

# Optimal Demand Response in DC Distribution Networks

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**Abstract**—Direct current (DC) power systems have recently been proposed as a promising technology for distribution networks and microgrids. By eliminating unnecessary conversion stages, DC distribution systems can enable seamless integration of natively DC devices such as photovoltaic cells and batteries. Moreover, using DC technologies can overcome several disadvantages of alternating current (AC) distribution systems, such as synchronization requirements, reactive power compensation, and harmonics. Therefore, in this paper, we take the first steps towards designing demand response programs for DC distribution networks. We seek to adjust the internal parameters of various power electronics loads to assure reliable and efficient operation of the DC distribution system. In this regard, we first present an optimization-based foundation for demand response in DC distribution networks. Then, we devise a pricing mechanism to enforce optimal demand response in a distributed fashion. Simulation results are presented to assess the performance and to gain insights into the proposed demand-response paradigm.

**Keywords:** DC distribution networks, power electronics load, demand response, pricing, convex optimization, fairness.

## I. INTRODUCTION

Demand response programs are implemented by utilities to control the resources at the consumer side of the meter in response to changes in the grid's operating conditions. In the United States, Department of Energy and National Institute of Standards and Technology have identified demand response as one of the priority areas to develop a smart grid [1].

The literature on demand response in smart grid can be divided into two general branches. On one hand, the majority of studies do *not* take into account the characteristics of the underlying physical power system. Instead, they mainly focus on balancing the load across time by shifting a portion of load from peak hours to off-peak hours, e.g., see [2]–[6]. On the other hand, there are also a few recent studies that have incorporated the impact of grid topology, power flow equations, and voltage control into the design of demand response programs. However, so far, the focus has been only on *alternating current* (AC) distribution networks, e.g., see [7]. In contrast, in this paper, we investigate designing demand response programs for *direct current* (DC) distribution networks.

Traditionally, DC power systems have been used in telecommunications, naval ships, and industrial systems [8]–[10]. More recently, they are also proposed for microgrids [11] and distribution networks [12]–[14]. DC distribution networks can offer several important advantages. For example, they can enable easier interconnection of renewable energy sources, because many of these sources such as photovoltaic systems are essentially DC sources. Battery storage devices are also easier

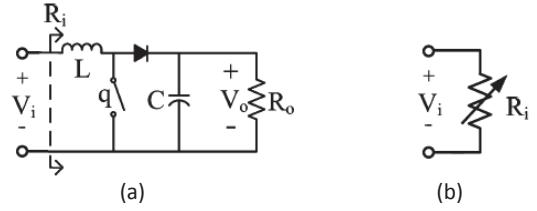


Fig. 1. A DC-DC boost converter can be seen as a variable resistor load. The external resistor  $R_i$  is controlled by setting the duty cycle for switch  $q$  [18].

to connect to a DC system. A similar argument applies to the integration of plug-in electric vehicles [15]. Finally, using DC technologies in microgrids and distribution networks can overcome several disadvantages of AC technologies, including synchronization requirements, reactive power flow, and circulating currents due to differences in voltage magnitude, phase angle, or DC offset in a multi-inverter system [14].

Given the benefits of DC distribution networks and because of the importance of demand response in the future smart grid, in this paper, we seek to take the first steps towards designing demand response programs for DC distribution systems. We combine the recent advancements in power electronics [16]–[18] with techniques from optimization theory [19] to build a new foundation for optimal demand response in DC distribution networks. Interestingly, we show that the formulated optimization problem is convex and therefore tractable. We also devise a pricing mechanism to enforce optimal demand response in a distributed fashion. Various simulation results are provided to assess the performance and to gain insights.

The rest of this paper is organized as follows. We explain the system model and the fundamental equations for DC distribution systems in Section II. The optimal demand response problem is formulated in Section III. Our pricing method is proposed in Section IV. Simulation results are presented in Section V. The paper is concluded in Section VI.

## II. SYSTEM MODEL

Loads in power systems are typically modeled with fixed parameters, such as a fixed impedance or a fixed power. However, with the increasing use of *power electronics loads*, i.e., loads that are equipped with power electronics converters, the internal load characteristics can be controlled to reflect a desirable effective input or output impedance [16], [17]. In particular, a power electronics load with a switch-mode converter can be seen by the rest of the grid as a *variable resistor* load, as shown in Fig. 1 [18]. For the boost converter example in this figure, the relationship between the internal resistance  $R_o$  and the effective resistance  $R_i$  can be written as

$$R_i = (1 - D)^2 R_o, \quad (1)$$

where  $D \in [0, 1)$  denotes the duty cycle of the active switch component  $q$ . A similar variable resistor load model can be

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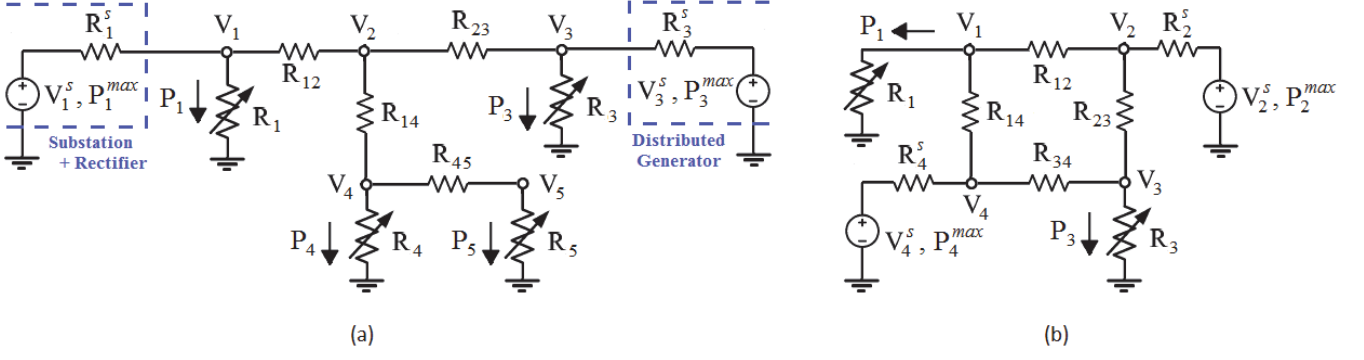


Fig. 2. Two examples for DC power systems: (a) A five-bus radial power distribution network with four power electronics loads. (b) A four-bus DC microgrid with two power electronics loads. Demand response is done by changing the variable resistors for the participating power electronics loads.

found for pulse-width modulation (PWM) rectifiers that feed DC distribution systems from AC sources [20, Chapter 18].

Treating power electronics loads as variable resistors and developing a distributed control system for such variable resistors is the foundation of our proposed demand response paradigm: *we seek to change the external resistance for each load in order to assure reliable and efficient operation of the DC distribution network*. For example, consider the DC power systems in Fig. 2. In both cases, we can use demand response to coordinate and control variable resistors to affect the amount of power delivered to each user, the amount of power drawn from each source, the amount of power loss on each distribution line, and the voltage level at each bus.

Let  $\mathcal{N}$  denote the set of all buses in the DC power system. The power source at each bus  $k$  is modeled using its Thevenin's representation with a fixed voltage  $V_k^s$ , a fixed internal resistor  $R_k^s$ , and a maximum power limit  $P_k^{\max}$ . If there does *not* exist a power source at a bus  $k$ , then we simply assume that  $R_k^s = \infty$ . Next, let  $\mathcal{L} \subset \mathcal{N}$  denote the set of *load* buses. Each load bus  $i \in \mathcal{L}$  serves a power electronics load with a variable resistor  $R_i$ . Using the Kirchhoff's current law, we have:

$$\frac{V_i}{R_i} + \frac{V_i - V_i^s}{R_i^s} + \sum_{k \in \mathcal{N}_i} \frac{V_i - V_k}{R_{ik}} = 0, \quad \forall i \in \mathcal{L}, \quad (2)$$

and

$$\frac{V_j - V_j^s}{R_j^s} + \sum_{k \in \mathcal{N}_j} \frac{V_j - V_k}{R_{jk}} = 0, \quad \forall j \in \mathcal{N} \setminus \mathcal{L}, \quad (3)$$

where  $\mathcal{N} \setminus \mathcal{L}$  denotes the set of all *non-load* buses and  $\mathcal{N}_k \subset \mathcal{N}$  denotes the set of neighboring buses of bus  $k$ . The equalities in (2) and (3) are the fundamental equations to understand DC distribution systems. From (2) and after reordering the terms, at each load bus  $i \in \mathcal{L}$  we have

$$\begin{aligned} \frac{V_i}{R_i} &= \frac{V_i^s - V_i}{R_i^s} + \sum_{k \in \mathcal{N}_i} \frac{V_k - V_i}{R_{ik}}. \\ \Rightarrow R_i &= V_i / \left( \frac{V_i^s - V_i}{R_i^s} + \sum_{k \in \mathcal{N}_i} \frac{V_k - V_i}{R_{ik}} \right). \end{aligned} \quad (4)$$

Also from (3), at each non-load bus  $j \in \mathcal{N} \setminus \mathcal{L}$ , we have

$$\frac{V_j - V_j^s}{R_j^s} + \sum_{k \in \mathcal{N}_j} \frac{V_j}{R_{jk}} - \sum_{k \in \mathcal{N}_j, k \notin \mathcal{L}} \frac{V_k}{R_{jk}} = \sum_{k \in \mathcal{N}_j, k \in \mathcal{L}} \frac{V_k}{R_{jk}}. \quad (5)$$

The left hand side in (5) is a linear combination of variables  $V_j$  for all  $j \in \mathcal{N} \setminus \mathcal{L}$  while the right hand side in (5) is a linear combination of variables  $V_i$  for all  $i \in \mathcal{L}$ . Therefore, the system of  $|\mathcal{N}| - |\mathcal{L}|$  equations in (5) can be solved to obtain the voltages  $V_j$  for all  $j \in \mathcal{N} \setminus \mathcal{L}$  in terms of voltages  $V_i$  for all  $i \in \mathcal{L}$ . More specifically, from (5) we can derive:

$$V_j = \sum_{i \in \mathcal{L}} a_{ji} V_i + b_j, \quad \forall j \in \mathcal{N} \setminus \mathcal{L}, \quad (6)$$

where parameters  $a_{ji}$  and  $b_j$  are constant. For example, consider the four-bus DC power system in Fig. 2(b). Here, we have  $\mathcal{L} = \{1, 3\}$  and  $\mathcal{N} \setminus \mathcal{L} = \{2, 4\}$ . We can show that

$$\begin{aligned} a_{21} &\triangleq 1 / (1 + R_{12} / R_{23} + R_{12} / R_2^s), \\ a_{23} &\triangleq 1 / (1 + R_{23} / R_{12} + R_{23} / R_2^s), \\ b_2 &\triangleq V_2^s / (1 + R_2^s / R_{12} + R_2^s / R_{23}). \end{aligned} \quad (7)$$

Parameters  $a_{41}$ ,  $a_{43}$ , and  $b_4$  can be derived similarly.

By replacing (6) in (4), at each load bus  $i \in \mathcal{L}$ , we have

$$\begin{aligned} R_i &= V_i / \left( \frac{V_i^s - V_i}{R_i^s} - \sum_{k \in \mathcal{N}_i} \frac{V_i}{R_{ik}} + \sum_{k \in \mathcal{N}_i, k \in \mathcal{L}} \frac{V_k}{R_{ik}} \right. \\ &\quad \left. + \sum_{k \in \mathcal{N}_i, k \notin \mathcal{L}} \left( \sum_{l \in \mathcal{L}} a_{kl} V_l + b_k \right) / R_{ik} \right). \end{aligned} \quad (8)$$

The key property of the above equation is that it expresses the variable resistor of each power electronics load in terms of the voltages at all *load* buses. Therefore, by using the expression in (8) together with the rest of the system model that we introduced in this section, we can formulate different objective functions in terms of bus voltages, which can be adjusted by changing the effective input resistor of converters / rectifiers.

### III. OPTIMAL DEMAND RESPONSE

The focus of our proposed demand response paradigm in DC distribution networks is to adjust the variable resistors of power electronics loads to assure DC power system reliability and efficiency. We are particularly concerned with two issues:

- The amount of power draw from the DC power source at each source bus should not exceed its supply limit.
- The power should be delivered *fairly* among the loads at different locations of the DC distribution network.

First, we consider the power draw limits from DC sources. For the power source at each source bus  $k$ , the amount of current draw can be calculated as  $(V_k^s - V_k)/R_k^s$ . Therefore, the power draw limit requirement can be formulated as  $V_k^s(V_k^s - V_k)/R_k^s \leq P_k^{\max}$ . In other words, we must have

$$V_k \geq V_k^s - P_k^{\max} R_k^s / V_k^s, \quad \forall k \in \mathcal{N}. \quad (9)$$

From (6), we can rewrite the above constraints as

$$V_i \geq e_i, \quad \forall i \in \mathcal{L}, \quad (10)$$

and

$$\sum_{i \in \mathcal{L}} a_{ji} V_i \geq e_j, \quad \forall j \in \mathcal{N} \setminus \mathcal{L}, \quad (11)$$

where for each  $i \in \mathcal{L}$ , we have  $e_i = V_i^s - P_i^{\max} R_i^s / V_i^s$  and for each  $j \in \mathcal{N} \setminus \mathcal{L}$  we have  $e_j = V_j^s - P_j^{\max} R_j^s / V_j^s - b_j$ .

Next, we note that from (8), at each load bus  $i \in \mathcal{L}$ , the amount of power delivered to the load is calculated as

$$P_i = \frac{V_i^2}{R_i} = V_i \left( \sum_{j \in \mathcal{L}} c_{ij} V_j + d_i \right), \quad (12)$$

where

$$c_{ii} = \sum_{k \in \mathcal{N}_i, k \notin \mathcal{L}} \frac{a_{ki}}{R_{ik}} - \sum_{k \in \mathcal{N}_i} \frac{1}{R_{ik}} - \frac{1}{R_i^s}, \quad (13)$$

$$d_i = \frac{V_i^s}{R_i^s} + \sum_{k \in \mathcal{N}_i, k \notin \mathcal{L}} \frac{b_k}{R_{ik}}, \quad (14)$$

and for each load bus  $k \in \mathcal{L} \setminus \{i\}$ , we have

$$c_{ik} = \sum_{l \in \mathcal{N}_i, l \notin \mathcal{L}} \frac{a_{lk}}{R_{ik}} + \frac{1_{k \in \mathcal{N}_i}}{R_{ik}}. \quad (15)$$

In the equation above,  $1_{k \in \mathcal{N}_i}$  is an indicator function. That is, if  $k \in \mathcal{N}_i$ , then  $1_{k \in \mathcal{N}_i} = 1$ ; otherwise,  $1_{k \in \mathcal{N}_i} = 0$ .

We are now ready to formulate the optimal demand response in DC distribution network. We use the notion of *proportional fairness* from the utility theory [21]. Proportional fairness is achieved in power delivery to users if we can maximize [22]:

$$\sum_{i \in \mathcal{L}} \log(P_i). \quad (16)$$

From this, together with (10)-(12), in order to assure fair power delivery to users while observing the power limits of DC sources, the voltages at load buses must be set according to the solution of the following optimization problem<sup>1</sup>:

$$\begin{aligned} & \textbf{maximize} \quad \sum_{i \in \mathcal{L}} \log(V_i) + \sum_{i \in \mathcal{L}} \log \left( \sum_{j \in \mathcal{L}} c_{ij} V_j + d_i \right) \\ & \textbf{subject to} \quad \sum_{i \in \mathcal{L}} a_{ji} V_i \geq e_j, \quad \forall j \in \mathcal{N} \setminus \mathcal{L}. \end{aligned} \quad (17)$$

The following theorem summarizes the key characteristics and the importance of the above optimization problem.

*Theorem 1:* (a) Problem (17) is a convex optimization problem. Therefore, it can be solved in polynomial time using

<sup>1</sup>The formulation of problem (17) can be justified also based on the concept of *Nash bargaining solution* in cooperative game theory [23].

standard convex programming techniques. (b) Given  $V_i^*$  for all  $i \in \mathcal{L}$  as the optimal solution of (17), the optimal values for variable resistors can be obtained accordingly using (8). The optimal duty cycles for the switch-mode converters can also be accordingly obtained, e.g., using (1) for a boost converter.

*Proof:* To prove part (a) we note that since logarithm is a concave function and because the expression inside the parenthesis for the second logarithmic term in the objective function in (17) is affine, the objective function in optimization problem (17) is concave. From this, together with the fact that all constraints are linear, the maximization problem in (17) is a convex program. The proof of part (b) is evident. ■

From Theorem 1, if direct load control is possible, then optimal demand response in DC distribution networks can be achieved by first solving the convex optimization problem in (17) and then setting the duty cycles for switch-mode converters of all power electronics loads using (8) and (1). If direct load control is *not* feasible, then optimal demand response can be enforced through pricing as we explain next.

#### IV. DISTRIBUTED IMPLEMENTATION AND PRICING

In this section, our focus is on designing a distributed power electronics load control algorithm together with a pricing method to achieve the optimal solution of problem (17).

##### A. General Case

From duality theory [19, Chapter 5], the Lagrangian associated with the *primal* optimization problem (17) becomes:

$$\begin{aligned} L(\mathbf{V}_{\mathcal{L}}, \boldsymbol{\lambda}_{\mathcal{N} \setminus \mathcal{L}}) = & \sum_{i \in \mathcal{L}} \log(V_i) + \sum_{i \in \mathcal{L}} \log \left( \sum_{j \in \mathcal{L}} c_{ij} V_j + d_i \right) \\ & - \sum_{j \in \mathcal{N} \setminus \mathcal{L}} \lambda_j \left( e_j - \sum_{k \in \mathcal{L}} a_{jk} V_k \right), \end{aligned} \quad (18)$$

where  $\lambda_j \geq 0$  denotes the Lagrange multiplier associated with the inequality constraint corresponding to each non-load bus  $j \in \mathcal{N} \setminus \mathcal{L}$ . Here,  $\mathbf{V}_{\mathcal{L}} = (V_i, \forall i \in \mathcal{L})$  and  $\boldsymbol{\lambda}_{\mathcal{N} \setminus \mathcal{L}} = (\lambda_j, \forall j \in \mathcal{N} \setminus \mathcal{L})$ . Next, we define the Lagrange dual function as

$$\begin{aligned} g(\boldsymbol{\lambda}_{\mathcal{N} \setminus \mathcal{L}}) = & - \sum_{j \in \mathcal{N} \setminus \mathcal{L}} \lambda_j e_j + \textbf{maximum}_{V_i \geq e_i, \forall i \in \mathcal{L}} \sum_{i \in \mathcal{L}} \log(V_i) \\ & + \sum_{i \in \mathcal{L}} \log \left( \sum_{j \in \mathcal{L}} c_{ij} V_j + d_i \right) \\ & + \sum_{i \in \mathcal{L}} \left( \sum_{j \in \mathcal{N} \setminus \mathcal{L}} \lambda_j a_{ji} \right) V_i. \end{aligned} \quad (19)$$

Finally, we can define the *dual* optimization problem as

$$\textbf{minimize}_{\lambda_j \geq 0, \forall j \in \mathcal{N} \setminus \mathcal{L}} g(\boldsymbol{\lambda}_{\mathcal{N} \setminus \mathcal{L}}). \quad (20)$$

Recall from Theorem 1 that the primal optimization problem (17) is a convex program. From this, together with the fact that the linear constraints in (17) satisfy the Slater's condition,

**Algorithm 1** : Optimal Demand Response in DC Networks.

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1: Initialize  $\mathbf{V}_{\mathcal{L}}$  and  $\lambda_{\mathcal{N} \setminus \mathcal{L}}$ .
2: Repeat
3:   Repeat
4:     Each load bus  $i \in \mathcal{L}$  updates  $V_i$  by solving (21).
5:     Broadcast  $V_i$  to all other buses.
6:   Until no entry of vector  $\mathbf{V}_{\mathcal{L}}$  changes.
7:   Each non-load bus  $j \in \mathcal{N} \setminus \mathcal{L}$  updates  $\lambda_j$  using (22).
8:   Broadcast  $\lambda_j$  to all other buses.
9: Until no entry of vector  $\lambda_{\mathcal{N} \setminus \mathcal{L}}$  changes.
10: Each bus load  $i \in \mathcal{L}$  updates  $R_i$  using (8).

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strong duality holds and the duality gap is zero and the Lagrange multipliers always exist [19, Section 5.2.3]. Therefore, if we can iteratively solve the maximization in (19) and the minimization in (20), then after convergence, the global optimal solution of the primal optimization problem (17) will be achieved. In this regard, we propose to use the *coordinate ascent method* [24, Section 3.2.4] to solve problem (19) and the *gradient method* [19, Section 9.3] to solve problem (20).

In order to solve problem (19) using the coordinate ascent method, we fix all variables  $V_k$  for  $k \in \mathcal{L} \setminus \{i\}$ , and then we solve the optimization problem with respect to  $V_i$  as follows:

$$\begin{aligned} \underset{V_i \geq e_i}{\text{maximize}} \quad & \sum_{i \in \mathcal{L}} \log(V_i) + \sum_{i \in \mathcal{L}} \log \left( \sum_{k \in \mathcal{L}} c_{ik} V_k + d_i \right) \\ & + \sum_{i \in \mathcal{L}} \left( \sum_{j \in \mathcal{N} \setminus \mathcal{L}} \lambda_j a_{ji} \right) V_i. \end{aligned} \quad (21)$$

This procedure is repeated for all  $i \in \mathcal{L}$ , leading to an iterative algorithm. Since problem (19) is convex, if the iterations are implemented in form of a Gauss-Seidel algorithm where users take turns, then the iterations are guaranteed to converge to the optimal solution of problem (19) [24, Proposition 2.5].

In order to solve problem (20) using the gradient method, at each iteration, we can update  $\lambda_j$  for each  $j \in \mathcal{N} \setminus \mathcal{L}$  as follows:

$$\lambda_j \leftarrow \left[ \lambda_j + \gamma \left( e_j - \sum_{i \in \mathcal{L}} a_{ji} V_i \right) \right]^+, \quad (22)$$

where  $\gamma > 0$  is a stepsize and  $[z]^+ = \max\{z, 0\}$ . If  $\gamma$  is small enough or diminishing, then the convergence of the iterations in (22) to the solution of problem (20) is guaranteed [25].

We are now ready to introduce our proposed distributed demand response algorithm in Algorithm 1. There are two loops in this algorithm. The inner-loop is designed to solve problem (19) using the coordinated ascent method. The outer loop is designed to solve problem (20) using the gradient method. As we explained earlier, since the duality gap is zero, Algorithm 1 will converge to the optimal solution of problem (17), as long as the load bus voltages in Line 4 are updated sequentially and the step size to update the Lagrange multipliers in Line 7 is small enough or diminishing.

### B. Special Case

An interesting special case in our analysis is the scenario where there is a power electronics load at *every* bus. That is,

the case where all buses are load buses and we have  $\mathcal{L} = \mathcal{N}$ . Then, the optimization problem (17) will be reduced to

$$\underset{V_i \geq e_i, \forall i \in \mathcal{L}}{\text{maximize}} \quad \sum_{i \in \mathcal{L}} \log(V_i) + \sum_{i \in \mathcal{L}} \log \left( \sum_{j \in \mathcal{L}} c_{ij} V_j + d_i \right), \quad (23)$$

where for each bus  $i \in \mathcal{N}$  we have

$$c_{ii} = - \sum_{k \in \mathcal{N}_i} \frac{1}{R_{ik}} - \frac{1}{R_i^s} < 0, \quad (24)$$

$$d_i = \frac{V_i^s}{R_i^s} \geq 0, \quad (25)$$

and

$$c_{ij} = \frac{1_{j \in \mathcal{N}_i}}{R_{ij}} \geq 0, \quad \forall j \in \mathcal{N} \setminus \{i\}. \quad (26)$$

Since problem (23) does not have any coupling constraint, it is easier to solve than problem (17). In fact, we can apply the coordinated ascending method directly to problem (23) and derive an algorithm similar to Algorithm 1 but only with a single loop as in Lines 3-6. The details are omitted for brevity.

### C. Pricing Interpretation

Consider the local problem (21) that each load bus (or user) must solve when we implement Algorithm 1. From (12), and after reordering the terms, we can show that problem (21) is *equivalent* [19, Section 4.1.3] to the following problem:

$$\underset{V_i \geq e_i}{\text{maximize}} \quad \log(P_i) + \delta_i V_i + \sum_{k \in \mathcal{L} \setminus \{i\}} \log(c_{ki} V_i + \beta_{ik}) \quad (27)$$

where

$$\delta_i = \sum_{j \in \mathcal{N} \setminus \mathcal{L}} \lambda_j a_{ji}, \quad (28)$$

and for each  $k \in \mathcal{N} \setminus \{i\}$ , we have

$$\beta_{ik} = \sum_{l \in \mathcal{L} \setminus \{i\}} c_{kl} V_l + d_k. \quad (29)$$

As far as solving problem (27) is concerned,  $\delta_i$  and  $\beta_{ik}$  are constant. They can take both positive and negative values.

The objective function in (27) can be interpreted as follows. The first term,  $\log(P_i)$ , can be seen as user  $i$ 's monotonically increasing and concave *utility function* that quantifies user  $i$ 's level of satisfaction when it draws power  $P_i$ , c.f. [26]. The second term,  $\delta_i V_i$ , is for *voltage regulation*. If  $\delta_i < 0$ , then user  $i$  is encouraged to reduce its voltage. If  $\delta_i > 0$ , then user  $i$  is encouraged to increase its voltage. Finally, the third term,  $\sum_{k \in \mathcal{L} \setminus \{i\}} \log(c_{ki} V_i + \beta_{ik})$ , can *enforce fairness*. The higher the value of  $\beta_{ik}$ , the stronger user  $i$  is encouraged to regulate its voltage to allow more power delivery to user  $k$ .

Based on the discussion above, we can interpret  $\delta_i$  as a *voltage regulating price* and  $\beta_{ik}$  as a *fairness enforcement price*. From (28), the voltage regulating price depends on  $\lambda_j$ , i.e., whether any DC source at a non-load bus  $j$  has reached its power delivery limit, and also on  $a_{ji}$ , i.e., the way that the operation of the variable resistor load at load bus  $i$  may affect the voltage at the terminal of a DC source at a non-load bus  $j$ . From (29), the fairness enforcement price depends on the amount of current that user  $k$  can draw from the DC distribution network if  $R_i \rightarrow 0$  and, accordingly,  $V_i \rightarrow 0$ .



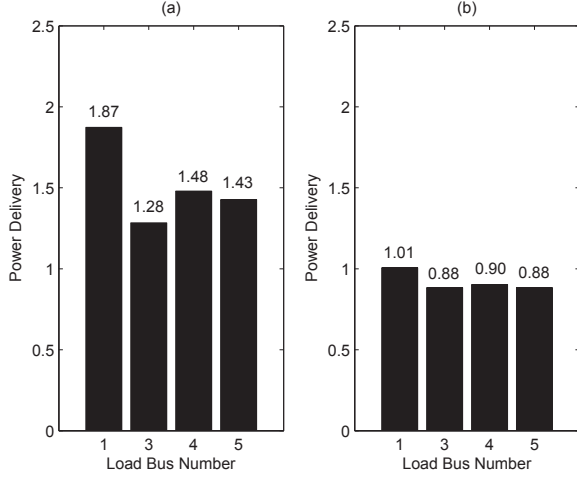


Fig. 3. Power delivery to end users based on the solution of problem (17) in the radial power distribution network of Fig. 2(a) with four power electronics loads: (a) With distributed generation. (b) Without distributed generation.

## V. PERFORMANCE EVALUATION

In this section, we assess the performance of the proposed optimal demand response framework for DC distribution networks. We examine both centralized and distributed designs. All parameters in this section are in a *per unit* system with a base voltage of 380 V DC and a base power of 10 kW [27].

First, consider the radial distribution network in Fig. 2(a). We assume that  $R_{12} = 0.01$ ,  $R_{23} = 0.03$ ,  $R_{24} = 0.01$ ,  $R_{45} = 0.01$ ,  $R_1^s = 0.01$ ,  $R_3^s = 0.01$ ,  $V_1^s = 1$ ,  $V_3^s = 1$ ,  $P_1^{\max} = 4$ , and  $P_3^{\max} = 2.5$ . In presence of the distributed generator at bus 3, the optimal values for the variable resistors become  $R_1^* = 0.4921$ ,  $R_3^* = 0.7404$ ,  $R_4^* = 0.5563$ , and  $R_5^* = 0.5565$ . The amount of power that is delivered to each load is shown in Fig. 3(a). At optimality, both DC sources reach their power limits  $P_1^{\max}$  and  $P_3^{\max}$ . Next, assume that the distributed generator at bus 3 is disconnected from the network. The optimal values for the variable resistors are updated accordingly and we have  $R_1^* = 0.9159$ ,  $R_3^* = 0.9192$ ,  $R_4^* = 0.9189$ ,  $R_5^* = 0.9190$ . The amount of power that is delivered to each load is shown in Fig. 3(b). We can see that our design can maintain efficiency and fairness under different grid conditions.

Next, we repeated the analysis of the previous paragraph for 100 different scenarios by randomly choosing the values of parameters  $R_{12}$ ,  $R_{23}$ ,  $R_{24}$ ,  $R_{45}$ ,  $R_1^s$ ,  $R_3^s$ ,  $P_1^{\max}$ , and  $P_3^{\max}$ . We focus only on the case *with* distributed generation. To have a base for comparison, we have considered the case where all variable resistors are equal. The simulation results are shown in Fig. 4(a). On average, the proportional fairness optimization objective increases by 11.2% when our proposed design is implemented compared with the best performance possible when  $R_1 = R_3 = R_4 = R_5$ . As an example, consider the performance improvement for the first scenario in Fig. 4(a) that is 10.8%. This result is obtained based on the extended analysis that is shown in Fig. 4(b). The comparison for the rest of the scenarios in Fig. 4(a) are made similarly.

Finally, consider the DC microgrid in Fig. 2(b), where  $R_{12} = 0.01$ ,  $R_{14} = 0.03$ ,  $R_{23} = 0.01$ ,  $R_{34} = 0.01$ ,  $R_2^s =$

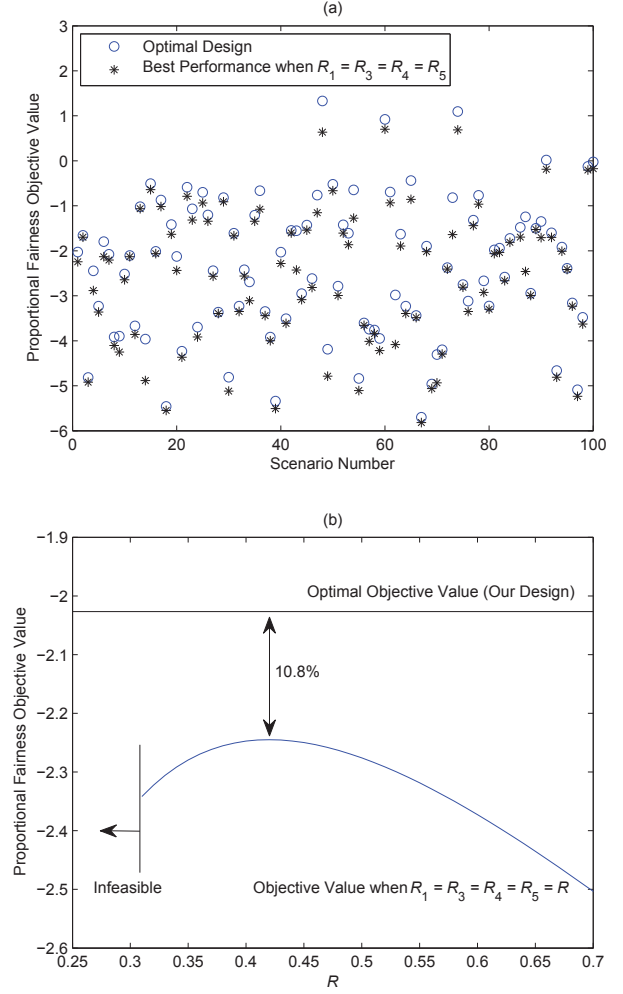


Fig. 4. Performance comparison between the proposed optimal design and the case where  $R_1 = R_3 = R_4 = R_5$  in the radial power distribution network of Fig. 2(a) with four power electronics loads: (a) Comparison across 100 random scenarios. (b) Detailed comparison based on scenario number 1.

$0.01$ ,  $R_4^s = 0.01$ ,  $V_2^s = 1$ ,  $V_4^s = 1$ ,  $P_2^{\max} = 4$ , and  $P_4^{\max} = 3$ . Note that, the two power sources are connected to non-load buses. The power draw constraints are  $0.3333V_1 + 0.3333V_3 \geq 0.6267$  and  $0.1429V_1 + 0.4286V_3 \geq 0.5414$ . The simulation results when we run Algorithm 1 are shown in Fig. 5. We can see that the distributed design results in the exact optimal performance as in the centralized design. In particular, both  $V_1$  and  $V_3$ , i.e., the voltages at the load buses, converge to their optimal values as shown in Fig. 5(a). The voltage regulation price signals are shown in Fig. 5(b). At optimality, only the DC source at bus 4 reaches its power limit  $P_4^{\max}$ . Therefore, at steady state, we have  $\lambda_4 > 0$  while  $\lambda_2 = 0$ . Note that, here we only show the iterations in the outer loop of Algorithm 1. Typically, the inner loop in this simulation converges within 3 or 4 iterations. We chose a fixed step size  $\gamma = 1$ .

## VI. CONCLUSIONS AND FUTURE WORK

Given the great benefits of DC distribution networks and because of the importance of demand response in the future smart grid, in this paper, we took the first steps towards

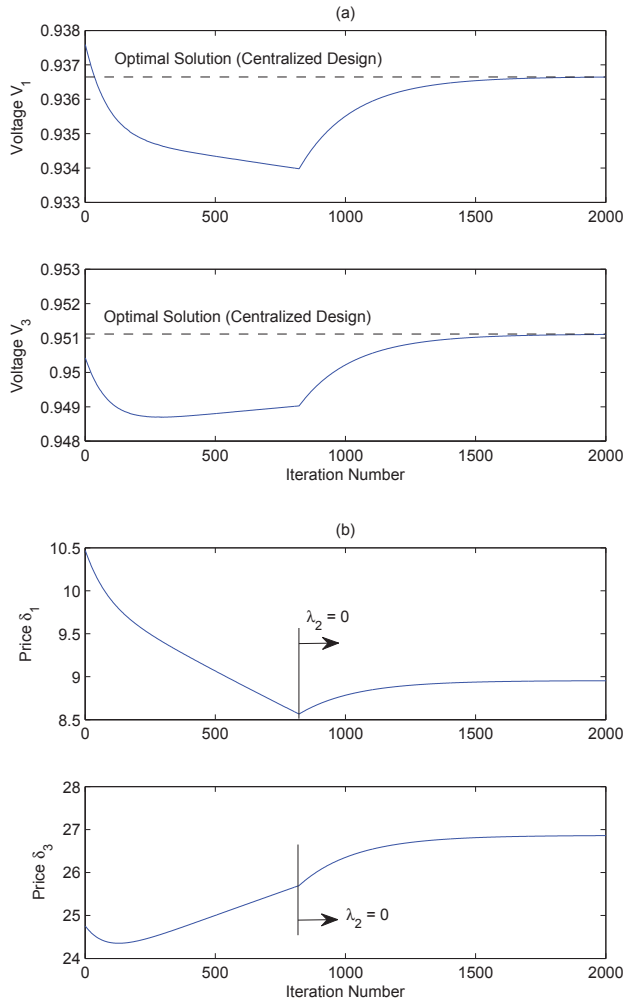


Fig. 5. Performance evaluation of distributed Algorithm 1 for the DC microgrid in Fig. 2(b): (a) The trends of the voltages at load buses 1 and 3. (b) The trends of the voltage regulation prices at load buses 1 and 3.

designing demand response programs for DC distribution systems. We combined the recent advancements in power electronics with techniques from optimization theory to develop an optimization-based DC demand response foundation to adjust the internal parameters of power electronics loads to assure reliable and efficient power system operation. We showed that the formulated optimization problem is convex and therefore tractable. We also devised a pricing mechanism to enforce optimal DC demand response in a distributed fashion. Various simulation results are presented to assess the performance.

The results in this paper can be extended in several directions. For example, we are planning to examine demand response design objective functions other than proportional fairness. Other constraints, e.g., with respect to the capacities of DC distribution lines, can also be considered. The proposed design may also be evaluated using hardware implementation.

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