Optimal and Autonomous Incentive-based Energy Consumption Scheduling Algorithm for Smart Grid

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***Abstract*— In this paper, we consider deployment of *energy consumption scheduling* (ECS) devices in smart meters for autonomous demand side management within a neighborhood, where several buildings share an energy source. The ECS devices are assumed to be built inside *smart meters* and to be connected to not only the power grid, but also to a local area network which is essential for handling two-way communications in a smart grid infrastructure. They interact automatically by running a *distributed* algorithm to find the *optimal* energy consumption schedule for each subscriber, with an aim at reducing the total energy cost as well as the *peak-to-average-ratio* (PAR) in load demand in the system. Incentives are also provided for the subscribers to actually use the ECS devices via a novel pricing model, derived from a *game-theoretic* analysis. Simulation results confirm that our proposed distributed algorithm significantly reduces the PAR and the total cost in the system.**

I. Introduction

According to a report by the U.S. Department of Energy in 2008 [1], 74% of the nation's electricity consumption occurs in buildings. This represents 39% of the total energy consumption among all sectors. Currently, the electricity consumption is not efficient in most buildings, leading to the waste of billions of dollars and a major amount of extra greenhouse gas emissions.

There are two general approaches for energy consumption management in buildings: *reducing consumption* and *shifting consumption* [2]. The former can be done through raising awareness among subscribers for more careful consumption patterns as well as constructing more energy efficient buildings, e.g., with better heat isolations, less energy consuming lighting, etc. However, there is also an important need for practical solutions to *shift* high-load household appliances to *off-peak* hours in order to reduce the *peak-to-average ratio* (PAR) in load demand. Appropriate load-shifting is foreseen to become even more crucial as *plug-in hybrid electric vehicles* (PHEVs) become more popular. Most PHEVs need 0.2 - 0.3 kWh of charging power for one mile of driving [3]. This will represent a significant new load on the existing distribution system. In particular, during the *charging time*, the PHEVs double the average household load. Unbalanced conditions resulting from an increasing number of PHEVs will drastically exacerbate the already high PAR of the load demand, leading to a degradation of the power quality, voltage problems, and even potential damage to utility and consumer equipment [3].

Load management, also known as *demand side management* [4]–[6], has been practiced since the early 1980s in different forms such as direct load control and small-scale voluntary load management programs, with varying degrees of success. However, thanks to the advancements in *smart metering* technologies [7] and the increasing interest in *smart grid* infrastructure (cf. [3], [8]–[10]) with two-way digital communication capability through computer networking, we can push a modernized load management system forward and introduce *energy consumption scheduling* (ECS) devices (e.g., as part of a smart meter) that can optimally coordinate the timing of household energy consumption in each neighborhood, a high-rise building, or a large PHEV parking lot, through communication among ECS devices and also between the ECS devices and the control and dispatch centers.

Despite the importance of an efficient energy consumption scheduling system, such large-scale scheduling plans cannot be implemented unless intelligent pricing schemes are used to provide *incentives* for the subscribers to follow them. The incentives can be in form of lower utility charges.

In this paper, we consider a scenario where a source of energy (e.g., a generator or a step-down substation transformer which is connected to the grid) is *shared* by several subscribers, each one equipped with an ECS device. The ECS devices are deployed inside the smart meters and are connected to not only the power line, but also to a communication network which is essentially needed to handle two-way data communications. The ECS devices interact automatically by running a *distributed* algorithm to find the *optimal* energy consumption schedule for each subscriber, with an aim at reducing the PAR in the system. Interestingly, we can show with a *game-theoretic* analysis (cf. [11]) that a simple pricing mechanism can provide the subscribers with the incentives to *cooperate* in order to not only improve the system overall performance, but also to *pay less* individually. In other words, through an appropriate pricing scheme, the *Nash equilibrium* of an *energy consumption game* among the subscribers who are sharing a common energy source will be the exact global optimal solution of a system-wide optimization problem, making our design optimal and practical.

The rest of this paper is organized as follows. We introduce the system model and notations in Section II. This includes an elaborate mathematical formulation of the energy consumption scheduling problem as a convex optimization problem. We discuss the requirements of a valid billing scheme and also introduce the concept of an energy consumption game in Section III. Our distributed algorithm to be executed by the ECS devices is presented in Section IV. Simulation results are given in Section V. The paper is concluded in Section VI. All analytical proofs are provided in the Appendices.

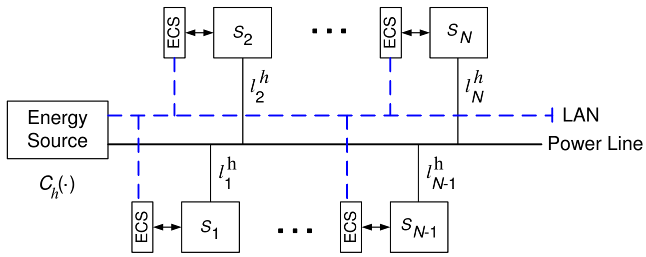

Fig. 1. A sample smart grid system with N load subscribers.

II. System Model

A. Power System

Consider a smart power system with several load subscribers and one source of energy, e.g., a generator or a step-down substation transformer connected to the electric grid. An example of such a system is illustrated in Fig. 1. We assume that each subscriber is equipped with an ECS device in its smart meter for scheduling the household energy consumption. The subscribers are all connected to the power line (solid line) coming from the energy source. The ECS devices are also connected to each other and also to the energy source through a local area network (LAN) (dashed line).

Let $\mathcal{N}$ denote the set of subscribers, where $N \triangleq |\mathcal{N}|$. For each subscriber $n \in \mathcal{N}$, let $\mathcal{A}_n$ denote the set of appliances: washer/dryer, refrigerator, PHEVs, etc. For each appliance $a \in \mathcal{A}_n$, we define *energy consumption scheduling vector*

$$\mathbf{x}_{n,a} \triangleq [x_{n,a}^1, \ldots, x_{n,a}^H], \tag{1}$$

where $H = 24$ hours. For each hour of the day $h \in \mathcal{H} \triangleq \{1, \ldots, H\}$, real-valued scalar $x_{n,a}^h$ denotes the corresponding one-hour energy consumption that is scheduled for appliance a from subscriber n. In this case, we can define the total hourly energy consumption for each subscriber $n \in \mathcal{N}$ as

$$l_n^h \triangleq \sum_{a \in \mathcal{A}_n} x_{n,a}^h, \qquad h \in \mathcal{H}. \tag{2}$$

We also define

$$E_{n,a} \triangleq \sum_{h=1}^{H} x_{n,a}^h \tag{3}$$

as the total *daily* energy consumption for appliance a from subscriber n. Here, we assume that $E_{n,a}$ is pre-determined and set by the load subscriber according to her needs. For example, $E_{n,a} = 16$ kWh for a plug-in hybrid electric sedan for a 40-mile daily driving range [3]. In fact, our designed scheduler aims *not* to change the amount of energy consumption, but instead to systematically *manage* and *shift* it, e.g., in order to reduce the PAR. In this regard, the subscriber also needs to select the *beginning* $\alpha_{n,a} \in \mathcal{H}$ and the *end* $\beta_{n,a} \in \mathcal{H}$ of a time interval that the energy consumption for appliance a is *valid* to be scheduled[1]. Clearly, $\alpha_{n,a} < \beta_{n,a}$. For example, a subscriber may select $\alpha_{n,a} = 1$ and $\beta_{n,a} = 8$ for her PHEV such that the battery charging finishes before 8:00 AM when she needs to use the vehicle. This imposes certain constraints on vector $\mathbf{x}_{n,a}$. In fact, it is required that

$$\sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h = E_{n,a}, \tag{4}$$

and

$$x_{n,a}^h = 0, \quad \forall\, h \in \{1, \ldots, \alpha_{n,a}-1\} \cup \{\beta_{n,a}+1, \ldots, H\}. \tag{5}$$

The time range set by the subscriber needs to be larger than or equal to the time interval needed to finish the charging. For example, for a single-phase PHEV the normal charging time is 3 hours [3]; therefore, it is required that $\beta_{n,a} - \alpha_{n,a} \geq 3$. We note that certain appliances may have very *strict* scheduling requirements, for example, a refrigerator, may require operation all the time. In that case, $\alpha_{n,a} = 1$ and $\beta_{n,a} = 24$.

Many home appliances may have some *maximum* power levels $\gamma_{n,a}^{\max}$, for each $a \in \mathcal{A}_n$. For example, a PHEV may be charged only up to $\gamma_{n,a}^{\max}$ = 3.3 kWh per hour [3]. This imposes the following upper-bound constraints on the choice of energy consumption scheduling vector $\mathbf{x}_{n,a}$ for each appliance a:

$$x_{n,a}^h \leq \gamma_{n,a}^{\max}, \qquad \forall\, h \in \{\alpha_{n,a}, \ldots, \beta_{n,a}\}. \tag{6}$$

Some appliances also have *minimum* stand-by power levels $\gamma_{n,a}^{\min}$, for each $a \in \mathcal{N}$. In that case, it is further required that

$$x_{n,a}^h \geq \gamma_{n,a}^{\min}, \qquad \forall\, h \in \{\alpha_{n,a}, \ldots, \beta_{n,a}\}. \tag{7}$$

In certain cases, we may require scheduling *discrete* power levels. This can be fulfilled by either rounding the continuous value of $x_{n,a}^h$ to the required discrete power levels, or redefining $x_{n,a}^h$ as a discrete variable with desired discrete levels. Discrete-level energy consumption scheduling is beyond the scope of this paper. The information needed to be set by subscriber n for appliance a is summarized in Table I.

TABLE I
System parameters to be set for each appliance $a \in \mathcal{A}_n$ by each subscriber $n \in \mathcal{N}$.

$E_{n,a}$	Total energy to be scheduled.
$\alpha_{n,a}$	Beginning of the time interval that consumption can be scheduled.
$\beta_{n,a}$	End of the time interval that consumption can be scheduled.
$\gamma_{n,a}^{\min}$	Minimum scheduled power level.
$\gamma_{n,a}^{\max}$	Maximum scheduled power level.

B. Energy Cost

Consider the *total* load at each hour of the day $h \in \mathcal{H}$:

$$L_h \triangleq \sum_{n \in \mathcal{N}} l_n^h. \tag{8}$$

We define a *cost function* $C_h(L_h)$ indicating the cost of generating or providing energy by the energy source at each hour $h \in \mathcal{H}$. We first notice that in general

$$C_{h_1}(L) \neq C_{h_2}(L), \qquad \forall\, h_1, h_2 \in \mathcal{H},\ h_1 \neq h_2. \tag{9}$$

In other words, the cost of the *same* load can be *different* at different times of the day. In particular, the cost can be less

[1]The model here can be easily extended to the case when a particular appliance is needed to be scheduled *multiple* times during the day. However, here we focus on single-time scheduling for the ease of exposition.

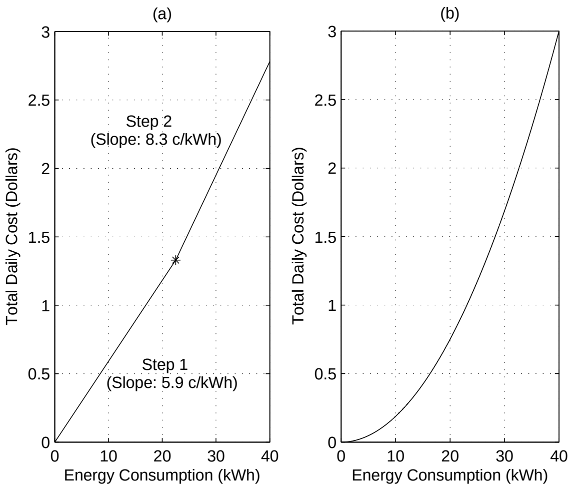

Fig. 2. Two sample convex and increasing cost functions: (a) Two-step conservation rate model used by BC Hydro [13]; (b) A quadratic cost function.

during the night compared to the day time. Furthermore, we make the following assumptions:

Assumption 1: The cost functions are *increasing* in total per-hour load. That is, for each $h \in \mathcal{H}$, we have

$$C_h(\hat{L}_h) \leq C_h(\tilde{L}_h), \quad \forall \hat{L}_h \leq \tilde{L}_h. \tag{10}$$

The inequality in (10) simply implies that the energy cost will always increase when the total load increases.

Assumption 2: The cost functions are *strictly convex*. That is, for each $h \in \mathcal{H}$, and any $\hat{L}_h, \tilde{L}_h \geq 0$, we have [12]

$$C_h(\theta \hat{L}_h + (1-\theta)\tilde{L}_h) \leq \theta C_h(\hat{L}_h) + (1-\theta) C_h(\tilde{L}_h), \tag{11}$$

where $0 \leq \theta \leq 1$. Examples of convex cost functions are shown in Fig. 2. A convex function can be a piece-wise linear function as in the *two-step conservation rate* used by British Columbia (BC) Hydro [13], as in Fig. 2(a); or a smooth differentiable quadratic function as in Fig. 2(b).

The cost function we assume is general and can represent either the actual energy cost or simply a cost model used by a utility company in order to impose a proper load-shifting.

C. Optimization Problem

Given complete knowledge about subscribers' needs and a centralized control of the system in Fig. 1, an efficient energy consumption scheduling to be implemented by ECS devices can be characterized as the solution of the following problem:

$$\begin{aligned} \min_{\mathbf{x}_1,\ldots,\mathbf{x}_N} \quad & \sum_{h=1}^{H} C_h\left(\sum_{n\in\mathcal{N}}\sum_{a\in\mathcal{A}_n} x_{n,a}^h\right) \\ \textbf{s.t.} \quad & \sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h = E_{n,a}, \qquad \forall\, a \in \mathcal{A}_n, n \in \mathcal{N}, \\ & \gamma_{n,a}^{\min} \leq x_{n,a}^h \leq \gamma_{n,a}^{\max}, \qquad \forall\, h \in \mathcal{H}_{n,a}, \forall a \in \mathcal{A}_n, n \in \mathcal{N}, \\ & x_{n,a}^h = 0, \qquad \forall\, h \in \mathcal{H} \backslash \mathcal{H}_{n,a}, \forall a \in \mathcal{A}_n, n \in \mathcal{N}. \end{aligned} \tag{12}$$

Here, for each $n \in \mathcal{N}$ and any $a \in \mathcal{A}_n$, we have

$$\mathcal{H}_{n,a} \triangleq \{\alpha_{n,a}, \ldots, \beta_{n,a}\}. \tag{13}$$

For each subscriber $n \in \mathcal{N}$, tensor $\mathbf{x}_n$ is formed by stacking up energy consumption scheduling vectors $\mathbf{x}_{n,a}$. The optimization problem in (12) is convex and can be solved using *convex programming* techniques such as the *interior point method* (IPM) [12] in a centralized fashion. However, we are interested in solving problem (12) *distributively* at the ECS devices with minimum amount of information exchanges among the ECS devices and the energy source. In particular, we would like each ECS device be able to schedule the energy consumption at the household according to the individual needs of the subscribers. It is also important to make sure that the subscribers have the incentive to actually use the ECS devices and follow the schedules they determine.

III. Energy Consumption Game

A. Pricing and Billing

For each registered subscriber $n \in \mathcal{N}$, let b_n denote the *daily* amount in dollars to be charged to subscriber n by the utility company which owns the energy source. In other words, b_n is the amount appears on the load subscriber n's bill at the end of the day. In general, we expect that the following two key properties hold for any billing model:

1) Property I: Clearly, we need to have

$$\sum_{n\in\mathcal{N}} b_n \;\geq\; \sum_{h=1}^{H} C_h\left(\sum_{n\in\mathcal{N}} l_n^h\right), \tag{14}$$

where the left hand side denotes the *total daily charge* to the subscribers and the right hand side denotes the *total daily cost*. In this regard, we can define

$$\kappa \triangleq \frac{\sum_{n\in\mathcal{N}} b_n}{\sum_{h=1}^{H} C_h\left(\sum_{n\in\mathcal{N}} l_n^h\right)} \geq 1. \tag{15}$$

If $\kappa = 1$, then the billing system is *budget balanced* and the utility company charges the subscribers only with the same amount that generating/providing energy costs for the utility.

2) Property II: It is expected that the charges for each subscriber to be *proportional* to the her total daily load. That is, we have

$$b_n \;\propto\; \sum_{h=1}^{H} l_n^h, \qquad \forall n \in \mathcal{N}. \tag{16}$$

In other words, a fair billing leads to the following equality:

$$\frac{b_n}{b_m} = \frac{\sum_{h=1}^{H} l_n^h}{\sum_{h=1}^{H} l_m^h}, \qquad \forall n, m \in \mathcal{N}. \tag{17}$$

After summing up (17) across all subscribers $m \in \mathcal{N}$, for each $n \in \mathcal{N}$, we have

$$\sum_{m\in\mathcal{N}} b_m = \sum_{m\in\mathcal{N}} \left(b_n \frac{\sum_{h=1}^{H} l_m^h}{\sum_{h=1}^{H} l_n^h}\right) = b_n \frac{\sum_{m\in\mathcal{N}} \sum_{h=1}^{H} l_m^h}{\sum_{h=1}^{H} l_n^h}. \tag{18}$$

Finally, from (2), (4), (15), and (18), we have

$$\begin{aligned} b_n &= \frac{\sum_{h=1}^{H} l_n^h}{\sum_{m\in\mathcal{N}}\sum_{h=1}^{H} l_m^h}\left(\sum_{m\in\mathcal{N}} b_m\right) \\ &= \frac{\kappa\sum_{h=1}^{H} l_n^h}{\sum_{m\in\mathcal{N}}\sum_{h=1}^{H} l_m^h}\left(\sum_{h=1}^{H} C_h\left(\sum_{m\in\mathcal{N}} l_m^h\right)\right) \\ &= \frac{\kappa\sum_{a\in\mathcal{A}_n} E_{n,a}}{\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} E_{m,a}}\left(\sum_{h=1}^{H} C_h\left(\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} x_{m,a}^h\right)\right). \end{aligned} \tag{19}$$

In other words, the only billing model that satisfies the *axiomatic* requirements in (14) and (17) is the model in (19).

B. Energy Consumption Game

From (19), the charge on each subscriber would depend on how she and *other subscribers* schedule their consumptions. This leads to the following game among subscribers:

Game 1 (Energy Consumption Game Among Subscribers):

- *Players*: Registered subscribers in set $\mathcal{N}$.
- *Strategies*: Energy consumption scheduling vectors $\mathbf{x}_n$ for all subscribers and appliances.
- *Payoffs*: $P_n(\mathbf{x}_n;\mathbf{x}_{-n})$ for each subscriber $n\in\mathcal{N}$, where

$$\begin{aligned} P_n(\mathbf{x}_n;\mathbf{x}_{-n}) &= -b_n \\ &= -\frac{\kappa\sum_{a\in\mathcal{A}_n} E_{n,a}}{\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} E_{m,a}} \\ &\quad\times\left(\sum_{h=1}^{H} C_h\left(\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} x_{m,a}^h\right)\right). \end{aligned}$$

Here, $\mathbf{x}_{-n}$ denotes the energy consumption scheduling vectors for all subscribers *other than* subscriber n.

In Game 1, the subscribers try to select their energy consumption schedule to *minimize* their payments to the utility.

Theorem 1: Suppose the cost functions $C_h(\cdot)$ are *increasing* and *strictly convex* for each $h\in\mathcal{H}$. The Nash equilibrium of Game 1 always exists and is unique.

The proof of Theorem 1 is given in Appendix A. Note that Nash equilibrium is a solution concept in game theory that characterizes how the players play a game [11]. The energy consumption scheduling variables $(\mathbf{x}_n^*,\ \forall n\in\mathcal{N})$ form a Nash equilibrium for Game 1 if and only if for each $n\in\mathcal{N}$,

$$P_n(\mathbf{x}_n^*;\mathbf{x}_{-n}^*) \geq P_n(\mathbf{x}_n;\mathbf{x}_{-n}^*), \qquad \forall\, \mathbf{x}_n \geq 0. \tag{20}$$

If the energy consumption game is at its unique Nash equilibrium, then none of the subscribers would try to deviate from schedule $(\mathbf{x}_n^*,\ \forall n\in\mathcal{N})$. Next we show the following key result on the *performance* at Nash equilibrium of Game 1.

Theorem 2: The unique Nash equilibrium of Game 1 is the optimal solution of problem (12).

The proof of Theorem 2 is given in Appendix B. From Theorem 2, as long as the cost functions $C_h(\cdot)$ are increasing and strictly convex for each $h\in\mathcal{H}$ and also the price model satisfies the axiomatic requirements (14) and (17), the subscribers have all the *incentives* to cooperate with each other in order to solve the energy consumption management problem in (12) leading to the best possible energy consumption scheduling with load-shifting and low PAR properties.

IV. Distributed Algorithm

From the results in Section III, the subscribers would be willing to cooperate and allow their ECS devices schedule their household energy consumption to pay less. In particular, we showed that the unique Nash equilibrium of the energy consumption game among the subscribers is indeed the same as the global optimal solution of energy consumption scheduling problem (12). In this section, we provide a simple algorithm to be implemented in each ECS device to reach the Nash equilibrium of Game 1 and achieve the optimal performance.

Consider an arbitrary subscriber $n\in\mathcal{N}$. Given $\mathbf{x}_{-n}$ and assuming that all other subscribers fix their energy consumption schedule according to $\mathbf{x}_{-n}$, subscriber n can maximize its *own* payoff by solving the following *local* problem:

$$\begin{aligned} \max_{\mathbf{x}_n}\quad & P_n(\mathbf{x}_n;\mathbf{x}_{-n}) \\ \text{s.t.}\quad & \sum_{h=\alpha_{n,a}}^{\beta_{n,a}} x_{n,a}^h = E_{n,a}, && \forall\, a\in\mathcal{A}_n, \\ & \gamma_{n,a}^{\min} \leq x_{n,a}^h \leq \gamma_{n,a}^{\max}, && \forall\, h\in\mathcal{H}_{n,a}, a\in\mathcal{A}_n, \\ & x_{n,a}^h = 0, && \forall\, h\in\mathcal{H}\backslash\mathcal{H}_{n,a}, a\in\mathcal{A}_n. \end{aligned} \tag{21}$$

Notice that here $\mathbf{x}_n$ is the only vector variable. Since $\frac{\kappa\sum_{a\in\mathcal{A}_n} E_{n,a}}{\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} E_{m,a}}$ is fixed and does not depend on the choice of $\mathbf{x}_n$, the maximization in (21) can be replaced by a *minimization* over $\sum_{h=1}^{H} C_h\left(\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} x_{m,a}^h\right)$. Therefore, we can replace the maximization in problem (21) equivalently with the following minimization

$$\min_{\mathbf{x}_n}\quad \sum_{h=1}^{H} C_h\left(\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} x_{m,a}^h\right). \tag{22}$$

We notice that

- Problems (22) and (12) have the *same* objective functions.
- Problem (22) has only *local* variables to subscriber n.
- Problem (22) is convex and can be solved by IPM [12].

The above observations motivate us to propose Algorithm 1 to solve problem (12). Algorithm 1 works based on the *coordinate ascent* method [14], where we fix scheduling variables across all subscribers *except* for the subscriber n, and minimize the total cost $\sum_{h=1}^{H} C_h\left(\sum_{m\in\mathcal{N}}\sum_{a\in\mathcal{A}_m} x_{m,a}^h\right)$ only with respect to $\mathbf{x}_n$ as in (22). This procedure is repeated, leading to an iterative algorithm across the subscribers.

Next, we explain how Algorithm 1 works. In Line 1, each subscriber starts with random initial conditions. Then, the loop in Lines 2 to 11 continues until the algorithm converges. Within this loop, each ECS device solves the *local* problem (22) using IPM in Lines 4 and 5 and then announces its updated schedule to other ECS devices in Line 6. It also updates its local memory whenever it receives a control message from other subscribers in Line 9. Let $\mathcal{T}_n$ denote the set of time instances at which subscriber $n\in\mathcal{N}$ solves local problem (22). We assume that:

Algorithm 1 : Executed by each subscriber $n \in \mathcal{N}$.

```
Initialization.
repeat
    if time t ∈ T_n then
        Solve local problem (22) using IPM [12].
        Update x_n according to the solution.
        Broadcast a control message to announce x_n to the
        other ECS devices across the LAN.
    end if
    if a control message is received then
        Update x_{-n} accordingly.
    end if
until no ECS device announces change of schedule.
```

(a) For any subscriber $n \neq m$, we have $\mathcal{T}_n \cap \mathcal{T}_m = \{\}$. That is, the iterative local maximizations are carried out *successively* as in the *Gauss-Seidel* mapping [14, p. 21].

(b) There is a constant $T^{\max}$ such that for each subscriber $n \in \mathcal{N}$, there exist time instances $t_1, t_2 \in \mathcal{T}_n$ such that $|t_1 - t_2| \leq T^{\max}$. In other words, all subscribers update their transmission probabilities at least once every $T^{\max}$ seconds.

These assumptions guarantee the *asynchronous convergence* of Algorithm 1 to *some* fixed point [14, Proposition 2.5, p. 208]. The convergence property is directly resulted from the coordinate ascent structure of the algorithm and the Gauss-Seidel updates. Now the questions are: (1) *Starting from different randomly selected initial schedules, does Algorithm 1 always converge to the same point?* (2) *What is the performance of a fixed point that Algorithm 1 may converge to?*

Since each subscriber updates its energy consumption scheduling variables in Algorithm 1 to maximize its *own* payoff $P_n(\mathbf{x}_n; \mathbf{x}_{-n})$, the fixed point of Algorithm 1 is the Nash equilibrium of Game 1. From Theorem 1, the Nash equilibrium of Game 1 is unique. This directly answers our first question: *Algorithm 1 always converges to the unique Nash equilibrium of Game 1*. Moreover, from Theorem 2, the unique Nash equilibrium of Game 1 is the optimal solution of problem (12). This also answers the second question: *Algorithm 1 reaches the optimal performance with respect to solving the energy consumption scheduling problem in (12).*

V. Simulation Results

In this section, we present the simulation results and assess the performance of our proposed algorithm. In our model, the example power system at Fig. 1 is assumed to home 10 load subscribers, $N = 10$. For the purpose of study, each subscriber is selected randomly to have between 10 to 20 appliances with *hard* energy consumption scheduling constraints. Such appliances include refrigerator-freezer (daily usage: 1.32 kWh), electric stove (daily usage: 1.89 kWh for self-cleaning and 2.01 kWh for regular), lighting (daily usage for 10 standard bulbs: 1.00 kWh), heating (daily usage: 7.1 kWh), etc. [15]. Moreover, each subscriber is selected randomly to also have between 10 to 20 appliances with *soft* energy consumption scheduling constraints. Recall that the ECS devices may schedule only the appliances with soft energy consumption scheduling constraints. Such appliances include dishwasher (daily usage: 1.44 kWh), clothes washer (daily usage: 1.49 kWh for energy-star 1.94 kWh for regular), clothes dryer (daily usage: 2.50 kWh), and PHEV (daily usage: 9.9 kWh), etc. [3], [15]. The scheduling durations (i.e., valid stop and ending scheduling times) are selected randomly within a 24 hour period. The simulation time is also 24 hours, starting from 7:00 AM in the morning at one day until 7:00 AM in the next morning.

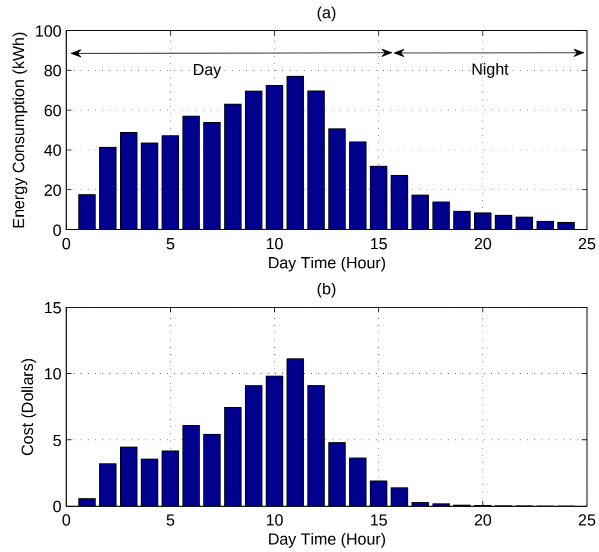

Fig. 3. Scheduled energy consumption and corresponding cost when ECS devices are not used. In this case, PAR is 2.1 and total daily cost is \$86.47.

As discussed in Section II-B, we assume that the cost functions are increasing and strictly convex as depicted in Fig. 2. We select the cost functions to be *quadratic*: $C_h(L_h) = \phi_{Day} \times (L_h)^2$ during the day and $C_h(L_h) = \phi_{Night} \times (L_h)^2$ at night, where $0 < \phi_{Night} \leq \phi_{Day}$ are constant. Without loss of generality, we select $\phi_{Night} = \frac{1}{2}\phi_{Day}$ and ϕ_{Day} = 0.1875 cents/kWh. In that case, the cost function during the day becomes as in Fig. 2(b). Assuming a budget-balanced system, we set parameter $\kappa = 1$. Last but not least, we assume that day-time cost applies to the first 16 hours of the simulation period (i.e., from 7:00 AM to 11:00 PM) and the night-time cost applies to the last 8 hours of the simulation period (i.e., from 11:00 PM to 7:00 AM on the next day).

The simulation results on total scheduled energy consumptions and total cost in the system *without* and *with* the deployment of ECS devices are shown in Figs. 3 and 4, respectively. As shown here, when the ECS devices are *not* used, the PAR is 2.1 and the total energy cost is \$86.47. At the same time, when the ECS devices are used, the PAR reduces to 1.3 (i.e., 38.1% less) and the total energy cost reduces to \$53.81 (i.e., 37.8% less). In fact, we have more *even load* in the latter case. Note that each subscriber consumes the *same* amount of energy in the two cases, but it simply schedules its consumption more efficiently in the case that the ECS devices are used. In this case, all subscribers will even pay less to the utility company as shown in Fig. 5. Therefore, the subscribers would be willing to participate in the proposed automatic demand side management system.

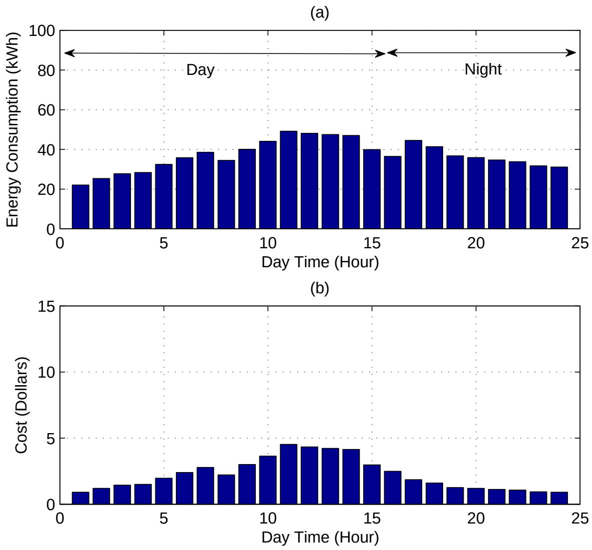

Fig. 4. Scheduled energy consumption and corresponding cost when ECS devices are deployed. In this case, PAR is 1.3 and total daily cost is \$53.81.

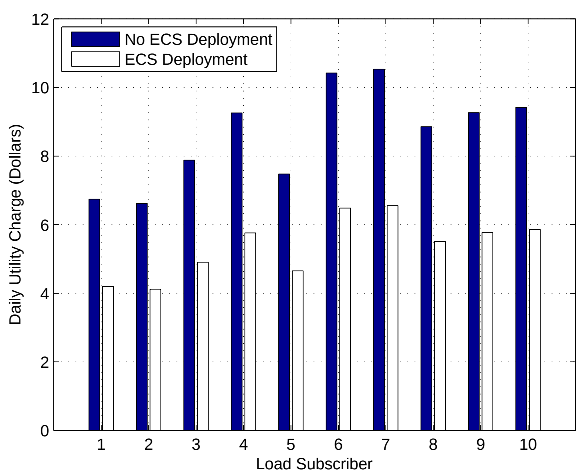

Fig. 5. Daily charges for each subscriber without and with ECS deployment.

VI. Conclusions

In this paper, we proposed an optimal, autonomous, and incentive-based energy consumption scheduling algorithm to balance the load among residential subscribers that share a common energy source. The proposed algorithm is designed to be implemented in energy consumption scheduling (ECS) devices inside smart meters in a smart grid infrastructure. We also proposed a simple pricing and billing model which provides the incentives for the subscribers encouraging them to actually use the ECS devices and run the proposed distributed algorithm in order to be charged less. Simulation results confirm that our proposed algorithm significantly reduces the PAR as well as the total energy cost in the system.

Appendix

A. Proof of Theorem 1

We first notice that since $C_h(\cdot)$ is *strictly convex* for each $h \in \mathcal{H}$, the payoff function $P_n(\mathbf{x}_n; \mathbf{x}_{-n})$ is *strictly concave* with respect to $\mathbf{x}_n$. Therefore, Game 1 is a strictly concave N-person game. In this case, the existence of a Nash equilibrium directly results from [16, Theorem 1]. Moreover, the Nash equilibrium is unique due to [16, Theorem 3]. ■

B. Proof of Theorem 2

We first show that the global optimal solution of problem (12) forms a Nash equilibrium for Game 1. For notational simplicity, let $\mathbf{x}_1^\star, \ldots, \mathbf{x}_N^\star$ denote the optimal solutions for problem (12). We also define

$$C^\star \triangleq \sum_{h=1}^{H} C_h \left(\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} x_{m,a}^{h\star} \right). \tag{23}$$

By definition of optimality, for each subscriber $n \in \mathcal{N}$ and for any arbitrary $\mathbf{x}_n \geq 0$, we have

$$C^\star \leq \sum_{h=1}^{H} C_h \left(\sum_{m \in \mathcal{N} \setminus \{n\}} \sum_{a \in \mathcal{A}_m} x_{m,a}^{h\star} + \sum_{a \in \mathcal{A}_n} x_{n,a}^{h} \right). \tag{24}$$

After multiplying both sides in (24) by negative constant $\frac{-\kappa \sum_{a \in \mathcal{A}_n} E_{n,a}}{\sum_{m \in \mathcal{N}} \sum_{a \in \mathcal{A}_m} E_{m,a}}$, it becomes

$$P_n(\mathbf{x}_n^\star; \mathbf{x}_{-n}^\star) \geq P_n(\mathbf{x}_n; \mathbf{x}_{-n}^\star), \quad \forall \mathbf{x}_n \geq 0. \tag{25}$$

Comparing (25) and (20), we can conclude that the optimal solution $\mathbf{x}_1^\star, \ldots, \mathbf{x}_N^\star$ forms a Nash equilibrium for Game 1. However, from Theorem 1, Game 1 has a unique Nash equilibrium. Thus, the optimal solution of problem (12) is equivalent to the Nash equilibrium of Game 1. ■

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