n\in\mathcal{N}$ over channel $c\in\mathcal{C}$ can be received correctly by node $m\in\mathcal{N}_n^{\text{out}}$ if *at least* one NIC $j\in\mathcal{I}_m$ is in the *receive mode* and *none* of the other NICs on node m are transmitting packets over channel c. From (3), (9), and (10), this happens with probability:

$$\begin{aligned} &1-\mathbb{P}\left(\tilde{A}_m^{(c)} \cup \breve{A}_m^{(-c)}\right) = \\ &\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right) - \prod_{j\in\mathcal{I}_m}\sum_{d\in\mathcal{C}\setminus\{c\}} P_m^{(j)(d)}. \end{aligned} \tag{11}$$

When the interference is taken into account, from (2), (7), and (11), for each link $(n,m)\in\mathcal{L}$, we have:

$$\begin{aligned} &r_{nm}(\boldsymbol{p}) = \\ &\sum_{i\in\mathcal{I}_n}\sum_{c\in\mathcal{C}} \gamma_{nm}^{(c)} p_{nm}^{(i)(c)} \left(\prod_{j\in\mathcal{I}_n\setminus\{i\}}\left(1-P_n^{(j)(c)}\right)\right) \\ &\left(\prod_{s\in\mathcal{N}\setminus\{n,m\}}\prod_{k\in\mathcal{I}_s}\left(1-\delta_{sm}P_s^{(k)(c)}\right)\right) \\ &\left(\prod_{j\in\mathcal{I}_m}\left(1-P_m^{(j)(c)}\right) - \prod_{j\in\mathcal{I}_m}\sum_{d\in\mathcal{C}\setminus\{c\}} P_m^{(j)(d)}\right). \end{aligned} \tag{12}$$

C. Network Utility Maximization Problem

The mathematical formulation in (1)-(12) models the link data rates for two possible multi-interface multi-channel random access scenarios. In this section, we formulate the random access problems in those scenarios according to the NUM framework (cf. [6]). In this regard, each wireless link $(n,m)\in\mathcal{L}$ is assumed to maintain a *utility* which is an increasing and concave function of its data rate r_{nm} and indicates the degree of satisfaction of link (n,m) on its average data rate. The utility of link (n,m) is denoted by $u(r_{nm}(\boldsymbol{p},\boldsymbol{Q}))$, which is also a function of all the persistent and listening probabilities. Assuming that the *single*-channel reception model is being used, we are interested in finding the optimal solution of the following NUM problem:

$$\max_{\langle\boldsymbol{p},\boldsymbol{Q}\rangle\in\Phi} \sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}_n^{\text{out}}} u(r_{nm}(\boldsymbol{p},\boldsymbol{Q})), \tag{NUM-S}$$

where the average data rates are as in (8) and we have:

$$\begin{aligned} \Phi = \{ &\langle\boldsymbol{p},\boldsymbol{Q}\rangle : p_{nm}^{(i)(c)}, P_n^{(i)(c)}, Q_n^{(i)(c)} \in [0,1], \\ &P_n^{(i)(c)} = \sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)},\ \sum_{d\in\mathcal{C}}(P_n^{(i)(d)} + Q_n^{(i)(d)}) = 1, \\ &\forall n\in\mathcal{N},\ m\in\mathcal{N}_n^{\text{out}},\ i\in\mathcal{I}_n,\ c\in\mathcal{C}\ \}. \end{aligned}$$

On the other hand, if the *multiple*-channel reception model is being used, then we would like to solve the following problem:

$$\max_{\boldsymbol{p}\in\Psi} \sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}_n^{\text{out}}} u(r_{nm}(\boldsymbol{p})), \tag{NUM-M}$$

where the average data rates are as in (12) and we have:

$$\begin{aligned} \Psi = \{ &\boldsymbol{p} : p_{nm}^{(i)(c)}, P_n^{(i)(c)} \in [0,1],\ P_n^{(i)(c)} = \sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{(i)(c)}, \\ &\sum_{d\in\mathcal{C}} P_n^{(i)(d)} \le 1,\ \forall n\in\mathcal{N},\ m\in\mathcal{N}_n^{\text{out}},\ i\in\mathcal{I}_n,\ c\in\mathcal{C}\ \}. \end{aligned}$$

Notice that Φ and Ψ are *convex* sets as all the constraints are *linear*. Various concave utility functions can also be considered to achieve different design objectives. For example, if $u(r_{nm}) = \log(r_{nm})$ for all $(n,m)\in\mathcal{L}$, then solving the NUM problems leads to *proportionally fair* resource allocation [11].

III. Distributed Multi-Interface Multi-Channel Random Access

In this section, we analytically investigate the properties of the optimization problems in (NUM-S) and (NUM-M).

Let $U_{Comb}^{\star}$ denote the maximum achievable network utility among *all possible* combinatorial channel assignment strategies. Recall from Section II that combinatorial channel assignments require each NIC to operate over exactly one channel. Thus, $U_{Comb}^{\star}$ can be obtained by solving the following *mixed-integer* optimization problem:

$$\begin{aligned} \max_{\substack{\langle\boldsymbol{p},\boldsymbol{Q}\rangle\in\Phi \\ \boldsymbol{x}\in\Upsilon}} \quad & \sum_{n\in\mathcal{N}}\sum_{m\in\mathcal{N}_n^{\text{out}}} u(r_{nm}(\boldsymbol{p},\boldsymbol{Q})), \\ \text{s.t.} \quad & \sum_{c\in\mathcal{C}} x_n^{(i)(c)} = 1,\ \forall n\in\mathcal{N}, i\in\mathcal{I}_n, \\ & P_n^{(i)(c)} \le x_n^{(i)(c)},\ \forall n\in\mathcal{N}, i\in\mathcal{I}_n, c\in\mathcal{C}, \\ & Q_n^{(i)(c)} \le x_n^{(i)(c)},\ \forall n\in\mathcal{N}, i\in\mathcal{I}_n, c\in\mathcal{C}, \end{aligned} \tag{NUM-C}$$

where for each node $n\in\mathcal{N}$, any NIC $i\in\mathcal{I}_n$, and each channel $c\in\mathcal{C}$, the *integer* variable $x_n^{(i)(c)}$ is defined as:

$$x_n^{(i)(c)} = \begin{cases} 1, & \text{if NIC } i \text{ operates over channel } c, \\ 0, & \text{otherwise.} \end{cases} \tag{13}$$

We also have $\boldsymbol{x} = (x_n^{(i)(c)},\ \forall n\in\mathcal{N},\ i\in\mathcal{I}_n\ c\in\mathcal{C})$ and

$$\Upsilon = \left\{\boldsymbol{x} : x_n^{(i)(c)} \in \{0,1\},\ \forall n\in\mathcal{N},\ i\in\mathcal{I}_n\ c\in\mathcal{C}\right\}.$$

Let $U_{SCR}^{\star}$ and $U_{MCR}^{\star}$ denote the optimal values (i.e., optimal network utilities) in problems in (NUM-S) and (NUM-M), respectively. We can show the following:

Theorem 1: a) Random access with multiple-channel reception outperforms random access with single-channel reception:

$$U_{SCR}^{\star} \le U_{MCR}^{\star}. \tag{14}$$

b) Random access with single-channel reception outperforms the optimal combinatorial channel assignment:

$$U^\star_{Comb} \le U^\star_{SCR}. \tag{15}$$

The proof of Theorem 1 is given in Appendix A. Notice that the inequality in (14) is quite intuitive. To better understand why the inequality in (15) also holds, consider an example network with a *ring* topology, $N=3$ nodes, $L=3$ links, $C=3$ channels, and logarithmic utilities. Each node has one NIC. We have: $\mathcal{N} = \{n, m, s\}$, $\mathcal{L} = \{(n,m), (m,s), (s,n)\}$, and $\mathcal{C} = \{1,2,3\}$. For any $c \in \mathcal{C}$, $\gamma^{(c)}_{nm} = \gamma^{(c)}_{ns} = \gamma^{(c)}_{sn} = 11$ Mbps. In this scenario, all the combinatorial channel assignment strategies in [1]–[4] can only assign the *same* frequency channel to all the NICs in the network. Otherwise, at least two of the nodes *cannot* communicate with each other. This leads to $U^\star_{Comb} = 3\log(11 \times \frac{1}{3} \times (1-\frac{1}{3}) \times (1-\frac{1}{3}) = 1.465$ where each link is optimally active with probability $\frac{1}{3}$. On the other hand, $U^\star_{SCR} = 3\log(11 \times \frac{1}{2} \times (1-0) \times (1-\frac{1}{2})) = 3.035$ where $P^{(1)(1)}_n = Q^{(1)(1)}_m = \frac{1}{2}$, $P^{(1)(2)}_m = Q^{(1)(2)}_s = \frac{1}{2}$, $P^{(1)(3)}_s = Q^{(1)(3)}_n = \frac{1}{2}$, $P^{(1)(2)}_n = P^{(1)(3)}_n = P^{(1)(1)}_m = P^{(1)(3)}_m = P^{(1)(1)}_s = P^{(1)(2)}_s = 0$, and $Q^{(1)(1)}_n = Q^{(1)(2)}_n = Q^{(1)(2)}_m = Q^{(1)(3)}_m = Q^{(1)(1)}_s = Q^{(1)(3)}_s = 0$. Performance improvement is in factor of $\frac{3.035}{1.465} = 2.1$.

Although the objective functions in problems (NUM-S) and (NUM-M) are concave in link rates $\boldsymbol{r} = (r_{nm}, \forall (n,m) \in \mathcal{L})$, they are *not* concave in persistent and listening probabilities $\boldsymbol{p}$ and $\boldsymbol{Q}$ due to the *product form* of the data rates in (8) and (12). Thus, finding the optimal solutions of these *non-convex* problems are not easy in general. Next, we discuss some of the interesting features of (NUM-S) and (NUM-M) which will help us to develop our algorithms in Section IV.

For each node $n \in \mathcal{N}$ and any of its NICs $i \in \mathcal{I}_n$, let $\boldsymbol{p}^{(i)} = (p^{(i)(c)}_{nm}, \forall m \in \mathcal{N}^{\text{out}}_n, c \in \mathcal{C})$, and $\boldsymbol{Q}^{(i)} = (Q^{(i)(c)}_n, \forall c \in \mathcal{C})$ denote the persistent and listening probabilities of NIC i, respectively. Consider the following *local* and *myopic* optimization problem in each NIC $i \in \mathcal{I}_n$:

$$\underset{\langle \boldsymbol{p}^{(i)}, \boldsymbol{Q}^{(i)} \rangle \in \Phi^{(i)}}{\text{maximize}} \; \textstyle\sum_{m \in \mathcal{N}} \sum_{s \in \mathcal{N}^{\text{out}}_m} u(r_{ms}(\boldsymbol{p}, \boldsymbol{Q})), \tag{LOCAL-NUM-S}$$

where the average data rates are as in (8) and we have:

$$\begin{aligned} \Phi^{(i)} = \{ & \langle \boldsymbol{p}^{(i)}, \boldsymbol{Q}^{(i)} \rangle : p^{(i)(c)}_{nm}, P^{(i)(c)}_n, Q^{(i)(c)}_n \in [0,1], \\ & P^{(i)(c)}_n = \textstyle\sum_{m \in \mathcal{N}^{\text{out}}_n} p^{(i)(c)}_{nm}, \; \sum_{d \in \mathcal{C}} (P^{(i)(d)}_n + Q^{(i)(d)}_n) = 1, \\ & \forall m \in \mathcal{N}^{\text{out}}_n, \; c \in \mathcal{C} \, \}. \end{aligned}$$

Similarly, consider the following local and myopic problem:

$$\underset{\boldsymbol{p}^{(i)} \in \Psi^{(i)}}{\text{maximize}} \textstyle\sum_{m \in \mathcal{N}} \sum_{s \in \mathcal{N}^{\text{out}}_n} u(r_{ms}(\boldsymbol{p})), \tag{LOCAL-NUM-M}$$

where the average data rates are as in (12) and we have:

$$\begin{aligned} \Psi^{(i)}_n = \{ & \boldsymbol{p}^{(i)} : p^{(i)(c)}_{nm}, P^{(i)(c)}_n \in [0,1], \; \textstyle\sum_{d \in \mathcal{C}} P^{(i)(d)}_n \le 1, \\ & P^{(i)(c)}_n = \textstyle\sum_{m \in \mathcal{N}^{\text{out}}_n} p^{(i)(c)}_{nm}, \; \forall m \in \mathcal{N}^{\text{out}}_n, \; c \in \mathcal{C} \, \}. \end{aligned} \tag{16}$$

The objective functions in (LOCAL-NUM-S) and (LOCAL-NUM-M) are the same as the objective functions in (NUM-S) and (NUM-M), respectively. However, the optimization variables in (LOCAL-NUM-S) and (LOCAL-NUM-M) are local to NIC i in node n.

Algorithm 1 - Distributed Multi-interface Multi-channel Random Access (DMMRA): Executed by each node $n \in \mathcal{N}$.

1: Allocate memory for $\boldsymbol{p}$ and $\boldsymbol{Q}$.
2: Randomly choose $\boldsymbol{p}$, and $\boldsymbol{Q}$ such that $\langle \boldsymbol{p}, \boldsymbol{Q} \rangle \in \Phi$.
3: **repeat**
4:     **for** each NIC $i \in \mathcal{I}_n$ **do**
5:         Either transmit to node $m \in \mathcal{N}^{\text{out}}_n$ on channel $c \in \mathcal{C}$ or listen to channel $c \in \mathcal{C}$ with probabilities $p^{(i)(c)}_{nm}$ and $Q^{(i)(c)}_n$, respectively.
6:     **end for**
7:     **if** $t \in \mathcal{T}^{(i)}_n$ for some $i \in \mathcal{I}_n$ **then**
8:         Solve problem (LOCAL-NUM-S) using IPM [12].
9:         Update $\boldsymbol{p}^{(i)}$, and $\boldsymbol{Q}^{(i)}$ according to the solution.
10:         Inform $\boldsymbol{p}^{(i)}$, and $\boldsymbol{Q}^{(i)}$ to all other nodes.
11:     **end if**
12:     **if** a message is received from another node **then**
13:         Update $\boldsymbol{p}$ and $\boldsymbol{Q}$ accordingly.
14:     **end if**
15: **until** node n leaves the network.

Consider the case where the single-channel reception model is being used. We define $\boldsymbol{p}^{(-i)}_n = (\boldsymbol{p}^{(j)}_n, \forall j \in \mathcal{I}_n \backslash \{i\}, \, \boldsymbol{p}^{(k)}_m, \forall k \in \mathcal{I}_m, m \in \mathcal{N} \backslash \{n\})$ and $\boldsymbol{Q}^{(-i)}_n = (\boldsymbol{Q}^{(j)}_n, \forall j \in \mathcal{I}_n \backslash \{i\}, \, \boldsymbol{Q}^{(k)}_m, \forall k \in \mathcal{I}_m, m \in \mathcal{N} \backslash \{n\})$ to be the respective persistent and listening probabilities corresponding to all NICs in the network *other than* NIC i in node n. By solving problem (LOCAL-NUM-S), we can select $\boldsymbol{p}^{(i)}_n$ and $\boldsymbol{Q}^{(i)}_n$ such that the total utility is maximized *assuming* that $\boldsymbol{p}^{(-i)}_n$ and $\boldsymbol{Q}^{(-i)}_n$ are *fixed*. Solving problem (LOCAL-NUM-M) leads to similar results if multi-channel reception model is being used.

Theorem 2: Local optimization problems in (LOCAL-NUM-S) and (LOCAL-NUM-M) are *convex* problems.

The proof of Theorem 2 is given in Appendix B. From Theorem 2, we can use various well-established and efficient *convex programming* techniques (cf. [12]) to solve problems (LOCAL-NUM-S) and (LOCAL-NUM-M). This motivates us to propose our distributed multi-interface multi-channel random medium access control algorithm in the next section.

IV. DMMRA Algorithm

Our proposed DMMRA algorithm, when the single-channel reception model is being used, is shown in Algorithm 1. The algorithm when the multiple-channel reception model is used is similar and is omitted for brevity. For each node $n \in \mathcal{N}$ and any of its NICs $i \in \mathcal{I}_n$, let $\mathcal{T}^{(i)}_n$ be an unbounded set of time slots at which node n updates NIC i's persistent and listening probabilities. We assume that the updates are *asynchronous* across the network. That is, $\mathcal{T}^{(i)}_n \cap \mathcal{T}^{(j)}_n = \{\}$ for all $j \in \mathcal{I}_n \backslash \{i\}$ and $\mathcal{T}^{(i)}_n \cap \mathcal{T}^{(k)}_m = \{\}$ for all $m \in \mathcal{N} \backslash \{n\}$ and $k \in \mathcal{I}_m$. In line 2 of Algorithm 1, node n randomly initiates all of its persistent and listening probabilities. Lines 4 to 14 are then executed repeatedly at every time slot until node n leaves the network or simply switches off. In lines 4 to 6 node n transmits and receives packets according to its persistent and listening probabilities. On the other hand, lines 8 to 10 are executed only if there exists an NIC $i \in \mathcal{I}_n$ such that $t \in \mathcal{T}_n^{(i)}$. That is, the *current* time slot is a time slot at which the persistent and the listening probabilities of NIC i need to be updated by solving problem (LOCAL-NUM-S). Recall from Theorem 2 that problem (LOCAL-NUM-S) is convex. Thus, it can be easily solved using *interior point method* (IPM) [12, Chapter 11]. In line 10, node n announces its updated persistent and listening probabilities to the rest of the nodes and then in line 13 it updates its local memory according to the announcements from other nodes. Algorithm 1 is fully distributed and allows each node to adjust its operation based on a few computationally simple tasks and some limited message exchanges with other wireless nodes.

Recall from Section II-C that we are interested in solving problems (NUM-S) and (NUM-M). At each time slot t, let $U_{SCR}(t)$ and $U_{MCR}(t)$ denote the current network utility achieved by running DMMRA algorithm. We can show that:

Lemma 1: At each time slot t, the aggregate network utilities $U_{SCR}(t)$ and $U_{MCR}(t)$ are both lower and upper bounded:

$$0 \leq U_{SCR}(t), U_{MCR}(t) \leq C\,\gamma^{\max}, \quad \forall\,(n,m) \in \mathcal{L}, \tag{17}$$

where $\gamma^{\max} = \max_{(n,m)\in\mathcal{L},\, c\in\mathcal{C}} \gamma_{nm}^{(c)}$.

Lemma 2: Aggregate network utilities $U_{SCR}(t)$ and $U_{MCR}(t)$ are *non-decreasing*. That is, for each $T \geq 2$,

$$U_{SCR}(1) \leq U_{SCR}(2) \leq \ldots \leq U_{SCR}(T), \tag{18}$$

$$U_{MCR}(1) \leq U_{MCR}(2) \leq \ldots \leq U_{MCR}(T). \tag{19}$$

The proof of Lemma 1 is evident. We proof Lemma 2 for single-channel reception model by *contradiction*. The proof for multiple-channel reception model is similar. Assume that at some time slot $t \in [2, T]$, $U_{SCR}(t-1) > U_{SCR}(t)$. In that case, there exists a node n and one of its NICs $i \in \mathcal{I}_n$ such that $t \in \mathcal{T}_n^{(i)}$ and solving problem (LOCAL-NUM-S) at NIC i reduces the aggregate network utility (i.e., the objective function of problem (NUM-S)). However, this is impossible as the objective functions in (LOCAL-NUM-S) and (NUM-S) are the same. Thus, $U_{SCR}(t-1) \leq U_{SCR}(t)$ and (18) holds.

Theorem 3: Let U_{SCR}^* and U_{MCR}^* denote the aggregate network utility at the fixed point of DMMRA algorithm when the single-channel reception and the multiple-channel reception models are being used, respectively. That is,

$$U_{SCR}^* = \lim_{t\to\infty} U_{SCR}(t) \quad \text{and} \quad U_{MCR}^* = \lim_{t\to\infty} U_{MCR}(t).$$

The fixed point aggregate utilities U_{SCR}^* and U_{MCR}^* are local maxima of problems (NUM-S) and (NUM-M), respectively.

Theorem 3 is directly resulted from Lemma 1 and Lemma 2. From Theorem 3, DMMRA algorithm converges to at least a *local* optimum of the formulated NUM problems. In some other practical cases (e.g., ring topology), the fixed points are even *globally* optimal.

V. Performance Evaluation

In this section, we evaluate the performance of our proposed DMMRA algorithm for both single-channel reception (SCR) and multiple-channel reception (MCR) scenarios. We compare DMMRA with optimal combinatorial interface and channel assignment (OCICA) in terms of achieved network utility and aggregate throughput. We consider ten different random topologies. In each topology, there exist $N = 10$ wireless nodes, randomly located in a 500 m × 500 m square field. Communication and interference ranges are 150 m and 250 m, respectively. Each node $n \in \mathcal{N}$ is equipped with $\mathcal{I}_n = 2$ NICs. The peak transmission rates (i.e., $\gamma_{nm}^{(c)}$ for all $(n,m) \in \mathcal{L}$ and $c \in \mathcal{C}$) are selected *randomly* between 6 Mbps and 54 Mbps. Utility functions are logarithmic (see Section II-C).

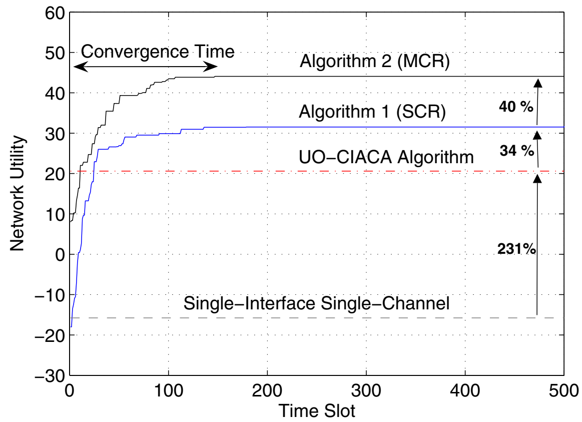

Fig. 2. Trend of the aggregate network utility versus time slots using our proposed DMMRA algorithm for the first simulated topology. Both single-channel reception and multiple-channel reception scenarios are considered.

The trend of the network utility for the first simulated topology when $C = 6$ and the DMMRA algorithm is being used is shown in Fig. 2. We can see that in both SCR and MCR scenarios, DMMRA converges to its fixed-point very fast, i.e., within 146 and 152 time slots, respectively. At steady state, DMMRA-SCR results in 34% higher utility compared to OCICA. The network utility for OCICA is calculated using *exhaustive search*, i.e., examining all possible channel combinations. On the other hand, if the NICs are upgraded to use MCR, the network utility is further increased by 40%.

The network utility and throughput when the number of channels varies from 1 to 6 are shown in Fig. 3 (a) and (b), respectively. Each point is the average of the measurements for all 10 topologies. We can see that utility and throughput increase as more channels become available. For the case when $C = 6$, DMMRA-SCR results in 36% and 23% higher utility and throughput, compared to OCICA, respectively (see Theorem 1b). On the other hand, using DMMRA-MCR leads to further 37% and 23% increase in utility and throughout compared to using DMMRA-SCR, respectively (see Theorem 1a). Further simulation results are available in [13].

VI. Conclusion

In this paper, we formulated a novel continuous multi-interface multi-channel random access model. Both single-channel reception and multi-channel reception scenarios were considered and the data rate models were obtained accordingly. We then formulated a multi-interface multi-channel network utility maximization problem and analytically proved that its

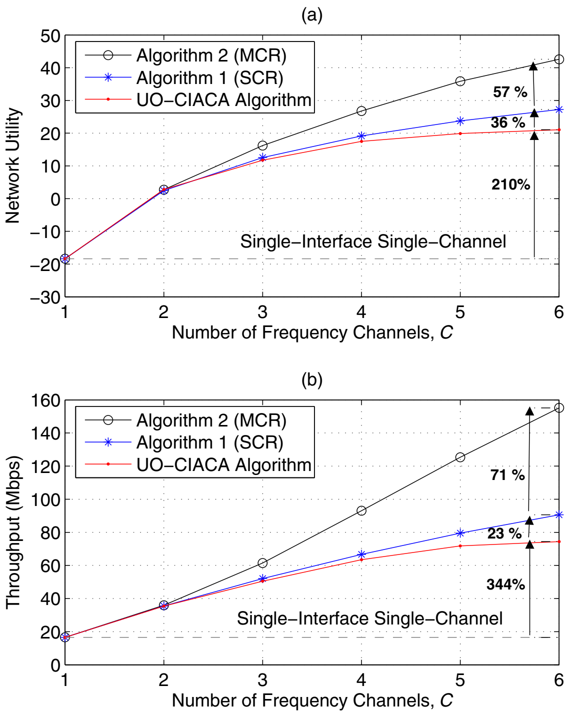

Fig. 3. Simulation results when the number of available frequency channels varies from 1 to 6: (a) Network utility, (b) Aggregate throughput.

optimal solution outperforms most of the previously proposed combinatorial channel assignment strategies. We also proposed a simple, fast, and distributed, algorithm, called DMMRA, to solve the formulated NUM problem. DMMRA requires each node to only iteratively solve a *local*, *myopic*, and *convex* optimization problem. The convergence and local optimality of the proposed algorithm were proved. Simulation results show that DMMRA algorithm with single-channel reception results in 36% and 23% higher network utility and aggregate throughput compared to combinatorial channel assignment. If multiple-channel reception is being used, then the utility and the throughput further increase by 57% and 71%, respectively.

APPENDIX

A. Proof of Theorem 1

Part a) For each $m \in \mathcal{N}$, any $j \in \mathcal{I}_m$, and each $c \in \mathcal{C}$,

$$1 - P_m^{(j)(c)} - Q_m^{(j)(c)} \overset{\text{by (1)}}{=} \sum_{d \in \mathcal{C}\setminus\{c\}} P_m^{(j)(c)} + \sum_{d \in \mathcal{C}\setminus\{c\}} Q_m^{(j)(c)} \geq \sum_{d \in \mathcal{C}\setminus\{c\}} P_m^{(j)(c)}.$$

Thus, $P(\hat{A}_m^{(c)}) \geq P(\breve{A}_m^{(c)})$ which implies that $(1 - P(\tilde{A}_m^{(c)} \cup \hat{A}_m^{(c)})) \leq (1 - P(\tilde{A}_m^{(c)} \cup \breve{A}_m^{(c)}))$. Hence, the average data rate in (12) is always greater than or equal to the average data rate in (8). Since the utility functions are increasing in data rates, greater data rates imply greater utilities. Aggregating the utilities of all links, inequality (14) is resulted.

Part b) Let Λ denote the feasible set of problem (NUM-C). It is clear that $\Lambda \subseteq \Phi$. Thus, any $\langle \bar{\boldsymbol{p}}, \bar{\boldsymbol{Q}} \rangle \in \Lambda$ is also a feasible solution for problem (NUM-S). Hence, *no* $\langle \bar{\boldsymbol{p}}, \bar{\boldsymbol{Q}} \rangle \in \Lambda$ can lead to an aggregate utility which is greater than the *optimal* utility $U_{SNR}^{\star}$ over set Φ. Therefore, inequality (15) holds. ■

B. Proof of Theorem 2

For each $n \in \mathcal{N}$ and any $i \in \mathcal{I}_n$, the objective function of problem (LOCAL-NUM-S) can be rewritten as:

$$\sum_{m \in \mathcal{N}_n^{\text{out}}} u\Big(\sum_{c \in \mathcal{C}} \big(\xi_{n,m}^{(i)(c)} p_{nm}^{(i)(c)} + \zeta_{n,m}^{(i)(c)} (1 - P_n^{(i)(c)})\big)\Big)$$
$$+ \sum_{m \in \mathcal{N}_n^{\text{in}}} u\Big(\sum_{c \in \mathcal{C}} \big((\theta_{n,m}^{(i)(c)} - \vartheta_{n,m}^{(i)(c)})(1 - P_n^{(i)(c)}) + \vartheta_{n,m}^{(i)(c)} Q_n^{(i)(c)}\big)\Big)$$
$$+ \sum_{m \in \mathcal{N}\setminus\{n\}} \sum_{s \in \mathcal{N}_m^{\text{out}}\setminus\{n\}} u\Big(\sum_{c \in \mathcal{C}} \beta_{n,ms}^{(i)(c)} (1 - \delta_{ns} P_n^{(i)(c)})\Big),$$

where for each node $m \in \mathcal{N}_n^{\text{out}}$ and any channel $c \in \mathcal{C}$,

$$\xi_{n,m}^{(i)(c)} = \gamma_{nm}^{(c)} \Big(\prod_{j \in \mathcal{I}_n\setminus\{i\}} (1 - P_n^{(j)(c)})\Big) \Big(\prod_{s \in \mathcal{N}\setminus\{n,m\}} \prod_{k \in \mathcal{I}_s} (1 - \delta_{sm} P_s^{(k)(c)})\Big) \Big(\prod_{l \in \mathcal{I}_m} (1 - P_m^{(l)(c)}) - \prod_{l \in \mathcal{I}_m} (1 - P_m^{(l)(c)} - Q_m^{(l)(c)})\Big),$$

$$\zeta_{n,m}^{(i)(c)} = \sum_{j \in \mathcal{I}_n\setminus\{i\}} \gamma_{nm}^{(c)} p_{nm}^{(j)(c)} \Big(\prod_{k \in \mathcal{I}_n\setminus\{i,j\}} (1 - P_n^{(k)(c)})\Big) \Big(\prod_{s \in \mathcal{N}\setminus\{n,m\}} \prod_{k \in \mathcal{I}_s} (1 - \delta_{sm} P_s^{(k)(c)})\Big) \Big(\prod_{j \in \mathcal{I}_m} (1 - P_m^{(j)(c)}) - \prod_{j \in \mathcal{I}_m} (1 - P_m^{(j)(c)} - Q_m^{(j)(c)})\Big).$$

Similarly, we can obtain $\theta_{n,m}^{(i)(c)}$ and $\vartheta_{n,m}^{(i)(c)}$ for any node $m \in \mathcal{N}_n^{\text{in}}$ and any channel $c \in \mathcal{C}$ as well as $\beta_{n,ms}^{(i)(c)}$ for any link $(m,n) \in \mathcal{L}\setminus(\mathcal{L}_n^{\text{in}} \cup \mathcal{L}_n^{\text{out}})$. Notice that $\xi_{n,m}^{(i)(c)}$, $\zeta_{n,m}^{(i)(c)}$, $\theta_{n,m}^{(i)(c)}$, $\vartheta_{n,m}^{(i)(c)}$, and $\beta_{n,ms}^{(i)(c)}$ are *constant* as far as problem (LOCAL-NUM-S) is concerned. Since the utility functions are concave, the objective function of problem (LOCAL-NUM-S) is a summation of *concave-affine* compositions over $\boldsymbol{p}^{(i)}$, and $\boldsymbol{Q}^{(i)}$. Therefore, it is also concave [12, pp. 84]. Clearly, the feasible set $\Phi^{(i)}$ is also a convex set. Together, these imply that problem (LOCAL-NUM-S) is a convex problem. The proof of convexity for problem (LOCAL-NUM-M) is similar. ■

REFERENCES

[1] A. H. Mohsenian Rad and V. W. S. Wong, "Joint channel allocation, interface assignment, and MAC design for multi-channel wireless mesh networks," in *Proc. of IEEE Infocom*, Anchorage, AK, May 2007.

[2] A. Raniwala and T. Chiueh, "Architecture and algorithms for an IEEE 802.11-based multi-channel wireless mesh network," in *Proc. of IEEE Infocom*, Miami, FL, March 2005.

[3] A. H. Mohsenian Rad and V. W. S. Wong, "Joint logical topology design, interface assignment, channel allocation, and routing for multi-channel wireless mesh networks," *IEEE Trans. on Wireless Communications*, vol. 6, pp. 4432–4440, Dec. 2007.

[4] X. Meng, K. Tan, and Q. Zhang, "Joint routing and channel assignment in multi-radio wireless mesh networks," in *Proc. of IEEE ICC*, Istanbul, Turkey, June 2006.

[5] H. Taha, *Operations Research: An introduction*, 7th ed. Prentice Hall, 2003.

[6] F. P. Kelly, "Charging and rate control for elastic traffic," *European Trans. on Telecommunication*, vol. 8, pp. 33–37, 1997.

[7] J. Lee, M. Chiang, and R. Calderbank, "Utility-optimal random-access control," *IEEE T. on W. Comm.*, vol. 25, pp. 1135–1147, Aug. 2007.

[8] A. H. Mohsenian Rad, J. Huang, M. Chiang, and V. W. S. Wong, "Utility-optimal random access: Reduced complexity, fast convergence, and robust performance," *IEEE T. on W. Comm. (submitted)*, Dec. 2007.

[9] D. P. Bertsekas and R. Gallager, *Data Communications*, 2nd ed. Prentice Hall, 1992.

[10] J. Proakis, *Digital Communications*. McGraw-Hill, 2000.

[11] J. Mo and J. Walrand, "Fair end-to-end window-based congestion control," *IEEE/ACM Trans. on Net.*, vol. 8, pp. 556–567, Oct. 2000.

[12] S. Boyd and L. Vandenberghe, *Convex Optimization*. Cambridge University Press, 2004.

[13] A. H. Mohsenian-Rad, "Optimal resource management in wireless access networks," Ph.D. Dissertation, University of British Columbia, Vancouver, BC, Canada, Aug. 2008.