Optimal Charging of Electric Vehicles with Uncertain Departure Times: A Closed-Form Solution

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***Abstract*—In this paper, we show that an uncertain departure time significantly changes the analysis in optimizing the charging schedule of electric vehicles. We also obtain a closed-form solution for the stochastic optimization problem that is formulated to schedule charging of electric vehicles with uncertain departure times in presence of hourly time-of-use pricing tariffs.**

***Keywords*: Electric vehicles, uncertain departure time, time-shiftable loads, stochastic optimization, closed-form solution.**

I. Background

Assume that time is divided into T time slots. Consider an electric vehicle (EV) charging station with multiple charging outlets, c.f. [1], [2]. At each charging outlet, let $\alpha \geq 1$ denote the time slot at which an electric vehicle plugs in to the charging outlet. Also let e denote the total energy needed to charge the electric vehicle. In this section, we assume that the driver indicates her departure time $\beta \leq T$. As an example, if a Chevrolet Volt with a depleted battery plugs in to the charging outlet at 10:00 AM and the driver indicates that she will depart at 2:00 PM and she expects a fully charged battery, then we have $\alpha = 10$, $\beta = 14$, and $e = 16$ kWh. Let $x[t]$ denote the scheduled energy usage for charging at time slot t. To finish charging before the departure time β, it is required that

$$\sum_{t=\alpha}^{\beta} x[t] = e. \tag{1}$$

Let $\mu[t]$ denote the price of electricity at time slot t. The cost to finish charging is $\sum_{t=\alpha}^{\beta} \mu[t]x[t]$. If all parameters are known, then the cost is minimized if we choose $x[\tau] = e$, where $\tau = \arg\min_{\alpha \leq t \leq \beta} \mu[t]$; and we set $x[t] = 0$ for $t \neq \tau$. Here, τ is the time slot between α and β with the lowest price. Note that, the above analysis works at each charging outlet, regardless of the number of charging outlets at the charging station. An exception is when the charging station is very large and thus price-maker, as we point out in Section IV.

II. Analysis under Uncertain Deadline

Next, assume that the deadline β is *not* known. Once an EV plugs in, its target charge level e and start time α are identified. However, the charging station may not know *when the EV will depart*. Nevertheless, the charging station needs to operate in a way that it minimizes the cost of electricity.

The system setup is as follows. At the beginning of each time slot $t \geq \alpha$, if the user indicates that the EV will depart at the end of the current time slot, then we must set

$$x[t] = e - X[t], \quad \text{where} \quad X[t] \triangleq \sum_{k=\alpha}^{t-1} x[k], \tag{2}$$

because we must assure finishing charging before the user's departure as required by (1). Otherwise, i.e., if the user does not indicate that it will depart at the end of the current time slot t, then the charging station faces the uncertainty that the EV may depart at any of the future time slots $t+1, \ldots, T$.

At each time slot $t \geq \alpha$, let $\pi_{k|t} \triangleq Pr\{\beta = k \mid \beta > t\}$ denote the conditional probability that the user will depart in a future time slot k, knowing that it will not depart by the end of the current time slot t. We have $\sum_{k=t+1}^{T} \pi_{k|t} = 1$. The *expected* cost of electricity to finish charging is calculated as

$$\begin{aligned} &\sum_{i=t+1}^{T} \pi_{i|t} \left(\sum_{j=t}^{i-1} \mu[j]x[j] + \mu[i]x[i] \right) \\ &= \sum_{i=t+1}^{T} \pi_{i|t} \left(\sum_{j=t}^{i-1} \mu[j]x[j] + \mu[i]\left(e - X[i]\right) \right), \end{aligned} \tag{3}$$

where the equality is due to the departure-time requirement in (2). The question is: how can we minimize the above cost?

Answering the above question is trivial at $t = T$, because from (2), we have no choice but setting $x[T] = e - X[T]$. However, to find the answer for $t = \alpha, \ldots, T-1$, we need to first define a new notation $\phi[t]$ in a *recursive* manner as

$$\phi[t] \triangleq \pi_{t+1|t}\mu[t+1] + (1-\pi_{t+1|t}) \min\{\mu[t+1], \phi[t+1]\}, \tag{4}$$

with terminal condition $\phi[T-1] = \mu[T]$. From this definition and following the method of *backward induction* in stochastic optimization [3], we can present the following key results.

Theorem 1: At the beginning of time slot $t = \alpha, \ldots, T-1$, if the user indicates that it will be departing at the end of time slot t, then $x[t]$ must be set as in (2); otherwise, we can minimize the expected cost of electricity by choosing

$$x^\star[t] = \mathbb{I}(\mu[t] \leq \phi[t])(e - X[t]), \tag{5}$$

where $\mathbb{I}(\cdot)$ is the 0-1 indicator function. If $\mu[t] \leq \phi[t]$, then $\mathbb{I}(\mu[t] \leq \phi[t]) = 1$; otherwise, $\mathbb{I}(\mu[t] \leq \phi[t]) = 0$. Furthermore, if we follow (5), then the expected cost during the remaining time slots $k = t, \ldots, T$ is obtained as

$$C[t] \triangleq \mathbb{E}\left\{ \sum_{k=t}^{T} x^\star[t]\mu[t] \right\} = \min\{\mu[t], \phi[t]\}(e - X[t]). \tag{6}$$

The proof of Theorem 1 is given in the Appendix. Here, $\phi[t]$ serves as a reference on whether the price $\mu[t]$ is low enough to schedule the load, or if we should rather *wait* for a future time slot. The value of $\phi[t]$ depends on the *probability mass function* of the user departure time, e.g., see [4].

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III. Case Study

Consider charging an EV with $\alpha = 1$ PM and $e = 16$ kWh. The departure time is characterized by the probability mass function in Fig. 1(a). The price of electricity is based on the time-of-use prices used by Ameren on March 1, 2014 [5].

The electricity price $\mu[t]$ and reference parameter $\phi[t]$ are shown in Fig. 1(b). We can see that $\mu[t] > \phi[t]$ at time slots 1, 2, 6, 7, 8, 9, 10, 11, and $\mu[t] \leq \phi[t]$ at time slots 3, 4, 5, 12. From Theorem 1, if $\mu[t] > \phi[t]$, then the price of electricity at time slot t is not low enough and we must wait for a future time slot before we schedule the load. As soon as $\mu[t]$ drops below $\phi[t]$, we proceed with scheduling the load. Therefore, if the user indicates that it will depart at the end of time slots 1 or 2, then the charging station schedules the entire load e at time slots 1 or 2, respectively. In all other cases, the load is scheduled at time slot 3. Note that, electricity is cheaper at time slots 10, 11, and 12, compared to time slot 3. Nevertheless, it is not optimal for the charging station to wait that long, because it is likely for the EV to depart earlier than those low-price hours. The expected cost of charging when our design is used is 49 cents. However, if we follow the deterministic design in Section I and wait until time slot $\tau = 12$, then the expected cost of electricity becomes 56 cents, i.e., %14.3 higher.

IV. Conclusions and Extensions

We proposed a closed-form solution to optimally schedule time-shiftable loads with uncertain deadlines, with focus on charging EVs with uncertain departure times. We showed that an uncertain deadline significantly changes the analysis.

This paper can be extended in several directions. First, one can extend the analysis to consider other electricity pricing models such as inclining block rates or real time prices. The latter requires taking into account price uncertainty. Second, we can revise the problem formulation to coordinate charging across multiple charging outlets, e.g., due to load prioritization. Third, we can also extend the analysis to the case where the charging station is large enough such that the price becomes correlated with the demand. Finally, other design factors may be considered such as the per-time-slot charging power limits.

Appendix: Proof of Theorem 1

Step 1: Suppose we are at the beginning of time slot $t = T-1$. Since $\pi_{T|T-1} = 1$, and from (3), the expected cost in the remaining two time slots $T-1$ and T becomes

$$\mu[T]\,(e - X[T-1]) + (\mu[T-1] - \mu[T])\,x[T-1]. \quad (7)$$

The first term depends only on the electricity usage *before* the current time slot $t = T-1$. Hence, to minimize the expected cost for the remaining time slots, we must set

$$x[T-1] = \mathbb{I}(\mu[T-1] \leq \mu[T])\,(e - X[T-1])\,. \quad (8)$$

Once we substitute (8) in (7), we have

$$\begin{aligned} C[T-1] &= \min\{\mu[T-1], \mu[T]\}\,(e - X[T-1]) \\ &= \min\{\mu[T-1], \phi[T-1]\}\,(e - X[T-1])\,. \end{aligned} \quad (9)$$

Therefore, Theorem 1 holds for time slot $t = T-1$.

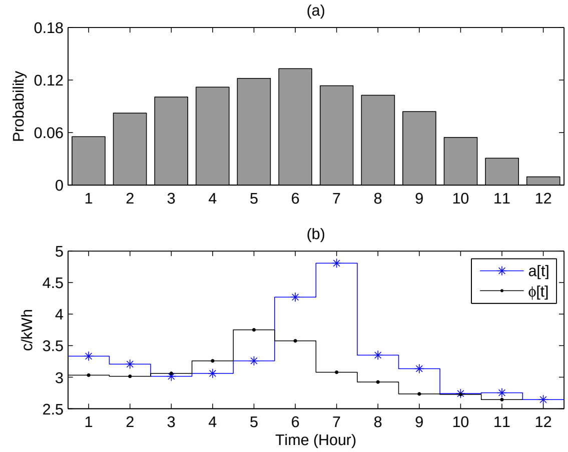

Fig. 1. The system parameters and numerical results for the case study.

Step 2: Suppose Theorem 1 holds for all time slot $t \geq i$, for a given $i \in [\alpha, T]$. That is, for any $t = i+1, \ldots, T$, suppose we set the energy usage $x[t] = x^{\star}[t]$ as in (5); and accordingly the expected cost in the remaining time slots $C[t]$ is calculated as in (6). Next, assume that we are at the beginning of time slot $t = i$ and suppose the user indicates that it will not depart at the end of the current time slot. In that case, the expected cost in the remaining time slots $i, i+1, \ldots, T$ is calculated as

$$x[i]\mu[i] + \mathbb{E}\left\{\sum_{k=i+1}^{T} x^{\star}[k]\mu[k]\right\}. \quad (10)$$

Note that, from (5), each term $x^{\star}[k]$ inside the summation in (10) depends on $X[k]$, which itself depends on $x[i]$. From (4)-(6), and after reordering the terms, we can rewrite (10) as

$$\begin{aligned} & x[i]\mu[i] + \pi_{i+1|i}\mu[i+1]\,(e - X[i] - x[i]) \\ & \quad + \left(1 - \pi_{i+1|i}\right) C[i+1] \\ & = x[i]\mu[i] + \pi_{i+1|i}\mu[i+1]\,(e - X[i] - x[i]) \\ & \quad + \left(1 - \pi_{i+1|t}\right) \min\{\mu[i+1], \phi[i+1]\}\,(e - X[i] - x[i]) \\ & = \phi[i]\,(e - X[i]) + (\mu[i] - \phi[i])\,x[i]. \end{aligned} \quad (11)$$

The first term in the last line in (11) is constant. The second term, however, is linear in $x[i]$. Therefore, to minimize the expected cost in (11), we must follow (5) and set $x[i] = \mathbb{I}\left(\mu[t] \leq \phi[t]\right)(e - X[i])$. By replacing this in (11) we have

$$\begin{aligned} C[i] &= (\phi[i] + \mathbb{I}(\mu[i] \leq \phi[i])(\mu[i] - \phi[i]))\,(e - X[i]) \\ &= \min\{\mu[i], \phi[t]\}(e - X[i]). \end{aligned} \quad (12)$$

This concludes the proof for any time slot $t = \alpha, \ldots, T-1$. ■

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