Two-Fold Pricing to Guarantee Individual Profits and Maximum Social Welfare in Multi-hop Wireless Access Networks

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***Abstract*— In a multi-hop wireless access network, where each node is an independent self-interested commercial entity, pricing is helpful not only to encourage collaboration but also to utilize the network resources efficiently. In this paper, we propose a market-based model with *two-fold pricing* (TFP) for wireless access networks. In our model, the *relay-pricing* is used to encourage nodes to forward packets for other nodes. Each node receives a payment for the relay service that it provides. We also consider *interference-pricing* to leverage optimal resource allocation. Together, the relay and interference prices incorporate both cooperative and competitive interactions among the nodes. We prove that TFP guarantees *positive* profit for each individual wireless node for a wide range of pricing functions. The profit increases as the node forwards more packets. Thus, the cooperative nodes are well rewarded. We then determine the relay and interference pricing functions such that the network social welfare and the aggregate network utility are maximized. Simulation results show that, compared to two recently proposed *single-fold pricing* models, where only the relay or only the interference prices are considered, our proposed TFP scheme significantly increases the total network profit as well as the aggregate network throughput. TFP also leads to more fair revenue sharing among the wireless relay nodes.**

***Index Terms*— Multi-hop wireless access networks, two-fold pricing, network optimization, social welfare, interference.**

I. Introduction

Various pricing schemes have recently been proposed either to encourage collaboration among the network elements or to utilize the network resources efficiently. Pricing as a tool for *resource allocation* was first proposed in [1], [2] for congestion control among elastic traffic sources. In this regard, the network is designed to solve a network utility maximization (NUM) problem across all traffic sources, subject to the link capacity constraints. The corresponding Lagrange multipliers are interpreted as the congestion prices. Each source which uses a link resource is *charged* with the link's congestion price. The transmission rates and the congestion prices are iteratively updated using the gradient projection method until the global optimal network utility is achieved. The work in [1] has been extended to other resource allocation problems such as medium access control, power control, frequency channel assignment, and spectrum sharing [3]–[9]. Recently, it has also been shown that the gradient updates can be replaced by the best-response updates to achieve faster convergence and more robust performance [10]–[12].

Another thread of research focuses on using pricing to encourage *collaboration* among the nodes [13]–[21]. In a multi-hop network, where the nodes need to forward packets for other nodes, the optimal network performance might be at the cost of performance degradation for some intermediate relay nodes. When the intermediate nodes have no incentive to collaborate, the well-known *forwarder's dilemma* (cf. [22]) can occur, where no node forwards the packets for other nodes. To resolve this problem, incentives can be offered to the relay nodes in the form of payments or rewards in turn for their help in forwarding other nodes's traffic. In general, achieving the *optimal* network performance may *not* be always guaranteed in the incentive-based strategies as they mainly take the individual profit objectives into consideration. The problem of designing pricing models for Internet service providers (ISPs) in a fixed *wired* network has been studied in [14]–[17]. In [16], Davoli *et al.* considered the pricing problem where the ISPs do not have any knowledge about users' utility functions.

The pricing models for wired networks cannot be easily extended to *wireless* networks. There are two main challenging issues that need to be addressed in wireless access networks: *channel imperfection* (e.g., wireless fading), and *interference*. In [18], Neely proposed an economic model for wireless ad-hoc networks, with *stochastic* channel states, within the general framework of *backpressure* algorithms [23], [24]. The relay prices are used to encourage packet forwarding. However, it is essentially assumed that the network is *interference-free*. Interference-free pricing is also studied in [25], [26].

In general, most of the previously proposed pricing models in the literature have one or more of the following performance bottlenecks: (1) network resources are not efficiently (i.e., optimally) allocated, (2) individual profits are not taken into consideration, and (3) interference among the wireless transmissions is not taken into account. In this paper, we address these performance bottlenecks in all three aspects. In particular, we extend the work by Neely [18] and propose a market-based network model with *two-fold pricing* (TFP) which fully incorporates the effect of interference. Our model uses *relay-pricing* to encourage nodes to collaborate and forward each other's packets. We also use *interference-pricing* to encourage

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the wireless relay nodes to properly share the common network resources. Together, the relay and interference prices incorporate both *cooperative* and *competitive* interactions among the nodes. We analytically prove that for a wide range of pricing functions, our proposed TFP scheme leads to a guaranteed *positive* profit for each individual node. The profit increases as the node forwards more packets. This better pays off the collaborative nodes. Finally, assuming the presence of slow-fading channels, we obtain the relay and interference pricing functions for a multi-hop wireless network such that not only the positive individual profits are guaranteed, but also the network social welfare and the network utility are maximized. Compared with the *single-fold relay pricing* model in [18] as well as the *single-fold interference pricing* model in [10], simulation results show that our TFP scheme increases the social welfare and the network throughput significantly. It also leads to more fair revenue sharing among the nodes.

The rest of this paper is organized as follows. Our proposed pricing model is described in Section II. The key properties of our model are analytically proved in Section III. Simulation results are presented in Section IV. The paper is concluded in Section V. All the proofs are given in the Appendices.

II. System Model

Consider a stationary wireless access network. Let $\mathcal{N}$, with size $|\mathcal{N}| = N$, denote the set of *wireless relay nodes* and $\mathcal{L}$, with size $|\mathcal{L}| = L$, denote the set of all unidirectional *wireless links*. For each node $n \in \mathcal{N}$, the set of all *incoming* and *outgoing* links are denoted by $\mathcal{L}_n^{\text{in}} \subset \mathcal{L}$ and $\mathcal{L}_n^{\text{out}} \subset \mathcal{L}$, respectively. We also define $\mathcal{N}_n^{\text{in}} = \{m : (m,n) \in \mathcal{L}_n^{\text{in}}\}$ and $\mathcal{N}_n^{\text{out}} = \{m : (n,m) \in \mathcal{L}_n^{\text{out}}\}$ as the set of *in-neighbors* and the set of *out-neighbors* of node n, respectively. Wireless relay nodes are assumed to be *independent* commercial entities. Together, they form a *wireless backbone* to provide connectivity among *wireless users* in a multi-hop manner. The set of users is denoted by $\mathcal{D}$, with size $|\mathcal{D}| = D$. Each relay node $n \in \mathcal{N}$ offers connectivity only to a subset of users, denoted by $\mathcal{D}_n \subset \mathcal{D}$. Each user is offered connectivity from exactly one wireless relay node. All users $i, j \in \mathcal{D}_n$ are able to communicate directly with each other. However, if any user $i \in \mathcal{D}_n$ wants to send data to another user $k \in \mathcal{D}_c$, where $c \in \mathcal{N} \backslash \{n\}$, it should first transfer the data to node n, and the data are then transferred to node c via the intermediate wireless relay nodes before delivering to user k. In turn, node n charges user i for its offered connectivity service. We assume that all wireless relay nodes communicate over the same frequency band which is different from those frequency bands used by the users to communicate with each other and their associated wireless relay nodes. This avoids interference between access and relay transmissions. However, the transmissions among the wireless relay nodes can still interfere with each other. A sample wireless access network is shown in Fig. 1. In this figure, there are $N = 6$ wireless relay nodes, labeled as n, m, s, a, b, and c. There are also $D = 15$ wireless users.

Each wireless relay node $n \in \mathcal{N}$ is assumed to have $N-1$ separate queues to store the incoming data according to their *final* destination. All data that are destined to any of the users

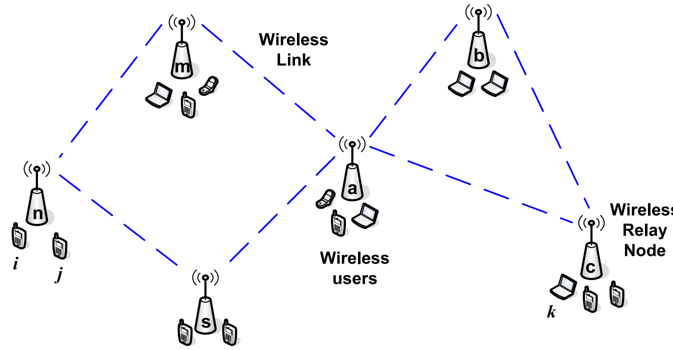

Fig. 1. A sample multi-hop wireless access network with six wireless relay nodes, labeled as n, m, s, a, b, c, and fifteen wireless users. Here $\mathcal{D}_n = \{i, j\}$ and $k \in \mathcal{D}_c$. Users i and j can directly communicate with each other. However, if user i (or user j) wants to send data to user k, it should first transfer data to its associated wireless relay node (i.e., node n), and the data are then transferred to wireless relay node c via the intermediate nodes (e.g., nodes s and a) in a multi-hop manner before being delivered to user k. In turn for the provided connectivity service, wireless relay node n and all the intermediate relay nodes are paid according to their offered relay prices.

of relay node $c \in \mathcal{N} \backslash \{n\}$ are stored in the c^{th} queue. The contents of the c^{th} queue are called *commodity c data*. For each commodity c data, node n maintains a set $\mathcal{H}_n^{(c)} \subseteq \mathcal{N}_n^{\text{out}}$, which includes its neighboring relay nodes with *minimum hop-counts* to node c and can potentially *relay* commodity c data towards node c. For example, $\mathcal{H}_n^{(c)} = \{m, s\}$, $\mathcal{H}_m^{(c)} = \{a\}$, $\mathcal{H}_s^{(c)} = \{a\}$, and $\mathcal{H}_a^{(c)} = \{c\}$ in Fig. 1.

Time is divided into equal-length slots $\mathcal{T} = \{0, 1, 2, \ldots\}$. For each link $(n,m) \in \mathcal{L}$, let Ω_{nm} denote the set of all possible *channel states*. Channel states can vary (e.g., due to wireless fading). At each time slot $t \in \mathcal{T}$, the current channel state is denoted by $\omega_{nm}(t) \in \Omega_{nm}$. We stack up the channel states of all links at time t and denote the obtained $L \times 1$ vector by $\boldsymbol{\omega}(t)$. That is, $\boldsymbol{\omega}(t) = (w_{nm}(t),\ \forall\, n, m \in \mathcal{N},\ (n,m) \in \mathcal{L})$. Let $\mathcal{T}_\omega \subseteq \mathcal{T}$ denote the set of time slots at which the channel state vector $\boldsymbol{\omega}$ changes. We assume that $\boldsymbol{\omega}$ has an independent and identical distribution (i.i.d.) over time slots $\mathcal{T}_\omega$. We also assume the *slow-fading* scenario such that

$$|t_2 - t_1| \geq \Lambda, \qquad \forall\, t_1, t_2 \in \mathcal{T}_\omega, \tag{1}$$

where $\Lambda \gg 1$. That is, two consecutive changes in channel states occur at least Λ time slots away. We will consider the *fast fading* case (i.e., when $\Lambda \to 1$) in Section IV.

For each wireless relay node $n \in \mathcal{N}$ and any of its neighboring nodes $m \in \mathcal{N}_n^{\text{out}}$, let $\mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t)) \geq 0$ denote the transmission rate offered to commodity c data over link (n,m) during time slot t. Here, $\boldsymbol{p}(t) = (p_{nm}^{(c)}(t),\ \forall\, n, m \in \mathcal{N},\ \forall c \in \mathcal{N} \backslash \{n\},\ (n,m) \in \mathcal{L})$ denotes the $L(N-1) \times 1$ vector of *transmission powers* for all links and all commodities. The scalar $p_{nm}^{(c)}(t) \geq 0$ denotes the transmission power corresponding to the transmission of commodity c data over link (n,m). At each time slot $t \in \mathcal{T}$ and for each wireless relay node $n \in \mathcal{N}$, the commodity $c \in \mathcal{N} \backslash \{n\}$ data transmission rate over wireless link $(n,m) \in \mathcal{L}_n^{\text{out}}$ can be modeled as [27]

$$\mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t)) = A_s \log\left(1 + \frac{K\, h_{nm}\, \omega_{nm}(t)\, p_{nm}^{(c)}(t)}{I_{nm}(\boldsymbol{p}_{-n}(t)) + \eta_m}\right), \tag{2}$$

where A_s denotes the channel symbol rate, K is the processing gain, η_m denotes the noise power at the receiver node m, h_{nm} is the *channel power gain* from relay node n to relay node m, $\boldsymbol{p}_{-n}(t) = (p_{ms}^{(d)}(t),\ \forall\, m \in \mathcal{N}\backslash\{n\},\ s \in \mathcal{N}_m^{\text{out}},\ d \in \mathcal{N}\backslash\{m\})$ denotes the transmission power of all nodes *other than* node n, and $I_{nm}(\boldsymbol{p}_{-n}(t))$ is the aggregate *interference power* on link (n,m). Notice that the term $K\, h_{nm}\, \omega_{nm}(t)\, p_{nm}^{(c)}(t)/(I_{nm}(\boldsymbol{p}_{-n}(t)) + \eta_m)$ is the *signal to interference plus noise ratio* (SINR) for commodity c data transmissions over link (n,m). We have

$$I_{nm}(\boldsymbol{p}_{-n}(t)) = \sum_{a \in \mathcal{N}\backslash\{n\}} h_{am} \left(\sum_{d \in \mathcal{N}\backslash\{n\}} \sum_{b \in \mathcal{N}_a^{\text{out}}} p_{ab}^{(d)}(t) \right). \quad (3)$$

Each node $n \in \mathcal{N}$ limits its total transmission power such that $\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)} \leq P_n^{\max}$, where $P_n^{\max} > 0$. Thus, the transmission rates are always *bounded*. We define

$$\mu_n^{\text{max, in}} = \max_{\boldsymbol{p},\boldsymbol{\omega}} \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{in}}} \mu_{mn}^{(c)}(\boldsymbol{p}, \boldsymbol{w}), \quad (4)$$

and

$$\mu_n^{\text{max, out}} = \max_{\boldsymbol{p},\boldsymbol{\omega}} \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} \mu_{nm}^{(c)}(\boldsymbol{p}, \boldsymbol{w}), \quad (5)$$

as the maximum data rate on any incoming and any outgoing link of node $n \in \mathcal{N}$, respectively.

A. Two-Fold Relay and Interference Pricing

1) Pricing among the wireless relay nodes: In our market-based model, at any time slot $t \in \mathcal{T}$, if wireless relay node $n \in \mathcal{N}$ transmits commodity c data with rate $\mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t))$ to its neighboring wireless relay node $m \in \mathcal{N}_n^{\text{out}}$, then it pays $\mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t))\, \phi_m^{(c)}(t)$ units of currency to node m as *relay service charge*. Here $\phi_m^{(c)}(t) \geq 0$ denotes the *relay price* corresponding to commodity c data, advertised by wireless relay node m. In total, at time slot t, node n pays

$$\sum_{c \in \mathcal{N}\backslash\{n\}} \mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t))\, \phi_m^{(c)}(t) \quad (6)$$

units of currency to any neighboring node $m \in \mathcal{N}_n^{\text{out}}$ as relay service charge. Similarly, in total, node n receives

$$\left(\sum_{c \in \mathcal{N}\backslash\{n\}} \mu_{mn}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t)) \right) \phi_n^{(c)}(t) \quad (7)$$

currency units from any node $m \in \mathcal{N}_n^{\text{in}}$ for *offered* relay service.

Besides the mutual relay services that the neighboring nodes offer to each other, they also affect each other's operation through *interference power* as shown in (2) and (3). From (3), for each node $n \in \mathcal{N}$, the higher the total transmission power $\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t)$, the greater is the interference power that relay node n causes on other nodes. In our pricing model, at each time slot t, wireless relay node n pays

$$\left(\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t) \right) \psi_a^{(n)}(t) \quad (8)$$

units of currency to each node $a \in \mathcal{N}\backslash\{n\}$ as *interference compensation charge*. Here $\psi_a^{(n)}(t) \geq 0$ denotes the *interference price* informed by node a to node n. Unlike the relay prices which vary depending on the commodity data, the interference prices are the same for all commodities as the contents of the transmissions do not affect their interference level. Instead, the interference prices may vary depending on the node locations. The closer two relay nodes are located, the higher is the corresponding channel gain. This results in higher interference power and consequently higher interference price. Similar to (8), at each time slot $t \in \mathcal{T}$, node n receives

$$\left(\sum_{d \in \mathcal{N}\backslash\{a\}} \sum_{b \in \mathcal{N}_a^{\text{out}}} p_{ab}^{(d)}(t) \right) \psi_n^{(a)}(t) \quad (9)$$

units of currency from node a as the compensation for the interference node a causes on the transmissions of node n.

For each wireless relay node $n \in \mathcal{N}$ and at any time slot $t \in \mathcal{T}$, let $U_n^{(c)}(t)$ denote the current commodity $c \in \mathcal{N}\backslash\{n\}$ queue backlog. We define

$$\boldsymbol{U}(t) = \left(U_n^{(c)}(t),\ \forall n \in \mathcal{N},\ \forall c \in \mathcal{N}\backslash\{n\} \right) \quad (10)$$

as the vector of *queue backlogs* in all wireless relay nodes at time slot t. For each $c \in \mathcal{N}\backslash\{n\}$, the corresponding relay price is assumed to be set as

$$\phi_n^{(c)}(t) = \Phi_n^{(c)}\left(\boldsymbol{U}(t-\Upsilon), \ldots, \boldsymbol{U}(t), \boldsymbol{p}(t-\Upsilon), \ldots, \boldsymbol{p}(t)\right). \quad (11)$$

Furthermore, for each $a \in \mathcal{N}\backslash\{n\}$, the corresponding relay price is assumed to be set as

$$\psi_n^{(a)}(t) = \Psi_n^{(a)}(\boldsymbol{U}(t-\Upsilon), \ldots, \boldsymbol{U}(t), \boldsymbol{p}(t-\Upsilon), \ldots, \boldsymbol{p}(t)). \quad (12)$$

Here, $\Phi_n^{(c)}(\cdot)$ and $\Psi_n^{(a)}(\cdot)$ are two non-negative real scalar *pricing functions* of all queue backlogs and all transmission powers at time slots $\{t-\Upsilon, t-\Upsilon+1, \ldots, t\}$. These pricing functions are *general*. We only make a few assumptions.

Assumption 1: If $U_n^{(c)}(t) > 0$, then $\Phi_n^{(c)}(\cdot) > 0$. That is, if wireless relay node n already has some backlogged commodity c data, then it will *not* offer free relay service.

Assumption 2: If

$$\sum_{c \in \mathcal{N}\backslash\{n\}} U_n^{(c)}(t) > 0 \quad (13)$$

and

$$\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t) > 0, \quad (14)$$

then

$$\Psi_n^{(m)}(\cdot) > 0. \quad (15)$$

That is, if relay node n has any backlogged data and it is currently transmitting some data on at least one of its outgoing links, it will *not* set its advertised interference prices to zero.

Assumption 3: Price $\Phi_n^{(c)}$ is *increasing* in $U_n^{(c)}(t)$.

Assumption 4: There exists a large enough but bounded constant $V_n^{\max}$ such that for any commodity $c \in \mathcal{N}\backslash\{n\}$ and any time slot $t \in \mathcal{T}$, $\phi_n^{(c)}(t) \leq V_n^{\max}\, U_n^{(c)}(t)$. In general, the unbounded sets of time slots at which the vector of relay prices

$$\boldsymbol{\phi}(t) = \left(\phi_n^{(c)}(t),\, \forall n \in \mathcal{N},\, c \in \mathcal{N}\backslash\{n\}\right) \tag{16}$$

and the vector of interference prices

$$\boldsymbol{\psi}(t) = \left(\psi_n^{(a)}(t),\, \forall n \in \mathcal{N},\, a \in \mathcal{N}\backslash\{n\}\right) \tag{17}$$

are being updated are denoted by $\mathcal{T}_\phi \subset \mathcal{T}$ and $\mathcal{T}_\psi \subset \mathcal{T}$, respectively. Here, $\mathcal{T}_\phi$ denotes the set of time slots at which the relay prices are updated and $\mathcal{T}_\phi$ denotes the set of time slots at which the interference prices are updated.

2) Pricing between each wireless relay node and its users: In our model, each relay node $n \in \mathcal{N}$ provides relay service for its associated wireless users according to its relay prices. At each time slot $t \in \mathcal{T}$, if user $i \in \mathcal{D}_n$ wants to send data to another user $k \in \mathcal{D}_c$ (for $c \neq n$) at rate $r_i^{(k)}(t)$, it needs to pay $r_i^{(k)}(t)\, \phi_n^{(c)}(t)$ units of currency to wireless relay node n as relay service charge. At time slot t, in total, user i pays

$$\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{k \in \mathcal{D}_c} r_i^{(k)}(t)\, \phi_n^{(c)}(t). \tag{18}$$

We assume that wireless relay node n assigns all its users with a maximum allowed sending rate $R_n^{\max}$ according to its processing capacity. Each user $i \in \mathcal{D}_n$ also maintains a non-negative, increasing, and strictly concave *utility function* $g_i^{(k)}(r_i^{(k)}(t))$ for any $k \in \mathcal{D}\backslash\mathcal{D}_n$ which indicates a *monetary measure* of user i's level of satisfaction from sending rate $r_i^{(k)}(t)$. Thus, user i adjusts its rates $\boldsymbol{r}_i = (r_i^{(k)}(t),\ \forall\, k \in \mathcal{D}\backslash\mathcal{D}_n)$ by solving the following *local* optimization problem

$$\begin{aligned} \max_{\boldsymbol{r}_i(t) \succeq \mathbf{0}} \quad & \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{k \in \mathcal{D}_c} \left(g_i^{(k)}(r_i^{(k)}(t)) - r_i^{(k)}(t) \phi_n^{(c)}(t) \right) \\ \text{s.t.} \quad & \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{k \in \mathcal{D}_c} r_i^{(k)}(t) \leq R_n^{\max}. \end{aligned} \tag{19}$$

Notice that the objective function in (19) is always *non-negative* as at least for $\boldsymbol{r}_i = \mathbf{0}$, it is equal to zero. We define user i's *profit* at each time slot $t \in \mathcal{T}$ as

$$\begin{aligned} \vartheta_i(t) = & \left(\sum_{n \in \mathcal{N}\backslash\{n\}} \sum_{k \in \mathcal{D}_c} g_i^{(k)}(r_i^{(k)}(t)) \right) \\ & - \left(\sum_{n \in \mathcal{N}\backslash\{n\}} (\sum_{k \in \mathcal{D}_c} r_i^{(k)}(t)) \phi_n^{(c)}(t) \right). \end{aligned} \tag{20}$$

From (19), user i adjusts $\boldsymbol{r}_i(t)$ to maximize its profit subject to the total rate constraint. Unlike the network model in [18], where each relay node can only support at most *one* user, here we allow each relay node to support *several* users.

B. Resource Allocation

At each time slot $t \in \mathcal{T}$, given the advertised relay prices from all its out-neighbors, node $n \in \mathcal{N}$ can compute *differential relay price* for any $m \in \mathcal{N}_n^{\text{out}}$ and each $c \in \mathcal{N}\backslash\{n\}$ as [18]

$$\delta_{nm}^{(c)}(t) = \phi_n^{(c)}(t) - \phi_m^{(c)}(t) - \phi^{\max}, \tag{21}$$

where $\phi^{\max} = V^{\max} U^{\max}$ denotes the largest possible change in *any* relay price during one time slot. Here,

$$V^{\max} = \max_n V_n^{\max} \tag{22}$$

and

$$U^{\max} = \max_n \left\{ \mu_n^{\text{max, out}}, \mu_n^{\text{max, in}} + R_n^{\max} \right\} \tag{23}$$

represent the largest possible change in any queue backlog, where $\mu_n^{\text{max, in}}$ and $\mu_n^{\text{max, out}}$ are defined in (4) and (5), respectively. At the beginning of each time slot $t \in \mathcal{T}$, relay node n measures $\omega_{nm}(t)$ for all of its outgoing wireless links $(n, m) \in \mathcal{L}_n^{\text{out}}$ and adjusts its transmission powers

$$\boldsymbol{p}_n(t) = \left(p_{nm}^{(c)}(t), \forall c \in \mathcal{N}\backslash\{n\},\, m \in \mathcal{N}_n^{\text{out}} \right) \tag{24}$$

by solving the following *local* optimization problem

$$\begin{aligned} \max_{\boldsymbol{p}_n(t) \succeq \mathbf{0}} \quad & \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} \mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{\omega}(t))\, \delta_{nm}^{(c)}(t) \\ & - \left(\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t) \right) \left(\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) \right) \\ \text{s.t.} \quad & \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t) \leq P_n^{\max}, \\ & p_{nm}^{(c)}(t) = 0, \qquad \begin{array}{l} \forall\, c \in \mathcal{N}\backslash\{n\}, \\ m \notin \mathcal{H}_n^{(c)} \text{ or } \ c \neq c_{nm}^{\star}(t) \\ \text{or } \delta_{nm}^{(c)}(t) \leq 0, \end{array} \end{aligned} \tag{25}$$

where $\mu_{nm}^{(c)}$ is as in (2), $\mathcal{H}_n^{(c)}$ is defined in Section II, and

$$c_{nm}^{\star}(t) = \arg \max_{c : m \in \mathcal{H}_n^{(c)}} \delta_{nm}^{(c)}(t), \quad \forall n \in \mathcal{N},\, m \in \mathcal{N}_n^{\text{out}}. \tag{26}$$

The *optimal* objective function in (25) is always *non-negative* since at least when $\boldsymbol{p}_n(t) = \mathbf{0}$, the objective function is equal to zero. Comparing to the resource allocation problem in [18], the objective function in (25) has an extra *negative* term

$$- \left(\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t) \right) \left(\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) \right), \tag{27}$$

which denotes the *total* interference compensation charge that wireless relay node n should pay to other relay nodes. By solving (25), node n finds the trade-off between *maximizing* $\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}^{\text{out}}} \mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{\omega}(t)) \delta_{nm}(t)$ (i.e., the original objective function in [18]) and *minimizing* its interference compensation cost. Each node then implements the same routing strategy as in [18]. That is, node n transmits commodity $c_{nm}^{\star}(t)$ data on link (n, m) as long as $\delta_{nm}^{(c_{nm}^{\star}(t))} > 0$. No commodity $c \neq c_{nm}^{\star}(t)$ data is sent on link (n, m) at time t.

Theorem 1: Let $\boldsymbol{p}_n^{\star}(t)$ denote the optimal solution of problem (25). Assuming that $K \gg 1$ and all links operate in the high SINR regime (cf. [4], [27]), for each neighboring relay node $m \in \mathcal{N}_n^{\text{out}}$ and any commodity $c \in \mathcal{N}\backslash\{n\}$, if $c = c_{nm}^{\star}(t)$, $\delta_{nm}^{(c)}(t) > 0$, and $m \in \mathcal{H}_n^{(c)}$, then

$$p_{nm}^{\star(c)}(t) = \min \left\{ \frac{\delta_{nm}^{(c_{nm}^{\star}(t))}(t)}{\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t)}, \frac{\delta_{nm}^{(c_{nm}^{\star}(t))}(t)\, P_n^{\max}}{\sum_{a \in \mathcal{N}_n^{\text{out}}} \delta_{na}^{(c_{na}^{\star}(t))}} \right\}; \tag{28}$$

otherwise, $p_{nm}^{\star(c)}(t) = 0$.

Theorem 1 provides a *closed-form* solution for the constrained optimization problem in (25). The proof of Theorem 1 is given in Appendix A. The key is to show that (28) satisfies all the *necessary* and *sufficient* Karush-Kuhn-Tucker (KKT) optimality conditions (cf. [28]).

III. Key Properties of Two-Fold Pricing

Recall from Section II that each relay node $n \in \mathcal{N}$ is an independent commercial entity who wants to *make money* out of its offered relay and connectivity services. In this regard, at each time slot $t \in \mathcal{T}$ we define node n's *profit* as

$$\begin{aligned}\chi_n(t) = &\sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{i \in \mathcal{D}_n} \sum_{k \in \mathcal{D}_c} r_i^{(k)}(t)\, \phi_n^{(c)}(t) \\ &+ \sum_{m \in \mathcal{N}_n^{\text{in}}} \sum_{c \in \mathcal{N}\backslash\{n\}} \mu_{mn}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t)) \phi_n^{(c)}(t) \\ &- \sum_{m \in \mathcal{N}_n^{\text{out}}} \sum_{c \in \mathcal{N}\backslash\{n\}} \mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{w}(t)) \phi_m^{(c)}(t) \\ &+ \sum_{a \in \mathcal{N}\backslash\{n\}} \psi_n^{(a)}(t) \sum_{d \in \mathcal{N}\backslash\{a\}} \sum_{b \in \mathcal{N}_a^{\text{out}}} p_{ab}^{(d)}(t) \\ &- \sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t).\end{aligned} \tag{29}$$

The first term in (29) is the *total* relay charges from all users $i \in \mathcal{D}_n$. The second and the third terms denote the *total* relay charges from and to all the neighboring relay nodes, respectively. The fourth and the fifth terms denote the *total* interference charges from and to all other nodes $a \in \mathcal{N}\backslash\{n\}$, respectively. We are now ready to present our first key result.

Theorem 2: For each $T \gg 1$ and any relay node $n \in \mathcal{N}$,

$$\sum_{t=0}^{T} \chi_n(t) \geq \sum_{t=0}^{T} \sum_{a \in \mathcal{N}\backslash\{n\}} \psi_n^{(a)}(t) \left(\sum_{d \in \mathcal{N}\backslash\{a\}} \sum_{b \in \mathcal{N}_a^{\text{out}}} p_{ab}^{(d)}(t) \right). \tag{30}$$

The proof of Theorem 2 is given in Appendix B. From Theorem 2, each relay node is guaranteed to obtain a profit which is at least as high as the right-hand side (RHS) of (30). All the terms in the RHS of (30) are *non-negative*. From Assumptions 1 to 4, the RHS of (30) is zero *only if* for the duration from time $t = 0$ to $t = T$, no relay node in set $\mathcal{N}\backslash\{n\}$ transmits any data and there is no data in any of the $N-1$ queues in node n. This happens only if *either* $N = 1$ and there is no other relay node in the network *or* node n has set its relay prices too high so that none of its users and neighboring relay nodes are interested in transferring their data to node n. The former is the case when there is no need to relay node n as all users in set $\mathcal{D}_n = \mathcal{D}$ can communicate with each other directly. The latter is the case when node n is reluctant to contribute as a part of the wireless access network.

Corollary 1: Each wireless relay node that contributes in relaying data is guaranteed to receive a positive-valued profit. The profit increases as the node forwards more packets.

Theorem 2 and Corollary 1 are general and apply to any choice of user utilities and pricing functions. Next, we determine the pricing functions $\Phi_n^{(c)}$ and $\Psi_n^{(m)}$ for all relay nodes $n \in \mathcal{N}$, any commodity $c \in \mathcal{N}\backslash\{n\}$, and any neighboring relay node $m \in \mathcal{N}_n^{\text{out}}$ to maximize the *network social welfare*; i.e., the aggregate profit across all relay nodes and users

$$\sum_{t=1}^{T} \sum_{n \in \mathcal{N}} \chi_n(t) + \sum_{t=1}^{T} \sum_{i \in \mathcal{D}} \vartheta_i(t). \tag{31}$$

Lemma 1: The social welfare model in (31) is equal to

$$\sum_{t=1}^{T} \sum_{n \in \mathcal{N}} \sum_{i \in \mathcal{D}_n} \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{k \in \mathcal{D}_c} g_i^{(k)}(r_i^{(k)}(t)). \tag{32}$$

The proof of Lemma 1 is given in Appendix C. From Lemma 1, the monetary exchanges among the relay nodes and the users cancel out each other. Eq. (32) is the *aggregate network utility* across all users. Thus, maximizing the network social welfare in our TFP model is equivalent to maximizing the network utility. Therefore, we can use the recent results from the literature on *backpressure* algorithms (cf. [18], [23], [24], [29], [30]) and obtain the interference and relay prices such that we can maximize the aggregate network utility. From [24] and [30], the network utility is maximized if we *periodically* solve the following *global* optimization problem

$$\begin{aligned}\max_{\boldsymbol{p}(t) \succeq \mathbf{0}} \quad & \sum_{n \in \mathcal{N}} \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} \mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{\omega}(t))\, \delta_{nm}^{(c)}(t') \\ \text{s.t.} \quad & \sum_{c \in \mathcal{N}\backslash\{n\}} \sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{(c)}(t) \leq P_n^{\max}, \qquad \forall n \in \mathcal{N} \\ & p_{nm}^{(c)}(t) = 0, \qquad \begin{array}{l}\forall n \in \mathcal{N},\ c \in \mathcal{N}\backslash\{n\}, \\ m \notin \mathcal{H}_n^{(c)} \text{ or } c \neq c_{nm}^{\star}(t') \\ \text{or } \delta_{nm}^{(c)}(t') \leq 0.\end{array}\end{aligned} \tag{33}$$

Problem (33) is a *maximum weight matching* problem. The objective function in (33) is a weighted summation of the link data rates for all links in the network. For each link (n, m), the weight is proportional (see the differential relay price model in (21)) to the difference between the queue length at the transmitter node n and the queue length at the receiver node m. By maximizing the objective function in (33), we aim to balance the queue lengths at the network nodes. This leads to *stabilizing* the network queues [30, Theorem 4]) and also reaching the maximum aggregate network utility [24]. From Lemma 1, it also leads to maximum network social welfare. Therefore, our job is to determine the interference and relay prices to solve the maximum weight matching problem (33).

Theorem 3: Given $\mathcal{T}$ (i.e., the set of time slots), $\mathcal{T}_\omega$ (i.e., the set of time slots at which the vector of channel states $\boldsymbol{\omega}$ changes), and $\Lambda \gg 1$ (i.e., the fading parameter), let

$$\mathcal{T}_\phi = \mathcal{T}_\omega, \quad \mathcal{T}_\psi = \mathcal{T}, \quad \text{and} \quad \Upsilon = \Lambda, \tag{34}$$

where $\mathcal{T}_\omega$, $\mathcal{T}_\psi$, and Υ are defined in Section II-A. The aggregate network utility and the network social welfare are maximized if each node $n \in \mathcal{N}$ at each time slot $t' \in \mathcal{T}_\phi$ sets

$$\Phi_n^{(c)} = V\, U_n^{(c)}(t'), \qquad \forall c \in \mathcal{N}\backslash\{n\}, \tag{35}$$

and at each time $t \in \{t', \ldots, t' + \Upsilon\}$ each node $n \in \mathcal{N}$ sets

$$\Psi_n^{(a)} = \sum_{m \in \mathcal{N}_n^{\text{out}}} h_{am} \frac{\max\{\delta_{nm}^{(c_{nm}^{\star}(t'))}(t'), 0\}}{I_{nm}(\boldsymbol{p}_{-n}(t-1)) + \eta_m}, \quad \forall a \in \mathcal{N}\backslash\{n\}, \tag{36}$$

Algorithm 1 *Executed by each wireless relay node $n \in \mathcal{N}$*

1: Randomly choose the prices and transmission powers.
2: **repeat**
3: Transmit commodity $c \in \mathcal{N}\backslash\{n\}$ data to node $m \in \mathcal{H}_n^{(c)}$ with power $p_{nm}^{(c)}$.
4: **if** $t \in \mathcal{T}_\phi$ **then**
5: **for** all commodity $c \in \mathcal{N}\backslash\{n\}$ **do**
6: Set $\phi_n^{(c)} = V\, U_n^{(c)}$.
7: **end**
8: Inform $\boldsymbol{\phi}_n = (\phi_n^{(c)},\ \forall c \neq n)$ to neighbors and users.
9: **for** all out-neighbors $m \in \mathcal{N}_n^{\text{out}}$ **do**
10: **for** all commodity $c \in \mathcal{N}\backslash\{n\}$ **do**
11: Set $\delta_{nm}^{(c)} = \phi_n^{(c)} - \phi_m^{(c)} - \phi^{\max}$.
12: **end**
13: Set $c_{nm}^\star = \arg\max_{c: m \in \mathcal{H}_n^{(c)}} \ \delta_{nm}(c)$.
14: **end**
15: **end**
16: **if** $t \in \mathcal{T}_\psi$ **then**
17: **for** all relay nodes $a \in \mathcal{N}\backslash\{n\}$ **do**
18: Set $\psi_n^{(a)} = \sum_{m \in \mathcal{N}_n^{\text{out}}} (\frac{h_{am}}{I_{nm}+\eta_m}) \max\{\delta_{nm}^{(c_{nm}^\star)}, 0\}$.
19: **end**
20: Inform $\boldsymbol{\psi}_n = (\psi_n^{(a)},\ \forall a \neq n)$ to all other nodes.
21: Set $\boldsymbol{p}_n = \mathbf{0}$.
22: **for** all out-neighbors $m \in \mathcal{N}_n^{\text{out}}$ **do**
23: **if** $\sum_{m \in \mathcal{N}_n^{\text{out}}} \delta_{nm}^{(c_{nm}^\star)} \leq P_n^{\max} (\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)})$ **then**
24: Set $p_{nm}^{(c)} = \delta_{nm}^{(c_{nm}^\star)} / (\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)})$.
25: **else**
26: Set $p_{nm}^{(c)} = (\delta_{nm}^{(c_{nm}^\star)} P_n^{\max}) / (\sum_{a \in \mathcal{N}_n^{\text{out}}} \delta_{na}^{(c_{na}^\star)})$.
27: **end**
28: **end**
29: **end**
30: Charge any node $m \in \mathcal{N}_n^{\text{in}}$ and any $i \in \mathcal{D}_n$ for relaying commodity $c \neq n$ data at price $\phi_n^{(c)}$.
31: Pay any node $m \in \mathcal{N}_n^{\text{out}}$ for relaying commodity $c \neq n$ data at price $\phi_m^{(c)}$.
32: Charge any other node $a \neq n$ for the interference it causes on node n at price $\psi_n^{(a)}$.
33: Pay any other node $a \neq n$ for the interference node n causes on it at price $\psi_a^{(n)}$.
34: **until** the wireless relay node n switches off.

where $V > 0$ is an arbitrary design parameter. Notice that here $I_{nm}(\boldsymbol{p}_{-n}(t-1))$ denotes the *most recent* measurement of the interference power at the receiver node of link (n, m).

The proof of Theorem 3 is given in Appendix D. The key is to show that our proposed two-fold pricing functions result in solving the maximum weight matching problem *periodically* (i.e., every $\Upsilon = \Lambda$ time slots). Together, Theorems 2 and 3 show that if the transmission powers, relay prices, and interference prices are set according to (28), (35), and (36), respectively, then not only each relay node receives a guaranteed positive profit, but also the social welfare and the network utility are maximized. The pseudocode of the pricing algorithms that each node and each user need to execute are given in Algorithms 1 and 2, respectively. In lines 5 to 7 and lines 17 to 19 of Algorithm 1, the relay and interference prices are adjusted according to Theorem 3, respectively. On the other hand, in lines 21 to 28, the transmission powers are set according to Theorem 1. Notice that Algorithm 2 simply adjusts the transmission rates of the users based on the optimal solutions of the profit maximization problem in (19).

Algorithm 2 *Executed by each wireless user $i \in \mathcal{D}_n$*

repeat
Set the rates $\boldsymbol{r}_i$ by solving problem (19).
Pay node n for commodity $c \neq n$ data at price $\phi_n^{(c)}$.
until the wireless user i switches off or leaves the network.

IV. Simulation Results

In this section, we evaluate the performance of our proposed TFP scheme and compare it with two recently proposed single-fold pricing strategies. In particular, we compared TFP with the *single-fold relay pricing* (SFRP) scheme in [18] and the *single-fold interference pricing* (SFIP) scheme in [10]. For the pricing model in [18], only the relay prices are taken into account and the network is assumed to be interference-free. On the other hand, for the pricing model in [10], only the interference prices are considered and it is assumed that all relay nodes are willing to relay data for other relay nodes free of charge. We consider three performance metrics: 1) network social welfare, 2) fairness index, and 3) aggregate network throughput. The fairness index is calculated among the *profits* that the wireless relay nodes achieve [31]

$$\frac{\left(\sum_{n \in \mathcal{N}} \sum_{t=1}^{T} \chi_n(t)\right)^2}{N \sum_{n \in \mathcal{N}} \left(\sum_{t=1}^{T} \chi_n(t)\right)^2}, \tag{37}$$

where $T = 5000$ is the simulation time. Clearly, for the case of SFRP, the profit for each node includes the balance of the received and paid prices only for relay prices. Similarly, for

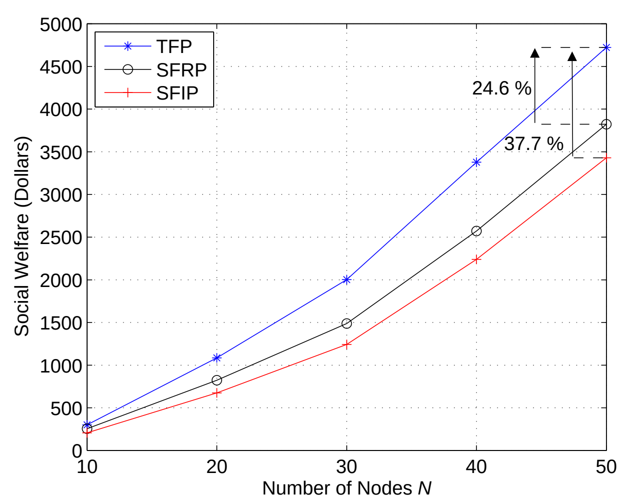

Fig. 2. Network social welfare when the number of wireless relay nodes N varies from 10 to 50. Each relay node provides network connectivity for 5 users. Each point is the average of the measurements for all ten topologies.

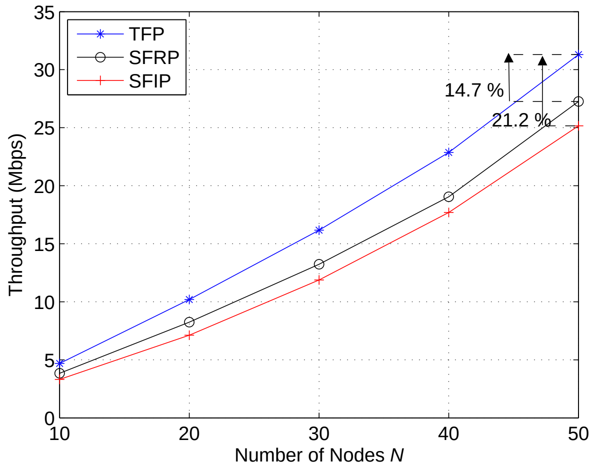

Fig. 3. Network throughput when the number of relay nodes N varies from 10 to 50. Each point is the average of the measurements for all ten topologies.

the case of SFIP, the profit for each node includes the balance of the received and paid prices only for interference prices. In the simulation model, each wireless relay node n provides the connectivity for $|\mathcal{D}_n| = 5$ wireless users. Each wireless user is interested in sending data to two other (randomly selected) users inside the network. We consider ten different *random* topologies. In each topology, the wireless relay nodes are randomly located in a 1 km $\times$ 1 km square field and the communication range is 200 m. There is a link between any two neighboring wireless relay nodes if they are within the communication range of each other. For each wireless relay node $n \in \mathcal{N}$, we have: $P_n^{\max} = 20$ W and $R_n^{\max} = 100$ kbps. The transmission power, relay prices, and the interference prices are set according to (28), (35), and (36), respectively. The unit of currency is selected such that for a unit queue backlog, relaying 1 Mbps data costs 1 *cent*, i.e., 0.01 *dollar*. Unless stated otherwise, we assume the presence of slow-fading channels with the fading parameter $\Lambda = 10$. The impact of fast-fading is also studied in Section IV-C.

A. Performance Comparison with Single-Fold Pricing

The network social welfare, where the number of relay nodes N varies from 10 to 50, is shown in Fig. 2. In this figure, each point is the average of the measurements for all 10 simulated topologies. We can see that the proposed TFP scheme always outperforms the SFRP and SFIP strategies and results in higher network social welfare[1]. Notice that, from Theorem 3, TFP indeed leads to achieving the *maximum* network social welfare. Considering the case where the number of relay nodes $N = 50$, TFP results in 24.6% and 36.7% higher network social welfare compared to SFRP and SFIP, respectively. We can also see that SFRP outperforms the SFIP scheme. This is due to

[1]Notice that from Lemma 1, the network social welfare when TFP is used is equal to aggregate network utility. We can easily show that a similar statement is true when SFRP is used. Therefore, to have a fair comparison, we also considered the aggregate network utility as the network social welfare when SFIP is used. Otherwise, the network social welfare for SFIP would be significantly less than the values shown in Fig. 2.

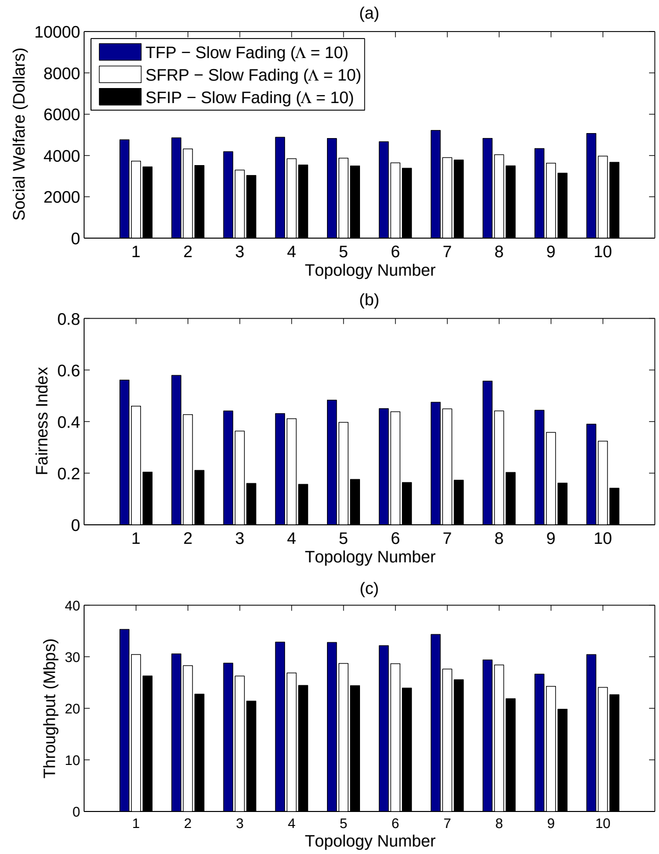

Fig. 4. Simulation results for each of the 10 random simulated topologies when $N = 50$ and the communication channels experience slow-fading (i.e., $\Lambda = 10$): (a) Network social welfare, (b) Fairness index among the profits achieved by wireless relay nodes, and (c) Aggregate network throughput.

the fact that SFIP does not take into account the information about the traffic load (e.g., queue backlog) and the link state information. In fact, SFIP simply assumes infinite backlog in all queues in the network. In contrary, our proposed TFP model takes into account the wireless interference, load information, and channel states leading to significantly better performance.

Next, we compare the throughout in TFP, SFRP, and SFIP. Results are shown in Fig. 3. We can see that TFP can increase the throughput significantly compared to both SFRP and SFIP schemes. Considering the case where $N = 50$, the proposed TFP results in 14.7% and 21.2% higher aggregate throughput compared to SFRP and SFIP, respectively.

The exact value of the social welfare, fairness index, and throughput for each of the 10 simulated topologies, where $N = 50$, are shown in Fig. 4 (a), (b), and (c), respectively. From Fig. 4 (a) and (c), TFP always results in higher social welfare and higher throughput compared to both SFRP and SFIP. From Fig. 4 (b), TFP also always acts more fair. Recall from Theorem 2 that TFP guarantees high positive profits for all relay nodes. In fact, compared to SFRP, having the interference prices in the TFP scheme helps those relay nodes that do not experience high traffic demand. Instead, they make some money out of the interference charges. This results in a more fair revenue distribution among the nodes. On average, TFP leads to 18.3%

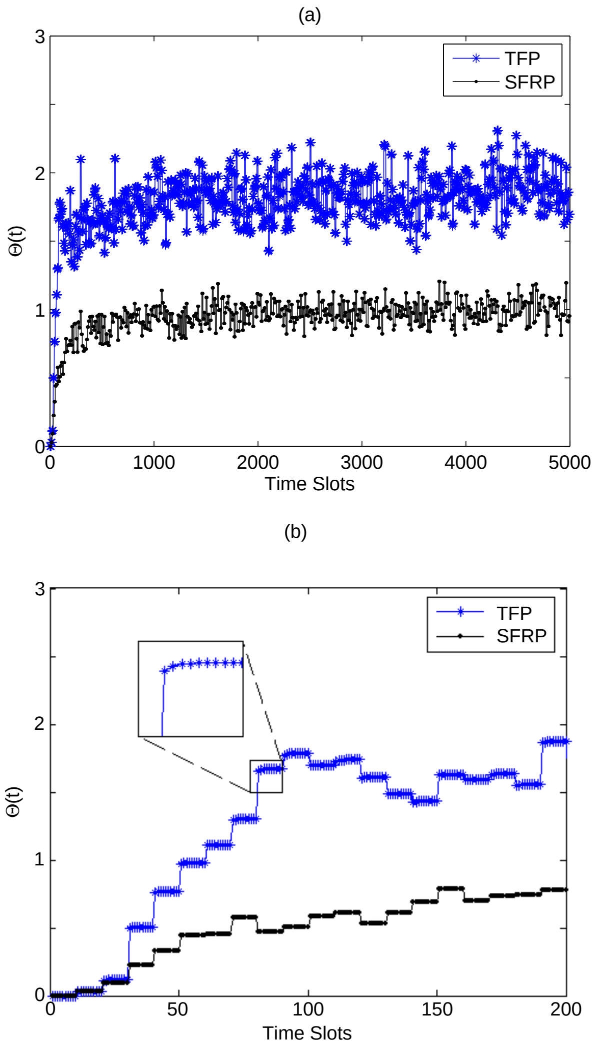

Fig. 5. Trend of the maximum weight matching objective $\Theta(t)$ versus time slots: (a) During the whole simulation time, i.e., 5000 time slots, (b) During the first 200 time slots. Notice that every $\Upsilon = \Lambda = 10$ time slots, the channel states change randomly and the maximum weight matching objective converges to its new optimal value accordingly.

higher fairness index compared to SFRP. We further notice that SFIP shows poor performance in terms of fairness. In fact, since SFIP is based on the assumption of free relay service, in this case the only monetary exchange among the relay nodes is the interference prices. The interference prices only depend on the network topology, not the traffic load relayed by each node. Therefore, when it comes to the profit made by each node, SFIP does not show an acceptable performance.

B. *Maximum Weight Matching*

Recall from Section III that both TFP and SFIP aim to solve the maximum weight matching problem in (33). Therefore, it is interesting to compare TFP and SFIP in terms of maximizing the objective value in (33). At each time slot $t \in \mathcal{T}$, we define

$$\Theta(t) = \sum_{n\in\mathcal{N}} \sum_{c\in\mathcal{N}\setminus\{n\}} \sum_{m\in\mathcal{N}_n^{\text{out}}} \mu_{nm}^{(c)}(\boldsymbol{p}(t), \boldsymbol{\omega}(t))\, \delta_{nm}^{(c)}(t'), \quad (38)$$

where t' is the *smallest* time slot in set $\mathcal{T}_\omega$ such that: $t \geq t'$. In other words, t' is the most recent time slot at which the vector of channel states $\boldsymbol{\omega}$ has changed. We notice that, $\Theta(t)$ is indeed the same as the objective function in the maximum weight matching problem in (33). From Theorem 3, TFP results in maximum network social welfare and maximum network utility by *periodically* solving the optimization problem (33), i.e., maximizing the values of $\Theta(t)$. This is illustrated in Fig. 5. In this figure, the trend of $\Theta(t)$ for topology number 1 is shown versus the time slots. Notice that, the fading parameter $\Lambda = 10$. Thus, the vector of channel states $\boldsymbol{\omega}$ changes randomly every 10 time slots. This implies that the optimal solution of the maximum weight matching problem also changes every 10 time slots[2]. For proper operation, TFP needs to converge to the *new* optimal solution accordingly. This is shown in the zoomed area in Fig. 5 (b). Clearly, the convergence is fast. From the results in Fig. 5 (a) and (b), we can also see that TFP always results in substantially higher maximum weight matching objective $\Theta(t)$, compared to SFRP. The higher the maximum weight matching objective, the higher is the aggregate network utility [24]. From Lemma 1, this also implies higher network social welfare.

C. *Impact of Fast-Fading*

In the previous experiments, we assumed that the channels experience slow-fading. In this section, we study the impact of fast-fading. Results for all the ten simulated topologies, where the number of relay nodes $N = 50$ and the fading parameter $\Lambda = 2$, are shown in Fig. 6. In this scenario, the channel states change randomly every 2 time slots. This implies that the optimal solution of the maximum weight matching problem also changes every 2 time slots. As a result, our proposed distributed transmission power adjustment mechanism (see lines 22 to 29 in Algorithm 1) does *not* have enough time to converge to the *new* optimal solution of the maximum weight matching problem after each change in the channel states. Thus, the optimal performance may *not* be achieved. Nevertheless, from the results in Fig. 6(a) and 6(c), TFP still results in 46.3% higher network social welfare and 32.4% higher aggregate throughput, compared to SFRP. On the other hand, from Fig. 6 (b), TFP is 35.2% more fair compared to SFRP in this scenario. Similar results are obtained compared to SFIP scheme. Notice that SFIP has slightly better performance than SFRP in presence of fast fading channels. That is due to the fact that SFIP assigns prices only based on the network topology, not the channel states. Thus, unlike SFRP, SFIP is not noticeably affected by fast fading.

In summary, assuming the presence of slow-fading channels, our proposed TFP scheme leads to not only higher aggregate profit across the nodes and users, but also more fair revenue distribution among the relay nodes. The former results from Theorem 3 while the latter results from Theorem 2. Both features are also confirmed through extensive simulation studies. Furthermore, TFP can also significantly increase the network aggregate throughput. When the underlying communication channels experience fast-fading, although TFP still results in substantially better performance compared to SFRP and

[2] That is why the interval for solving problem (33) must be at least 10.

SFIP as the simulation results indicate, achieving the optimal performance may not always be guaranteed.

V. Conclusion

In this paper, we proposed a market-based wireless access network model with two-fold pricing (TFP), where several self-interested wireless relay nodes provide connectivity for a number of wireless users. The relay-prices are used as incentives to encourage nodes to collaborate and forward each other's packets. The interference-prices are also used to leverage optimal resource allocation. Together, the relay and interference prices incorporate both *cooperative* and *competitive* interactions among the nodes. The positive profit for each individual wireless relay node is guaranteed for a wide range of pricing functions. The relay and interference pricing functions are then determined to maximize the network social welfare and aggregate network utility. Compared with the single-fold relay pricing (SFRP) scheme in [18], where only the relay prices are taken into account, as well as the single-fold interference pricing (SFIP) scheme in [10], where only the interference prices are considered, simulation results show that TFP significantly increases the network social welfare and aggregate throughput. TFP also leads to more fair revenue sharing and better profit distribution among the wireless relay nodes. Therefore, TFP leads to a wireless access network with multiple independent service providers which not only operates at optimal performance, but also is beneficial for each individual wireless service provider.

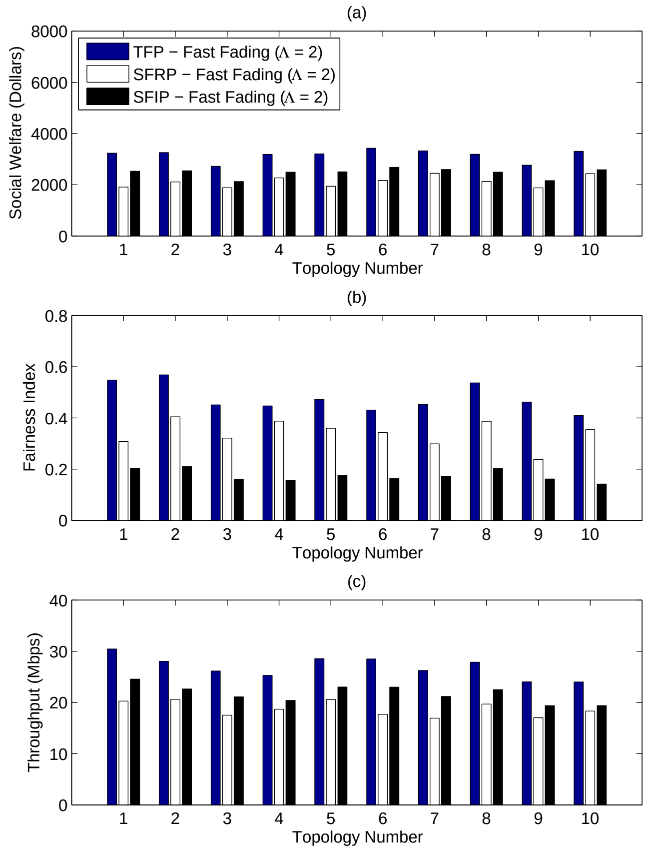

Fig. 6. Simulation results for each of the 10 random simulated topologies when $N = 50$ and the communication channels experience fast-fading (i.e., $\Lambda = 2$): (a) Network social welfare, (b) Fairness index among the profits achieved by wireless relay nodes, and (c) Aggregate network throughput.

Appendix

A. Proof of Theorem 1

Knowing that all the constraints are linear and the objective function is concave, problem (25) is a *convex* optimization problem. Therefore, it has a *unique* local (thus global) optimal solution. In a high SINR regime, the optimal solution should satisfy the following *necessary* and *sufficient* Karush-Kuhn-Tucker (KKT) optimality conditions [32, Proposition 3.3.1] for each out-neighbor node $m \in \mathcal{N}_n^{\text{out}}$:

$$\frac{\delta_{nm}^{(c^\star_{nm}(t))}(t)}{p_{nm}^{\star(c^\star_{nm}(t))}(t)} - \sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) = \lambda_n^\star - \sigma_{nm}^\star, \tag{39}$$

$$\lambda_n^\star \left[\sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{\star(c^\star_{nm}(t))}(t) - P_n^{\max} \right] = 0, \tag{40}$$

$$\sigma_{nm}^\star p_{nm}^{\star(c^\star_{nm}(t))}(t) = 0, \tag{41}$$

$$\lambda_n^\star \geq 0, \tag{42}$$

$$\sigma_{nm}^\star \geq 0, \tag{43}$$

where $\lambda_n^\star$ denotes the Lagrange multiplier corresponding to constraint $\sum_{m \in \mathcal{N}_n^{\text{out}}} p_{nm}^{\star(c^\star_{nm}(t))}(t) \leq P_n^{\max}$ and $\sigma_{nm}^\star$ denotes the Lagrange multiplier corresponding to constraint $p_{nm}^{\star(c^\star_{nm}(t))}(t) \geq 0$ for each wireless link $(n,m) \in \mathcal{L}_n^{\text{out}}$. We can show that if

$$\sum_{m \in \mathcal{N}_n^{\text{out}}} \delta_{nm}^{(c^\star_{nm}(t))}(t) \; < \; P_n^{\max} \left(\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) \right), \tag{44}$$

then the KKT conditions (39)-(43) are satisfied by setting $\lambda_n^\star = 0$ and $\sigma_{nm}^\star = 0$ for all links $(n,m) \in \mathcal{L}_n^{\text{out}}$. In this case, for each link $(n,m) \in \mathcal{L}_n^{\text{out}}$ and any commodity $c = c^\star_{nm}(t)$ such that $\delta_{nm}^{(c)}(t) > 0$, we have $p_{nm}^{\star(c)}(t) = \delta_{nm}^{(c^\star_{nm}(t))} / (\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t))$. On the other hand, if

$$\sum_{m \in \mathcal{N}_n^{\text{out}}} \delta_{nm}^{(c^\star_{nm}(t))}(t) \; \geq \; P_n^{\max} \left(\sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) \right), \tag{45}$$

then the KKT conditions are satisfied by setting $\sigma_{nm}^\star = 0$ for all links $(n,m) \in \mathcal{L}_n^{\text{out}}$ and

$$\lambda_n^\star = \frac{1}{P_n^{\max}} \sum_{m \in \mathcal{N}_n^{\text{out}}} \delta_{nm}^{(c^\star_{nm}(t))}(t) - \sum_{a \in \mathcal{N}\backslash\{n\}} \psi_a^{(n)}(t) \overset{\text{by (45)}}{\geq} 0. \tag{46}$$

Notice that, in this case, for each link $(n,m) \in \mathcal{L}_n^{\text{out}}$ and any commodity $c = c^\star_{nm}(t)$ such that $\delta_{nm}^{(c)}(t) > 0$, we have $p_{nm}^{\star(c)}(t) = \delta_{nm}^{(c^\star_{nm}(t))}(t) \; P_n^{\max} / \left(\sum_{a \in \mathcal{N}_n^{\text{out}}} \delta_{na}^{(c^\star_{na}(t))}(t) \right)$. Thus, since $\boldsymbol{p}_n(t)$ is the only point that satisfies the KKT conditions in (39)-(43), it is indeed the unique global optimal solution for the local transmission power control problem in (25). ■

B. Proof of Theorem 2

From (21) and (29), for any node $n\in\mathcal{N}$ and at any time $t\in\mathcal{T}$,

$$
\begin{aligned}
&\chi_n(t)\\
&=\chi_n(t)+\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\delta_{nm}^{(c)}(t)\\
&\quad-\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\left(\phi_n^{(c)}(t)-\phi_m^{(c)}(t)-\phi^{\max}\right)\\
&=\Bigg[\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\;\delta_{nm}^{(c)}(t)\\
&\quad-\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_a^{(n)}(t)\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}p_{nm}^{(c)}(t)\Bigg]\\
&\quad+\Bigg[\sum_{c\in\mathcal{N}\backslash\{n\}}\Bigg(\sum_{i\in\mathcal{D}_n}\sum_{k\in\mathcal{D}_c}r_i^{(k)}(t)+\\
&\qquad\sum_{m\in\mathcal{N}_n^{\text{in}}}\mu_{mn}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\Bigg)\phi_n^{(c)}(t)\\
&\quad-\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\;\left(\phi_n^{(c)}(t)-\phi^{\max}\right)\Bigg]\\
&\quad+\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_n^{(m)}(t)\sum_{d\in\mathcal{N}\backslash\{a\}}\sum_{b\in\mathcal{N}_a^{\text{out}}}p_{ab}^{(d)}(t).
\end{aligned}
\tag{47}
$$

Since the optimal objective function in (25) is non-negative,

$$
\begin{aligned}
&\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\;\delta_{nm}^{(c)}(t)\\
&-\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_a^{(n)}(t)\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}p_{nm}^{(c)}(t)\;\ge\;0.
\end{aligned}
\tag{48}
$$

Following the proof of [18, Theorem 2b], we can also show that

$$
\begin{aligned}
&\sum_{c\in\mathcal{N}\backslash\{n\}}\Bigg(\sum_{i\in\mathcal{D}_n}\sum_{k\in\mathcal{D}_c}r_i^{(k)}(t)+\sum_{m\in\mathcal{N}_n^{\text{in}}}\mu_{mn}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\Bigg)\phi_n^{(c)}(t)\\
&-\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{\omega}(t))\;\left(\phi_n^{(c)}(t)-\phi^{\max}\right)\ge 0.
\end{aligned}
\tag{49}
$$

By replacing (48) and (49) in (21) we have

$$
\chi_n(t)\ge\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_n^{(m)}(t)\sum_{d\in\mathcal{N}\backslash\{a\}}\sum_{b\in\mathcal{N}_a^{\text{out}}}p_{ab}^{(d)}(t).
\tag{50}
$$

Adding up both sides for any time slot $t=1,\ldots,T$, the inequality in (30) is resulted. ■

C. Proof of Lemma 1

Replacing $\vartheta_i(t)$ and $\chi_n(t)$ in (31) by (20) and (29),

$$
\begin{aligned}
&\sum_{t=1}^{T}\sum_{n\in\mathcal{N}}\chi_n(t)+\sum_{t=1}^{T}\sum_{i\in\mathcal{D}}\vartheta_i(t)\\
&=\sum_{t=1}^{T}\sum_{n\in\mathcal{N}}\Bigg(\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{i\in\mathcal{D}_n}\sum_{k\in\mathcal{D}_c}r_i^{(k)}(t)\phi_n^{(c)}(t)\\
&\quad+\sum_{m\in\mathcal{N}_n^{\text{in}}}\sum_{c\in\mathcal{N}\backslash\{n\}}\mu_{mn}^{(c)}(\boldsymbol{p}(t),\boldsymbol{w}(t))\phi_n^{(c)}(t)\\
&\quad-\sum_{m\in\mathcal{N}_n^{\text{out}}}\sum_{c\in\mathcal{N}\backslash\{n\}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{w}(t))\phi_m^{(c)}(t)\\
&\quad+\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_n^{(a)}(t)\sum_{d\in\mathcal{N}\backslash\{a\}}\sum_{b\in\mathcal{N}_a^{\text{out}}}p_{ab}^{(d)}(t)\\
&\quad-\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_a^{(n)}(t)\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}p_{nm}^{(c)}(t)\Bigg)\\
&\quad+\sum_{t=1}^{T}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{D}_n}\Bigg(\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{k\in\mathcal{D}_c}g_i^{(k)}(r_i^{(k)}(t))\\
&\quad-\sum_{n\in\mathcal{N}\backslash\{n\}}\sum_{k\in\mathcal{D}_c}r_i^{(k)}(t)\phi_n^{(c)}(t)\Bigg)\\
&=\sum_{t=1}^{T}\sum_{n\in\mathcal{N}}\Bigg(\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{i\in\mathcal{D}_n}\sum_{k\in\mathcal{D}_c}r_i^{(k)}(t)\phi_n^{(c)}(t)-\\
&\qquad r_i^{(k)}(t)\phi_n^{(c)}(t)\Bigg)\\
&\quad+\sum_{t=1}^{T}\Bigg(\Bigg(\sum_{n\in\mathcal{N}}\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{in}}}\mu_{mn}^{(c)}(\boldsymbol{p}(t),\boldsymbol{w}(t))\phi_n^{(c)}(t)\Bigg)\\
&\quad-\Bigg(\sum_{n\in\mathcal{N}}\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{w}(t))\phi_m^{(c)}(t)\Bigg)\Bigg)\\
&\quad+\sum_{t=1}^{T}\Bigg(\Bigg(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_n^{(a)}(t)\sum_{d\in\mathcal{N}\backslash\{a\}}\sum_{b\in\mathcal{N}_a^{\text{out}}}p_{ab}^{(d)}(t)\Bigg)\\
&\quad-\Bigg(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{N}\backslash\{n\}}\psi_a^{(n)}(t)\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}p_{nm}^{(c)}(t)\Bigg)\Bigg)\\
&\quad+\sum_{t=1}^{T}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{D}_n}\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{k\in\mathcal{D}_c}g_i^{(k)}(r_i^{(k)}(t))\\
&=\sum_{t=1}^{T}\sum_{n\in\mathcal{N}}\sum_{i\in\mathcal{D}_n}\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{k\in\mathcal{D}_c}g_i^{(k)}(r_i^{(k)}(t)).
\end{aligned}
\tag{51}
$$

The last line in (51) is indeed the same as (32). Notice that

$$
\begin{aligned}
&\sum_{n\in\mathcal{N}}\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{in}}}\mu_{mn}^{(c)}(\boldsymbol{p}(t),\boldsymbol{w}(t))\phi_n^{(c)}(t)\\
&\quad=\sum_{n\in\mathcal{N}}\sum_{c\in\mathcal{N}\backslash\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\mu_{nm}^{(c)}(\boldsymbol{p}(t),\boldsymbol{w}(t))\phi_m^{(c)}(t),
\end{aligned}
\tag{52}
$$

and

$$\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{N}\setminus\{n\}}\sum_{d\in\mathcal{N}\setminus\{a\}}\sum_{b\in\mathcal{N}_a^{\text{out}}}\psi_n^{(a)}(t)\,p_{ab}^{(d)}(t) = \sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{N}\setminus\{n\}}\sum_{c\in\mathcal{N}\setminus\{n\}}\sum_{m\in\mathcal{N}_n^{\text{out}}}\psi_a^{(n)}(t)\,p_{nm}^{(c)}(t). \tag{53}$$

In (52), the left hand side denotes the aggregate relay price that *all* wireless relay nodes *receive* while the right hand side denotes the aggregate relay price *all* wireless relay nodes *pay*. Similarly, in (53), the left hand side is the aggregate interference price that *all* wireless relay nodes *receive* while the right hand side is the aggregate interference price *all* wireless relay nodes *pay*. ■

D. Proof of Theorem 3

Given $t' \in \mathcal{T}_\phi$, for each time slot $t \in \{t', \ldots, t'+\Upsilon\}$, consider two arbitrary non-negative valued transmission power vectors $\tilde{\boldsymbol{p}}(t)$ and $\hat{\boldsymbol{p}}(t)$ such that

$$\tilde{\boldsymbol{p}}(t) \preceq \hat{\boldsymbol{p}}(t), \tag{54}$$

where the inequality is interpreted coordinate-wise. That is, for any wireless link $(n,m) \in \mathcal{L}$ and each commodity $c \in \mathcal{N}\setminus\{n\}$, we have $\tilde{p}_{nm}^{(c)}(t) \leq \hat{p}_{nm}^{(c)}(t)$. From (3), we can show that for each $n \in \mathcal{N}$ and any $m \in \mathcal{N}_n^{\text{out}}$ we have

$$I_{nm}(\tilde{\boldsymbol{p}}_{-n}(t)) \leq I_{nm}(\hat{\boldsymbol{p}}_{-n}(t)), \tag{55}$$

$$\frac{1}{(I_{nm}(\tilde{\boldsymbol{p}}_{-n}(t))+\eta_m)} \geq \frac{1}{(I_{nm}(\hat{\boldsymbol{p}}_{-n}(t))+\eta_m)}, \tag{56}$$

$$\psi_n^{(a)}(\tilde{\boldsymbol{p}}(t)) \geq \psi_n^{(a)}(\hat{\boldsymbol{p}}(t)), \tag{57}$$

Thus, for each $n \in \mathcal{N}$, we have

$$\frac{1}{\sum_{a\in\mathcal{N}\setminus\{n\}}\psi_a^{(n)}(\tilde{\boldsymbol{p}}(t))} \leq \frac{1}{\sum_{a\in\mathcal{N}\setminus\{n\}}\psi_a^{(n)}(\hat{\boldsymbol{p}}(t))}. \tag{58}$$

Replacing (58) in (28), we have

$$\tilde{\boldsymbol{p}}(t+1) \preceq \hat{\boldsymbol{p}}(t+1). \tag{59}$$

From (54) and (59), the update formulation in (28) forms a *monotone mapping* [33]. Monotone mappings satisfy both *synchronous convergence* and *box* conditions [33, pp. 431]. Thus, from the *asynchronous convergence theorem* [33], the transmission powers will converge to a fixed point, assuming that $\Upsilon = \Lambda$ is large enough. By definition, $\boldsymbol{p}^\star$ should denote the optimal solution of the local optimization problem in (25) for *all* relay nodes. Next, we show that $\boldsymbol{p}^\star$ also denotes the unique optimal solution of the maximum weight matching problem in (33). Notice that the objective function in (33) is different from the objective function in (25) as it is the weighted summation of the data rates over *all* links. Problem (33) is indeed the key resource allocation problem to be solved by the *backpressure* algorithms [23], [24]. Using the *logarithmic* change of variables (cf. [4, Theorem 1]), we can transform problem (33) to an equivalent *convex* problem. Thus, problem (33) has a unique global optimal solution (cf. [32]). Let $\boldsymbol{p}^*$ denote the unique optimal solution of problem (33). From KKT conditions, for each $n\in\mathcal{N}$ and $m\in\mathcal{N}_n^{\text{out}}$ we have

$$\frac{\delta_{nm}^{(c^\star_{nm}(t'))}(t')}{p_{nm}^{*(c^\star_{nm}(t'))}(t)} - \sum_{\substack{a\,\in\,\mathcal{N}\setminus\{n\},\\ b\,\in\,\mathcal{N}_a^{\text{out}}(t)}} h_{nb}\frac{\delta_{ab}^{(d^\star_{ab}(t'))}(t')}{I_n(\boldsymbol{p}^*_{-n}(t))+\eta_b} = \rho_n^* - \varrho_{nm}^*, \tag{60}$$

$$\rho_n^*\left(\sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{*(c^\star_{nm}(t'))}(t) - P_n^{\max}\right) = 0, \tag{61}$$

$$\varrho_{nm}^* p_{nm}^{*(c^\star_{nm}(t'))}(t) = 0, \tag{62}$$

$$\rho_n^\star \geq 0, \quad \varrho_{nm}^* \geq 0, \tag{63}$$

where for each node $n \in \mathcal{N}$, ρ_n^* denotes the Lagrange multiplier corresponding to constraint $\sum_{m\in\mathcal{N}_n^{\text{out}}} p_{nm}^{*(c^\star_{nm}(t'))}(t) \leq P_n^{\max}$ and ϱ_{nm}^* denotes the Lagrange multiplier corresponding to constraint $p_{nm}^{*(c^\star_{nm}(t'))}(t) \geq 0$. Comparing with (40)-(43), the KKT conditions (61)-(63) hold if we set $\boldsymbol{p}^* = \boldsymbol{p}^\star$, $\rho_n^* = \lambda_n^\star$, and $\varrho_{nm}^* = \sigma_{nm}^\star$, for all nodes $n \in \mathcal{N}$ and all links $(n,m)\in\mathcal{L}_n^{\text{out}}$. In this case, since

$$\sum_{a\in\mathcal{N}\setminus\{n\}}\sum_{b\in\mathcal{N}_a^{\text{out}}} h_{nb}\,\frac{\delta_{ab}^{(d^\star_{ab}(t'))}(t')}{I_n(\boldsymbol{p}^*_{-n})+\eta_b} \overset{\text{by (36)}}{=} \sum_{a\in\mathcal{N}\setminus\{n\}}\psi_a^{(n)}(t),$$

the KKT condition in (60) is also resulted from (39). Thus, $\boldsymbol{p}^* = \boldsymbol{p}^\star$ is indeed the unique optimal solution of the maximum weight matching problem in (33). In other words, given the interference pricing model in (36), problem (33) is solved every $\Upsilon = \Lambda$ time slots. This, together with (35), results in achieving maximum network utility (cf. [24] and [30, Theorem 4]). From Lemma 1, obtaining the maximum network utility also implies achieving maximum social welfare. ■

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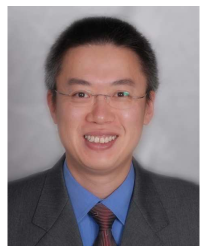

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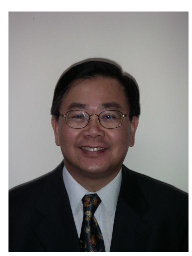

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