Inter-Session Network Coding with Strategic Users: A Game-Theoretic Analysis of the Butterfly Network

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***Abstract*— We analyze *inter-session network coding* in a wired network using game theory. We assume that users are *selfish* and act as *strategic* players to maximize their own utility, which leads to a resource allocation *game* among users. In particular, we study a *butterfly* network, where a bottleneck link is shared by network coding and routing flows. We assume that network coding is performed using pairwise XOR operations. We prove the existence of Nash equilibrium for a wide range of utility functions. We also show that the number of Nash equilibria can be large (even *infinite*) for certain choices of parameters. This is in sharp contrast to a similar game setting with traditional packet forwarding, where the Nash equilibrium is always unique. We characterize the worst-case efficiency bound, i.e., the *Price-of-Anarchy* (PoA), compared to an *optimal* and *cooperative* network design. We show that by using a *discriminatory pricing* scheme which charges encoded and forwarded packets differently, we can improve the PoA in comparison with the case where a *single* pricing scheme is used. However, even when a discriminatory pricing scheme is used, the PoA is still worse than for the case when network coding is not applied. This implies that, although inter-session network coding can improve performance compared to routing, it is *much more sensitive* to users' strategic behavior.**

***Keywords*: Inter-session network coding, butterfly network, game theory, Nash equilibrium, price-of-anarchy, efficiency bound.**

I. Introduction

Network coding is performed by *jointly* encoding multiple packets either from the *same* user or from *different* users. The former is *intra-session* network coding [1] while the latter is *inter-session* network coding [2], [3]. A common assumption in most prior network coding literature is that users are *cooperative* and do *not* pursue their own interests. However, this assumption can be violated in practice. Therefore, assuming that the users are *selfish* and *strategic*, in this paper we ask the following key questions: (a) *What is the impact of users' strategic behavior on network performance?* (b) *How does this impact change with different link pricing schemes?*

It is widely accepted that *pricing* can improve the efficiency of network resource allocation in *distributed* settings. In [4], Kelly *et al.* showed that if users are *price-taker* (i.e., they treat network prices as *fixed*), efficient resource allocation is achieved by properly setting *congestion prices* on each shared link. Recently, Johari *et al.* studied how the results can change in capacity-constrained [5] and capacity-unconstrained [6] networks if users are *price-anticipator* who realize that the price is impacted by each user's behavior. In this case, users play a *game*, and the efficiency of resource allocation is characterized by the Nash equilibrium. A key performance metric is the *Price-of-Anarchy (PoA)*, which measures the *worst-case efficiency loss* at a Nash equilibrium due to users' price anticipating behavior. The PoA equals 1 if there is *no* efficiency loss. A smaller PoA indicates more efficiency loss.

The game theoretic analysis of network coding has received limited attention, e.g., in [7]–[13]. The results in [7]–[10] focus on *intra-session* network coding. In [12], the authors calculated the PoA for a class of inter-session network coding games that use reverse carpooling. Their analysis is specific to wireless networks while our focus is on wired networks. Moreover, in [12], users' strategies are their choices of unicast routes. Here, users' strategies are rather defined as their data rates. Users can also decide on whether and at what rate they want to participate in network coding. Since we take into account the links' cost functions and the users' utility functions, the PoA is evaluated with respect to the optimal solution of a network surplus maximization problem. The authors in [13] considered a game theoretic analysis of inter-session network coding between *two users* that share a link. It is shown that a rate allocation mechanism can enforce cooperation among users. In this paper, we assume that there are $N \geq 2$ users in a wired network, two of which can perform network coding via pairwise XOR operations, while the rest only use routing. This setting helps us better understand the interaction between network coding and routing flows. Moreover, we consider the impact of the *utility functions* of users, the cost of side links, price anticipation, price discrimination, and the PoA which are all not addressed in [13]. Our contributions are as follows:

- *New problem formulation*: We formulate the problem of maximizing the *network surplus* for inter-session network coding. This problem has not been studied before.
- *Innovative pricing*: We introduce a two *discriminatory pricing* scheme that charges network coding and routing packets differently. This new pricing is a better choice in reflecting the *actual load* generated by each user.
- *Characterization of Nash equilibria*: We prove that a Nash equilibrium always exists but it may not be unique.
- *PoA calculation with zero-cost side links*: Even with the new pricing method, the PoA is still smaller (i.e., worse) than the case without network coding. In fact, the PoA can be as low as 25%, which is less than the well-known 67% worst-case efficiency in [6] for routing networks.
- *PoA Calculation with non-zero-cost side links*: We show that if the side links in the butterfly network have non-zero cost, then the PoA can further reduce to only 20%, where no user is willing to participate in network coding.

The key results of this paper together with a comparison with the related state-of-the-art results for the case *without*

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TABLE I

SUMMARY OF THE RESULTS VERSUS THE STATE-OF-THE-ART IN [6].

Network Setup	Routing	Network Coding and Routing: *Zero*-Cost Side Link	Network Coding and Routing: *Non-zero*-Cost Side Link
Optimization	Problem 1	Problem 2	Problem 3
Game	Game 1	Game 2	Game 3
Nash Equilibria	One	Can be *infinite*	One
Price-of Anarchy	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{1}{5}$
Theorem	Theorem 1	Theorem 8	Theorem 10
Reference	[6]	This Paper	

network coding in [6] are summarized in Table I.

II. BACKGROUND: RESOURCE ALLOCATION GAME WITH ROUTING FLOWS

We first review a resource allocation game described in [4]–[6], where multiple end-to-end users compete to send packets through a shared link as in Fig. 1. No inter-session network coding is performed in this case. We will briefly summarize the results in [6], which serves as benchmark for our later discussions. In Fig. 1, a set of users $\mathcal{N} = \{1, \ldots, N\}$ *shares* the bottleneck link (i, j) between nodes i and j. All packets that arrive at node i are simply *forwarded* to node j through link (i, j). For each user $n \in \mathcal{N}$, we denote the transmitter and receiver nodes by s_n and t_n, respectively. Let x_n denote the transmission rate of user $n \in \mathcal{N}$. We assume that each user $n \in \mathcal{N}$ has a *utility function* U_n, representing its satisfaction about its data rate x_n. On the other hand, the shared link has a *cost function* C, which depends on the total rate (i.e., $\sum_{n\in\mathcal{N}} x_n$). As in [6], we make the following assumptions:

Assumption 1: For each $n \in \mathcal{N}$, $U_n(x_n)$ is *concave*, *non-negative*, *increasing*, and *differentiable*.

Assumption 2: The cost and price functions for link (i, j) are chosen such that $C(q) = \int_0^q p(z)\, dz$. In particular, we assume that the link cost function is quadratic, $C(q) = \frac{a}{2} q^2$, and the link price function is linear, $p(q) = aq$. Quadratic cost functions and linear price functions are the only cost and price functions that satisfy the four axioms of rescaling, consistency, additivity, and positivity in cost-sharing systems [14].

Assumption 1 is used to model applications with *elastic* traffic, e.g., file transfer protocol (FTP) [4]. Examples of utility functions that satisfy Assumption 1 include the α-fair utility functions with $\alpha \in (0, 1)$ [15]. Assumption 2 is also common in the network resource management (cf. [16]). In practice, cost function C may reflect the *actual cost* of transmitting units of data over link (i, j) or simply an *approximate of the delay* that the packets experience over link (i, j). The more the aggregate data on the link, the higher is the average delay.

Let $\boldsymbol{x} = (x_1, \ldots, x_N)$. Given *complete* knowledge and *centralized* control of the network, an efficient rate allocation can be reached as a solution of the following problem:

Problem 1:

$$\begin{aligned} \underset{\boldsymbol{x}}{\text{maximize}} \quad & \textstyle\sum_{n=1}^N U_n(x_n) - C\left(\sum_{n=1}^N x_n\right) \\ \text{subject to} \quad & x_n \geq 0, \quad n = 1, \ldots, N. \end{aligned}$$

The objective function in Problem 1 is the *network aggregate surplus* [16], [17]. Problem 1 is a *convex* program. Therefore, if link (i, j), or another network authority, has full control over the end-users, then optimal resource management can be achieved by forcing users to set their rates according to the centrally obtained optimal solutions of Problem 1. However, in practice, users may have full control over their own transmission rates. As a result, a distributed approach is more desirable. To implement a distributed resource management, link (i, j) can use *pricing*. In particular, following the price-based design in [4], link (i, j) may introduce a single price:

$$\mu(\boldsymbol{x}) = p\left(\textstyle\sum_{n=1}^N x_n\right) \tag{1}$$

for each unit of data rate it carries. Each user $n \in \mathcal{N}$ then pays $x_n \mu(\boldsymbol{x})$ for its data rate x_n that goes into the shared link.

Next, we analyze how the users set their rates based on the price set by link (i, j). If users are *price takers*, then each user $n \in \mathcal{N}$ selects its rate x_n to maximize its *own* surplus (utility minus payment) by solving the following *local* problem [4]:

$$\max_{x_n \geq 0} \; (U_n(x_n) - x_n \mu) \quad \Rightarrow \quad x_n = {U_n'}^{-1}(\mu). \tag{2}$$

From the first fundamental theorem of welfare economics, if each user $n \in \mathcal{N}$ selects its rate as in (2), then the network aggregate surplus is maximized at equilibrium [17, p. 326].

Next, we consider *price anticipating* users, where each user anticipates the effect of its data rate on the price. In this case, each user $n \in \mathcal{N}$ no longer selects its rate as in (2). Instead, it *strategically* selects x_n to maximize its surplus given the knowledge that the price $\mu(\boldsymbol{x})$ is set according to (1) and is *not* fixed; rather it depends on user n's strategy x_n, as well as all other users' strategies $\boldsymbol{x}_{-n}$. Clearly, the decision made by user n also depends on the rates selected by other users, leading to a *resource allocation game* among all users:

Game 1:
- *Players*: Users in set $\mathcal{N}$.
- *Strategies*: Transmission rates $\boldsymbol{x}$ for all users.

TABLE II

LIST OF KEY NOTATIONS

$\mathcal{N}, N$	Set of all users in the network and its cardinality.
s_n, t_n	Transmitter and receiver nodes of user $n \in \mathcal{N}$.
(i, j)	Shared link between intermediate nodes i and j.
x_n	Data rate of user $n \in \mathcal{N}$.
$\boldsymbol{x}_{-n}$	Vector of data rates of all users other than user n.
$\boldsymbol{x}$	Vector of data rates of all users.
U_n	Utility function of user $n \in \mathcal{N}$.
γ_n	The slope of linear utility function of user $n \in \mathcal{N}$.
C, p	Cost and price functions of shared bottleneck link (i, j).
a	Price parameter, $p(q) = aq$.
μ, δ	Price values for routed and network coding packets.
β	Price discrimination parameter.
P_n, Q_n	Payoff of user $n \in \mathcal{N}$ in Game 1 and Game 2.
$\boldsymbol{x}^S$	Optimal solution for Problems 1 and 2.
$\boldsymbol{x}^*$	Nash equilibrium for Games 1 and 2.
X_1, X_N	Packets sent from source nodes s_1 and s_N, respectively.
$X_1 \oplus X_N$	Packet obtained by joint encoding of packets X_1 and X_N.
C_1, C_N	Cost functions of side links (s_1, t_N) and (s_N, t_1).
p_1, p_N	Price functions of side links (s_1, t_N) and (s_N, t_1).
a_1, a_N	Price parameters, $p_1(q) = a_1 q$ and $p_N(q) = a_N q$.
y_n	Data rate for routed packets of user $n \in \mathcal{N}$.
z_1, z_N	Data rate for encoded packets of users 1 and N.
v_1, v_N	Data rate for remedy packets of users 1 and N.
$\boldsymbol{y}_{-n}$	Data rates for routed packets of all users other than user n.
W_n	Payoff function of user $n \in \mathcal{N}$ in Game 3.
$\boldsymbol{y}^S, \boldsymbol{z}^S, \boldsymbol{v}^S$	Optimal solution for Problem 3.
$\boldsymbol{y}^*, \boldsymbol{z}^*, \boldsymbol{v}^*$	Nash equilibrium for Game 3.

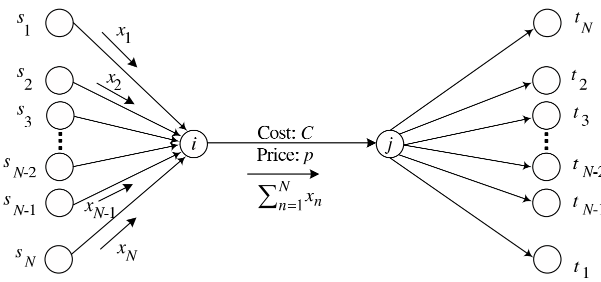

Fig. 1. A single bottleneck link shared by N *routing* flows [6].

- *Payoffs*: $P_n(x_n;\boldsymbol{x}_{-n})=U_n(x_n)-x_np(x_n+\sum_{r=1,r\neq n}^{N} x_r)$, where $\boldsymbol{x}_{-n}=x_1,\ldots,x_{n-1},x_{n+1},\ldots,x_N$.

In Game 1, each user $n\in\mathcal{N}$ selects its rate $x_n\geq 0$ to *maximize* its payoff $P_n(x_n;\boldsymbol{x}_{-n})$. At a Nash equilibrium $\boldsymbol{x}^*=(x_1^*,\ldots,x_N^*)$, no user $n\in\mathcal{N}$ can increase its payoff by *unilaterally* changing its strategy x_n. We note that, Game 1, as well as all other games that we define in this paper are games with complete information, where users are aware of the cost models and other users' utility functions.

Definition 1: Let $\boldsymbol{x}^S=(x_1^S,\ldots,x_N^S)$ be an optimal solution for Problem 1 and $\boldsymbol{x}^*$ be a Nash equilibrium for Game 1 for the *same* choice of system parameters. We can define:

$$\text{Efficiency at } \boldsymbol{x}^*=\frac{\sum_{n=1}^{N}U_n\left(x_n^*\right)-C\left(\sum_{n=1}^{N}x_n^*\right)}{\sum_{n=1}^{N}U_n\left(x_n^S\right)-C\left(\sum_{n=1}^{N}x_n^S\right)}. \tag{3}$$

Definition 2: The *price-of-anarchy* is the *worst-case* efficiency of a Nash equilibrium of Game 1 among *all* possible choices of system parameters (i.e., number of users, utility, cost, and price functions) under Assumptions 1 and 2.

The following key result is based on [6, Theorem 3]:

Theorem 1: Game 1 has a *unique* Nash equilibrium and

$$\text{PoA}\left(\text{Game 1},\text{Problem 1}\right)=\frac{2}{3}\approx 67\%. \tag{4}$$

The PoA indicates how bad the network performance can become due to strategic behavior of end-users.

III. Inter-Session Network Coding Games with Zero Side Link Costs

In this section, we reformulate Problem 1 and Game 1 for a network with both routing and inter-session network coding flows. We show that the new game may have *multiple* Nash equilibria and the PoA will significantly reduced to 25%.

A. Problem Formulation

Consider the *butterfly* network in Fig. 2 [18]. Compared to Fig. 1, it has two direct *side* links (s_1,t_N) and (s_N,t_1). The source node of user 1 is located closer to the destination node of user N than to its own destination node (and vice versa). Thus, users 1 and N can perform *inter-session* network coding. In this regard, we must distinguish two types of users:

- *Network Coding Users*: Users 1 and N, who *can* perform inter-session network coding.
- *Routing Users*: Users $2,\ldots,N-1$, who *cannot* perform inter-session network coding.

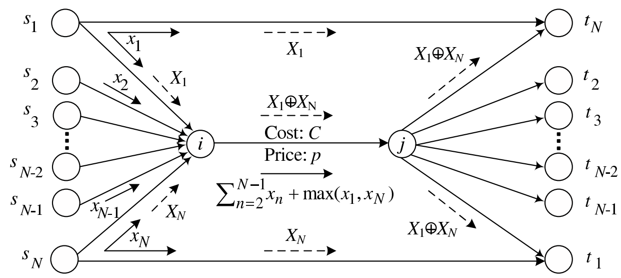

Fig. 2. A butterfly network with one shared two side links. The side links (s_1,t_N) and (s_N,t_1) are assumed to be free of charge in this setting.

Let X_1 and X_N denote packets sent from source nodes s_1 and s_N, respectively. Node i can jointly encode packets X_1 and X_N using pairwise XOR operations, and then send out the resulting encoded packet, denoted by $X_1\oplus X_N$, towards node j (and from there towards t_1 and t_N). Given the *remedy* data X_1 from the side link (s_1,t_N) and the *remedy* data X_N from the side link (s_N,t_1), nodes t_N and t_1 can again use XOR operation to *decode* the encoded packets that they receive. In fact, nodes t_1 and t_N can decode both X_1 and X_N. The benefit of network coding is to reduce the load on link (i,j) (thus reducing the cost) while achieving the *same* data rates compared to the case that no network coding is performed.

Assumption 3: Side links (s_1,t_N) and (s_N,t_1) in Fig. 2 have *zero* cost and impose *zero* prices.

For example, if the link cost is used to model the link *delay* and the side links (s_1,t_N) and (s_N,t_1) have a higher capacity than the shared link (i,j), then the costs of the side links can be neglected. The case where the side links have *non-zero* cost is studied in Section IV. For the network in Fig. 2, the network aggregate surplus maximization problem becomes:

Problem 2:

$$\begin{array}{ll}\underset{\boldsymbol{x}}{\text{maximize}} & \displaystyle\sum_{n=1}^{N}U_n\left(x_n\right)-C\left(\sum_{n=2}^{N-1}x_n+\max(x_1,x_N)\right)\\ \text{subject to} & x_n\geq 0,\quad n=1,\ldots,N.\end{array}$$

The intuition behind the objective in Problem 2 is as follows. Since x_1 and x_N are selected *independently* by users 1 and N, in general, $x_1\neq x_N$. Thus, regardless of the choice of an *efficient* network coding scheme, node i can network code *only* at rate $\min(x_1,x_N)$. Those packets which are *not* encoded (e.g., at rate $x_1-\min(x_1,x_N)$ if $x_1\geq x_N$, and at rate $x_N-\min(x_1,x_N)$ if $x_1\leq x_N$) are simply *forwarded*, leading to a total rate $\sum_{n=2}^{N-1}x_n+\max(x_1,x_N)$ on link (i,j). If $x_1=x_N$, then *all* packets from users 1 and N are jointly encoded.

Theorem 2: Let $\boldsymbol{x}^S=(x_1^S,\ldots,x_N^S)$ be an *optimal* solution for Problem 2. We have $x_1^S=x_N^S$.

The proof is based on solving the Karush-Kuhn-Tucker (KKT) optimality conditions. Problem 2 can be solved in a distributed fashion again via pricing. Following the same pricing scheme in Section II, the shared link may apply a *single* price for *all* (i.e., coded and routed) packets:

$$\mu(\boldsymbol{x})=p\left(\sum_{n=2}^{N-1}x_n+\max(x_1,x_N)\right). \tag{5}$$

Each user n pays $x_n\mu(\boldsymbol{x})$. However, this causes *double charging* for *encoded* packets. Thus, the *single* pricing model in (5) leads to more payment from users than the actual link cost. This can be avoided by *price discrimination*, i.e., charging the routed and network-coded packets with *different* prices.

Let $\mu(\boldsymbol{x})$ in (5) denote the price to be charged for routed packets. Under the discriminatory pricing scheme, we define another price $\delta(\boldsymbol{x})$ for network coded packets. We have

$$\delta(\boldsymbol{x}) = \beta\, \mu(\boldsymbol{x}), \tag{6}$$

where $0<\beta\leq 1$. Since encoded packets carry data from users 1 and N, they are both charged for the delivery of an encoded packet. As a result, if $\beta > \frac{1}{2}$, then the combined payment from users 1 and N for delivery of an encoded packet becomes *higher* than the payment that each user makes for the delivery of a routed packet. Similarly, if $\beta < \frac{1}{2}$, then the combined payment from users 1 and N for delivery of an encoded packet becomes *lower* than the payment that each user makes for the delivery of a routed packet. Therefore, in this paper, we focus on the case of $\beta = \frac{1}{2}$ because this is the only choice of β that avoids *over*- or *under*-charging with two network coding flows. Based on the this pricing scheme, user 1 pays $\min(x_1, x_N)\delta(\boldsymbol{x}) + (x_1 - \min(x_1, x_N))\, \mu(\boldsymbol{x})$. That is, it pays for transmission of its encoded packets at a price of $\delta(\boldsymbol{x})$ and for transmission of its forwarded (not coded) packets at a price of $\mu(\boldsymbol{x})$. From (6), the *total* payment by user 1 is

$$(x_1 - (1-\beta)\min(x_1, x_N))\, \mu(\boldsymbol{x}). \tag{7}$$

A similar payment model applies to user N. Each routing user $n = 2, \ldots, N-1$ pays $x_n \mu(\boldsymbol{x})$.

We are now ready to define a resource allocation game for the network setting in Fig. 2, when users can anticipate prices μ and δ according to (5) and (6), respectively:

Game 2:

- *Players*: Users in set $\mathcal{N}$.
- *Strategies*: Transmission rates $\boldsymbol{x}$ for all users.
- *Payoffs*: For network coding users 1 and N, we have

$$Q_1(x_1; \boldsymbol{x}_{-1}) = U_1(x_1) - (x_1 - (1-\beta)\min(x_1, x_N)) \times p\left(\textstyle\sum_{r=2}^{N-1} x_r + \max(x_1, x_N)\right),$$

$$Q_N(x_N; \boldsymbol{x}_{-N}) = U_N(x_N) - (x_N - (1-\beta)\min(x_1, x_N)) \times p\left(\textstyle\sum_{r=2}^{N-1} x_r + \max(x_1, x_N)\right),$$

and each routing user $n \in \mathcal{N}\backslash\{1, N\}$ has

$$Q_n(x_n; \boldsymbol{x}_{-n}) = U_n(x_n) - x_n p(\textstyle\sum_{r=2}^{N-1} x_r + \max(x_1, x_N)).$$

In the rest of this section, we answer the following questions:

1) Does Game 2 always (i.e., for *any* choice of system parameters) have a Nash equilibrium?
2) If a Nash equilibrium exists for Game 2, is it unique?
3) What is the *worst-case* efficiency (i.e., the PoA) at a Nash equilibrium of Game 2?

B. Existence and Non-uniqueness of Nash Equilibria

A Nash equilibrium of Game 2 is a non-negative data rate vector such that for all users $n \in \mathcal{N}$, we have $Q_n(x_n^*; \boldsymbol{x}_{-n}^*) \geq Q_n(\bar{x}_n; \boldsymbol{x}_{-n}^*)$ for any choice of $\bar{x}_n \geq 0$.

Theorem 3: Game 2 has *at least* one Nash equilibrium.

The proof of Theorem 3 is the direct application of the Rosen's existence theorem for N-person games [19, Theorem 1] and is omitted. It is based on showing that for *all* users $n \in \mathcal{N}$, the payoff function $Q_n(x_n; \boldsymbol{x}_{-n})$ is a *concave* function with respect to x_n, even though Q_1 and Q_N are *not* differentiable due to the max and min functions. Regarding the second question in Section III-A, we will see in Section III-C that Game 2 may have multiple Nash equilibria.

C. Users' Best Responses

The strategic behavior of users can be modeled based on their *best responses*. In this regard, each user $n \in \mathcal{N}$ selects its data rate as x_n^B to *maximize* its own payoff Q_n, given $\boldsymbol{x}_{-n}$:

$$x_n^B(\boldsymbol{x}_{-n}) \;=\; \operatorname*{arg\,max}_{x_n \geq 0}\; Q_n(x_n; \boldsymbol{x}_{-n}), \quad \forall n \in \mathcal{N}. \tag{8}$$

Since problem (8) is *concave*, for each *routing* user $n \in \mathcal{N}\backslash\{1, N\}$, $x_n^B(\boldsymbol{x}_{-n})$ is the solution of

$$U_n'(x_n) - a\left(\sum_{r=2, r\neq n}^{N-1} x_r + \max(x_1, x_N)\right) - 2a x_n = 0. \tag{9}$$

However, the best response for *network coding* users 1 and N is more complex, due to the non-differentiability of the payoff functions $Q_1(x_1; \boldsymbol{x}_{-1})$ and $Q_N(x_N; \boldsymbol{x}_{-N})$. In fact, network coding user 1 should *separately* examine two scenarios:

(a) Selecting its strategy x_1 to be *greater* than or equal to x_N:

$$\tilde{x}_1^B(\boldsymbol{x}_{-1}) = \operatorname*{arg\,max}_{x_1 \geq x_N} U_1(x_1) - (x_1 - (1-\beta)x_N) \times a\left(\sum_{n=2}^{N-1} x_n + x_1\right). \tag{10}$$

(b) Selecting its strategy x_1 to be *less* than or equal to x_N:

$$\hat{x}_1^B(\boldsymbol{x}_{-1}) = \operatorname*{arg\,max}_{0\leq x_1 \leq x_N}\; U_1(x_1) - \beta x_1\; a\left(\sum_{n=2}^{N-1} x_n + x_N\right). \tag{11}$$

In (10), since $x_1 \geq x_N$, we have: $\min(x_1, x_N) = x_N$ and $\max(x_1, x_N) = x_1$. Thus, $Q_1(x_1; \boldsymbol{x}_{-1})$ reduces to the objective function in (10). In (11), since $x_1 \leq x_N$, we have: $\min(x_1, x_N) = x_1$, $\max(x_1, x_N) = x_N$, and $x_1 - (1-\beta)\min(x_1, x_N) = \beta x_1$. Thus, $Q_1(x_1; \boldsymbol{x}_{-1})$ reduces to the objective function in (11). Given $\tilde{x}_1^B(\boldsymbol{x}_{-1})$ and $\hat{x}_1^B(\boldsymbol{x}_{-1})$, if $Q_1(\tilde{x}_1^B(\boldsymbol{x}_{-1}); \boldsymbol{x}_{-1}) \geq Q_1(\hat{x}_1^B(\boldsymbol{x}_{-1}); \boldsymbol{x}_{-1})$, then user 1 selects its best response $x_1^B(\boldsymbol{x}_{-1}) = \tilde{x}_1^B(\boldsymbol{x}_{-1})$; otherwise, it selects $x_1^B(\boldsymbol{x}_{-1}) = \hat{x}_1^B(\boldsymbol{x}_{-1})$. The best response for user N is obtained similarly. For user 1, the data rate $\tilde{x}_1^B(\boldsymbol{x}_{-1})$ is obtained as the value of $x_1 \geq x_N$ that satisfies

$$U_1'(x_1) - a\left(\sum_{n=2}^{N-1} x_n + x_1\right) + a(1-\beta)x_N - a x_1 = 0. \tag{12}$$

If $U_1(x_1)$ is *non-linear*, then $\hat{x}_1^B(\boldsymbol{x}_{-1})$ is obtained as the value of $x_1 \in [0, x_N]$ that satisfies

$$U_1'(x_1) - \beta a\left(\sum_{n=2}^{N-1} x_n + x_N\right) = 0. \tag{13}$$

When the utility function $U_1(x_1)$ is *linear* (i.e., $U_1'(x_1)$ is a *constant* for all $x_1 \geq 0$), we have $\hat{x}_1^B(\boldsymbol{x}_{-1}) = x_N$, if $U_1'(x_1) > \beta a(\sum_{n=2}^{N-1} x_n + x_N)$; and $\hat{x}_1^B(\boldsymbol{x}_{-1}) = 0$, if $U_1'(x_1) < \beta a(\sum_{n=2}^{N-1} x_n + x_N)$. If $U_1'(x_1) = \beta a(\sum_{n=2}^{N-1} x_n + x_N)$, then $\hat{x}_1^B(\boldsymbol{x}_{-1})$ can be *any* value between 0 and x_N.

D. Nash Equilibrium and Price-of-Anarchy

Let $\mathcal{X}^*$ denote the set of *all* Nash equilibria of Game 2. Recall that set $\mathcal{X}^*$ has *at least* one member as shown in Theorem 3. By definition, for any Nash equilibrium $\boldsymbol{x}^* \in \mathcal{X}^*$, given $\boldsymbol{x}^*_{-n}$, we have $x^B_n(\boldsymbol{x}^*_{-n}) = x^*_n$ for all $n \in \mathcal{N}$. Thus, all Nash equilibria of Game 2 can be obtained using (11), (12), (13) that only depend on the *first derivatives* of the utility functions. Therefore, for each Nash equilibrium $\boldsymbol{x}^* \in \mathcal{X}^*$, if we define the following *linear* utility functions:

$$\bar{U}_n(x_n) \;=\; U'_n(x^*_n)\, x_n, \qquad \forall n \in \mathcal{N}, \tag{14}$$

then $\boldsymbol{x}^*$ continues to be a Nash equilibrium for a *new* game with *new* utilities $\bar{U}_1(x_1), \ldots, \bar{U}_N(x_N)$. In fact, $\boldsymbol{x}^*$ is a Nash equilibrium for the *family* of games with utility functions $U_1(x_1), \ldots, U_N(x_N)$ having their first derivatives equal to $U'_1(x^*_1), \ldots, U'_N(x^*_N)$ at Nash equilibrium, respectively [6].

Theorem 4: Let $\sigma = \max\big\{U'_2(x^*_2), \ldots, U'_{N-1}(x^*_{N-1}), U'_1(x^*_1) + U'_N(x^*_N)\big\}$. For each Nash equilibrium $\boldsymbol{x}^* \in \mathcal{X}^*$ of Game 2 and any optimal solution $\boldsymbol{x}^S$ of Problem 2, we have:

$$\begin{aligned}&\frac{\sum_{n=1}^{N} U_n(x^*_n) - C\left(\sum_{n=2}^{N-1} x^*_n + \max(x^*_1, x^*_N)\right)}{\sum_{n=1}^{N} U_n(x^S_n) - C\left(\sum_{n=2}^{N-1} x^S_n + \max(x^S_1, x^S_N)\right)} \\ &\quad \geq \frac{\sum_{n=1}^{N} \bar{U}_n(x^*_n) - C\left(\sum_{n=2}^{N-1} x^*_n + \max(x^*_1, x^*_N)\right)}{\max_{\tilde{q} \geq 0}\left[\,\sigma\,\tilde{q} - C(\tilde{q})\,\right]}.\end{aligned} \tag{15}$$

The proof of Theorem 4 is similar [6, Lemma 4]. Note that $\max_{\tilde{q} \geq 0}\left[\,\sigma\,\tilde{q} - C(\tilde{q})\,\right]$ is the optimal objective of Problem 2 when utilities are *linear*. Thus, the right hand side in (15) is the efficiency for *linear* utility functions while the left hand side is the efficiency for *any* utility function, assuming that other parameters are fixed. We can rewrite Theorem 4 as:

Theorem 5: The *worst-case* efficiency at a Nash equilibrium of Game 2 occurs when the utility functions are *linear*. That is, $U_n(x_n) = \gamma_n x_n$, where $\gamma_n > 0$ for all users $n \in \mathcal{N}$.

From Theorem 5, the efficiency at Nash equilibrium depends on the concavity (i.e., the second derivative) of the utility functions. Note that, a liner utility is a least concave utility function that satisfies Assumption 1. Next, we obtain the value(s) of the Nash equilibrium(s) and PoA for Game 2.

Theorem 6: Suppose the utility functions are *linear*. Assume that $N \geq 2$ and let $\boldsymbol{x}^*$ denote the Nash equilibrium for Game 2. Without loss of generality, assume that $\gamma_1 \geq \gamma_N$. For notational simplicity, we define $q^* = \sum_{n=2}^{N-1} x^*_n$.

(a) If $\gamma_N \leq \gamma_1 \leq \left(1 + \frac{1}{\beta}\right)\gamma_N - \beta a q^*$, then

$$\max\left\{0, \frac{\gamma_1 - a q^*}{a(1+\beta)}\right\} \leq x^*_1 = x^*_N \leq \max\left\{0, \frac{\gamma_N - \beta a q^*}{\beta a}\right\}. \tag{16}$$

(b) If $\left(1 + \frac{1}{\beta}\right)\gamma_N - \beta a q^* \leq \gamma_1 \leq \frac{2}{\beta}\gamma_N - a q^*$, then

$$x^*_1 = \frac{\gamma_N}{\beta a} - q^*, \qquad x^*_N = \frac{\frac{2}{\beta}\gamma_N - \gamma_1}{a(1-\beta)} - \frac{q^*}{1-\beta}. \tag{17}$$

(c) If $\gamma_1 \geq \frac{2}{\beta}\gamma_N - a q^*$, then

$$x^*_1 = \max\left\{0, \frac{\gamma_1}{2a} - \frac{q^*}{2}\right\}, \qquad x^*_N = 0. \tag{18}$$

(d) For any choice of system parameters in (a)-(c), each routing user $n = 2, \ldots, N-14$ has the following rate

$$x^*_n = \begin{cases} 0, & \text{if } \gamma_n \leq a(q^* + x^*_1), \\ \frac{\gamma_n}{a} - q^* - x^*_1, & \text{otherwise.}\end{cases} \tag{19}$$

The proof of Theorem 6 is given in Appendix A. From Theorem 6(a), if the slopes of the linear utility functions for users 1 and N (i.e., γ_1 and γ_N) are identical or close, then users 1 and N choose equal rates and there is an *infinite* number of Nash equilibria. Theorem 6(b) and 6(c) show that if γ_1 and γ_N are *not* close, then users 1 and N choose *different* rates at the Nash equilibrium. Comparing this with the results in Theorem 2, we shall expect a drastic efficiency loss, especially if $\gamma_1 \geq \frac{2}{\beta}\gamma_N - a q^*$ as it results in $x^*_N = 0$.

To study the properties of Nash equilibria of Game 2, we consider two different cases:

1) Two Users Case: Assume that $N = 2$. In this case, the butterfly network includes two network coding users and *no* routing user. We can obtain the Nash equilibria using Theorem 6 by setting $q^* = 0$ and show the following:

Theorem 7: In a network as in Fig. 2 with $N = 2$, under the *single* pricing scheme ($\beta = 1$),

$$\mathrm{PoA}\,(\text{Game 2}, \text{Problem 2}) = \frac{1}{3}, \tag{20}$$

and under the *discriminatory* pricing scheme with $\beta = \frac{1}{2}$,

$$\mathrm{PoA}\,(\text{Game 2}, \text{Problem 2}) = \frac{12}{25}. \tag{21}$$

The proof of Theorem 7 is given in Appendix B. For this simple two-user scenario, inter-session network coding with no price discrimination can reduce the PoA from 0.67 in Theorem 1 to $\frac{1}{3} \approx 0.33$. Even if we use price discrimination by setting $\beta = \frac{1}{2}$, i.e., users 1 and N split the price of encoded packets, the PoA improves only to $\frac{12}{25} = 0.48$. This implies that inter-session network coding is *very sensitive* to strategic users.

Note that, these results do *not* imply superiority of routing over network coding. For example, we can numerically verify that at *any* Nash equilibrium of Game 2, the surplus is no less than the surplus at the Nash equilibrium of Game 1 for the *same* choices of system parameters. That is, the *absolute* performance of non-cooperative network coding is *no worse* than the *absolute* performance of non-cooperative routing. However, the *relative* performance in non-cooperative network coding compared to optimal cooperative network coding is worse than the relative performance in routing case.

Numerical results on efficiency of the Nash equilibrium of Game 2 for 200 *randomly* generated scenarios with different choices of system parameters in the two-user case are shown in Fig. 3. In particular, in each scenario, the utility functions of the users are chosen to be α-fair (cf. [15]) with a randomly selected utility parameter $\alpha \in (0, 1)$. We can see that by using price discrimination with parameter $\beta = \frac{1}{2}$, the guaranteed worst-case efficiency bound (i.e., the PoA) improves from 0.33 to 0.48. For the rest of this paper, we focus on the case with $\beta = \frac{1}{2}$. That is, the network coding users *split* the charge of transmitting their jointly encoded packets.

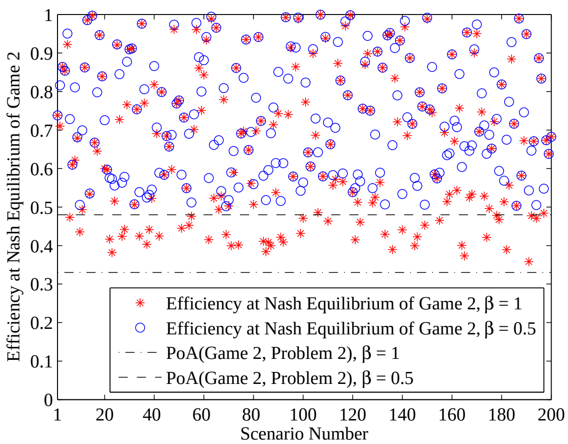

Fig. 3. Efficiency at a Nash equilibrium of 200 random game scenarios when the network topology is as in Fig. 2 and the number of users is $N = 2$.

2) General Case: Next, consider the case where $N > 2$ users in the network. The presence of both network coding and routing users makes the analysis more complex. To see this, consider the network in Fig. 2 and assume that $N = 3$, $a = 1$, $\beta = \frac{1}{2}$, $\gamma_1 \geq \gamma_3$, $\gamma_3 = 1$, and $\gamma_2 = 3$. In this case, users 1 and 3 are the network coding users and user 2 is a routing user. From Theorem 6, the Nash equilibria are obtained as shown in Fig. 4. We can numerically verify that in this scenario, the *worst-case* efficiency at Nash equilibrium of Game 2 is 46.5%. Comparing this with the results in Theorem 7, we can expect that adding routing users will further reduce the PoA. This is shown in the next theorem for a *general* case:

Theorem 8: Assume that $N \geq 2$. (a) If the price discrimination parameter $\beta = \frac{1}{2}$, we have

$$\text{PoA}\left(\text{Game } 2, \text{Problem } 2\right) = \frac{1}{4}. \tag{22}$$

(b) The worst-case efficiency occurs when $N \to \infty$.

The proof of Theorem 8 is given in Appendix C. Comparing Theorems 1, 7, and 8 we can see that a resource allocation game with *both* network coding and routing users has a *worse* PoA than the *routing only* and *network coding only* cases.

IV. Inter-Session Network Coding Games with Non-Zero Side Link Costs

In this section, we study the case where the side links have *non-zero* cost and show that the network coding users are no longer interested in participating in network coding in this case. This can further reduce the PoA to only 20%.

A. Problem Formulation

Consider the network in Fig. 5. In this figure, the side link (s_1, t_N) has price p_1 and cost C_1 while the side link (s_N, t_1) has price p_N and cost C_N. Suppose that Assumption 2 also holds for the price and cost functions of both side links. In addition, we make the following assumption.

Assumption 4 (Non-Zero Cost for Side Links): The side links (s_1, t_N) and (s_N, t_1) in Fig. 5 always have *non-zero* cost and impose *non-zero* prices. In particular, the side link

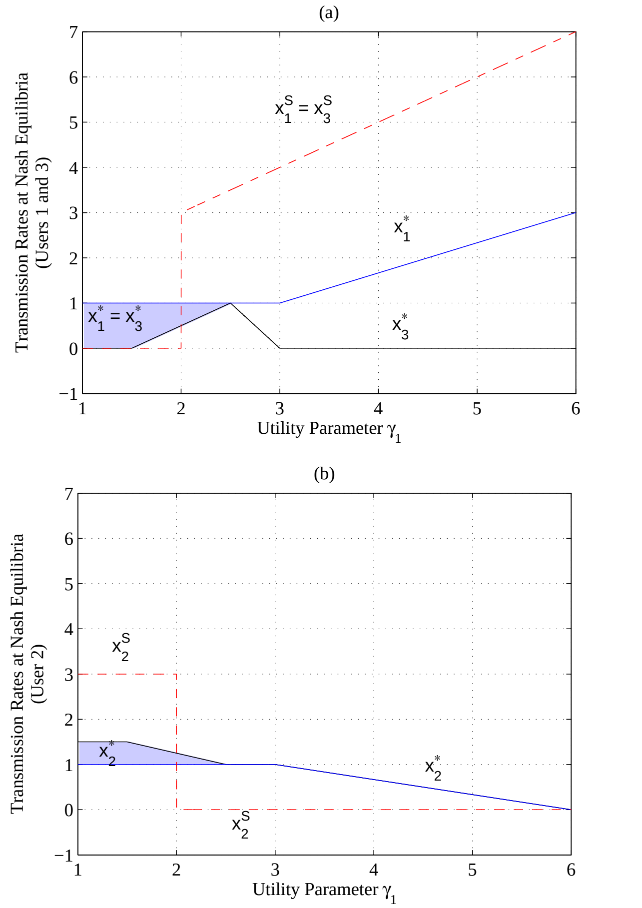

Fig. 4. Nash equilibria for the resource allocation game in Fig. 2 when $N = 3$, $a = 1$, $\beta = \frac{1}{2}$, $\gamma_1 \geq \gamma_3$, $\gamma_3 = 1$, and $\gamma_2 = 3$. (a) Data rates for network coding users 1 and 3, (b) Data rates for routing user 2.

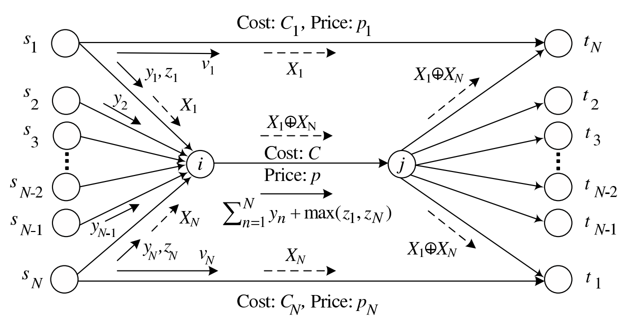

Fig. 5. A link shared by N flows. Users 1 and N perform inter-session network coding. The side links (s_1, t_N) and (s_N, t_1) have non-zero cost.

(s_1, t_N) has price $p_1(v_1) = a_1 v_1$ for $a_1 > 0$ and the side link (s_N, t_1) has price $p_N(v_N) = a_N v_N$ for $a_N > 0$.

Clearly, by sending remedy packets over side link (s_1, t_N), user 1 is *helping* user N to decode the encoded packets it may receive. However, due to non-zero cost at the side links, user 1 will be *charged* for sending these remedy packets. A similar statement is true for user N. Therefore, users 1 and N may decide to *reduce* the rate at which they send the remedy packets. Users 1 and N can inform node i about their decision via *packet marking*. Let y_1 and z_1 denote the rate at which source s_1 sends data to node i *marked* for routing and network coding. Data rates y_N and z_N are defined for user N similarly. Node i may encode only those packets which are marked for network coding, at rate $\min(z_1, z_N)$. Node i simply forwards the rest of packets[1], at rate $\sum_{n=1}^{N} y_n + |z_1 - z_N|$. Therefore, the total rate on link (i, j) becomes $\sum_{n=1}^{N} y_n + \max(z_1, z_N)$. At destination node t_1, a packet coming from node i that is marked for network coding is collected and assumed to carry useful information only if it is accompanied by a remedy packet from node s_N; otherwise, such packet is dropped. Similarly, at destination node t_N, a packet coming from node i that is marked for network coding is collected only if it is accompanied by a remedy packet from node s_1; otherwise, such packet is dropped. Finally, we denote v_1 and v_N as the rates at which sources s_1 and s_N send remedy packets on side links (s_1, t_N) and (s_N, t_1). The routing users $2, \ldots, N$–1 send routing packets at rates $y_2, \ldots, y_{N-1}$. Let $\boldsymbol{y} = (y_1, \ldots, y_N)$, $\boldsymbol{z} = (z_1, z_N)$, and $\boldsymbol{v} = (v_1, v_N)$. For the network in Fig. 5, the network aggregate surplus maximization problem becomes

Problem 3:

$$\begin{aligned}\underset{\boldsymbol{y},\boldsymbol{z},\boldsymbol{v}}{\text{maximize}} \quad & \sum_{n=2}^{N-1} U_n(y_n) + U_1(y_1 + \min(z_1, v_N)) \\ & + U_N(y_N + \min(z_N, v_1)) \\ & - C\left(\sum_{n=1}^{N} y_n + \max(z_1, z_N)\right) - C_1(v_1) - C_N(v_N) \\ \text{subject to} \quad & y_n \geq 0, \quad n = 1, \ldots, N, \quad z_1, z_N, v_1, v_N \geq 0.\end{aligned}$$

Following a discriminatory pricing model as in Section III-A, we can define a resource allocation game for the network setting in Fig. 5, when users are price anticipators:

Game 3:
- *Players*: Users in set $\mathcal{N}$.
- *Strategies*: Transmission rates $\boldsymbol{y}$, $\boldsymbol{z}$, and $\boldsymbol{v}$.
- *Payoffs*: $W_n(\cdot)$ for each user $n \in \mathcal{N}$, where

$$\begin{aligned} W_1(y_1, z_1, v_1; \boldsymbol{y}_{-1}, z_N, v_N) &= U_1(y_1 + \min(z_1, v_N)) \\ & - v_1 p_1(v_1) - (y_1 + z_1 - (1-\beta)\min(z_1, z_N)) \\ & p\left(\textstyle\sum_{r=1}^{N} y_r + \max(z_1, z_N)\right), \end{aligned}$$

$$\begin{aligned} W_N(y_N, z_N, v_N; \boldsymbol{y}_{-N}, z_1, v_1) &= U_N(y_N + \min(z_N, v_1)) \\ & - v_N p_N(v_N) - (y_N + z_N - (1-\beta)\min(z_1, z_N)) \\ & p\left(\textstyle\sum_{r=1}^{N} y_r + \max(z_1, z_N)\right), \end{aligned}$$

and for each $n \in \mathcal{N} \backslash \{1, N\}$, we have

$$\begin{aligned} W_n(y_n; \boldsymbol{y}_{-n}) = U_n(y_n) - y_n \\ \times p\left(\textstyle\sum_{r=1}^{N} y_r + \max(z_1, z_N)\right). \end{aligned}$$

Here, $\boldsymbol{y}_{-n} = (y_1, \ldots, y_{n-1}, y_{n+1}, \ldots, y_N)$.

B. Users' Best Responses

For network coding user 1, the best response is denoted by $(y_1^B(\boldsymbol{y}_{-1}, z_N, v_N), z_1^B(\boldsymbol{y}_{-1}, z_N, v_N), \; v_1^B(\boldsymbol{y}_{-1}, z_N, v_N))$, which is obtained as the solution of the following problem

$$\begin{aligned} \left(y_1^B(\boldsymbol{y}_{-1}, z_N, v_N), z_1^B(\boldsymbol{y}_{-1}, z_N, v_N), v_1^B(\boldsymbol{y}_{-1}, z_N, v_N)\right) = \\ \underset{y_1 \geq 0,\, z_1 \geq 0,\, v_1 \geq 0}{\arg\max} \; W_1(y_1, z_1, v_1; \boldsymbol{y}_{-1}, z_N, v_N). \end{aligned}$$

[1] Alternatively, node i can unmark any packet that was marked for network coding by users 1 and N but it did not participate in network coding. However, we can show that this approach has no advantage over the scenario considered.

The best response for network coding user N, denoted by $\left(y_N^B(\boldsymbol{y}_{-N}, z_1, v_1), \; z_N^B(\boldsymbol{y}_{-N}, z_1, v_1), \quad v_N^B(\boldsymbol{y}_{-N}, z_1, v_1)\right)$ can be obtained similarly. Next, we can show the following.

Proposition 1: Users 1 and N *always* send zero remedy packets at the best responses of Game 3. That is, we always have $v_1^B(\boldsymbol{y}_{-1}, z_N, v_N) = 0$ and $v_N^B(\boldsymbol{y}_{-N}, z_1, v_1) = 0$.

Proposition 1 can be proved by noticing that the payoff $W_1(y_1, z_1, v_1; \boldsymbol{y}_{-1}, z_N, v_N)$ is *decreasing* in v_1 and $W_N(y_N, z_N, v_N; \boldsymbol{y}_{-N}, z_1, v_1)$ is *decreasing* in v_N. Clearly, if the network coding users do *not* receive the remedy data from the side links, they *cannot* decode any encoded packet they may receive through the shared link (i, j). In fact, we can further show the following.

Proposition 2: Users 1 and N *always* send zero network coding packets to node i as the best responses of Game 3. That is, $z_1^B(\boldsymbol{y}_{-1}, z_N, v_N) = 0$ and $z_N^B(\boldsymbol{y}_{-N}, z_1, v_1) = 0$.

Notice that if $v_N = 0$, then $\min(z_1, v_N) = 0$ and the payoff function for user 1 reduces to $U_1(y_1) - v_1 p_1(v_1) - (y_1 + z_1 - (1-\beta)\min(z_1, z_N))\, p(\sum_{r=1}^{N} y_r + \max(z_1, z_N))$. In that case, the payoff function is *decreasing* in z_1. A similar statement is true for network coding user N.

C. Nash Equilibrium and Price-of-Anarchy

Given the results on the users' best responses in Propositions 1 and 2, we can conclude that at any Nash equilibrium of Game 3, denoted by $(\boldsymbol{y}^*, \boldsymbol{z}^*, \boldsymbol{v}^*)$, we should indeed have

$$z_1^* = z_N^* = v_1^* = v_N^* = 0. \tag{23}$$

In other words, at a Nash equilibrium of Game 3, *no* users performs network coding. In that case, the Nash equilibria of Game 3 would be closely related to the Nash equilibria of Game 1. In fact, for any choice of parameters, if $\boldsymbol{x}^*$ is a Nash equilibrium of Game 1, then $\boldsymbol{y}^* = \boldsymbol{x}^*$, $\boldsymbol{z}^* = \boldsymbol{0}$, and $\boldsymbol{v}^* = \boldsymbol{0}$ would be a Nash equilibrium of Game 3 for the same choice of system parameters. From this, together with the results in Theorem 1(a), we can conclude that Game 3 always has a *unique* Nash equilibrium. This leads to the following theorem.

Theorem 9: The *worst-case* efficiency of Game 3 occurs when the utility functions are *linear*.

The proof of Theorem 9 is similar to that of [6, Lemma 4]. From Theorem 9, to obtain the PoA for Game 3 for *arbitrary* utility functions (as long as they satisfy Assumption 1), it is *sufficient* to only analyze the case where all utility functions are *linear*. Furthermore, if the side links have a very large cost compared to the cost of the bottleneck link, the optimal performance is achieved with *no* network coding. In that case, the efficiency can be obtained by using Theorem 1 and the *optimal* network aggregate surplus for Problem 3 is the same as the *optimal* network aggregate surplus for Problem 1. In addition, the network aggregate surplus is the same at the Nash equilibrium of Game 3 and Game 1. However, for general choices of $a_1 > 0$ and $a_N > 0$, obtaining the PoA requires further investigation of the optimal solution of Problem 3.

Theorem 10: Consider the network coding system in Fig.

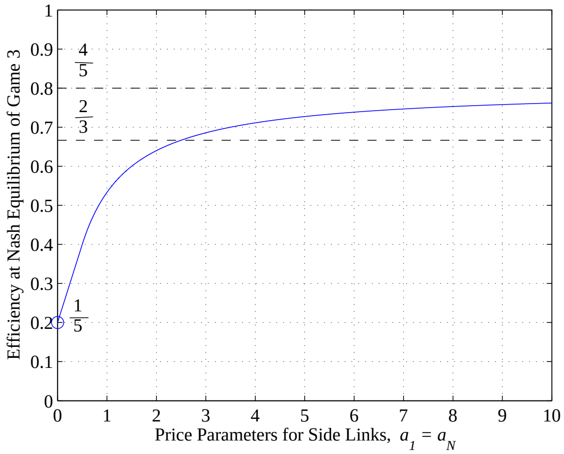

Fig. 6. Efficiency at Nash equilibrium of Game 3 for the network in Fig. 5, where $N \to \infty$, $\gamma_1 = \gamma_N = 1$, $\gamma_n = \frac{4}{5}$ for all $n \in \mathcal{N} \backslash \{1, N\}$, and $a = 1$. Side link price parameters $a_1 = a_N$ vary from 0 (*non-inclusive*) to 10.

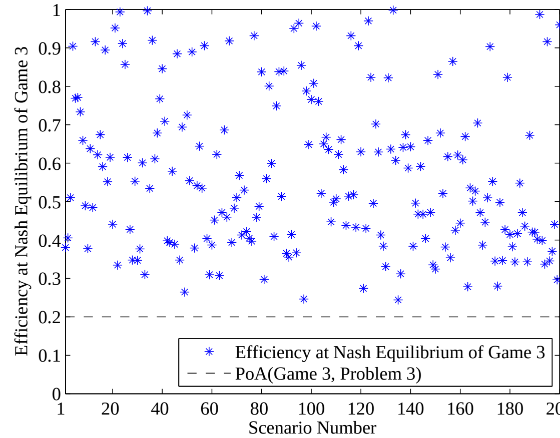

Fig. 7. Efficiency at Nash equilibrium of 200 randomly generated resource allocation game scenarios for the network in Fig. 5 with $N = 2$. Efficiency of Game 3 is lower bounded by the PoA = $\frac{1}{5} = 0.2$ in Theorem 10.

5 with $N \geq 2$ users. (a) We have

$$\text{PoA}\left(\text{Game } 3, \text{Problem } 3\right) = \frac{1}{5}. \tag{24}$$

(b) The worst-case efficiency occurs when $N \to \infty$.

The proof of Theorem 10 is given in Appendix D. Comparing Theorem 10 and Theorem 8, we can see that a non-zero cost at the side links can further reduce the PoA in a network resource allocation game as the users do *not* perform network coding in this case. If the side link price parameters a_1 and a_N are significantly greater than the bottleneck link price parameter a, then network coding is not an optimal solution and the efficiency loss follows from the results in Theorem 1. This is shown in Fig. 6. For the results in this figure, the network topology is assumed to be as in Fig. 5, where $N \to \infty$, $\gamma_1 = \gamma_N = 1$, $a = 1$, and $\gamma_n = \frac{4}{5}$ for all $n \in \mathcal{N} \backslash \{1, N\}$. The side link price parameters $a_1 = a_N$ vary from 0 (*non-inclusive*) to 10. If $a_1 > 0$ and $a_N > 0$ tend to zero, the efficiency becomes as low as $\frac{1}{5} = 0.2$ as Theorem 10 suggests. As $a_1 = a_N$ increase and tend to infinity, Problem 3 becomes equivalent to Problem 1 (in terms of the optimal network aggregate surplus) and Game 3 becomes equivalent to Game 1 (in terms of network aggregate surplus at Nash equilibrium) which leads to an efficiency higher than $\frac{2}{3} \approx 0.67$ as Theorem 1 suggests (for the choice of parameters in Fig. 6, the efficiency approaches $\frac{4}{5} = 0.8$). Numerical results on the efficiency of the Nash equilibrium of Game 3 for 200 random scenarios with different choices of system parameters in the *two-user* case are shown in Fig. 7. We can see that the simulations confirm Theorem 10.

V. More General Network Topologies

Although the butterfly networks in Figs. 2 and 5 are simple, they can be used as building blocks for more general networks. In fact, as shown in [20], [21], many networks can be modeled as *superposition* of multiple butterfly networks. As an example, consider the grid topology in Fig. 8 with nine nodes, 12 links, and six users. All links have *non-zero cost* and incur *non-zero prices*, as in Section IV. The pricing parameter for each link $l \in \{1, \ldots, 12\}$ is denoted by $a_l > 0$. Users 1

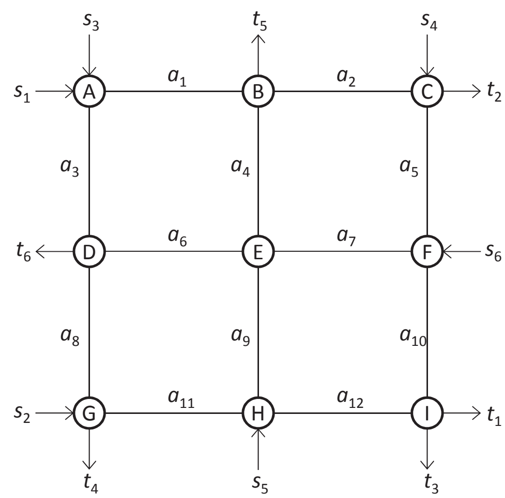

Fig. 8. A grid topology as a *superposition* of multiple butterfly networks.

and 2 can form a network coding pair over a butterfly network with shared links 6 and 7, side links 1 and 2 between s_1 and t_2, and side links 11 and 12 between s_2 and t_1. Node D can act as an intermediate node to encode packets from s_1 and s_2. Similarly, users 3 and 4 can form a network coding pair over a butterfly network with shared links 4 and 9, side links 3 and 8 between s_3 and t_4, and side links 5 and 10 between s_4 and t_3. Node B can act as an intermediate node to encode packets from s_3 and s_4. Users 5 and 6 are routing users.

Following similar steps as in formulating Problem 3 and Game 3, we can formulate the network surplus maximization problem and the resource allocation game for the network in Fig. 8. Although it is difficult to obtain the PoA analytically, we can still estimate the PoA numerically. Note that, we only need to calculate y_n^* for $n = 1, \ldots, 6$, because we already know from Proposition 1 that at Nash equilibrium, all network coding rates are zero. This is done as follows. First, we randomly select an initial strategy y_n for each user n. Then, users take random turns and each user n individually updates y_n given the most updated $\boldsymbol{y}_{-n}$ from other users. The successive calculation of the best response strategies will continue until no user changes its strategy, i.e., no user can improve its payoff

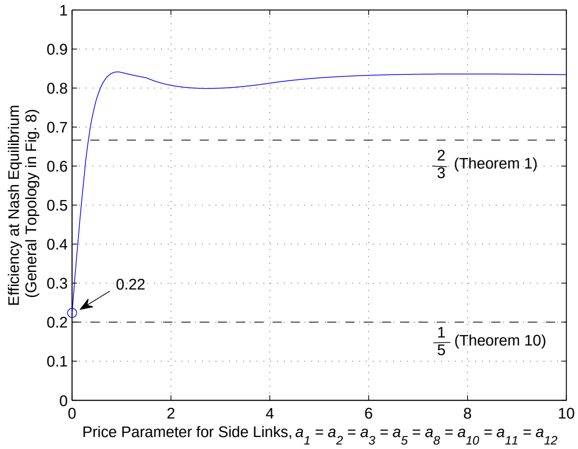

Fig. 9. Efficiency at Nash equilibrium for the grid network in Fig. 8.

by unilaterally changing its transmission rate. Since all the best response dynamics converged in the numerical examples that we considered, the users' transmission rates at convergence are used as Nash equilibrium in our numerical results.

The numerical results are shown in Fig. 9. Here, we assume that the price parameters on the side links $a_1=a_2=a_3=a_5=a_8=a_{10}=a_{11}=a_{12}$ vary from 0 to 10. The price parameters on the inner links are $a_4=a_6=a_7=a_8=1$. Utility functions are linear and $\gamma_1=\gamma_2=\gamma_3=\gamma_4=3$ and $\gamma_5=\gamma_6=1$. When the side links have non-zero costs, no user participates in network coding. We can see that efficiency can be as low as 0.22, when $a_1=a_2=a_3=a_5=a_8=a_{10}=a_{11}=a_{12}\to 0$ and network coding is the optimal solution. This result is close to the PoA = 0.2 in Theorem 10. When $a_1=a_2=a_3=a_5=a_8=a_{10}=a_{11}=a_{12}\to\infty$, network coding is no longer an optimal solution and thus the efficiency at the Nash equilibrium increases, approaching the results in Theorem 1.

We must emphasize that the possibility for generalizing the results in this section is only a conjecture, as the observations are limited to the very specific example of the topology in Fig. 8. The special trend in Fig. 9 and its resemblance to the trend in Fig. 6 may not apply to different inter-session network coding topologies. Furthermore, we note that the results here are limited to the case when network coding in a grid topology is performed similarly as in a butterfly topology. However, when it comes to a large network, such as the grid network in Fig. 8, there can be other options to perform network coding, such as the construction of grail topologies [22].

VI. Conclusion

In this paper, we studied the impact of strategic network coding users on the efficiency of network resource allocation in a butterfly network, where a bottleneck link is shared by several users. Two of the users have the capability of performing inter-session network coding, and the rest perform routing. Even with this simple setup, the results are dramatically different from the routing-only case. In particular, there can be many (even infinite) Nash equilibria in the resulting resource allocation game. This is in sharp contrast to a similar game setting with traditional packet forwarding where the Nash equilibrium is always unique. Furthermore, the efficiency loss can be more severe than for the case without network coding. In a butterfly network when the side links have *zero* cost, the efficiency at Nash equilibrium can be as low as 25%. If the side links have *non-zero* cost, then the efficiency at Nash equilibrium can further reduce to only 20%. These results generalize the well-known result of guaranteed 67% worst-case efficiency for packet forwarding networks in [6].

Appendix

A. Proof of Theorem 6

Due to $\gamma_1 \geq \gamma_N$, we have $x_1^* \geq x_N^*$. We prove this by contradiction. Assume $x_1^* < x_N^*$. Since $U_1'(x_1)=\gamma_1$, from (13), $x_1^*<x_N^*$ implies that $\gamma_1 \leq \beta a\,(q^*+x_N^*)$. Furthermore, since $U_N'(x_N)=\gamma_N$, from (12), we have $\gamma_N = aq^*+2ax_N^*-a(1-\beta)x_1^*$. Since $\gamma_N\leq\gamma_1$, it is required that

$$\begin{aligned} &q^*+2ax_N^*-a(1-\beta)x_1^* \leq \beta a(q^*+x_N^*)\\ &\quad\Rightarrow\quad q^*(1-\beta)+x_N^*(2-\beta)-(1-\beta)x_1^*\leq 0. \end{aligned} \tag{25}$$

If $\beta=1$, then the inequality in (25) reduces to $x_N^*\leq 0$ which *contradicts* the assumption that $0\leq x_1^*<x_N^*$. On the other hand, if $0<\beta<1$, then we can further show that

$$\begin{aligned} &q^*(1-\beta)+x_N^*(2-\beta)-(1-\beta)x_1^*\\ &\qquad\geq\; q^*(1-\beta)+(x_N^*-x_1^*)(1-\beta)\;>\;0, \end{aligned} \tag{26}$$

where the last inequality is because $x_1^*<x_N^*$. It is clear that (26) *contradicts* (25). Thus, for any $0<\beta\leq 1$, x_N^* *cannot* be greater than x_1^* and we *always* have $x_N^*\leq x_1^*$.

Part (a): To show $x_1^*=x_N^*$, assume that $x_1^*\neq x_N^*$. Since $x_1^*\geq x_N^*$, then $x_1^*>x_N^*$. From (12), we have

$$x_1^*=\frac{\gamma_1-aq^*+a(1-\beta)x_N^*}{2a}>x_N^* \;\Rightarrow\; \gamma_1>(1+\beta)ax_N^*+aq^*, \tag{27}$$

and

$$\gamma_N\leq\beta ax_1^*+\beta aq^*. \tag{28}$$

From (27) and (28), we also have

$$\begin{aligned} \gamma_N &\leq \beta aq^*+\frac{\beta}{2}\left(\gamma_1-aq^*+a(1-\beta)x_N^*\right)\\ &<\frac{\beta}{1+\beta}\left(\gamma_1+\beta aq^*\right). \end{aligned} \tag{29}$$

Therefore, $(1+1/\beta)\gamma_N-\beta aq^*<\gamma_1$. However, this contradicts the assumption that $\gamma_1\leq(1+1/\beta)\,\gamma_N-\beta aq^*$. Thus, we indeed have $x_1^*=x_N^*$. From this, together with (12), we have

$$\begin{aligned} &\gamma_1\leq\beta ax_1^*+aq^*+ax_1^*=aq^*+(1+\beta)ax_1^*\\ &\qquad\Rightarrow\quad\frac{\gamma_1-aq^*}{a(1+\beta)}\leq x_1^*=x_N^*, \end{aligned} \tag{30}$$

and

$$\gamma_N\geq\beta aq^*+\beta ax_1^* \quad\Rightarrow\quad x_N^*=x_1^*\leq\frac{\gamma_N-\beta aq^*}{\beta a}. \tag{31}$$

Part (b): The condition in this scenario holds if and only if

$$\left(1+\frac{1}{\beta}\right)\gamma_N-\beta aq^*\leq\frac{2}{\beta}\gamma_N-aq^* \quad\Rightarrow\quad \gamma_N\geq\beta aq^*. \tag{32}$$

Moreover, since $\gamma_1\leq\frac{2}{\beta}\gamma_N-aq^*$, we have $\frac{2}{\beta}\gamma_N-\gamma_1-aq^*\geq 0$ and x_N^* in (17) is *non- negative*. Since $x_1^*\geq x_N^*$, this also implies non-negativity of x_1^*. Next, we consider two cases:

Case I) Assume that $x_N^* > 0$. Similar to Part (a), we can show that $x_1^* > x_N^*$. From this, together with (12), we have $x_1^* = (\gamma_1 - aq^* + a(1-\beta)x_N^*)/(2a)$ and $\gamma_N = \beta a q^* + \beta a x_1^*$. The latter further results in $x_1^* = (\gamma_N - \beta a q^*)/(\beta a) = \gamma_N/(\beta a) - q^*$. Thus, we finally have:

$$\begin{aligned} \frac{\gamma_N}{\beta a} - q^* &= \frac{\gamma_1 - aq^* + a(1-\beta)x_N^*}{2a} \\ \Rightarrow \qquad x_N^* &= \frac{\frac{2}{\beta}\gamma_N - \gamma_1 - aq^*}{a(1-\beta)}. \end{aligned} \tag{33}$$

Case II) Assume that $x_N^* = 0$. In that case, from (12), we have $x_1^* = (\gamma_1 - aq^*)/(2a)$ and $\gamma_N \le \beta a q^* + \beta a x_1^*$. Replacing the former in the latter, we have

$$\gamma_N \le \frac{\beta}{2} + \frac{\beta a q^*}{2} \qquad \Rightarrow \qquad \gamma_1 \ge \frac{2}{\beta}\gamma_N - aq^*. \tag{34}$$

From (34) and since $\gamma_1 \le \frac{2}{\beta}\gamma_N - aq^*$, we have $\gamma_1 = \frac{2}{\beta}\gamma_N - aq^*$. From (12), $x_1^* = \gamma_N/(\beta a) - q^*$.

Part (c): The proof is similar to Part (b). Two cases of $x_1^* > x_N$ and $x_1^* = x_N^*$ are considered.

Part (d): For each node $n \in \mathcal{N}\backslash\{1, N\}$, at each Nash equilibrium $\boldsymbol{x}^*$ of Game 2, we have $x_n^B(\boldsymbol{x}_{-n}^*) = x_n^*$. Thus, for linear utilities, the derivative of the objective function in (9) in x_n is $\gamma_n - a(q^* + x_1^*) - x_n a$. If $\gamma_n \le a(q^* + x_1^*)$, the derivative is always *non-positive* and the objective function is *decreasing* in x_n. In that case, $x_n^* = 0$. If $\gamma_n \ge a(q^* + x_1^*)$, then since the objective is *convex*, we have $x_n^* = \frac{\gamma_n}{a} - q^* - x_1^*$. Together, these two cases result in (19). ■

B. Proof of Theorem 7

At optimality, we have $x_1^S = x_2^S = (\gamma_1 + \gamma_2)/a$. Thus, the optimal network surplus becomes

$$\gamma_1 x_1^S + \gamma_2 x_2^S - \frac{a}{2}\left(\max\{x_1^S, x_2^S\}\right)^2 = \frac{(\gamma_1+\gamma_2)^2}{2a}. \tag{35}$$

Next, we examine the efficiency for all the scenarios in Theorem 6(a), (b), (c), where $q^* = 0$.

Case I) If $\gamma_2 \le \gamma_1 \le (1+1/\beta)\gamma_2$, then the Nash equilibria are as in (16). Since there are *multiple* Nash equilibria, the *worst-case* efficiency for Game 2 is obtained by solving

$$\begin{aligned} \underset{x_1^*}{\text{minimize}} \quad & \left((\gamma_1+\gamma_N)\,x_1^* - \frac{a}{2}x_1^{*2}\right) \Big/ \left(\frac{(\gamma_1+\gamma_2)^2}{2a}\right) \\ \text{subject to} \quad & \frac{\gamma_1}{(1+\beta)a} \le x_1^* \le \frac{\gamma_2}{\beta a}. \end{aligned} \tag{36}$$

We can show that if $\beta = 1$, then the optimal objective value of problem (36) becomes $1/2 - 1/16 = 7/16 \approx 0.438$. On the other hand, if $\beta = \frac{1}{2}$, then the optimal objective value of problem (36) becomes $6/9 - 1/9 = 5/9 \approx 0.556$.

Case II) If $(1+1/\beta)\gamma_2 < \gamma_1 \le \frac{2}{\beta}\gamma_2$ (note: this may hold only if $\beta < 1$), then x_1^* and x_N^* are as in (17) where $q^* = 0$. The worst-case efficiency is obtained by solving

$$\begin{aligned} \underset{\gamma_1}{\text{minimize}} \quad & \frac{\gamma_2}{(\gamma_1+\gamma_2)^2}\left(\frac{2(1-2\beta)}{\beta(1-\beta)}\gamma_1 + \frac{5\beta-1}{\beta^2(1-\beta)}\gamma_2\right) \\ \text{subject to} \quad & (1+1/\beta)\,\gamma_2 < \gamma_1 \le \frac{2}{\beta}\gamma_2. \end{aligned}$$

By applying the KKT conditions, the optimal objective of the above optimization problem when $\beta = \frac{1}{2}$ becomes $\frac{12}{25} = 0.48$.

Case III) We assume that $\frac{2}{\beta}\gamma_2 < \gamma_1$. From Theorem 6(c), the Nash equilibrium is as in (18) and the worst-case efficiency is obtained by solving the following optimization problem

$$\begin{aligned} \underset{\gamma_1,\gamma_2}{\text{minimize}} \quad & \left(\gamma_1\frac{\gamma_1}{2a} - \frac{a}{2}\left(\frac{\gamma_1}{2a}\right)^2\right) \Big/ \left(\frac{(\gamma_1+\gamma_2)^2}{2a}\right) \\ \text{subject to} \quad & 0 \le \frac{2}{\beta}\gamma_2 < \gamma_1. \end{aligned} \tag{37}$$

For $\beta = 1$, the optimal objective value becomes $\frac{1}{3} \approx 0.33$. For $\beta = \frac{1}{2}$, the optimal objective value becomes $\frac{12}{25} = 0.48$.

From Cases I and III, if $\beta = 1$, $\text{PoA}\,(\text{Game } 2, \text{Problem } 2) = \min\left\{\frac{7}{16}, \frac{1}{3}\right\} = \frac{1}{3}$. From Cases I, II, and III, if $\beta = \frac{1}{2}$, $\text{PoA}\,(\text{Game } 2, \text{Problem } 2) = \min\left\{\frac{5}{9}, \frac{12}{25}, \frac{12}{25}\right\} = \frac{12}{25}$. ■

C. Proof of Theorem 8

The optimal surplus for linear utilities is $\sigma^2/(2a)$.

Case I) We assume that $\gamma_1 + \gamma_N = \sigma$. Similar to the proof of Theorem 7, here we obtain the PoA by examining all the scenarios in Theorem 6(a), (b), (c). First, assume that

$$\gamma_N \le \gamma_1 \le (1 + 1/\beta)\,\gamma_N - \beta a q^*, \tag{38}$$

and $\gamma_1 \ge aq^*$. To obtain the worst-case efficiency for this scenario, we need to solve the following optimization problem:

$$\begin{aligned} \underset{\boldsymbol{x}^*,\boldsymbol{\gamma},a,N,q^*}{\text{minimize}} \quad & \frac{\sigma x_1^* + \sum_{n=2}^{N-1}\gamma_n x_n^* - \frac{a}{2}(q^* + x_1^*)^2}{\sigma^2/(2a)} \\ \text{subject to} \quad & \gamma_n = a\,(q^* + x_n^* + x_1^*)\,, \text{ if } x_n^* > 0,\ n \ne 2, N-1, \\ & \gamma_n \le a(q^* + x_1^*), \qquad \text{if } x_n^* = 0,\ n \ne 2, N-1, \\ & \textstyle\sum_{n=2}^{N-1} x_n^* = q^*, \\ & \gamma_1 + \gamma_N = \sigma, \\ & \gamma_1 \ge aq^*, \\ & 0 < \gamma_n \le \sigma, \qquad n \ne 2, N-1, \\ & \gamma_N \le \gamma_1 \le (1+1/\beta)\,\gamma_N - \beta a q^*, \\ & \frac{\gamma_1 - aq^*}{a(1+\beta)} \le x_1^* = x_N^* \le \frac{\gamma_N - \beta a q^*}{\beta a}. \end{aligned}$$

We can show that for any choice of β the optimal objective value of the above optimization problem is $\frac{1}{4} = 0.25$. Next, assume that (38) holds and we have

$$\gamma_1 \le aq^*. \tag{39}$$

From Theorem 6(a), the Nash equilibria are obtained as $0 \le x_1^* = x_N^* \le \frac{\gamma_N - \beta a q^*}{\beta a}$. We can show that, in this scenario, the worst-case efficiency occurs if $N \to \infty$ and we have $x_1^* = x_N^* = 0$ and $aq^* = \frac{1+2\beta\sigma}{2\beta^2+4\beta+3}$. Thus, the worst-case efficiency when (38) and (39) hold is obtained as

$$\frac{2}{2\beta^2 + 4\beta + 3}. \tag{40}$$

If $\beta = \frac{1}{2}$, then (40) becomes $\frac{4}{11} \approx 0.36$. Finally, we assume that $\left(1 + \frac{1}{\beta}\right)\gamma_N - \beta a q^* \le \gamma_1 \le \frac{2}{\beta}\gamma_N - aq^*$. We can show that the worst-case efficiency in this scenario is still as in (40).

Case II) We assume that $\gamma_1 + \gamma_N < \sigma$. Following similar steps as in Case I and also using [6, Theorem 3], the worst-case efficiency in this scenario becomes $\frac{2}{3} \approx 0.67$.

From Cases I and II, if $\beta = 1$, $\text{PoA}\,(\text{Game } 2, \text{Problem } 2) = \min\left\{\frac{1}{4}, \frac{2}{9}, \frac{2}{3}\right\} = \frac{2}{9}$. On the other hand, if $\beta = \frac{1}{2}$, then $\text{PoA}\,(\text{Game } 2, \text{Problem } 2) = \min\left\{\frac{1}{4}, \frac{4}{11}, \frac{2}{3}\right\} = \frac{1}{4}$. ■

D. Proof of Theorem 10

Let $\boldsymbol{y}^S = (y_1^S, \ldots, y_N^S)$, $\boldsymbol{z}^S = (z_1^S, z_N^S)$, and $\boldsymbol{v}^S = (v_1^S, v_N^S)$ be the solution for Problem 3. Define $\gamma_{\max} = \max_{n\in\mathcal{N}} \gamma_n$ and $\mathcal{M} = \{n : \gamma_n = \gamma_{\max}\}$ with size $M = |\mathcal{M}|$. We can verify that (a) If $\gamma_1 + \gamma_N \geq \left(1 + \frac{a_1+a_N}{a}\right)\gamma_{\max}$, then

$$z_1^S = z_N^S = v_1^S = v_N^S = (\gamma_1 + \gamma_N)/(a + a_1 + a_N), \tag{41}$$

and for each $n \in \mathcal{N}$, we have $y_n^S = 0$. (b) If $\gamma_{\max} \leq \gamma_1 + \gamma_N \leq (1 + (a_1 + a_N)/a)\,\gamma_{\max}$, then

$$z_1^S = z_N^S = v_1^S = v_N^S = \frac{\gamma_1 + \gamma_N - \gamma_{\max}}{a_1 + a_N}, \tag{42}$$

and for each $n \in \mathcal{M}$, we have

$$y_n^S = \frac{(a + a_1 + a_N)\gamma_{\max} - a(\gamma_1 + \gamma_N)}{aM(a_1 + a_N)}, \tag{43}$$

while for each $n \in \mathcal{N}\backslash\mathcal{M}$, we have $y_n^S = 0$. (c) If $\gamma_{\max} \geq \gamma_1 + \gamma_N$, then $z_1^S = z_N^S = v_1^S = v_N^S = 0$ and for each $n \in \mathcal{M}$, we have $y_n^S = \frac{\gamma_{\max}}{aM}$ while for each $n \in \mathcal{N}\backslash\mathcal{M}$, we have $y_n^S = 0$.

Next, from (23), for each user $n \in \mathcal{N}$, we have

$$y_n^* = \begin{cases} \frac{1}{2a}\left(\gamma_n - a\sum_{r=1,r\neq n}^N y_r^*\right), & \text{if } \gamma_n > a\sum_{r=1,r\neq n}^N y_r^*, \\ 0, & \text{if } \gamma_n \leq a\sum_{r=1,r\neq n}^N y_r^*. \end{cases}$$

Case I) If $\gamma_1 + \gamma_N \geq \left(1 + \frac{a_1+a_N}{a}\right)\gamma_{\max}$, then the maximum network surplus is $(\gamma_1 + \gamma_N)^2/(2(a + a_1 + a_N))$. The worst-case efficiency is obtained by solving the following problem

$$\begin{aligned} \underset{\boldsymbol{y}^*,\boldsymbol{\gamma},a,a_1,a_N,N,q^*}{\text{minimize}} \quad & \left(\sum_{n=1}^N \gamma_n y_n^* - \frac{a}{2}q^{*2}\right) \Big/ \left(\frac{(\gamma_1+\gamma_N)^2}{2(a+a_1+a_N)}\right) \\ \text{subject to} \quad & \gamma_n = aq^* + ay_n^*, \quad \text{if } y_n^* > 0,\ n \in \mathcal{N}, \\ & \gamma_n \leq aq^*, \qquad\quad \text{if } y_n^* = 0,\ n \in \mathcal{N}, \\ & \textstyle\sum_{n=1}^N y_n^* = q^* \geq 0, \\ & 0 \leq \gamma_n \leq \gamma_{\max}, \quad n \in \mathcal{N}, \\ & \gamma_1 + \gamma_N \geq \left(1 + \frac{a_1 + a_N}{a}\right)\gamma_{\max}, \\ & y_n^* \geq 0, \qquad\qquad n \in \mathcal{N}. \end{aligned}$$

We can show the optimal objective value is $\frac{1}{5} = 0.2$.

Case II) If $\gamma_{\max} \leq \gamma_1 + \gamma_N \leq \left(1 + \frac{a_1+a_N}{a}\right)\gamma_{\max}$ or $\gamma_{\max} \geq \gamma_1 + \gamma_N$, then the worst-case efficiency is *equal* to or *higher* (i.e., better) than $\frac{1}{5}$. In particular, if $\gamma_{\max} \geq \gamma_1 + \gamma_N$, then the worst-case efficiency is $\frac{2}{3}$ which resembles the results in [6].

Combing the results above in Cases I and II, we have $\text{PoA}\,(\text{Game } 3, \text{Problem } 3) = \min\left\{\frac{1}{5}, \frac{2}{3}\right\} = \frac{1}{5}$. ■

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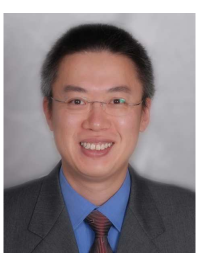

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